ADMIRALTY

MANUAL OF

HYDROGRAPHIC

SURVEYING



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Preface to the Second Edition

The necessity of reprinting the "Admiralty Manual of Hydrographic Surveying" has provided the opportunity for adding chapters on Grids, Surveying from Air Photographs, and Reproduction and Printing of Charts.

The chapter on Surveying from Air Photographs is very condensed and deals only with methods of plotting topography from air photographs, using such simple equipment as may be found in H.M. Surveying Ships; it borrows largely from "A Simple Method of Surveying from Air Photographs," by Lieutenant J. S. Salt, R.E., to which further reference should be made. For a fuller exposition of the whole subject, "Air Photography applied to Surveying," by Major C. A. Hart, R.E., and "Surveying from Air Photographs," by Captain M. Hotine, R.E., should be consulted.

All the text has been revised and, in a few cases, entirely re-written; notably the brief section on the Nature of the Ocean Bottom, which has been brought into line with the comprehensive study of marine sedimentation given in "The Oceans" (Sverdrup, Johnson and Fleming, Prentice-Hall Inc., New York, 1942); to which further reference should be made.

The Tables have been revised and several additions have been made; Table IV of the First Edition, containing values in feet of one second of arc measured along parallels of latitude has, however, been omitted since these values may be easily calculated from columns 4 and 5 of the enlarged Table III.

The notation used throughout the volume (see page x) has been altered to conform more generally with common geodetical and astronomical practice; the most radical change being the use of the symbols ϕ and ω for latitude and longitude (instead of the λ and L used in the First Edition).

A. G. N. WYATT,

Rear Admiral, and Hydrographer of the Navy.

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Extract from Preface to the First Edition

The " Admiralty Manual of Hydrographic Surveying " has been written to provide a text book in which modern methods of hydrographic surveying and instruments are described. In order to keep the size of the volume within reasonable compass, it has been necessary to refer the reader to other publications for more detailed information on certain subjects which by their importance or complexity require separate study for a proper exposition. In particular, mention is made of the following subjects and publications which the reader may consult :—

Tides and Tidal Streams—" Admiralty Tide Tables, Part III,"* and the " Admiralty Manual of Tides."

. . . . . .

This manual has been prepared for publication by Commander R. M. Southern, Royal Navy, and in its compilation it has been assumed that the reader is acquainted with the general principles and practice of Navigation—such knowledge is, indeed, essential before a proper understanding of the methods and processes of hydrographic surveying can be acquired. For this reason subjects such as astronomical observations at sea, are only discussed in so far as ordinary navigational practice is modified to fulfil the requirements of the hydrographic surveyor, and reference to ordinary navigational instruments, *e.g.* the sextant and chronometer, is curtailed to a minimum.

That part of Chapter XVI dealing with the Least Square Method of treating astrolabe observations has been written by Mr. T. J. Richmond, and the whole of Chapter XXII by Mr. G. R. Hayes, both of the Hydrographic Department, Admiralty.

Acknowledgment is due to Messrs. Cooke, Troughton & Simms, Ltd., of York and London, for the loan of blocks illustrating the Tavistock theodolite, and to Messrs. Henry Hughes & Son, Ltd., of London, for the loan of photographs illustrating the Wollaston current meter.

Also to the Controller, His Majesty's Stationery Office, for permission to reproduce Tables V, VII, X and XII from Close's " Text Book of Topographical Surveying " and to Messrs. John Murray for permission to reproduce Tables XIII, XVI and XVIII from Wharton and Field's " Hydrographical Surveying."

. . . . . .

* Shortly to be republished under the title " Admiralty Handbook of Tidal Prediction and Analysis for Seamen."

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NOTATION USED IN THIS VOLUME

Geodetic notation

E East
N North
O Origin of rectangular co-ordinates
RO Referring object
S South
W West

Astronomic notation

E East
H Altitude of heavenly body
N North
S South
W West

a Semi-major axis of spheroid
b Semi-minor axis of spheroid
f Compression
h Height of a station
p Polar distance
r Mean radius of spheroid
s Distance between two points
x Eastings of rectangular co-ordinates
y Northings of rectangular co-ordinates

h Hour angle
n a number
p Polar distance
t Clock time
v a residual
z Zenith distance

α Azimuth
Δ, $\propto$ or γ Convergency of meridians
Δ difference
η Ordinate of curvature of a parallel
ν Radius of curvature at right angles to the meridian
π 3.141592653 . . . . . .
ρ Radius of curvature in the meridian
ϕ Latitude
ω Longitude

α Azimuth
δ Declination
Δ difference
Φ or ϕ Latitude
Ω or ω Longitude

Suffixes

a assumed, as ω_a—assumed Longitude
m mean, as ω_m—mean Longitude
o origin, as ω_o—Longitude of origin

CHAPTER I

INTRODUCTION

The science of Hydrography originated in the need for the production of maps specially designed for the use of the mariner. Nothing has been of more importance to the foundation and expansion of sea-borne trade among the nations than the production of the charts which represent the results of the hydrographic surveyor's work.

The seafarer has been provided with hydrographic information of some description for many centuries. Early Greek and Roman writers do not allude to the existence of charts in their day, but descriptions of the Mediterranean coasts, evidently for the seaman's use, were published in the years before Christ and are still extant ; it is possible that these were intended to be used in conjunction with maps now no longer in existence. The compass was applied to nautical use as early as the twelfth century, and from that time onwards voyages became longer and the need of special types of maps for nautical purposes more essential. Charts were drawn in Italy in the fourteenth century ; these were of the loxodromic type, being covered with loxodromes (or rhumb lines) radiating from a number of points and extending all over the map. At that time there were no very fixed ideas as to the shape of the earth and charts were not drawn on any projection which could represent more than a small part of the earth's surface with any degree of accuracy. The cartographers of those days endeavoured to show the various known parts of the land in some approximately true relation to each other : soundings were not given though sometimes shallows might be indicated, and small islands and points were usually exaggerated out of their true proportions.

Charts were first printed towards the end of the fifteenth century and were being engraved on copper by 1560. Nine years later Mercator published his well-known " Universal Map " of the world, the first real attempt to produce a map on a mathematical projection. In the following centuries a great number of charts of a private or semi-official character were produced. These were usually accompanied by written Sailing Directions and in most cases were remarkably inaccurate though the titles and letterpress did not hesitate to extol their merits. They were, of course, based on very sketchy and imperfect information, the results of mariners' observations on voyages of commerce and exploration. By the middle of the seventeenth century the Dutch were publishing charts under Government authority and in 1720 the French also established for this purpose a central chart office in Paris. The exploratory voyages of Cook, Vancouver and many others soon provided a wealth of hydrographic information and material on which the publication of more dependable charts could be based. Towards the end of the eighteenth century this material had accumulated to such an extent at the Admiralty that in 1795 the Hydrographic Department was founded " to take charge and custody of such plans and charts as then were, or should thereafter be, deposited in the Admiralty, and to select and compile such information as might appear to be requisite for the purpose of improving navigation." Little systematic surveying, as such, had been attempted up to this time, hydrographic work being merely an ancillary part of ships' instructions when they were engaged on voyages of exploration or other nature. In 1801, however, Captain Flinders on sailing for Australia received supplementary instructions regarding his work from the Hydrographer and this may be said to have constituted a precedent for the organisation of systematic marine surveys.

During the nineteenth century nearly every civilised maritime country founded a department for the sole purpose of dealing with the issue and publication of charts and for the co-ordination and execution of marine surveys, and immense progress has been achieved in the production of charts. As long as people and goods continue to be carried by sea, hydrographic work must continue, for even to-day only a comparatively small proportion of the seas is adequately charted and constant changes in the sea-bed and coasts in many parts of the world necessitate frequent revision of existing work. Furthermore, the increasing size and draught of ships and the commercial importance of speedy voyages now require far more exacting standards of work than were contemplated in years past. Hydrographic data on many of the present-day charts still depend on work done by vessels under sail, equipped with out-of-date instruments and incapable of carrying out a detailed and accurate survey. The need for charts was so urgent in the last century that until perhaps fifty or sixty years ago the hydrographic surveyor was sometimes obliged to subordinate completeness and accuracy to speed, for almost any chart is better than none at all. Although by modern standards many of the charts of those days are sketchy and incomplete, the reputation of our naval surveyors was second to none in the volume of work which they produced, and the accuracy of this work, often done in the face of great difficulties, has stood the test of time.

The Surveyor's Work.—The work of the hydrographic surveyor includes many branches of the science of hydrology and is of constant interest. No two surveys are precisely similar in character and new problems to test the surveyor's skill and ingenuity are constantly being presented. The work in the field is of an arduous and often trying nature, but, unlike many other occupations in life, it results in a tangible and permanent record of the surveyor's labours and is often subsequently of historical interest. Moreover, the information which he gathers is essential for the furtherance of the scientific knowledge of the world in which we live. The hydrographic surveyor must possess some knowledge of algebra and trigonometry, but the mathematical processes, with which he has to deal, are not of a very abstruse character. Some aptitude for draughtsmanship is to be desired, but experience has shown that a sufficiently high standard can usually be acquired without much difficulty. The principal qualifications, perhaps, of an efficient surveyor are unlimited capacity for taking pains and conscientious devotion to accuracy of detail in his work, coupled with a methodical and orderly mind. The standards set in these respects by hydrographic surveyors of previous years are an example for all time. In these days, when the great majority of surveys must be of the most complete and detailed character, it is necessary to lavish every care on the smallest detail of the work in order that the accuracy of the final result may be beyond question. The printed word has a powerful influence over the human mind and the printed chart just as much so. It may be said that in general the mariner places boundless confidence in the published chart and almost invariably assumes that, where no dangers are shown, none in fact exists, and it is the surveyor's business to see that this trust is not misplaced. The introduction of steam vessels was an epoch-making event in the progress of hydrography. For the first time it became possible to carry out detailed surveys of large areas of the ocean. Modern inventions, in particular echo sounding, have increased the speed and accuracy of marine surveys to a notable extent, and our knowledge of the earth's features below the surface of the sea is being extended with great rapidity. These inventions have also relieved to a great extent physical exertion in the field, and sounding under oars and the use of the hand lead will soon be memories of the past.

General

Projection.—A chart represents on a reduced scale the positions of various physical features of some portion of the earth's surface. For convenience in use

and construction, charts must be drawn on plane surfaces, *i.e.*, flat sheets of paper, and the cartographer is confronted with the problem of " projecting " on to a plane the positions of objects which in nature lie on the surface of a sphere. It is impossible to make a flat sheet of paper touch the surface of a sphere at more than one point and, even if the paper is bent (not creased), it can only make contact along a selected line. It follows from this that a truly accurate representation on a flat sheet of paper of the positions of objects on the earth's surface is not possible. When dealing with small areas, however, such as those embraced by charts of harbours or coastal sheets of moderate extent, the errors of any sound method of projection will be so small that relative bearings and distances are reproduced with all measurable precision on the paper. When larger areas are considered, as in charts showing whole seas or oceans, distortion of the true shape of the earth's physical features is inevitable, but by the choice of a suitable projection particular features of value to the purpose in view may be accurately represented whilst others of lesser importance are not unduly distorted. Small scale charts covering large portions of the earth's surface are the product of collating the results of a number of small surveys and consequently the types of projection employed and their errors are matters of importance, but the problems involved in fitting the various surveys together are rather the concern of the cartographer than of the hydrographic surveyor.

The positions of points on the earth's surface are usually defined in terms of their geographical co-ordinates, latitude being measured north or south of the equator and longitude east or west from the meridian of Greenwich. No chart, except possibly a plan of a very small area, can be called complete unless it is " graduated " in latitude and longitude in order that the geographical position of any point may be readily seen. Once out of sight of land the mariner defines his position in terms of latitude and longitude and a graduated chart is essential to him for plotting his position relative to the land and for finding his way from port to port. An orderly arrangement of meridians and parallels correctly disposed in relation to the physical features represented on the chart, is a natural and essential feature of any map projection.

The number of projections on which a chart may be drawn is legion, but two which have properties of special value to the mariner may be mentioned here ; these are the Mercator and gnomonic. The Mercator projection was invented in 1569 and is still used extensively for maps and charts of various types, embracing large areas. It is a cylindrical projection on which the meridians are straight and equidistant parallel lines and the parallels are straight lines at right angles to them but whose distance apart varies with the latitude, increasing towards the poles. It has certain mechanical advantages in construction and its special property is that loxodromes or rhumb lines (lines of constant course) are represented by straight lines, and this is a great convenience to the mariner. In high latitudes this projection cannot be used in its normal form owing to the great distortion introduced. For charts of the polar regions, for those of large oceans where great circle passages may be made, and for many charts of small areas the gnomonic projection has been largely used. It is a projection of the zenithal type and is formed by projecting from the centre of the sphere onto a plane in contact with the surface at some particular point. The special property of the gnomonic projection of value to seamen is that all lines of sight (which are also parts of great circles) are represented by straight lines. This feature is a convenience in enabling bearings, whether visual or wireless, to be plotted as straight lines, and in determining the shortest route between any two points. The scale on this projection is not constant and increases from the centre of the sheet outwards, but for small areas, up to, say, 50 miles square, there is no measurable variation in it on any single sheet which can be constructed.

Scale.—The first consideration, when a marine survey is to be made, is to decide on the scale, *i.e.*, the proportion to be maintained between actual distances on the earth's surface and those distances as represented on the chart. Obviously, the larger the scale adopted, the more detail will it be possible to show on the chart. Harbours and important anchorages must therefore be surveyed on very large scales in order that the maximum amount of detail may be clearly incorporated in the final drawing and that the survey may be as complete as possible. Practical considerations of time and expense necessitate the use of smaller scales for coastal sheets and those of more open waters. British Admiralty charts are now generally produced from surveys on certain standard scales, varying from 1/2,500 (nearly 30 inches to the nautical mile) to 1/12,500 for harbour surveys, and from 1/25,000 to 1/120,000 (about 3 inches and 0·6 of an inch respectively to the nautical mile) for surveys of harbour approaches and coastal sheets.

Usually the surveyor receives instructions as to the scale he is to use for any particular survey, but, in the absence of these, he must be guided by the nature and importance of the area he is going to survey. Supposing, for instance, a survey has to be made of a navigable river some 200 yards broad. It is unlikely that the shipping channel and dangers could be clearly shown unless this distance of 200 yards is represented by a length of at least an inch on the chart and it will therefore be necessary to survey on a scale of at least 10 inches to the nautical mile. The more intricate the channel the larger must be the scale of the survey in order that it may be clearly shown. Again, open coastal surveys would generally be undertaken on a scale of about 1½ inches to the mile, but if an important shipping channel existed within the survey area or if it was known or suspected to be very irregular in depth and to be strewn with numbers of small isolated rocks and dangers, it would be difficult, if not impossible, to carry out a sufficiently detailed examination on such a small scale, and it might be necessary to double or even quadruple the scale in order to ensure that no dangers remained undetected. Lines of soundings may be run very close together over the ground, but in practice their distance apart is controlled by the ability to plot them and show the soundings clearly. The normal procedure is to run lines of soundings 1/5th of an inch apart on the plotted sheet; naturally, their actual distance apart over the ground depends on the scale adopted. For instance, on a scale of 1/12,500, or about 6 inches to the nautical mile, the lines will be actually about 70 yards apart, whilst on a scale of 1/50,000, or about 1½ inches to the mile, they will be nearly 270 yards apart. Obviously, the examination of the ground can be much more complete on the 1/12,500 scale and the probability of detecting a danger will be correspondingly greater. A final consideration in deciding the scale must be the ability to plot the work within the limits of the plotting table and with the instruments and paper provided.

Triangulation.—The surveyor's first step in the field is to fix the number of marks, known as trigonometrical stations, which will provide control points for the survey. These marks must be plotted on the chart in the same relative position that they occupy on the earth's surface thus providing a fixed framework by which every detail in the survey may be accurately located. The survey will be plotted on some predetermined scale and it is therefore necessary that the distances between the various trigonometrical stations should be calculated. If, for instance, the survey was to be carried out on a scale of 1/25,000, two stations 1,000 feet apart would be plotted exactly 1/25th of a foot, or 0·48 inches, from each other on the sheet. In general, the precise measurement of the distance between points on the earth's surface is a matter of some difficulty and it would be impossible to make direct measurements between all the various points of the framework. If, however, the distance between any two of the stations can be found and if horizontal angular measurements are made from these to other stations in their vicinity and thence to the remainder,

thereby connecting all the points of the framework by a system of triangles, it is possible to calculate the distances between all other points in the survey. In the absence, therefore, of any known trigonometrical data it will be necessary to measure a "base," the distance between two selected stations or points which are marked in a convenient position where the measurement can be carried out over ground as level and free from obstruction as possible. The ends of the base are then connected to other selected stations by observing horizontal angles to and from them, and these in their turn are similarly connected to the remainder, by observing angles between all that are intervisible. The result will be to provide a system of geometrical figures, known as the triangulation, which embraces the area to be surveyed. From these main stations of the triangulation, horizontal angles are observed to secondary stations and points, natural and artificial, of lesser importance which can thereby be fixed and will provide additional marks for locating all the detail of the survey. For the purpose of fixing the soundings it is usually necessary for the surveyor to see at least three fixed points in the survey from every part of the water area, and this is an important consideration in selecting the sites for the main and secondary stations and sounding marks.

When the base has been measured and all the angles observed, the lengths of the sides can be computed by plane trigonometry and the relative positions of the various main and secondary stations determined. These calculations enable the stations to be plotted and from them sounding and other marks can be laid down by protraction of the observed angles. A series of fixed points is now available on the sheet for accurately locating and plotting all the detail of the survey. The methods of doing this will be discussed in subsequent chapters.

Azimuth.—Just as the lengths of all the sides can be computed from that of a known side or base, so their bearings can be deduced from the known azimuth, or true bearing, of one side. This azimuth, which is required also for the correct orientation of the entire work, is obtained by astronomical observations. It may here be noted that the true meridian, as derived by correcting magnetic compass observations for the known or estimated variation, is not sufficiently accurate for the purposes of a detailed survey though it may often be used as an approximation in the earlier stages of the work. Finally it is necessary to determine the geographical position of one or more of the fixed points of the triangulation in order that the survey as a whole may be correctly located on the earth's surface and to enable the chart to be graduated accordingly; this must be done by astronomical observations of suitable heavenly bodies.

Requirements of a Hydrographic Survey

The finished chart should be a complete, accurate, intelligible and up-to-date representation of the whole area which it sets out to portray. In addition to the detail which actually appears on the chart, a large variety of supplementary information is needed for inclusion in Sailing Directions and other nautical publications. The requirements of a hydrographic survey to put this into effect may be briefly stated as follows:

(*a*) The entire water area in the survey seaward of the high-water line must be closely sounded by running regular evenly-spaced lines of sounding over it and a rigorous examination of the soundings must be made in the field to ensure that no irregularities in depth are overlooked and that, when such are found, the localities in which they occur are closely examined by taking additional soundings or, if necessary, by sweeping.

(*b*) The position of, and least depth over, every shoal, including submerged reefs, rocks, banks, wrecks and other obstructions to safe navigation, must be found and clearly recorded on the chart.

(*c*) The positions of breakers, tide-rips, eddies, weed, fishing stakes and similar objects of interest and importance to the navigator must be fixed and shown on the chart. The survey must also show in their correct positions all light-vessels, buoys and other navigational marks, and the characteristics of these must be fully investigated.

(*d*) The nature of the sea bottom must be ascertained at frequent and regular intervals over the water area and especially in depths where vessels may anchor, and samples of the bottom should be secured for the advancement of our knowledge of the sea-bed.

(*e*) Leading and clearing lines must be indicated on the chart and these should be sounded over with particular care.

(*f*) Both the high and low water line must be accurately fixed and the nature of the foreshore ascertained in order that it may be shown by the use of appropriate symbols.

(*g*) All topographical features, which may be of value to the mariner in helping him to recognise the coast and to determine his position, must be carefully fixed, special attention being paid to hill summits and other conspicuous objects. The heights of all such objects must also be determined and recorded. The topographical detail must, as far as circumstances admit, include contours or form lines of the land in the vicinity of the coast.

(*h*) Observations of the rise and fall of the tide must be made for the purpose of reducing soundings to a common datum, and if possible, they should be extended over a sufficiently long and unbroken period to enable them to be analysed for the determination of tidal constants.

(*i*) Measurements of tidal streams and currents must be made at various points of the survey in order that this information may be available to the mariner. If time permits, sufficient observations should be made to enable the results to be analysed and embodied in the appropriate publications.

(*j*) Opportunity should be taken to secure views of the coast by means of sketches or photographs as these are often of use to the mariner in helping him to recognise the land features of a locality with which he is not acquainted. Views of leading lines from near the position where they will first be used are of special value.

(*k*) Observations for magnetic variation must be made both ashore and afloat to determine its value and to discover unsuspected deviations from the normal variation to be expected in that part of the globe. These observations also serve to check the annual or secular change in the variation.

(*l*) The names must be ascertained and recorded of towns, villages, rivers, streams, points, hills, shoals, rocks and islands, and of any prominent and important features of the coast. Correct nomenclature and spelling are matters of great importance on the published chart and no efforts should be spared to trace acknowledged and native names which generally exist for all prominent places and objects though they are sometimes hard to discover. Local maps usually form the most reliable guide in this direction and in the absence of other information should be used as the best authority.

(*m*) All information which may be of use in the compilation and correction of Sailing Directions must be collected during the course of the survey. This information must include a general description of the coast and off-lying islands and dangers, directions for entering and leaving harbours, full descriptions of lights and other navigational aids and details of the facilities offered to shipping by ports.

(*n*) The survey must be located on the earth's surface and the true meridian determined by astronomical observations of great precision.

(*o*) Permanent marks must be placed in the positions of any of the trigonometrical stations which may be of subsequent use in extending the work or re-surveying the locality, and bench-marks must be established to enable chart datum to be recovered for use in future surveys.

(*p*) A comprehensive record of base measurements and triangulation, and of all observations taken in the field must be prepared. Too much stress cannot be laid on the necessity for careful and methodical compilation of *all* data obtained during the course of the survey as this provides the only real check on the accuracy of the chart. Furthermore, unless the records of the field work are kept in the clearest and most intelligible manner, they will be nearly valueless in after years.

(*q*) A continuous record of meteorological conditions in the survey area should be kept during the whole period of the work.

(*r*) The surveyor should endeavour to collect any information likely to be of value to the advancement of scientific knowledge generally, and, in particular, that of a hydrological character, such as the temperature and salinity of the ocean at various depths.

CHAPTER II

THE FRAMEWORK OF THE SURVEY

Use of Existing Data.—The land features of a great part of the civilised world are now shown on accurate and reliable maps. The surveys on which these maps depend are controlled by triangulations of the most precise and elaborate character; in them, no efforts are spared to secure every refinement of observation made possible by the use of the highest class of surveying instruments and by the subordination, if necessary, of time and expense to accuracy. The Ordnance Survey of Great Britain, the great Trigonometrical Survey of India, the United States Coast and Geodetic Survey, and those of most European countries are examples. They have as their object not only the mapping of great countries but also the determination of the true size and shape of the earth.

The main purpose of a chart is to guide the mariner from one piece of land to another and, except in a few isolated instances every chart shows some part or parts of the land. It follows, therefore, that the hydrographic surveyor should whenever possible connect his surveys directly to those of the land surveyor and, when land surveys of known reliability exist, he should be prepared to make use of their trigonometrical data as a basis for his own work. Many countries are also covered with a system of precise levelling, the datum for which depends on many years' tidal observations, and it is equally important that the hydrographic surveyor should, whenever possible, connect his datum to that of the land survey.

When available, trigonometrical data is usually supplied direct to the surveyor at the same time as orders for the survey are issued. If this is not done, the data can often be obtained from the local land survey office. The supply of this information will frequently relieve the hydrographic surveyor of the necessity of measuring a base and of making astronomical observations for geographical position and azimuth.

When a resurvey is to be carried out of an area which has already been charted, it may be possible to make use of the trigonometrical data of the old survey if one or more of the points of that survey can be accurately located. It is necessary for the surveyor to use his discretion in using such data as the old triangulation may have been lacking in the accuracy which would be expected to-day. Nevertheless it is important that the new work should be connected directly to the old at as many points as possible, if only for the purpose of checking the reliability of the previous survey. Furthermore, it should be remembered that it is often necessary to incorporate part of the old work in the published chart embracing the new survey and a rigid connection between them is therefore most desirable.

The Scale of the Survey.—A chart shows certain features of part of the earth's surface on a reduced scale. A distance on the earth is represented by some corresponding and proportional distance on the chart. If a mile on the earth is represented by a length of 3 inches on the chart, two miles will be represented by a length of 6 inches, and so on. The constant proportion between the true distance and that on the paper is called the Natural Scale and is usually given as a fraction. For instance, a natural scale of 1/1,000 means that 1,000 feet on the earth is represented by 1 foot on the chart, 1,500 feet by 1·5 feet, and so on in proportion.

The scale to be used is usually stated when instructions are issued for the survey. If not, it must be decided in the field before any work can be plotted. Except in special cases, hydrographic surveys are carried out on standard scales, such as 1/12,500, 1/25,000, etc., and in the absence of special instructions the surveyor should adopt that which is suitable for the work in view.

Before points on the earth's surface can be plotted in correct relation to each other on a reduced scale, the distances between them must be known. All these distances can be determined if the distance between *two* of the points is measured and if these and the remainder are connected by observing horizontal angles between them. In the absence of any known trigonometrical data, then, the distance between two points, *i.e.*, one side of the triangulation, must be measured directly on the ground. This side is called the Base.

Base Measurement

Bases can be measured with various degrees of accuracy and may be roughly classed as follows :

(1) **Base-lines for Geodetic Triangulation.**—These are of great precision and are usually measured with metallic tapes or wires suspended in catenary. The apparatus required is expensive and the degree of accuracy obtained is far in excess of the normal requirements of a hydrographic survey. Full descriptions of the method are given in Close's "Text Book of Topographical Surveying"* and other works.

(2) **Base-lines for Secondary Triangulation and Topographical Work, including Hydrographic Surveys.**—These are usually measured with steel tapes along the surface of the ground. This is the method employed in hydrographic surveys whenever circumstances permit and it will be fully described.

(3) **Base-lines by other Methods.**—These include bases obtained by taut wire measurements, sound-ranging, range-finder, subtended angle, astronomical observations and other less accurate means which may sometimes be used.

Bases by Steel Tape.

Apparatus.—The steel tapes usually supplied are 100 feet long and consist of a flat steel band about 5/8ths of an inch wide with a swivel and brass handle fitted at each end : they are graduated at every foot on one side and at every link on the other. The handles should be fitted with extension pieces, as shown in Fig. 1, to enable a spring balance to be attached and tension put on the tape during measurement.

The exact length of a tape can only be determined after certain corrections have been applied. In some tapes, as in that shown in Fig. 1, the distance of 100 feet (approx.) is that between the outer ends of the brass handles ; in others the zero and 100 foot marks are on the tape itself some inches from the handles. Three steel tapes are normally carried in surveying ships, two for use in the field and the third, a reference tape, solely for use as a standard of comparison ; the latter has its exact length, as determined at the National Physical Laboratory, at a certain temperature and tension and its coefficient of expansion stamped on a brass plate attached to its reel.

*Text Book of Topographical and Geographical Surveying. Close & Winterbotham. H.M. Stationery Office.

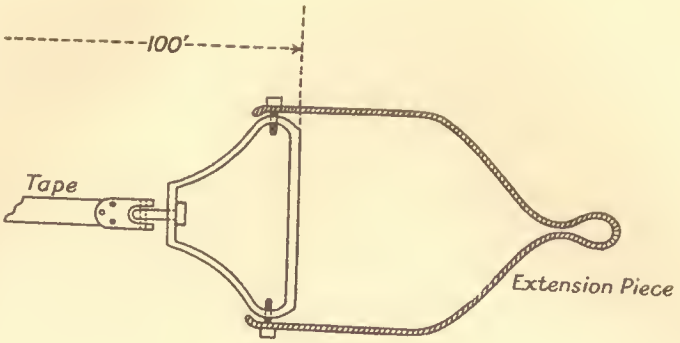


FIG. 1

A thermometer in a copper guard is provided for ascertaining the temperature of the tape during measurement, and a small spring balance registering to 20 lb. by ½ lb. for putting a pre-arranged tension on the tape. A number of metal base staves or ranging poles are also part of the equipment. During the measurement it is necessary to mark the two ends of the tape in each position it occupies on the ground. This can be conveniently done on hard wood blocks which should be pegged into the ground and a number of these should be prepared. Alternatively, zinc slabs, as supplied to ships for use in the boilers, are very suitable: these are about 16 ins. by 8 ins. and weigh about 9 lbs. each; their weight is such that they can generally be firmly bedded down on the ground without the need for pegs and the surface of the slab takes a fine knife-cut very well.

Length of Base.—No hard-and-fast rules can be laid down as to the length of base which should be measured. As will be apparent later, it is more important to extend the base by means of well-conditioned triangles than to measure a very long one. For the survey of a small harbour, a base of 1,000 feet or even less may suffice, but for larger areas and exterior coastal surveys a base of 5,000 feet or more is often desirable. In many cases the length of the base will be governed by the natural features of the ground in the locality.

Selection of Site.—In selecting a site for a base measurement the following are the most important considerations:

(1) The ground should be reasonably level and as free from obstructions as possible. Moderate slopes and changes in slope can be accepted provided that the ground as a whole has an even surface.

(2) The two terminals of the base must be intervisible and should have the best all-round view that can be obtained.

(3) The base terminals should be sited so that they can be permanently marked.

(4) If possible, the whole measurement should be made in direct alignment between the terminals.

Preparation.—*Clearing the Site.*—All obstructions must first be cleared from the line of the base and small cuttings and embankments made so that the tape may lie evenly and be supported on the ground throughout its length.

Base Terminals.—A concrete block or mark-stone with a metal plug carrying a fine mark to denote the exact position of each terminal should be buried so that the mark is flush with the ground.

Alignment.—Base staves or ranging poles should be erected at intervals of not more than 500 feet along the line, the correct alignment being secured by means of theodolites erected over the base terminals. When direct alignment throughout cannot be secured, the turning points must also be marked by staves.

The Measurement.—About ten men are required in a measuring party. Starting from one base terminal, all available marking slabs or blocks are bedded down on the line of the base at intervals of 100 feet (this distance need only be correct to within, say, a couple of inches). The alignment of the slabs, with the base staves already in position can be made with sufficient accuracy by eye. If insufficient marking slabs are available to extend along the whole length of the base, the rear used ones may be fleeted forward from time to time as the measurement progresses.

To measure the first fleet, the tape is laid out carefully along the line with its rear end (or zero mark) held down close to the base terminal; no attempt should be made to secure absolute coincidence with the terminal mark. The spring balance is hooked into the leading end of the tape and a pre-arranged tension (usually 12 lb.) put on. At a given signal a fine knife cut is made on the terminal brass and first slab respectively to mark the position of the two ends (or zero and 100 foot marks) of the tape. At the same time an assistant notes the tape temperature, taking care to shield the thermometer from the direct rays of the sun. When this has been done, the short distance between the cut showing the position occupied by the rear end (or zero mark) of the tape and the fine mark indicating the actual base terminal is set on a small beam compass or pair of dividers, and measured on a metal scale. The tape is then fleeted forward, four carriers keeping it clear of the ground. The second and subsequent fleets are measured in a similar manner. No effort is made to secure absolute coincidence between the cuts showing the position of the leading end (or 100 foot mark) of the tape on one fleet and that showing the rear end (or zero mark) on the following fleet. The short distances between these marks on each slab are measured and recorded as the work proceeds and the sum of all the measurements is applied as a correction to the whole base length. Each slab will be marked in the following manner:

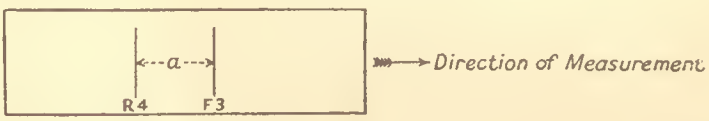


Fig. 2

This slab is the one between the third and fourth fleets of a base; the knife cut showing the position of the leading end (or 100 foot mark) of the tape on the third fleet is marked F3, that showing the rear end (or zero mark) of the tape on the fourth fleet is marked R4. The small measurement "a" will be a correction to the whole base measurement and in this instance, as there has been an overlap between the front and rear ends of the tape, will be subtractive.

A levelling party follows or precedes the base-measuring party to determine the slope of the ground over the whole line. When changes of alignment are necessary, horizontal angles must be observed from the terminals to the turning points and from the turning points themselves.

The base should be measured at least once in each direction, preferably with different field tapes, and, if possible, the operation should be carried out in cloudy weather when temperature changes are likely to be small and the true temperature of the tape is more accurately shown by the thermometer.

Corrections to be applied to Base Measurements.

To obtain the true length of the base after the measurements have been made, corrections for the following must be applied :

(i) *Standard.*—The length of the standard or reference tape, as shown on its reel, is accepted as being correct. The length of the field tape must be determined by comparisons with the standard tape before and after the base measurement.

(ii) *Temperature.*—A steel tape expands as the temperature rises. The coefficient of expansion of the standard tape is stamped on its reel and is generally about 0·000 006 per degree Fahrenheit, *i.e.*, its change in length is 0·000 006 units for each unit of its length with a change in temperature of 1° F. The coefficient of expansion of the field tape may be assumed to be the same as that of the standard tape.

(iii) *Slope.*—Distances on the earth are reduced to their horizontal components before they are used in the survey. It is nccessary, therefore, to correct the base measurement for slope. (*Note.*—A slope of 1° will reduce a measurement of 1,000 ft. by about 1·8 ins.)

(iv) *Alignment.*—The direct distance between the two ends of the base is required. If a change or changes in alignment during measurement are necessary, the direct distance must be deduced trigonometrically from the indirect base measurements and horizontal angles.

(v) *Tension.*—If the tension on the tape is varied, its length will change, but if the tension used throughout the measurement is constant and the same as that used when comparison is made with the standard tape a correction for this will not be necessary.

(vi) *Height above Sea-Level.*—The various points of a survey are plotted in the positions they would occupy if they all lay at sea-level. The base measurement must, therefore, be adjusted to what it would have been if it was at sea-level. Up to a height of 300 feet this correction is practically negligible and will rarely be required in any case by the hydrographic surveyor who usually measures his base close to the sea. the value of this correction is given by the expression :

$$a \times \frac{h}{r + h}$$

where a = length of the base,
h = mean height above sea-level,
r = mean radius of the earth at that place.

It is clearly negative when the base is above sea-level.

(vii) *Sag.*—It may occasionally happen that the base measurement has to be carried across a ravine or gully, or that owing to the uneven nature of the ground the tape has to be supported on pickets at intervals. A correction (always negative) for sag is then necessary and is given by the formula :

$$\text{Correction} = \frac{lW^2}{24t^2} \text{ (nearly)}$$

where l = length of the tape in feet between supports.
W = weight of the tape in lb.
t = tension applied in lb.

Example of Correcting a Base Measurement.

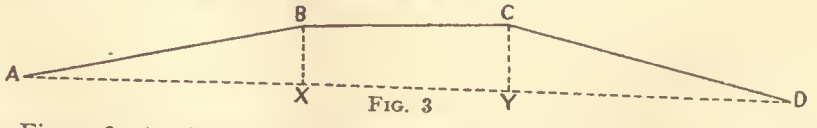


FIG. 3

In Figure 3, the distances AB, BC and CD have been measured. The true length of AD is required.

Data.—Standard tape No. 122 is 100·004 feet long at 62° F. and 12 lb. tension: coefficient of expansion 0·000 0062.

Extracts from Field Records.

(i) Field Tape No. 129—100 feet long.

(ii) Taped distance AB = 2130 + 1·123* = 2131·123 ft.
,, ,, BC = 1219 − 0·344* = 1218·656 ,,
,, ,, CD = 2276 + 0·778* = 2276·778 ,,

The figures marked * represent the algebraic sums of the small measurements at the end of each fleet. It can be seen from the figures that AB was measured with 21 whole tape lengths and one part-tape length, BC with 12 whole and one part-tape lengths, and CD with 22 whole and one part-tape lengths. In all, then, there were 58 fleets in the measurement.

(iii) *Horizontal Angles.*

DAB = 12° 20′ 05″
ABC = 169° 59′ 40″
BCD = 164° 50′ 20″
CDA = 12° 49′ 55″

(iv) *Temperatures.*

On fleets 1–10, tape temperature = 80° F.
,, ,, 11–18, ,, ,, = 82° F.
,, ,, 19–31, ,, ,, = 80° F.
,, ,, 32–39, ,, ,, = 78° F.
,, ,, 40–51, ,, ,, = 77° F.
,, ,, 52–58, ,, ,, = 75° F.

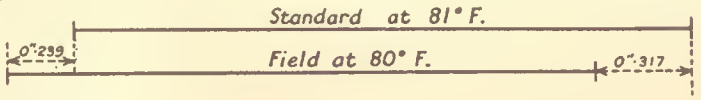
(v) *Slopes.*

On fleets 1–17, slope = 1° 50′
,, ,, 18–30, ,, = 0° 40′
,, ,, 31–58, ,, = 1° 35′

(vi) *Height:* negligible. *Tension:* constant. *Sag:* nil.

(vii) *Comparison of Field and Standard Tapes.*

1st Comparison.



2nd Comparison.

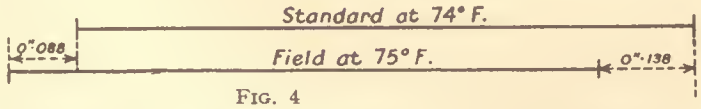


FIG. 4

Before measurement, field tape at 80° F. is short of standard tape at 81° F. by 0·078 in.

After measurement, field tape at 75° F. is short of standard tape at 74° F. by 0·050 in.

Computation.

At 62° F., true length of standard tape = 100·004 ft.

First Comparison.

True length of standard tape at 81° F.
= 100·004 + (19 × 100 × 0·000 0062) = 100·0158 ft.
Field Tape at 80° is short of standard by 0·078 in. = 0·0065 ft.
∴ True length of field tape at 80° F = 100·0158 − 0·0065
= 100·0093 ft.....(1)

Second Comparison.

True length of standard tape at 74° F.
= 100·004 + (12 × 100 × 0·000 0062) = 100·0114 ft.
Field tape at 75° is short of standard by 0·050 in. = 0·0042 ft.
∴ True length of field tape at 75° F. = 100·0114 − 0·0042
= 100·0072 ft.....(2)

(1) and (2) will not as a rule be in perfect agreement even allowing for the difference in temperature. Accept a mean as the correct length of the field tape for the measurement, *i.e.*, at 77·5° F. true length of field tape = 100·0083 ft. The range of temperature experienced during the measurement was from 75° to 82° F. and the field tape will change in length 100·0083 × 0·000 0062 = 0·0006 ft. for each degree change of temperature. Its true lengths throughout the range can now be tabulated as follows :

At 75° F. length of field tape	..	..	= 100·0068 ft.
,, 77° F. ,, ,, ,, ,,	..	..	= 100·0080 ,,
,, 78° F. ,, ,, ,, ,,	..	..	= 100·0086 ,,
,, 80° F. ,, ,, ,, ,,	..	..	= 100·0098 ,,
,, 82° F. ,, ,, ,, ,,	..	..	= 100·0110 ,,

In the distance AB there are :

10 whole tape lengths at 80° F.			= 1000·0980 ft.
8 ,, ,, ,, 82° F.	..	..	= 800·0880 ,,
3 ,, ,, ,, 80° F.	..	..	= 300·0294 ,,
1 30-foot length ,, 80° F.	..	..	= 30·0029 ,,
Total	..	..	= 2130·2183 ft.
Add algebraically sum of small measurements			= + 1·1230 ft.
True length of AB	..	..	= 2131·341 ft.
Similarly the true length of BC is found to be			1218·772 ft.
and *the true length of CD is found to be*			2276·954 ,,

The measurements must now be corrected for slope ; it will be quite accurate enough to use distances to the nearest foot between the points where the slope changes.

Corrections to distance AB :

(1) 1700 − 1700 cos 1° 50′ = 1700 − 1699·130 = 0·870 ft.

log 1700	3·230 4489
L. cos 1° 50′	9·999 7776
Sum	3·230 2265

(2) 431 − 431 cos 0° 40′ = 431 − 430·971 = 0·029 ft.

log 431	2·634 4773
L. cos 0° 40′	9·999 9706
Sum	2·634 4479

Similarly, the corrections to distance BC are :

(1) For 800 feet at a slope of 0° 40′ − 0·054 ft.
(2) ,, 419 ,, ,, 1° 35′ − 0·160 ,,

And the correction to distance CD is :

for 2277 ft. at a slope of 1° 35′ − 0·869 ft.

Collecting these results :

	AB	BC	CD
True length in feet	2131·341	1218·772	2276·954
Corrections for slope	0·899	0·214	0·869
Horizontal Distance	2130·442	1218·558	2276·085

Using the horizontal distances AB, BC and CD, calculate the length of the side AD as follows :

$$AD = AX + XY + YD \qquad \text{(See Fig. 3)}$$
$$= AB \cos \angle BAD + BC \cos [(180° - ABC) - BAD)]$$
$$+ CD \cos \angle CDA$$

Logarithmic computation :

log 2130·442	3·328 4697	
L.cos 12° 20′ 05″	9·989 8574	
log AX	3·318 3271	: AX = 2081·264 ft.
log 1218·558	3·085 8462	
L.cos 2° 19′ 45″	9·999 6411	
log XY	3·085 4873	: XY = 1217·551 ft.
log 2276·085	3·357 1884	
L.cos 12° 49′ 55″	9·989 0161	
log YD	3·346 2045	: YD = 2219·241 ft.
	AX + XY + YD = 5518·056 ft.	

Hence, accepted length AD = 5518·056 ft.

Importance of Accuracy in Base Measurements.

Any error in the base measurement is reproduced in the whole triangulation and affects the entire survey. For instance, if the base is in error by one part in 1,000 every side in the triangulation will be in error to a similar extent. For this reason an accurate base is of great importance. It must also be remembered that, though moderate errors of measurement may not be apparent when the work is plotted on a small scale, part of the triangulation may be subsequently used for surveys on much larger scales and the errors in length of the sides may then be appreciable on the plotted sheet. Bases measured with steel tapes should have an accuracy of about one part in 50,000 if the measurement can be carried out over an even surface. The different measurements of the base should easily agree to within half an inch per thousand feet of length. Changes of alignment should always be avoided if possible, but, provided they are small and the angles are carefully observed, there should be no error of moment in the final result. In the example given there are two changes of alignment, one about 10° and the other of about 15°. If the angles at B and C were each 1′ smaller than as observed, it would only affect the whole base length by about 3½ inches.

Base by Mast-head Angle.

Measurement of base lines with steel tapes or other accurate means is not always feasible without the expenditure of more time and trouble than the nature of the survey would justify. Sometimes the land features are of so small an extent or of such a character that it is quite impossible to measure a base on land in the ordinary way. In such cases recourse must be had to less accurate methods of determining the scale of the work. One of these is measurement of the base from observations of the vertical angle subtended by a ship's mast. For this operation the ship must be on an even keel and the vertical distance from the truck (or some other suitable and easily-seen part of the mast head) to the upper deck, the upper edge of the boot topping, or the waterline must be known. Example :

In Fig. 5, a ship is at anchor and it is desired to carry out a survey of the river entrance. Low mud banks, backed by mangroves, make the measurement of a base ashore impracticable. Two stations A and B are selected on the banks and simultaneous observations are made of the horizontal angles BAC and

ABC and of the vertical angles subtended by the mast at A and B. As the ship is not perfectly stationary, it is essential that all angles are observed

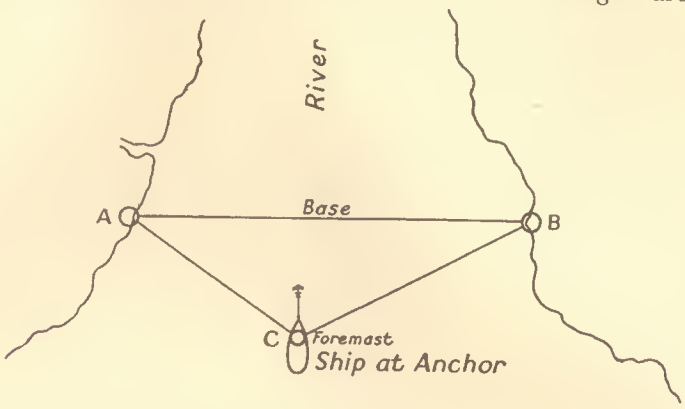


FIG. 5

at the same instant, a pre-arranged system of signals being used for the purpose. The horizontal angles should if possible be observed with theodolites and the vertical angles with observing sextants. In the following example a simultaneous set of observations gave these results :

At A : horizontal angle BAC = 35° 26′ 00″
vertical ,, to C = 3° 06′ 00″

At B : horizontal angle ABC = 28° 10′ 15″
vertical ,, to C = 2° 32′ 10″

The height of the mast was 121·5 ft. = h.

Computation.

By deduction, horizontal angle ACB = 116° 23′ 45″

Now, AC = h cot 3° 06′ 00″,
and BC = h cot 2° 32′ 10″.

By the sine rule,

AB = BC sin C cosec A = AC sin C cosec B, which provides two values for AB.

Substituting h cot 3° 06′ 00″ for AC and h cot 2° 32′ 10″ for BC :

AB = 121·5 × cot 2° 32′ 10″ × sin 116° 23′ 45″ × cosec 35° 26′ 00″

and

AB = 121·5 × cot 3° 06′ 00″ × sin 116° 23′ 45″ × cosec 28° 10′ 15″

121·5	log	2·08458	121·5	log	2·08458
2° 32′ 10″	L. cot	11·35367	3° 06′ 00″	L. cot.	11·26634
116° 23′ 45″	L. sin	9·95218	116° 23′ 45″	L. sin	9·95218
35° 26′ 00″	L. cosec	0·23676	28° 10′ 15″	L. cosec	0·32596
AB	log	3·62719	AB	log	3·62906
	AB = 4238·3 ft.			AB = 4256·6 ft.	

Meaning these values, AB = 4247 ft.

The observations should be repeated two or three times and the results worked out separately. The method is not susceptible of any great accuracy and errors as great as 1/500 may be expected. Five-figure logarithms for the calculations give all necessary accuracy when this type of base measurement is employed.

It is evident that this method can also be used with other objects whose height can be carefully measured, *e.g.*, lighthouses.

Base Measurement by Subtense Method.

If the horizontal angle is observed between two points whose distance apart is known, and which are at the same level and equidistant from the observer, a short base of fair precision can be obtained when circumstances preclude more accurate forms of measurement. Example :

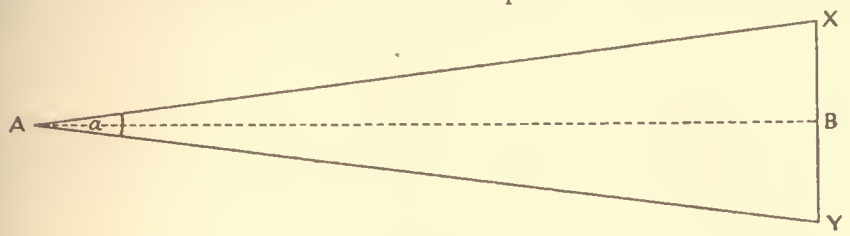


Fig. 6

In Fig. 6, let XY be a pole on which two marks are fixed or painted exactly 30 ft. apart. A is an observer with a theodolite and a sight is fitted at the centre of the pole at B so that it can be accurately placed perpendicular to the line of sight AB.

Then, if the observed angle XAY = a

AX = AY = $\frac{1}{2}$ XY cosec $\frac{1}{2}a$

Let a = 1° 42′ 18″

Then AX = AY = 15 cosec 0° 51′ 09″	15	log	1·176 0913
	0° 51′ 09″	L. cosec	11·827 4443
= 1008·2 ft.	AX	log	3·003 5356

Great care is necessary in measuring the angle a. An error of 5″ in $\angle a$ would in this instance produce an error of nearly a foot in AX or AY. An accuracy of about one in 1,000 should generally be attainable with this method.

Base Measurement by Tachymetry.

The telescopes of some theodolites and levels are fitted with two horizontal wires fixed in the diaphragm at equal distances from the axis of the telescope. If a graduated staff is held vertically and the divisions intersected by the fixed wires are read, the difference of the readings multiplied by a constant factor (usually 100) gives the distance of the staff from the observer. The accuracy obtainable by this method will vary with the instrument employed and with conditions of visibility during the observation. An error not exceeding 1/500 should normally be obtainable. A distance of about 1,000 ft. is the most that can be accurately measured by this means ; at longer distances the readings of the intersections cannot usually be seen with sufficient clearness to give good results.

Base Measurement by Range-Finder.

The accuracy of direct measurements by range-finder depends on the type employed and is influenced by temperature changes and other uncertain factors. With a 12-foot range-finder, a base of 2,000 yards could be measured with an error not likely to exceed 5 yards, *i.e.*, one in 400. In the example illustrated in Fig. 5, it would be possible to observe the horizontal angles at A and B to a range-finder at C and obtain direct measurements of AC and BC, thence calculating the distance AB as before. Smaller range-finders naturally give less accurate results and, in general, it is not advisable to use this method if any other can be applied.

Base Measurement by Astronomical Observations.

When the latitude and longitude of two points on the earth's surface have been accurately determined by astronomical observations, the distance between

them can be calculated provided certain dimensions are accepted for the shape and size of the earth. If the two positions are connected by a system of triangulation, the lengths of all the sides of that triangulation can be deduced. Astronomical observations do not, however, give an accuracy greater than 1″ or 2″ of arc (say, 100 to 200 feet) with the instruments normally available to the hydrographic surveyor, and there is a possible further uncertainty in the results due to unknown local deviation of the plumb line at the places of observation. For these reasons, this method of determining distances is rarely used.

Measurement of Distance by Taut Wire.

When a base cannot be measured ashore, use may be made of the Taut Wire Measuring Gear which is designed for measuring distance over the ground at sea. This apparatus is illustrated in Plate A, Fig. 7.

Components.—The apparatus consists of two separate parts :

(*a*) A drum standard, bolted to the deck, carries a fixed drum holding when fully wound about 140 miles of ungalvanised piano wire. It is fitted with a spindle passing through the centre of the drum, which has a band brake at its fore end and unwinding mechanism at its after end. The brake is operated by two butterfly nuts and is used to adjust the tension on the wire. The unwinding mechanism consists of a revolving arm secured to the spindle, carrying a " flyer " wheel at one end and a counterpoise weight at the other.

(*b*) A metal frame carries two sets of rollers, two leading wheels for guiding the wire, a cyclometer or counter wheel and a dynamometer. An indicator registers each whole turn of the counter wheel. The dynamometer A consists of a freely revolving wheel which is fitted in a frame carrying a pointer and is free to move up and down vertical guides. When a strain is put on the wire, the dynamometer wheel is lifted and the pointer indicates on a scale attached to the guides the approximate tension on the wire.

To Prepare and Run the Gear.—The lead of the wire can be clearly seen on Plate A, Fig. 7 ; it will be noted that there are about three-quarters of a turn of the wire on the counter wheel, the circumference of which is 6·080 ft. (or very nearly this) so that 1,000 revolutions are equivalent to one nautical mile. The foremost leading wheel is slightly off-set from the counter wheel and the vertical part of the wire between these two wheels is brought up on the *port* side of the horizontal part already led onto the counter wheel. The wire is spliced at intervals of a few miles and the off-setting of the fore leading wheel prevents fouling where the two parts of the wire cross each other. The wire has a breaking strain of about 200 lb. with a direct pull but, if kinked, breaks easily. To prevent this happening during the process of reeving, the brake should be eased off only just sufficiently for the flyer arm to be pushed round by hand. When the wire has been rove, an improvised anchor, *e.g.*, firebars, of 50 to 80 lb. weight, on a few feet of rope stray-line is secured to its outer end. The anchor should be lowered by hand till just clear of the water and the slack of the wire taken up by pushing backwards on the flyer arm. The brake is then set up and the gear is ready for running. At the end of a run the wire may be easily cut by holding a piece of iron beneath a moving part of it and giving it a smart blow with a cold chisel.

The whole apparatus is very simple and, provided all moving parts are kept well greased or oiled, nothing should go wrong with it. When the gear is idle, the flyer arm should be left in such a position that the oil hole in the lubricating ring is at the top. The brake drum should be lubricated with two or three drops of oil only and, to ensure its smooth working, should be protected from the rain. A good deal of grease is thrown off whilst working and hoses should be kept running to wash it away.

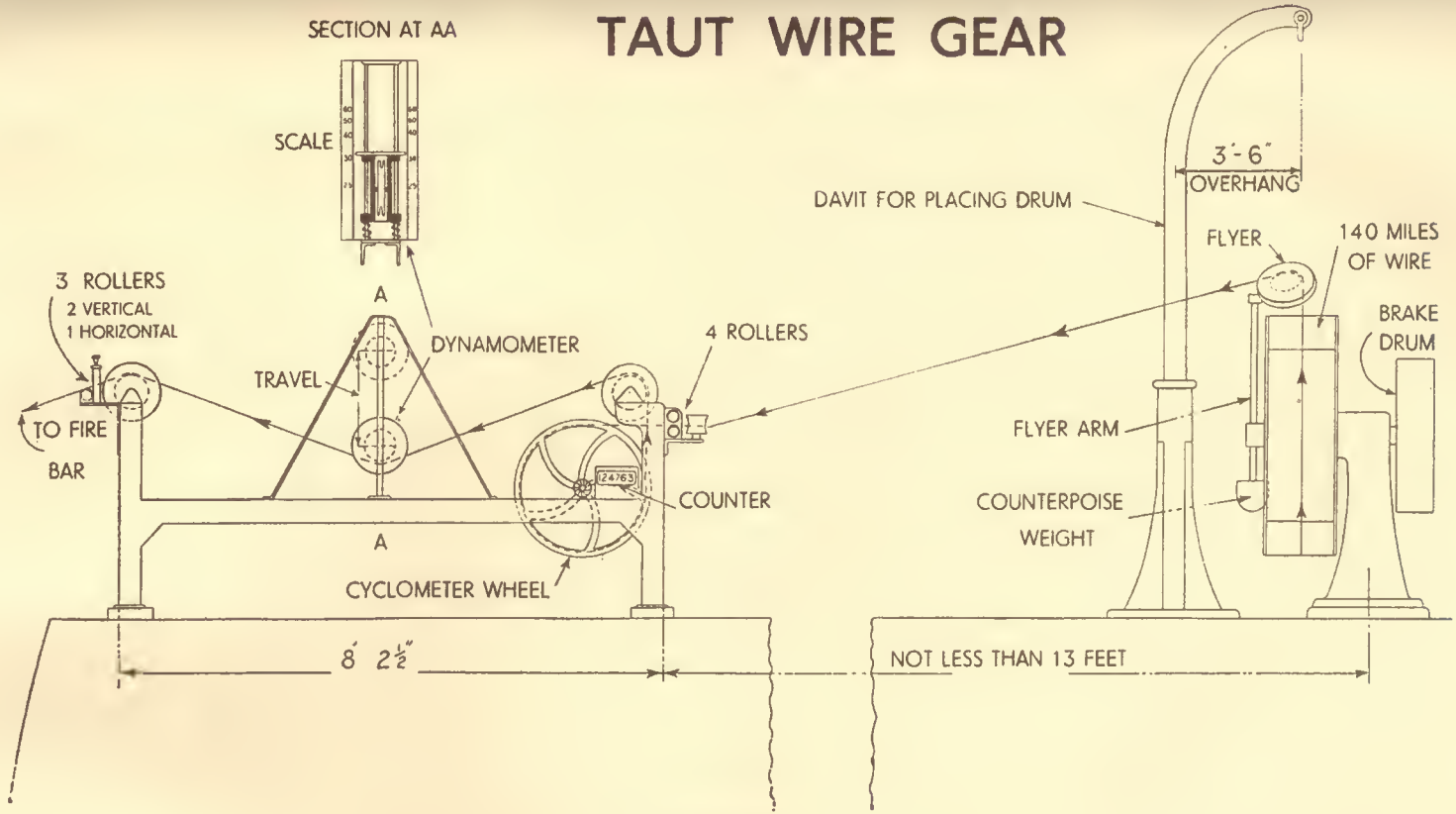

TAUT WIRE GEAR
SECTION AT AA
SCALE
A
3 ROLLERS
2 VERTICAL
1 HORIZONTAL
TRAVEL
DYNAMOMETER
4 ROLLERS
TO FIRE
BAR
COUNTER
A
CYCLOMETER WHEEL
8' 2½"
DAVIT FOR PLACING DRUM
3'-6"
OVERHANG
FLYER
140 MILES
OF WIRE
BRAKE
DRUM
FLYER ARM
COUNTERPOISE
WEIGHT
NOT LESS THAN 13 FEET

PLATE A, FIG. 7

The wire rusts rapidly if it is not well greased; if there is any doubt as to the condition of the outer turns on the drum it is as well to run off a mile or so and get rid of them rather than risk a break early in the run proper. When the anchor first enters the water, the wash from the propellers exercises a considerable force on it and for this reason speed at the start of a run should be reduced to "slow." When running, the best speed for the ship is about 8 to 10 knots. The gear will function satisfactorily at much higher speeds, but at over 10 knots the counter wheel revolves so fast that it is difficult to read off the indicator accurately at any given moment.

Method of Use.—One man is sufficient to work the gear, his duty being to tend the brake, and an observer should be posted to read the indicator. When it is desired to measure a distance between two fixed marks at sea, *e.g.*, moored beacons, the procedure is as follows. The "run" must be started 1½ to 2 miles *before* the first mark is reached and a straight course over the ground must be maintained over the whole distance. The ship therefore proceeds to a point on an extension of the line joining the two marks and 2 miles, say, from the nearer one. Speed is then reduced to slow and the ship is put on the course for running the line. The brake is then eased off a little and the wire allowed to run out slowly. As soon as the anchor has reached the bottom, the ship's speed is increased gradually to the desired maximum and the brake adjusted so that the dynamometer shows a tension of about 38 lbs. When abeam of the first mark, as noted by the compass, the indicator is read and similarly on passing the second mark. The signal for reading off should be given from the bridge as the observer at the indicator cannot have his eyes on the mark as well as the indicator and would in any case be unable to determine when it was exactly abeam or on the required bearing. The difference between the two readings gives the number of revolutions of the counter wheel and hence the distance between the marks. When passing each mark its distance from the ship should be measured by range-finder or other means; it may happen that, owing to an unsuspected current, the ship is set a little off the course truly parallel to the line of the marks. In such cases the distance run must be corrected as in the following example.

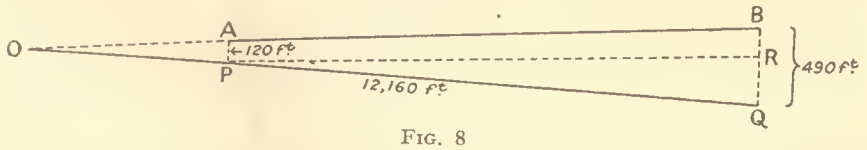


FIG. 8

In Fig. 8, the run was started at O and followed the line OPQ. Passing A and B, their distances were noted to be 120 feet and 490 feet respectively. Measured distance PQ was 12,160 feet.

Then distance AB = PR = PQ cos ∠ RPQ

Now, $\sin \angle RPQ = \frac{RQ}{PQ} = \frac{490 - 120}{12160} = \frac{370}{12160}$

Logarithmic Computation.

log 370		2·568 2017
log 12160		4·084 9336
L.sin ∠ RPQ		8·483 2681
∴ ∠ RPQ	= 1° 44′ 37″	
log PQ		4·084 9336
L. cos ∠RPQ		9·999 7989
log AB		4·084 7325
∴ AB	= 12154 ft.	

It may sometimes occur that a distance has to be measured which is not a straight line. The following methods may be used but the accuracy will be of a lower order; it should, however, be within 0·2 miles in 50.

(*a*) Where the position of course alteration is marked by a buoy, the ship is steered to pass very close to it. A reading is taken at the instant that it is abeam and the wheel is put hard over. Although the ship turns on a wide arc the bight of wire is soon pulled straight and taut against the mooring of the buoy, so that the reading taken when the buoy was abeam may be accepted as the reading of the buoy on the new course.

(*b*) A weight of 3 hand leads or a sinker on a stout line secured in a loop around the wire is let go by signal from the bridge when the wheel is put over. A similar weight is let go as the ship's head nears the new course. The distance between the two weights may be taken from data of "advance" and "transfer" for the ship or, the bearing and distance of a floating object, dropped with the first weight, may be observed when the second weight is let go.

Examples of Taut Wire Measurement.

(*a*) *To find the distance between two moored beacons at sea.*

This is a straightforward measurement and only requires taut wire runs between the beacons as laid down in the previous paragraph. It is always best to run the distance three times and take a mean if this can be arranged. If x is the number of revolutions of the counter wheel over the run between the beacons, their distance apart is $6{\cdot}080x$.

(*b*) *To fix the position of a mark out of sight of land.*

When it is necessary to fix the position of a mark or to carry out a survey out of sight of land, a reliable position can often be obtained by running two or more taut wire distances from positions near the land, where the ship can

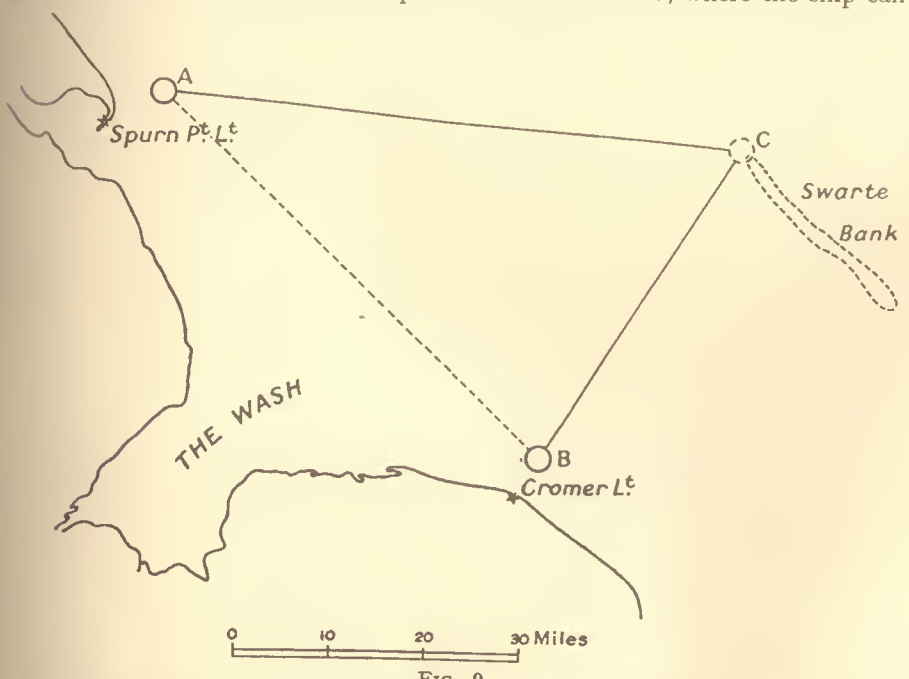


FIG. 9

be accurately fixed, to the position of the mark or one of the beacons of the survey. Supposing, for example, a survey of the Swarte Bank, which lies about 40 miles from the nearest part of the English coast, is to be made: see Fig. 9.

A beacon is moored near the north-west end of the bank and to fix it two taut wire distances are run :

(1) Between the beacon " C " and a position " A " off Spurn Point where the ship can be accurately fixed relative to Spurn Point lighthouse at the beginning or end of the run.

(2) Between the beacon " C " and a position " B " off Cromer where the ship can be accurately fixed relative to Cromer lighthouse at the beginning or end of the run.

The geographical positions of Spurn Point lighthouse and Cromer lighthouse are known and from these the positions of A and B and the bearing and distance of the side AB can be calculated. The angles of the triangle ABC can now be calculated from the sides by plane trigonometry. These angles will, however, be *plane* angles and their true values on the earth's surface must be found by adding to each a small correction equal to one-third of the spherical excess of the whole triangle. The spherical excess depends on the area of the triangle, which can be deduced from a rough plot, and is very nearly equal to 1″ of arc for each 60 sq. nautical miles of area, see Table II. Applying the corrected angles CAB and CBA to the true bearings AB and BA respectively, we have the true bearings of the sides AC and BC. Accepting the measured distances AC and BC, the geographical position of C can now be calculated from either. Both values should be obtained and the mean accepted. The errors to which measurements of this type are subject are discussed later.

(*c*) *To find the distance between two points on shore from a Taut Wire Measurement at sea.*

Suppose a survey is required of a harbour on a part of the coast where it is impossible to secure a suitable site for the measurement of a base on land.



FIG. 10

Two intervisible stations, A and B (see Fig. 10) are selected on shore and manned by observers with theodolites. Two beacons, C and D, are moored off

the coast and the ship runs a taut wire distance between them, thereby obtaining the length of the side CD. As the ship is abeam of each beacon on the run, the observers at A and B take horizontal angles to it, a system of signals being arranged from the ship to enable the observers to obtain their angles simultaneously with the taking of the taut wire indicator readings at each beacon. The theodolite readings will then be applicable to the positions actually occupied by each beacon as the ship passed, and no sensible errors will be introduced by previous or subsequent movements of the beacons round their anchors. It is required to find the distance AB.

In the quadrilateral ABDC, the angles 2, 3, 4, and 5 and the side CD have been measured. Let the observed values be as follows:

$\angle 2 = 39° 58' 55''$
$\angle 3 = 59° 03' 10''$
$\angle 4 = 37° 15' 30''$
$\angle 5 = 43° 20' 25''$
Side CD = 31,840 ft.

By trigonometry, the following formulae are true for the quadrilateral:

(1) $x + y = 3 + 4$

(2) $\dfrac{\sin x}{\sin y} = \dfrac{\sin 1 \sin 3 \sin 5.}{\sin 2 \sin 4 \sin 6.}$

(3) $\tan \dfrac{x - y}{2} = \tan \dfrac{x + y}{2} \times \dfrac{\sin x - \sin y}{\sin x + \sin y}$

Let $P = \dfrac{\sin x}{\sin y}$

Then $\tan \dfrac{x - y}{2} = \tan \dfrac{3 + 4}{2} \times \dfrac{P - 1}{P + 1}$

Solving for observed values, we have:

$\angle 1 = 180° - \angle \cdot (2 + 3 + 4)$
$= 180° - 136° 17' 35''$
$= 43° 42' 25''$

$\angle 6 = 180° - \angle \cdot (3 + 4 + 5)$
$= 180° - 139° 39' 05''$
$= 40° 20' 55''$

$x + y = 3 + 4$
$= 96° 18' 40''$

$\dfrac{x + y}{2} = 48° 09' 20''$

$$P = \frac{\sin x}{\sin y} = \frac{\sin 43° 42' 25'' \times \sin 59° 03' 10'' \times \sin 43° 20' 25''}{\sin 39° 58' 55'' \times \sin 37° 15' 30'' \times \sin 40° 20' 55''}$$

L. sin 1	9·839 4592	L. sin 2	9·807 9043
,, 3	9·933 3057	,, 4	9·782 0494
,, 5	9·836 5329	,, 6	9·811 1973
S_1	9·609 2978	S_2	9·401 1510
S_2	9·401 1510		
Log P	0·208 1468		

$\therefore P = 1·614\,904$

Hence $P - 1 = 0·614\,904$ and $P + 1 = 2·614\,904$

$$\therefore \tan \frac{x-y}{2} = \tan 48^\circ\ 09'\ 20'' \times \frac{0{\cdot}614\,904}{2{\cdot}614\,904}$$

L. tan	10·047 9344
log	9·788 8073
log	9·836 7417
log	0·417 4558
L. tan	9·419 2859

$$\frac{x-y}{2} = 14^\circ\ 42'\ 48''$$

$$\text{But } \frac{x+y}{2} = 48^\circ\ 09'\ 20''$$

$$\therefore x = 62^\circ\ 52'\ 08''$$

$$\text{and } y = 33^\circ\ 26'\ 32''$$

The distance AB can now be found from either of the formulae :

$$AB = AC \sin 1.\text{cosec } 4 = CD \sin y \sin 1 \text{ cosec } 4 \text{ cosec } 2 \ldots\ldots\ldots (1)$$

$$\text{or, } AB = BD \sin 6.\text{cosec } 3 = CD \sin x \sin 6 \text{ cosec } 3 \text{ cosec } 5 \ldots\ldots\ldots (2)$$

Logarithmic computation for (1) and (2)

(1)		(2)	
log CD	4·502 9731	log CD	4·502 9731
L. sin y	9·741 2271	L. sin x	9·949 3731
L. sin 1	9·839 4592	L. sin 6	9·811 1973
L. cosec 4,	0·217 9506	L. cosec 3,	0·066 6943
L. cosec 2,	0·192 0957	L. cosec 5,	0·163 4671
log AB	4·493 7057	log AB	4·493 7049

$\therefore$ AB = 31,168 ft.

(d) To find the distance between two inaccessible marks on shore from a Taut Wire Measurement at sea.

If in the previous example A and B (see Fig. 10) are inaccessible, the angles at D (6 and y) and at C (x and 1) must be measured by sextant. As before, CD is measured by taut wire and the sides AC and AD, and BC and BD can be found directly from the triangles ACD and BCD respectively. AB can now be calculated from either of the triangles ABC or ABD in each of which two sides and the included angle are known. Naturally, the accuracy of the deduced length of AB will not be as great as it would have been if angles could be observed from A and B, since in this case sextant angles only are depended on and any movement of the beacons during the taut wire run will also be a source of error.

Accuracy of Taut Wire Measurements.—Errors in taut wire measurements may be caused as follows :

(1) *Inaccuracy of Indicator Reading.*—The indicator only shows each *whole* revolution of the counter wheel which corresponds to a distance of 6·08 ft. It may be anticipated that readings will sometimes be in error to this extent.

(2) *Circumference of the Indicator Wheel.*—This may not be exactly 6·08 ft. It can be tested on board by measuring on deck the precise amount of wire run off in, say, 20 revolutions of the wheel. It is not easy, however, to reproduce the same tension as would be experienced in actual running, and tests of this sort are of doubtful value.

(3) *Elasticity of the Wire.*—Measurements will obviously be affected by changes in the tension of the wire. If the dynamometer is watched carefully and the brake adjusted as necessary to preserve a tension as constant as possible, there should be little error from this cause.

(4) *Irregularities of the sea-bed.*—These cannot be allowed for except by an arbitrary correction. The aim should be to run the distance over as even a bottom as can be found.

(5) *Deviations of the ship's course due to unsuspected changes in tidal streams or currents, and due to yawing.*—To minimise these a straight course should be kept either by the use of a back transit mark (a moored beacon or mark ashore) or by running the distance in the line of the stream and fixing frequently so that the line may be kept or recovered by very small alterations of course. In good weather it should be possible to reduce yawing to an immaterial amount.

Taking all sources of error into consideration, the errors of taut wire measurements under good conditions should not exceed about one part in 500 of the measured length. Whenever possible, two or more runs should be made over the distance to be measured.

Calibration of Taut Wire Gear.—Undoubtedly the best way to determine the error of a particular set of taut wire gear in any ship is to run the gear several times over a known measured distance at sea. By this method, all the actual conditions of use can be reproduced. In many places measured distances exist and can be utilised for this purpose. Alternatively, the error can be found by running a distance between two moored beacons which are "shot up" from two fixed stations ashore at the same time as the ship passes each beacon. This, in fact, is the reverse of the problem shown in Fig. 10: if the side AB is known, and angles are observed to C and D, CD can be calculated.

Measurement of Distance by Sub-aqueous Sound Ranging.

The velocity of sound through sea water varies within comparatively small limits and is much more constant than in the atmosphere. This property of sound may be made use of in determining the positions of marks out of sight of land and for measuring distances between beacons or other control marks of a hydrographic survey. If the velocity of the sound wave can be accurately determined for the conditions prevailing in any given area, measurement of the time interval for the passage of the wave between two points will enable their distance apart to be calculated with considerable precision. Two distinct methods of sub-aqueous sound ranging have been employed and will now be briefly described.

(1) In the first method, a number of hydrophones are anchored off shore in positions where they can be accurately fixed relative to the land. The hydrophones are connected by cable to an electrical recording apparatus ashore. If an explosive signal is fired under water at some point out to sea, the sound wave travels to the hydrophones and the moment of its reception at each is automatically recorded in the apparatus ashore. The time interval elapsing from the moment of the explosion till the moment the sound wave reaches a hydrophone depends on the distance of the hydrophone from the explosion. This cannot be measured, but the *differences* of the time intervals for the various hydrophones are shown in the recording apparatus, and from these the bearing and distance of the explosion from any of the hydrophones, and hence from the land, can be calculated. The explosive signal is, of course, fired in close proximity to a beacon or other fixed mark of the survey. Other points in the survey can, if necessary, be fixed in a similar manner and hence the scale of the survey determined.

The mathematical theory involved in calculating the position of the explosion from the time intervals at the hydrophones is rather involved, and this method has not been brought into general use.

(2) The second method is considerably simpler as it makes use of direct measurements of the time interval between the explosion and its reception at

a hydrophone. Suppose it is required to fix the position of a moored beacon out of sight of land. Two or more hydrophones are anchored off shore in suitable positions where they can be accurately fixed relative to the land, and they are connected by cables to wireless transmitters on shore. A charge is exploded close to the beacon to be fixed and the moment of the explosion is automatically recorded on board the ship which is stationed in the vicinity. The sound wave travels to the hydrophones and its time of reception at each is automatically transmitted by the shore wireless stations back to the ship. The wireless signal may be assumed to travel instantaneously and the time intervals between the explosion and the receipt of the wireless signals represent the time taken for the sound wave to travel through the water to each hydrophone. If the velocity of the sound wave is assumed to be some definite quantity, the distances from the explosion to each hydrophone can be calculated at once, and hence the position of the beacon found. An alternative method of obtaining the same result would be to transmit a wireless signal from the ship simultaneously with the explosion of the charge and record its reception and that of the sound wave at the hydrophones at the shore wireless stations. In a similar manner the distance between two beacons at sea can be found by sending a simultaneous wireless and sound signal from a ship stationed at one to a ship stationed at the other.

Sub-aqueous sound ranging depends for its accuracy on a knowledge of the velocity and path of the sound wave through the sea. The velocity varies with the salinity, temperature and depth of the water and reliable results can only be obtained by comprehensive observations of these factors in the area where the work is being undertaken. At one time it was assumed that the sound wave would invariably travel by the shortest path through the water between any two points, but recent observations seem to indicate that this is by no means always the case and small errors due to insufficient knowledge of the sound wave path must be anticipated. Improved technique and much experimental work have now brought the sub-aqueous sound ranging method to a standard of accuracy which is adequate for small-scale surveys out of sight of land, but it is doubtful whether results are as good as those obtained with taut wire measuring gear.

Inaccessible Base.

It is sometimes necessary to use a base measurement deduced from horizontal angles observed to two inaccessible points whose positions are accurately known. The problem of calculating the length of such a base is obviously identical with that given under Base Measurement by Taut Wire in paragraph (*c*) of Examples of Taut Wire Measurement and illustrated in Fig. 10, and the equations for solving it are the same.

Base Measurement by Radar.

The technique of base measurement by radar has not yet been fully developed, but as ranges of over 20 miles can now (1947) be measured to an accuracy of about 10 yards by this means, it is probable that, in the future radar bases will be much used for sketch and running surveys. (See page 386.)

CHAPTER III

THEODOLITES

Description, Adjustment and Use.

Theodolites are portable instruments devised for the accurate measurement of horizontal and vertical angles.

A transit theodolite is one in which the telescope can be rotated vertically end for end, and which has the vertical circle graduated throughout its circumference. In older types of theodolite this was not always the case, but all modern theodolites are transits.

Theodolites are made in various designs and sizes, and are usually described by:

(*a*) The diameter in inches of the engraved circles.
(*b*) The means provided for reading the graduations of the circles.
(*c*) The name of the maker.

As regards (*b*), the two principal methods of reading the engraved circles, are:—(1) by means of verniers, and (2) by means of micrometer microscopes (generally known simply as "micrometers"). With the latter much more accurate results can be obtained, but micrometer instruments are more expensive and delicate than those fitted with verniers.

The theodolite to be described here is a 3½" Tavistock double-reading model by Messrs. Cooke, Troughton & Sims, Ltd. Other modern theodolites are similar in principle, though differing in size and detail, and if one pattern is thoroughly mastered, the differences in others will quickly be apparent after a little handling, and will present no difficulty.

In the Tavistock model the means provided for reading off the circles are somewhat dissimilar to those normally employed. The principle of the "vernier" and the type of micrometer found in many other instruments, which are in common use, are fully explained in Close's "Text-Book of Topographical Surveying," and will not be described here.

It is essential that the surveyor should have a thorough knowledge of the features of the instrument he is using, and should know how to adjust and clean it, and effect minor repairs when necessary. Unless he feels absolute confidence in his knowledge of the details and in the theory of the adjustments, he will be apt, if he finds errors occurring in his work, to lose faith in his instrument.

Description of Parts, etc.

The instrument consists of two parts: (1) The Tripod; (2) The Theodolite.

The tripod is of an open-framed pattern with mahogany legs to the bottoms of which metal shoes are fitted. The head of the tripod is constructed to allow a centring movement of about 1½ in. to be made, and a circular bubble is fitted for rough levelling. The hinged pins which connect the legs to the centring head are self-adjusting and require no attention. A screw thread on the centring head enables the theodolite to be attached.

The theodolite is packed in one piece in a metal box and held in position by a frame, to which are attached two hinged metal straps. The Figs. 1 and 2 engraved on the straps correspond with Figs. 1 and 2 on the tribrach and transit axis of the theodolite, and indicate the position the instrument must occupy when stowed in the box; it will not stow in any other position.

The following are the names of the various parts of the instrument: see plates B and C.

A Cover plate containing optical system for horizontal circle.
B Box containing optical system for vertical circle.
D Vertical circle.

E Reading eyepiece for vertical circle.
E′ Micrometer milled head for vertical circle.
F Reading eyepiece for horizontal circle.
F′ Micrometer milled head for horizontal circle.
G Object glass.
H Focusing head for telescope.
H′ Screw focusing eyepiece.
J Cover to diaphragm adjusting screws.
K Plate spirit level.
L and L′ Screwed caps covering optical adjustments in A and B.
M Altitude or upper spirit level.
M′ ,, ,, ,, ,, ,, prism reader.
N Zero setting adjustment for vertical circle.

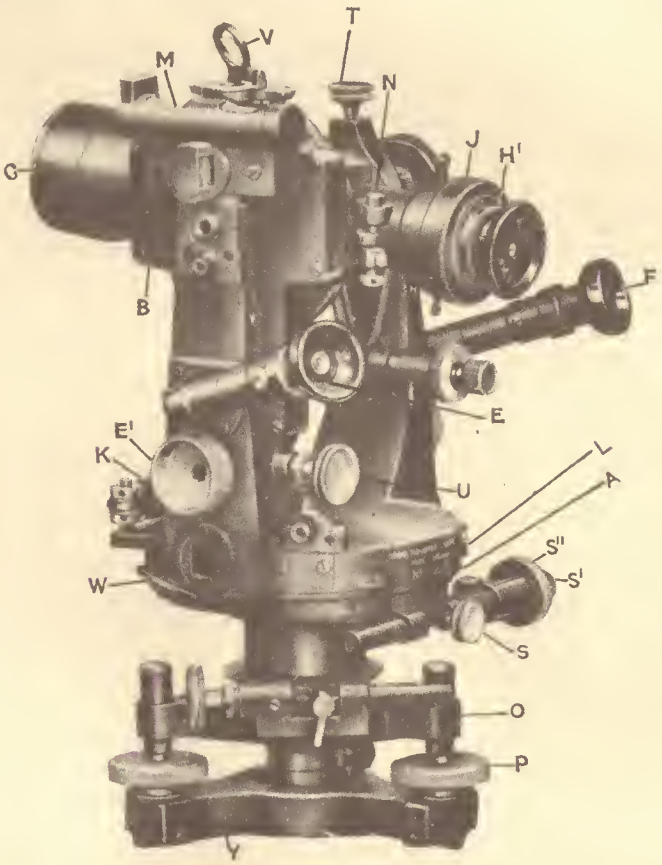


PLATE B

O Tribrach.
P Footscrews.
Q Two pin sockets for leads from battery.
R Clamp for lower plate.
R′ Slow-motion screw for lower plate.
S Clamp for upper plate.
S′ Slow-motion screw for upper plate.
S″ Fine-motion ,, ,, ,, ,,
T Clamp for telescope.
T′ Slow-motion screw for telescope.
T″ Fine-motion ,, ,, ,,
U Setting screw for altitude spirit level.
V Illumination reflector for vertical circle.
V′ Electric illumination for vertical circle.
W Illumination reflector for horizontal circle.

W Electric illumination for horizontal circle.
X Telescope diaphragm electric illumination.
Y Lower tribrach.

The following accessories, tools and spares are fitted in the box :

Diagonal eyepiece.
Spare lower-power eyepiece.
Tubular compass (not always provided).
Spare interchangeable diaphragm.
Extractor for withdrawing diaphragm.
Two coloured sun-glasses for eyepieces.
Rayshade for object glass.
Gunmetal plummet.
Spare electric bulbs.
Screw-driver and other tools.
Camel hair brush.
Metal bottle containing oil for centres.
Shower-proof cloth cover for instrument.

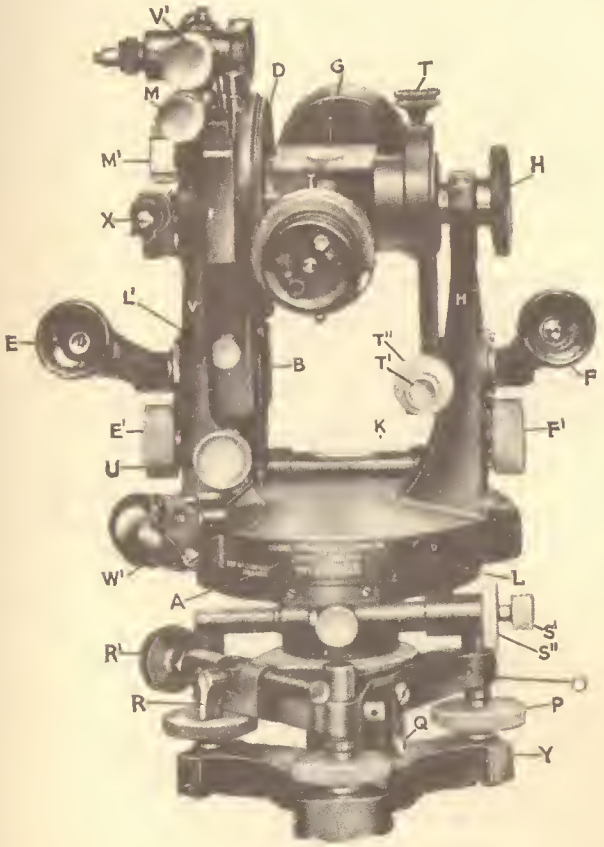


PLATE C

The electric illumination fittings, when not in use, can also be stowed in holders in the base of the box.

The *tribrach* is in two parts, connected together by three foot-screws (P) which enable the upper part (O) to be tilted relative to the lower (Y). The freedom of movement of the footscrews can be adjusted by capstan-headed screws on the upper part of the tribrach. The lower part is fitted with a screw-thread to engage with the thread on the tripod head. The upper part carries the axis of the *lower plate* which in its turn is hollow and carries the axis of the *upper plate*.

The *lower plate* carries the horizontal circle which is engraved on silvered

glass at every 20′ of arc, clockwise from 0° to 360°. It can be rotated round the tribrach or clamped rigidly to it by means of the clamp R.

The *upper plate* carries the optical system for reading the horizontal circle, the plate level K, and two standards with bearings for the telescope trunnions or transit axis. It can be rotated about the lower plate or rigidly clamped to it by the clamp S. The standards carry the microscopes for reading the vertical and horizontal circles.

The *telescope* is internal focusing; the distance, therefore, between the eyepiece and the object glass is constant, the focusing of objects at varying distances being accomplished by the movement of a subsidiary lens, situated between the object glass and the eyepiece, movable along the optical axis, and actuated by the milled head H. The telescope is also anallactic, that is to say, there is no constant to add for tachymetrical purposes. The diaphragm is of spider's web, with a simple cross, with the addition of stadia lines, 1 : 100, fitted in a removable cell. The spare diaphragm is engraved on glass. Open sights are fitted on the top and bottom of the telescope. The vertical circle is attached to the telescope but may be given a small movement relative to the latter by means of the capstan-headed screw N. This circle is engraved on silvered glass at every 20′ of arc from 0° to 180° and 0° to 180°, anti-clockwise from the zenith and nadir. The telescope (and vertical circle) can be rotated

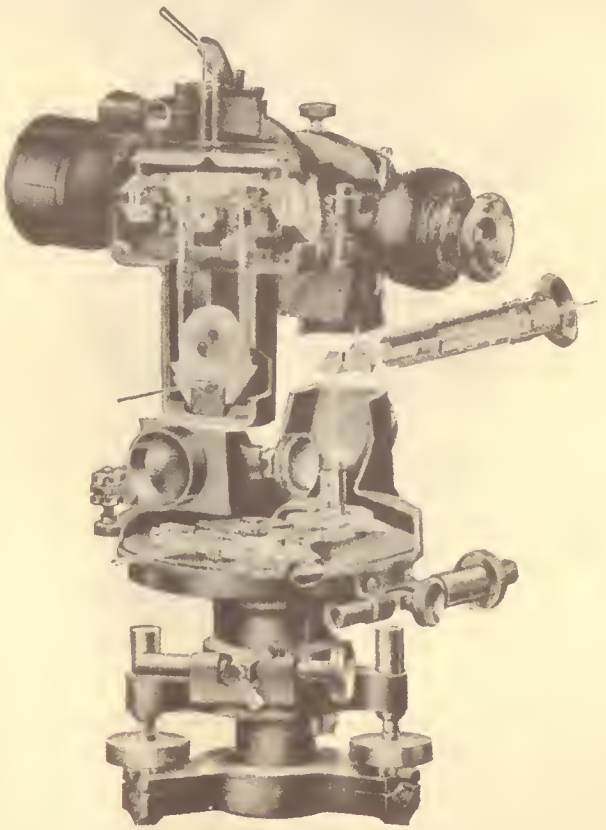


PLATE D

about the trunnion axis or clamped with the clamp T to one of the upper plate standards.

The *vertical arm* consists of the box B, containing the optical system for reading the vertical circle and carrying the altitude spirit level M and its

prism reader M′. The vertical arm can be rotated through a small angle about the trunnion axis by means of the setting screw U.

The *spirit levels*, or bubble tubes, K and M, are provided for the purpose of "levelling" the instrument, or, in other words, for making its axis vertical. The lower level K has a sensitivity of 40″ per 2/mm. run and is provided with adjusting nuts. The upper level M has a sensitivity of 22″ per 0·1 of an inch run. These two levels are at right angles to each other.

A reversible prism reader M′ is provided for the altitude spirit level. This enables the two ends of the bubble to be seen simultaneously from the eye-end of the telescope. By manipulating the setting screw U, the two ends of the bubble can be made to coincide, and the bubble is then in the centre of the tube and the vertical arm is "levelled."

The Optical System.—Each circle is read by an independent micrometer, and the circle reading eyepieces (E and F) are mounted one on each standard. These eyepieces are pivoted and can be swung into a convenient reading position accessible to an observer using the theodolite in either the "face left" or "face right" position. The instrument is said to be in the "face left" position, commonly expressed as F.L., when the vertical circle is on the *left* of the telescope when the latter is directed on any object, and *vice versa*. The process of reversal from F.L. to F.R. is called "changing face." The optical systems of the two circles differ only in detail.

Light from the engraved circle is reflected through two small apertures 180° apart into a box-like compartment where it is diverted through a system of prisms and lenses (see plates E and F) until it reaches the eyepiece. This compartment also contains a circular glass disc near the perimeter of which are engraved 1,200 divisions, corresponding to 20′ of arc (= 1200″ of arc) : this is the fine reading scale. Turning the milled head (E′ or F′) revolves the corresponding disc and at the same time moves two prisms which are placed in the path of the rays. The movement of these prisms gives an apparent lateral displacement to the images of the circle graduations (as seen in the eyepiece) corresponding to the angle through which the disc turns.

Fig. 11 shows typical views of the circle divisions and the fine reading scale as seen through the eyepieces. It will be noted that three rectangular apertures are seen. The coarse reading (to nearest 20′) is obtained from aperture 1, whilst the fine reading (minutes and seconds) appears in aperture 2. The fine reading, which must *always be added* to the coarse reading to obtain the complete circle reading, is invariably less than 20′. In aperture 3, which is really two apertures separated by a broad line (afterwards referred to as the setting index), two circle graduations are shown, one on each side of the setting index, and it will be observed that in all the diagrams these two circle graduations are so disposed about the setting index that the narrow light gaps on either side are of equal width. "Setting the micrometer" consists in equalising these two narrow light gaps, so that the graduations are positioned symmetrically about the setting index, and this operation must always be done before the circle is read.

"Setting the micrometer" is done by rotating the appropriate milled head (E′ or F′) on the side of the standard, or by rotating the circle by means of its slow-motion screw. The former method will be used if it is required to measure some unknown angle, the latter if it is required to set the circle to some pre-determined reading.

Referring again to Fig. 11, diagrams (a) to (f), it will be seen that :

(1) Aperture 1 shows about 1° of the circle which is engraved at every 20′ of arc. The coarse reading (to nearest 20′) is obtained from this aperture.

(2) Aperture 2 shows about 15″ of the fine reading scale, the figures nearer to the divisions representing the minutes, and those farther from the divisions the seconds, of arc.

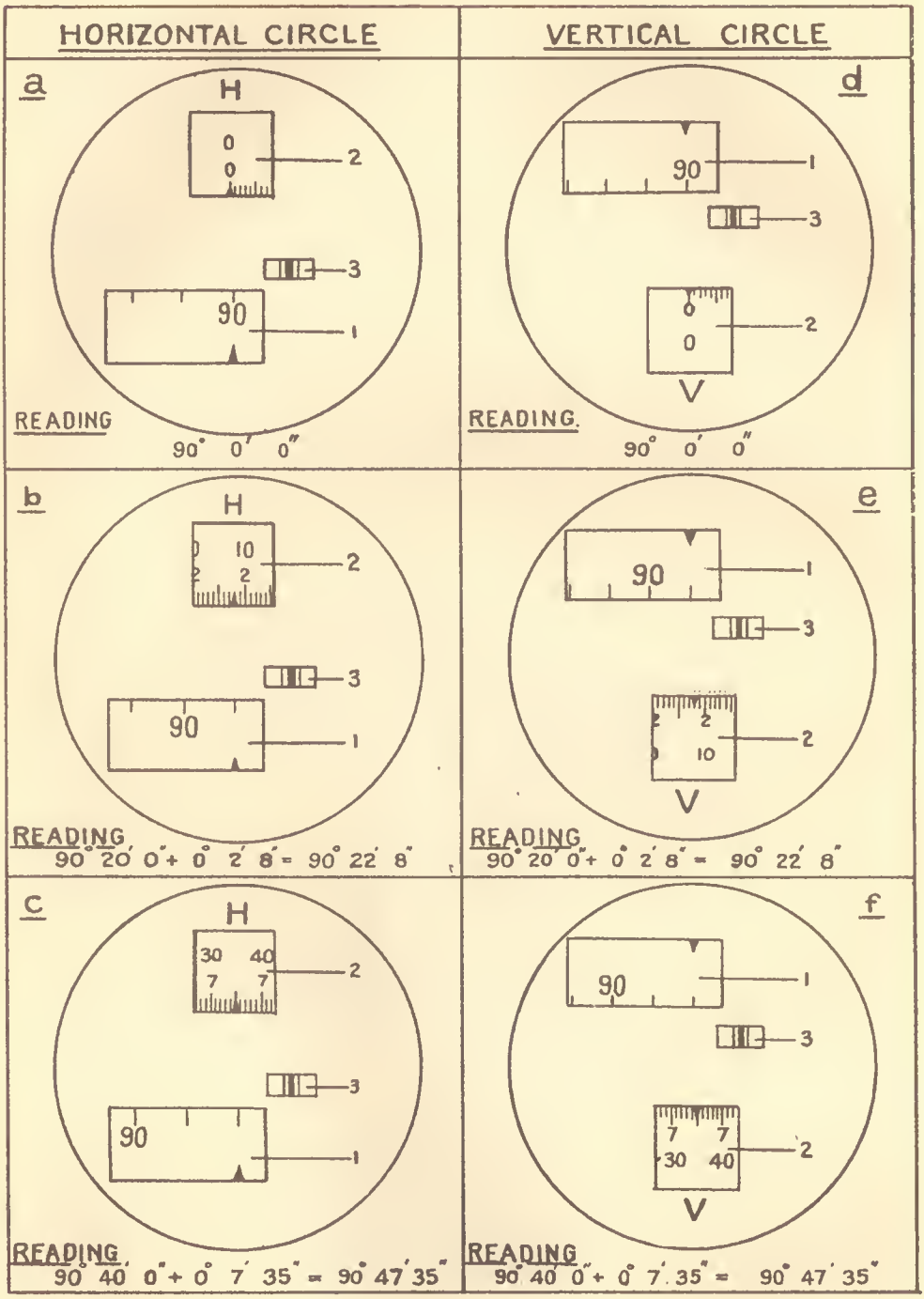

HORIZONTAL CIRCLE
VERTICAL CIRCLE
a
H
0
0
2
3
90
1
READING
90° 0′ 0″
d
90
1
3
0
0
2
V
READING.
90° 0′ 0″
b
H
10
2
2
3
90
1
READING
90° 20′ 0″ + 0° 2′ 8″ = 90° 22′ 8″
e
90
1
3
2
2
10
2
V
READING
90° 20′ 0″ + 0° 2′ 8″ = 90° 22′ 8″
c
H
30
40
7
7
2
3
90
1
READING
90° 40′ 0″ + 0° 7′ 35″ = 90° 47′ 35″
f
90
1
3
7
7
30
40
2
V
READING
90° 40′ 0″ + 0° 7′ 35″ = 90° 47′ 35″

FIG. 11

(3) Aperture 3 shows the setting index (a fixed black line) and two circle graduations 180° apart. Before the micrometer is "set" neither of these circle graduations may be in the field of view, but they will be brought into it by turning the appropriate milled head. By this method of reading off, both sides of the circle are read simultaneously and automatically meaned, thereby eliminating errors of eccentricity in the graduation.

Illumination.—For daylight use, movable reflectors (V and W) are provided for illuminating the circles. For night use, a 4·5 volt battery, complete with rheostat, is strapped to the tripod and three lamp holders can be fitted to the instrument, one on the lower part to illuminate the horizontal circle (W'), one on a standard to illuminate the diaphragm through the transit axis (X), and one on the vertical arm to illuminate the vertical circle and bubble (V'). The daylight reflector must be removed before the last-named lamp holder is fitted. The leads from the battery are plugged into sockets on the tribrach. The rheostat acts as a switch and controls the intensity of the lighting. The electric illumination of the circles is superior to that obtained from the daylight reflectors and should be used, if possible, when great accuracy is essential.

Setting up and Levelling the Theodolite.

The following is the procedure for setting up and levelling the theodolite:

(1) Press the legs, splayed well apart, firmly into the ground so that the head of the tripod is as nearly as possible over the station mark. On marshy ground it may be necessary to drive in stakes to support the feet of the tripod. See that the tripod head is approximately level by the circular bubble level.

(2) Remove the cap, and screw on the theodolite. Make sure that it is screwed up tight. Remove telescope cap.

(3) Attach plummet and centre the instrument over the station mark. Clamp the tripod head and remove plummet.

(4) Attach battery and electrical equipment, if required.

(5) Focus micrometers and telescope.

(6) Loosen all clamps and with the aid of the footscrews level the lower plate roughly by eye.

(7) The preliminary levelling is done with the aid of the *lower* plate level which is less sensitive than the upper level.

(8) Calling the three footscrews A, B and C, rotate the instrument until the plate level is parallel to two of them, say, A and B. Turn A and B in opposite directions until the bubble is centred in the plate level.

(9) Turn the instrument through 180° until it is again parallel to the line of A and B. If the bubble is no longer central turn A and B in opposite directions until it occupies a mean position which can be found by noting the scale reading of one end of the bubble when it was centred in the tube after operation (8) and the reading of the *same* end when the instrument has been rotated through 180°.

(10) Rotate the instrument 90° and bring the bubble to this mean position (not necessarily central) by means of the third footscrew C.

(11) As it is impossible to be certain that the level is in exact alignment with the footscrews in operations (8) and (9), it will be necessary to repeat (8), (9) and (10) once or twice to make the bubble occupy a constant position in its tube.

(12) The final levelling is done with the aid of the altitude spirit level M.

(13) Rotate the theodolite until the altitude level is parallel to one pair of footscrews, say, A and B, and bring the two ends of the bubble into coincidence (as seen through the prism reader) by means of the setting screw U.

(14) Rotate the instrument through 180° until the altitude level is again parallel to A and B, and bring the two ends of the bubble into coincidence again, making *half* the necessary adjustment by turning A and B in opposite directions and *half* by turning the setting screw U.

(15) Rotate the instrument through 90°, and bring the ends of the bubble into coincidence by means of the footscrew C.

(16) Owing to the difficulty of ensuring that the altitude level was parallel to the footscrews A and B—operations (13) and (14)—it will be necessary to repeat (13), (14 and (15) once or twice to steady the bubble in all three positions.

(17) The ends of the bubble should now remain in coincidence in any pointing of the theodolite, and the instrument is "levelled," *i.e.*, the axis is vertical.

(18) If the lower plate bubble is not now central, bring it to the centre of its run by means of the capstan screw on the bubble tube holder.

Optical Adjustment of the Telescope.

The theodolite having been set up, the telescope is adjusted as follows:

(*a*) Adjustment for distinct vision of the cross-wires.

This depends on the observer's eyesight and will vary with the individual. The eyepiece must be moved in or out until the cross-wires of the diaphragm are sharply defined. In the Tavistock model this movement is effected by twisting the eyepiece, and a locking screw is provided to keep it in position when set. This adjustment is most easily effected by turning the telescope on the sky to show up the cross-wires and then moving the eyepiece until they are sharply defined.

(*b*) Focusing the telescope, and Elimination of Parallax.

When the adjustment of the eyepiece is satisfactory, the telescope should be directed onto some distant and well-defined object and the focusing head H is turned until the object appears in sharp focus. When this has been done, the image formed by the object glass should be on the plane of the cross-wires.

To test whether this is the case, move the eye slightly from side to side, or up and down, and note if the object appears to move relatively to the wires. If it does not move there is no parallax, and the focus is perfect. But if there is movement it is due to imperfect focusing or parallax, and this must be eliminated by a further small alteration in the focus. It will be found that parallax can be eliminated by a small movement of the focusing head without throwing the object out of focus.

Adjustments (*a*) and (*b*) are always necessary when using the instrument. (Operation (5) above.)

Adjustments of Optical System and Micrometers.

The adjustments of each of the micrometers are three in number:

(*a*) Elimination of parallax.

(*b*) Adjustment of "light gap."

(*c*) Adjustment of micrometers for "run."

(*a*) The elimination of parallax is carried out on similar lines to those employed with the telescope. The eye-end of the micrometer can be rotated and the optical system is so arranged that the figures and graduations in the three rectangular apertures should be brought to a focus simultaneously. A locking screw is provided to prevent accidental rotation of the eye-end after the adjustment has been made.

(*b*) The adjustment of the "light gap" on either side of the setting index (aperture **3** in Fig. **11**) is effected by moving a small glass plate interposed in the path of one of the rays from the engraved circle. This plate can be rotated thereby giving an apparent lateral displacement to this ray. A special tool is

provided in the box for turning these deflecting plates: in the case of the vertical circle, access to the plate holder is obtained by unscrewing a small cover plate (L') on the side of the vertical arm; in the case of the horizontal circle access to the plate holder is obtained by removing a screw cap (L) on the side of the compartment containing the optical system.

The best width of the light gap is about 30″ as read in the micrometer by bringing first one circle graduation and then the other up to the setting index and noting the difference in the micrometer reading. To carry out the adjustment, proceed as follows (the operations are almost the same for each circle):

(1) Tighten all clamps.

(2) "Set" the micrometer. If the light gaps are too wide or too narrow, turn the plate a little with the tool provided.

(3) This will displace the image of one of the graduations: the amount of movement can be seen in the eyepiece as the operation is being performed.

(4) Reset the micrometer. If the light gaps are still unsatisfactory, give a little more, or less, movement to the plate until they are right.

This adjustment takes very little time and should not often be necessary.

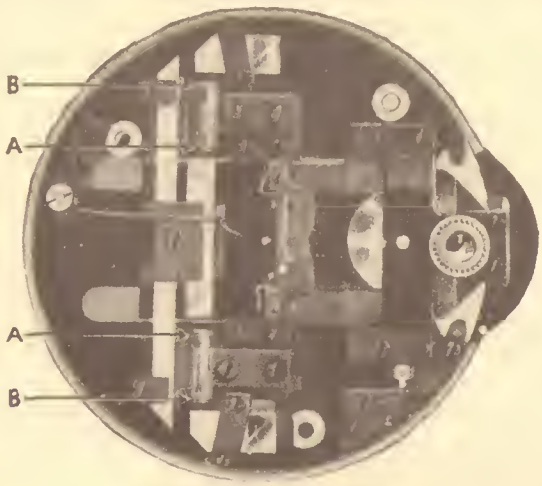


PLATE E

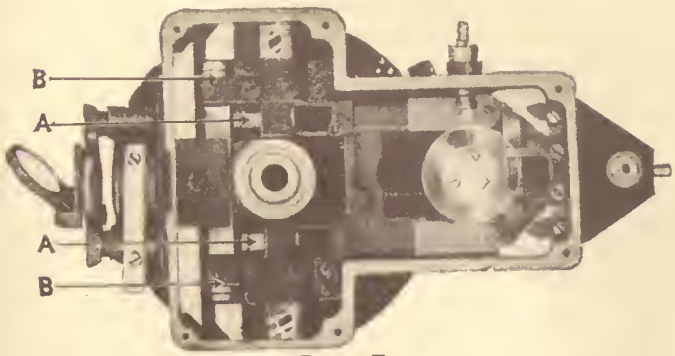


PLATE F

(*c*) Two objective systems each consisting of two parts, A and B (plates E and F), are mounted in each prism box, their purpose being to image the circle graduations at each end of a circle diameter in the correct focal plane at the correct magnification. The objectives are so designed that the portion

A controls the magnification without having much influence on the plane in which the image is formed ; a movement towards B increases the magnification and *vice versa*. The portion B controls the plane in which the image is found and a movement towards A causes the image plane to move inwards and fall short of the focal plane.

The correct image plane is that of the setting index and fine reading circle and, when these are clearly focused by the micrometer eyepiece, the circle graduations should also be clearly defined.

One revolution of the disc carrying the fine reading scale corresponds to 20′ of arc. If the image of a circle graduation happened to coincide exactly with the setting index at the beginning of the revolution, the image of the next circle graduation should make exact coincidence at the end of the revolution. If this is the case, the run of the micrometer for that side of the circle is correct.

To test the run of the micrometer both circle divisions should be examined independently as follows :

(1) Set the fine reading circle to zero (milled head E′ or F′) and bring the left-hand circle graduation close up to the setting index using the fine motion screw S″ or T″. Rotate E′ or F′ until the next circle division is close up to the setting index. The fine reading circle should now indicate 20′ within $\pm$ 5″. This tests the run of the graduations on one side of the circle.

(2) Set the fine reading circle to 20′ (milled head E′ or F′) and bring the right-hand circle graduation close up to the setting index, using the fine motion screw S″ or T″. Rotate E′ or F′ until the next division is close up to the setting index. The fine reading circle should now indicate zero within $\pm$ 5″. This tests the run of the graduations on the other side of the circle.

(3) If (1) and (2) are approximately correct a final test utilising both circle graduations should be made as follows :
Set the light gap to about 30″ as already described, set the fine reading circle to zero (with E′ or F′) and " set the micrometer " by means of the fine motion screw (S″ or T″). Rotate E′ or F′ until the second setting position is obtained (with the next pair of circle graduations) when the fine reading circle should read 20′ within $\pm$ 2″.

It is possible to have too low a magnification on one side of the setting index and too high on the other in which case the width of the light gaps will vary with different micrometer settings, although the run may read correctly. This cannot be the case if all three tests give satisfactory results.

If when making tests (1) and (2) an error in run is found on testing the circle division on the left of the setting index, the left-hand optical system is at fault in the case of the vertical circle and the right-hand in the case of the horizontal circle.

To take an example, suppose tests (1) and (2) are applied to the horizontal circle micrometer and that (1) gives a result in *excess* of 20′ while (2) is approximately correct. It is clear that the magnification of the left-hand circle spaces is too *great* and, in the case of the horizontal circle, the right-hand optical system in the prism box must be adjusted. To lower the magnification it is necessary to remove the small grub screw clamping the mount of the lens A and to insert a pointed tool through the screw hole in order to move the cell A away from B. If more than a very slight movement is required it will be necessary to move cell B also to bring the image of the circle divisions back into the correct focal plane.

Testing the run of the micrometers is a simple matter, but the adjustment, if it is required, is a slow and laborious process. Fortunately it is not often necessary as, once in adjustment, there is no reason for any error to creep in.

To gain access to the vertical circle and prism box :

(1) Remove altitude spirit level and reader (M and M') (3 screws).

(2) Unscrew spring boxes to vertical arm slow-motion (T') and altitude level (U) screws.

(3) Remove vertical circle micrometer milled head (E') (1 screw).

(4) Remove caps from top of standards (2 screws each).

(5) Lift vertical arm and telescope off the standards.

(6) Unscrew knurled ring retaining prism box.

(7) Withdraw prism box from axis : this will expose the face of the glass circle.

(8) Remove lid of prism box (6 screws). The optical system will now be exposed and the fine reading disc and mount can be drawn off its supporting pin for cleaning purposes, but care must be taken in reassembly to see that the relative positions of the rack and pinion are unaltered.

To gain access to the horizontal circle and prism box :

(1) Remove upper slow motion bracket (2 screws).

(2) Detach guard ring from cover plate (5 screws).

(3) Remove two-pin socket on tribrach (2 screws).

(4) Unscrew cap at base of centre socket on tribrach.

(5) Take out the centre screw with tommy pin.

(6) Tighten lower clamp, hold clamp arm and boss to prevent lifting and withdraw the upper part of the instrument, thereby exposing the horizontal glass circle.

(7) Holding the upper part of the instrument upside-down, remove the three recessed screws on the base of the prism box.

(8) The base of the prism box, to which the optical system is attached, can now be prised away from the upper part of the instrument. If the fine reading disc is removed for cleaning purposes, it must be replaced so that the relative positions of the rack and pinion are unaltered.

Theory of Levelling.

Theodolite Axis.—If all the clamps of the theodolite, with the exception of that of the lower plate, be tightened, the theodolite system is free to revolve about one axis only, *i.e.*, the bearing axis of the hollow cone attached to the lower plate. If this free clamp be now tightened and the clamp of the upper plate be released, the theodolite system is again free to revolve about one axis only, *i.e.*, the bearing axis of the spindle attached to the upper plate. It is the business of the maker to ensure that these two axes are coincident, and assuming that they are so, this common axis is called the theodolite axis. It is also the business of the maker to ensure that the lower plate which carries the horizontal graduated arc, is at right angles to this axis.

Trunnion Axis.—This is the bearing axis of the trunnions of the telescope. In many types of theodolite one end of this axis can be raised or lowered a small amount so that it can be adjusted to horizontality when the theodolite axis has been made vertical, but in the Tavistock instrument the bearings are rigidly attached to the upper plate standards and it is therefore the maker's business to ensure that the trunnion axis is at right angles to the theodolite axis.

Bubble Tubes.—A bubble tube consists of a hollow glass cylinder, closed at both ends and bent into the arc of a circle. The tube is almost filled with spirit, a small air-space being left. If the tube be tilted the air will travel along the tube in the form of a bubble, and the centre of the bubble will always occupy the highest part of the tube. The bubble tubes are provided for the purposes

of "levelling" the theodolite, *i.e.*, the process of making the theodolite axis vertical.

Reasons for Levelling.—The angular measurements required in a survey are those taken in the horizontal and vertical planes.

If, for instance, the angle between two objects, A and B (Fig. 12) lying at different levels is required, the angle subtended by the line AB′ (where B′ is a point vertically below B and on the same level as A) must be measured, and *not* the angle subtended by the line AB. If the elevation of A or B is required, the angle in the vertical plane between A or B and the horizontal must be measured.

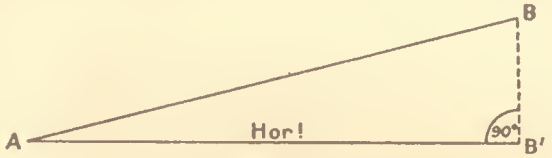


FIG. 12

Before a theodolite can measure angles in the horizontal and vertical planes, it is necessary to make the axis of revolution of the theodolite system vertical and the trunnion axis, about which the telescope turns, horizontal.

Suppose, for instance, it is required to measure the angle between two points, A and B (Fig. 13), which are on the same level.

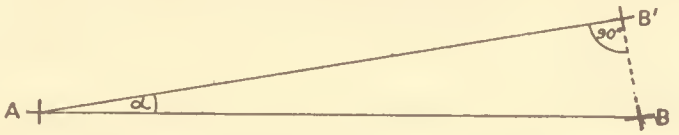


FIG. 13

(i) Assume that the theodolite axis is out of the vertical and inclined at an angle α in the direction of A, and that the trunnion axis is at right angles to the theodolite axis. If A is intersected with the cross-wires and the instrument is then revolved in the direction of B, the cross-wires will traverse the line AB′. At B′ the object end of the telescope is depressed and B is intersected, but the angle measured on the "horizontal" circle is that subtended by the line AB′, not that subtended by AB. If the theodolite axis had been vertical, the cross-wires would have traversed the line AB as the instrument was turned, and correctly measured the angle subtended by this line.

(ii) Suppose that the theodolite axis is vertical, but that the left-hand bearing of the trunnion axis (as seen from the eyepiece end of the telescope) is higher than the right-hand bearing and that it is required to measure the horizontal angle between two objects, A and B (Fig. 14) lying at different levels.

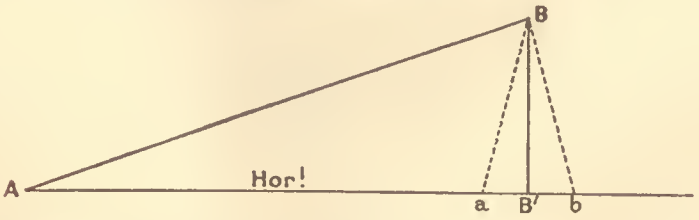


FIG. 14

If A is intersected with the cross-wires and the instrument is then revolved in the direction of B, the cross-wires will trace a horizontal path in the direction of B′, a point vertically below B and on the same level as A. Elevating the

object end of the telescope, B is intersected, but owing to the slope of the trunnion axis, the cross-wires trace the line *a*B and the angle subtended by the line A*a* has been measured, instead of that subtended by AB′. Now "change face," *i.e.*, transit the telescope through 180°, and repeat the process, the *right*-hand trunnion will now be the higher and the angle subtended by the line A*b* will be measured, B′*a* being equal to B′*b*. The *mean* of the two angles will therefore be correct.

It is clear from the figure that, if the difference in the elevation of A and B is required, the angle subtended by B*a* or B*b* will be measured since the trunnion axis is out of adjustment, and not the required angle which is that subtended by BB′.

Theory of Levelling.

If the theodolite axis be made to lie in any two vertical planes simultaneously, it will be truly vertical in every sense. If, of the three footscrews A, B and C, A and B are utilised for tilting the theodolite into a vertical plane at right-angles to AB by operating these footscrews antagonistically, then C can be utilised for tilting the theodolite in a second plane approximately at right-angles to the first, and the second tilting will not seriously disturb the effect of the first.

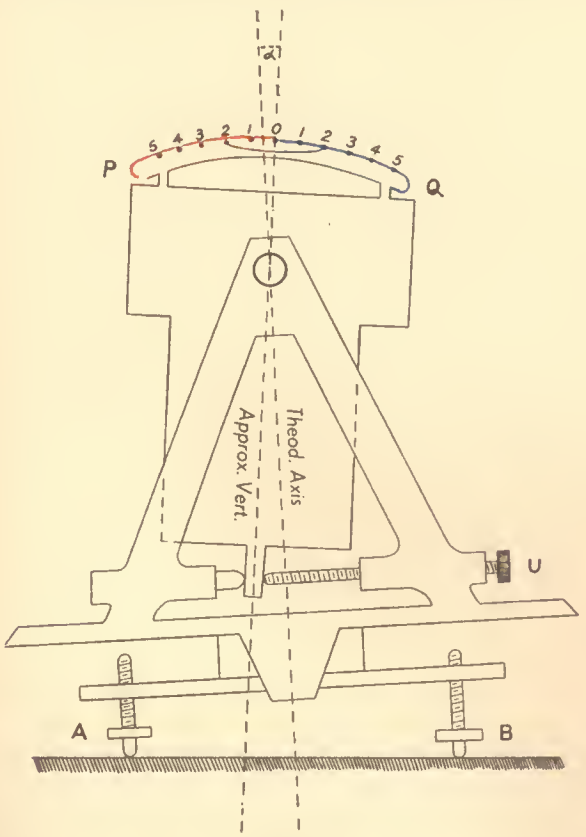


FIG. 15

Fig. 15 represents the upper part of a theodolite in which the bubble tube PQ is rigidly attached to the vertical arm, as is the case in the Tavistock instrument. The theodolite has been rotated until PQ is parallel to the line of the footscrews A and B, and the bubble has been brought to the centre of its run by means of the clip screw U. The two ends of the tube are shown in

different colours and for convenience divisions at equal distances apart are shown on them. (The upper bubble tube is not so engraved in the Tavistock model). Since the tube is an arc of a circle and since the bubble occupies the highest point of the tube, a normal to the tube through the zero mark will indicate the vertical. The theodolite axis is inclined at some, as yet unknown, angle α to the vertical and the problem is to make the two coincide.

Turn the theodolite through 180° so that PQ again lies parallel to AB, but end for end, and if the position of the observer remains unaltered, everything on the movable part of the instrument except the position of the bubble will be symmetrically reversed about the theodolite axis. This is shown in Fig. 16.

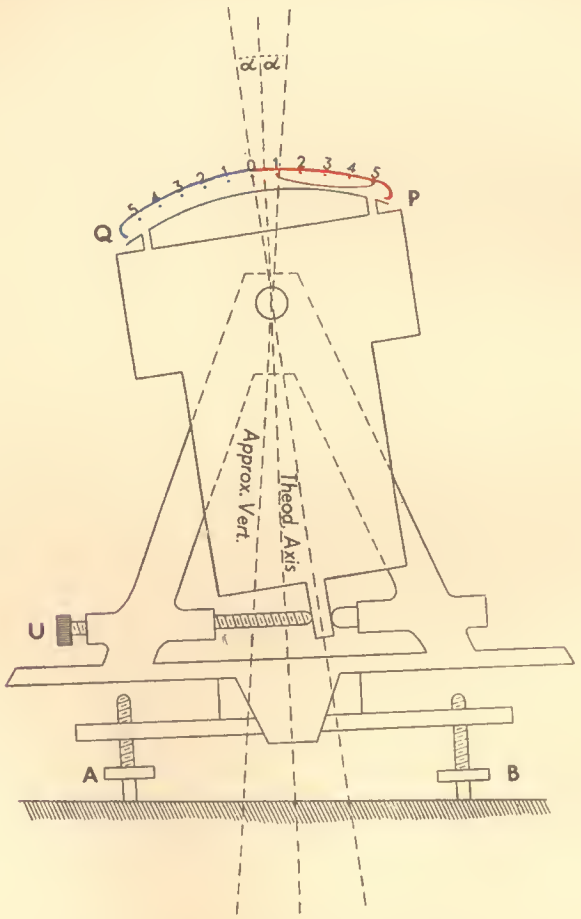


FIG. 16

Suppose that in the new position, the bubble is found to occupy the space between the 1 and 5 marks on the P end of the tube, as shown in Fig. 16. The centre of the bubble, therefore, is at the 3 mark on the P end of the tube, whereas in the first position it was at the zero mark. It is clear that the normal to the tube through the zero mark is now inclined at an angle 2α to the vertical and the movement of the bubble provides a means of measuring this angle. The reversal of the theodolite had the effect of moving the bubble 3 divisions along the tube towards the P end. If the clip screw U is now operated until the bubble has moved $1\frac{1}{2}$ divisions (*i.e.*, *half*-way) back towards Q, the normal to the tube through the zero mark will coincide with the theodolite

axis. Then, operate the footscrews A and B antagonistically until the bubble is moved a further $1\frac{1}{2}$ divisions towards Q; the theodolite axis will now coincide with the vertical in a plane parallel to AB and the bubble will be central again. If the instrument is now turned 90°, the bubble can be centred by means of the third footscrew C and the theodolite axis will be vertical in every plane. As explained in the procedure for levelling, it will probably be necessary to repeat these operations once or twice to steady the bubble owing to the difficulty of ensuring that the bubble tube is placed exactly parallel to any pair of footscrews.

It has been assumed for convenience that the theodolite axis passes through the centre of curvature of the bubble tube, but the investigation remains the same if a normal to the circumference of the tube can be drawn parallel to that axis.

Instrumental Adjustments.

It is impossible to make an absolutely perfect instrument which would always give perfect results, and every theodolite is subject to various errors of adjustment. These errors, in the order in which they should be corrected, are as follows:

1. Micrometer Errors.
2. Horizontal Collimation.
3. Verticality and Horizontality of Diaphragm Wires.
4. Dislevelment of the Transit Axis.
5. Vertical Collimation and Index Setting Error.

1. Micrometer Errors.—These have already been discussed and the methods of testing and eliminating them have been explained.

2. Dislevelment of the Transit Axis.—When the theodolite axis is vertical, the transit axis, about which the telescope turns, should be horizontal, *i.e.*, at right-angles to the theodolite axis. To test whether this is the case, proceed as follows:

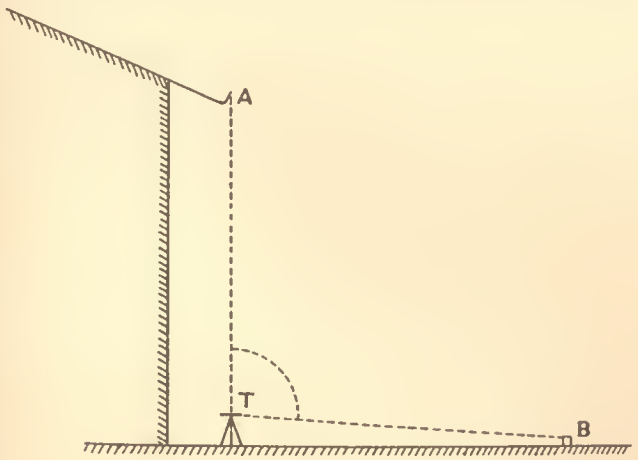


Fig. 17

(*a*) Set up the theodolite as close as possible to a tall building, insert the diagonal eyepiece in the telescope and level the instrument carefully.

(*b*) Sight on some sharply defined point A on the eaves of the building above, and adjust the telescope for parallax at this range.

(*c*) Bring the intersection of the cross-wires approximately on to A, clamp both plates and make absolute coincidence with the slow-motion screw.

(*d*) Rotate the telescope in its trunnions until it is roughly horizontal and points away from the building.

(*e*) Place a levelling staff B horizontally across the line of sight and at such distance that it is clearly in focus without the necessity of altering the adjustment of the telescope.

(*f*) Bring the cross-wires onto the staff and note the reading. Suppose it is 2·85.

(*g*) Without changing face, unclamp the lower plate, turn the instrument through 180° and intersect A again.

(*h*) Rotate the telescope in its trunnions, intersect the staff B again and note the reading on *this* face. If the two readings are the same the transit axis is horizontal: if not, it is out of adjustment. Suppose the reading on this face was 3·01, then the vertical plane through A must cut the staff at the mean reading 2·93.

(*i*) Raise or lower the adjustable trunnion by means of the capstan screw on the standard until by further repetitions of operations (*c*), (*d*), (*f*), (*g*) and (*h*) the instrument is found to give the correct reading. In the Tavistock model the trunnion *cannot* be adjusted, and if there is serious dislevelment of the trunnion axis, the instrument should be returned to the makers.

3. Verticality and Horizontality of Diaphragm Wires.—The correct positions for these wires are cut on the diaphragm by the maker so that the lines cut each other at right-angles. Therefore, if one is adjusted, the other should be correct.

Level the instrument and intersect a small well-defined object with the vertical wire. If it continues to be bisected by the wire as the telescope is elevated or depressed, no adjustment is necessary. If not, remove the diaphragm cover J (plate B) and slack off the holding screws sufficiently to enable the diaphragm to be revolved until this condition is secured, when they can be tightened up again, and the cover replaced. If a sea horizon is available, the test may be made by laying the horizontal wire on it and noting whether the wire makes contact throughout its length.

4. Horizontal Collimation.—The " line of collimation " may be considered as the imaginary line from the intersection of the cross-wires to the optical centre of the object glass and it should cut the axis of vertical motion (the transit axis) at right angles. The above definition of the " line of collimation " is not mathematically correct except for an infinitely thin lens, but it serves the present purpose.

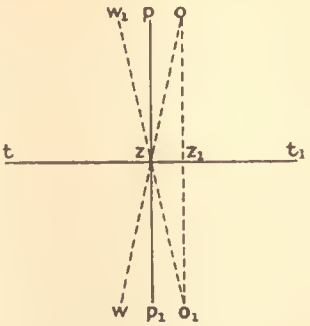


FIG. 18

In Fig. 18 tt_1 denotes the transit axis; when the line of collimation coincides with the perpendicular to tt_1, or the line pp_1, the instrument is in collimation, and if the telescope be made to describe a semi-circle in its bearings, its line of collimation produced will pass through the zenith, describing a great circle. But if the line of collimation be represented by *ow*, then the semi-circle traced

out will be presented by oo_1 the false zenith being z_1, while the arc described will be that of a small circle. In other words, by changing face, the object end, or o, will arrive at o_1, while the wires, w, will describe a small circle and reach w_1. Thus, if an object on the horizon, when intersected on face left, gives a horizontal circle reading of $a°$, when face is changed and the object intersected again (by turning the telescope in azimuth), the second reading, instead of being exactly $a° + 180°$, will differ from the first reading by $180° \pm ow_1$.

The error in collimation will, therefore, be :

$$\frac{ow_1}{2} = op.$$

To test and eliminate horizontal collimation error :

(*a*) Carefully level the theodolite and clamp the lower plate.

(*b*) Intersect a well-defined object, approximately on the same level as the theodolite, with the vertical wire. Read and book the angle, *e.g.*

F.R.	79°	15′	38″	
Add	180	00	00	
	259	15	38	.....(i)

(*c*) Change face and intersect the object again.

(*d*) Read and book the angle, *e.g.* :

F.L.	259°	36′	40″	.....(ii)
F.R.	259	15	38	(from (i))
2)	518	52	18	
Mean	259	26	09	.....(iii)

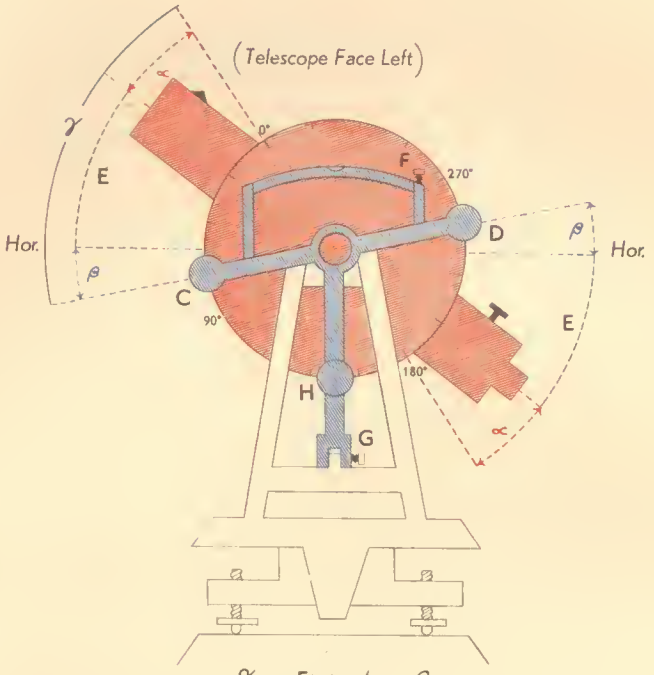
(*e*) With the milled head F′ (plate C) bring the circle reading (as seen in apertures 1 and 2, Fig. 11) to 259° 26′ 09″, and reset the micrometer by means of the slow motion screws S′ and S″. This will throw the vertical wire off the object.

(*f*) Unmask the diaphragm adjusting screws by removing cover J (Plate B) and by operating the two side screws antagonistically, bring the vertical wire onto the object again. Before replacing cover J make sure that the diaphragm has not been rotated during the process of adjustment, by laying the horizontal cross-wire on the sea horizon and noting whether it coincides throughout its length or alternatively whether the vertical wire coincides with a plumb line intersected at a convenient distance from the theodolite. (See **3** above.)

(*g*) It may be necessary to repeat the process of adjustment to eliminate the error entirely.

5. Vertical Collimation and Index Setting Error, or Dislevelment of Line of Vertical Micrometer Zeros.—Before considering the adjustments necessary to eliminate the effects of the above, the relations between those parts of the theodolite which revolve about the trunnion axis, should be clearly understood. Referring to plates B and C of the Tavistock theodolite, the vertical arm, containing the optical system for reading the vertical circle and to which the altitude spirit level M is rigidly attached, forms one unit which can be given a small angular movement by means of the setting screw U. Any movement of the vertical arm alters the position of the setting index, which acts as the reference mark for vertical angles, in relation to the vertical circle and telescope. The two latter parts form another and independent unit which can be rotated through 360° round the trunnion axis ; normally they move as one, but for the purposes of adjustment a small movement of the vertical circle in relation to the telescope can be given by means of the zero setting adjustment screw N.

The line of collimation should coincide with the instrumental axis of the



$\gamma = E + \alpha + \beta$

FIG. 19

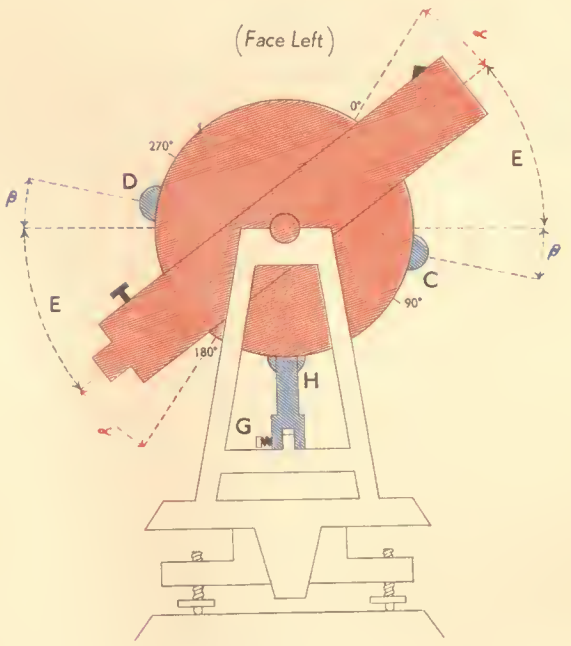


FIG. 20

telescope, and if, when the instrument has been levelled, the axis of the telescope could by some means be made horizontal, the position of the setting index relative to the engraved circle should be such that the vertical circle micrometer reads 90° exactly. If this reading is found to be in error, there is no ready method of finding out in the field what proportion of the error is due to faulty collimation, *i.e.*, the horizontal cross-wire not being coincident with the instrumental axis of the telescope and what proportion is due to an incorrect relationship between the positions of the vertical circle and its setting index. This difficulty, however, can be overcome by a method of adjustment which panders to any existing collimation error by throwing a compensating error into another portion of the instrument, thus eliminating the *effect* of vertical collimation and ensuring that the micrometer readings give the correct values of the required angles.

The relations between the telescope, vertical circle, vertical arm and the upper plate are shown in Figs. 19–22.

These figures represent a theodolite fitted with two micrometers, a vertical circle graduated from 0° to 360° and a bubble tube which can be given a small adjustment relative to the vertical arm, but the general principles covering the measurement of vertical angles are the same in the Tavistock instrument.

If the line of sight be directed on some object after the theodolite has been levelled and with the bubble central in its tube, as in Fig. 19, the angle which is read as the elevation or depression of the object is that between the line joining the scale zero to the scale division diametrically opposed (line of scale zeros) and the line joining the indices of the micrometers (line of micrometers.) In Fig. 19 this angle is $\alpha + E + \beta$, where α is the vertical collimation, E is the true elevation and β is the dislevelment of the line of micrometers. The angle read is burdened with two errors, the scale or collimation error α, and the dislevelment of the line of micrometers β. Now if the view point remains unchanged, but the theodolite be rotated 180° about its vertical axis, the telescope will assume the position shown in Fig. 20, still face left. The line of micrometers will still be inclined at an angle β to the horizontal. If the telescope is now rotated in its trunnions until the reference object is again intersected by the cross-wires, the position will be as shown in Fig. 21, the telescope now being on face right. The line of sight will still be inclined at the same angle E to the horizontal, but the circle reading, which is still the angle between the line of the scale zeros and the line of the micrometers, will be $E - \alpha - \beta$. If, therefore, the elevation is read on each face, it will be seen that the mean is E, the true elevation of the object. The process of adjustment for vertical collimation and dislevelment of the line of scale zeros consists in making the angle α equal but opposite in sign to the angle β, so that, when the adjustment is completed, the state of affairs is as shown in Fig. 22. It will be seen that the two errors then cancel out, and the true elevation is read on the circle.

Procedure for testing and eliminating the effect of vertical collimation :

(*a*) Set up and level the theodolite carefully.

(*b*) Select a distant object with a well-defined horizontal edge, such as the roof of a house.

(*c*) Bring the horizontal cross-wire onto the object, making sure that the bubble in the altitude spirit level is central. Read and book the vertical circle reading : suppose it to be 50° 10′ 30″ : call this E_1.

(*d*) Change face and repeat (*c*). Suppose the vertical circle reading is now 129° 49′ 56″ : call this E_2.

(*e*) Then $E_1 + E_2 = 180° 00′ 26″$. If there was no error, $E_1 + E_2$ would equal 180° exactly, so they must be made to do so by moving the vertical circle in relation to the telescope and setting index, thereby reducing each reading by half the total error.

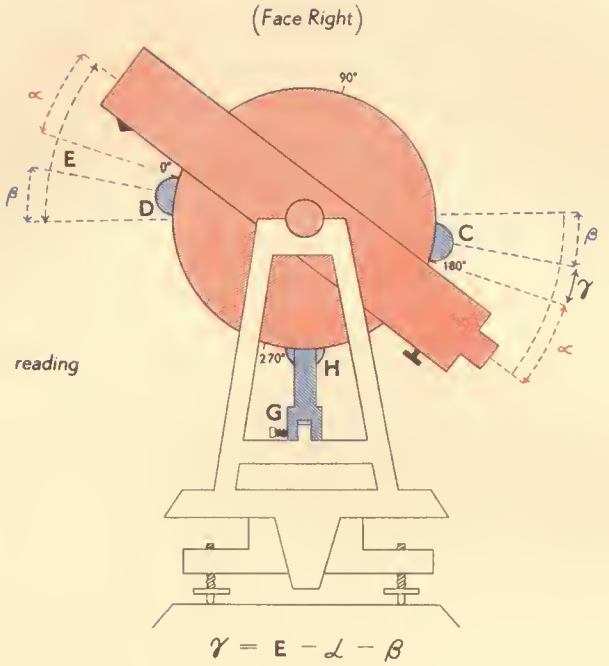

(Face Right)
90°
E
0°
D
C
180°
reading
270°
H
G
$\gamma = E - \alpha - \beta$

FIG. 21

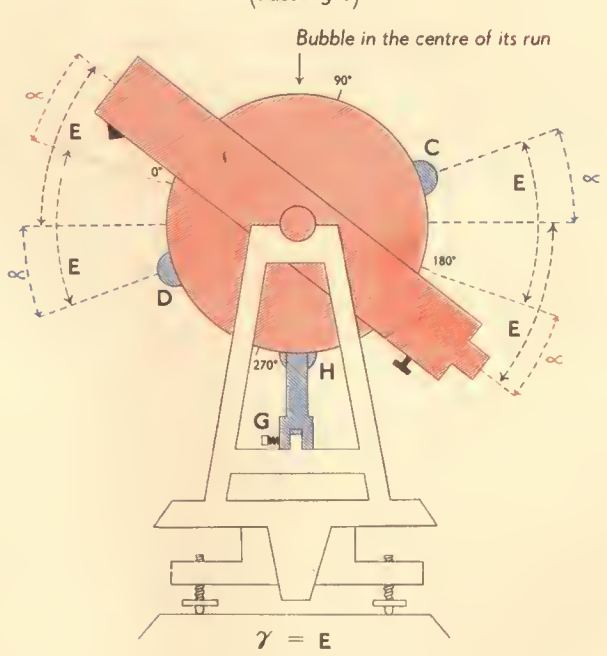

(Face Right)
Bubble in the centre of its run
90°
E
0°
C
E
E
D
180°
E
270°
H
G
$\gamma = E$

FIG. 22

(*f*) Keeping the telescope clamped, make the micrometer reading 129° 49′ 43″, using the milled head E′ (Plate B). "Reset" the micrometer by operating the capstan-headed screw N on the telescope barrel.

(*g*) Repeat operations (*c*) and (*d*) to check the adjustment.

The above procedure refers to the Tavistock model theodolite. In other types the vertical circle is usually rigidly attached to the telescope and the style of graduation is different. In instruments such as the 5″ micrometer transit theodolite, the adjustment for vertical collimation may be made as follows :

(i) Carry out operations (*a*) to (*d*) as above.

(ii) Take the *mean* of the readings E_1 and E_2 : call this E.

(iii) By means of the vertical circle slow-motion screw move the vertical circle in relation to the micrometers until the reading of the micrometer, which has been used in (*c*) and (*d*) shows the mean elevation E. This operation will move the horizontal wire off the object.

(iv) Bring the horizontal wire on the object again by means of the clip-screw. This operation will throw the bubble off the centre of the tube.

(v) Bring the bubble back to the centre by means of the capstan screws on the end of the bubble tube.

Cleaning and Repairs.

The theodolite should always be thoroughly dried before being stowed away in its box if it has got wet during use by rain or sea-water. Beyond this, it normally requires very little attention. A little olive oil should be smeared on the upper and lower plate axes and on the telescope trunnion bearings from time to time. Access to these parts can be gained by following out the initial stages of the instructions in the paragraphs dealing with the adjustments of the Optical System. The footscrew sockets and the movable portions of the clamping brackets also require a little oil occasionally:

The diaphragm wires are liable to be broken or to become slack through damp, and every surveyor should be able to replace them if required. The spare glass diaphragm can be temporarily used but there is some loss of light with this fitting and it is not so efficient as the ordinary "webbed" diaphragm. The best material for the purpose is spider's web—the garden spider produces the finest variety—or a fibre of raw silk.

To replace broken webs, remove the diaphragm ring and the old wires, and thoroughly clean out the engraved cuts with spirits of wine and warm water. Prepare a few rectangular cardboard frames, about 3 ins. by 2 ins. Place a spider on the edge of one of them and shake gently to detach it so that it will hang by its web. Wind up the frame as the spider falls, with the turns wide apart, and secure the end in a notch on the cardboard. If the web is too thick, starve the spider for a day or so. The process may then be repeated for as many cards as are likely to be required. With the aid of a magnifying glass, select a clean, uniform piece of web from one of the cards and holding the frame over the diaphragm ring, lower it onto a table so that the web falls into the proper cuts. The weight of the frame will keep the web taut and a drop of balsam or copal varnish, dropped onto the cuts and allowed to dry, will hold the web in position.

Methods of Observing.

The accuracy of the whole survey depends on the horizontal angles which connect the stations forming the framework, and the measurement of these angles is therefore a matter of the greatest importance.

All methods of observation are designed to eliminate instrumental errors as far as possible, for however carefully the various adjustments are carried out, it is impossible to make them absolutely correct.

HORIZONTAL ANGLES.—To eliminate the effect of dislevelment of the transit axis and want of adjustment of the line of collimation, an equal number of observations should be taken on each " face " of the instrument, that is, with the vertical arm on the right (F.R.) and left (F.L.) of the telescope, as seen from the eyepiece end. To eliminate any twist or strain of the instrument, each series of observations should be taken alternately on opposite *swings*, that is, all the objects are intersected *seriatim* by turning the theodolite alternately clockwise and anti-clockwise. Errors due to eccentricity of the graduated circle are automatically eliminated by the optical system in the Tavistock and some other theodolites; in other models it is eliminated by meaning the readings obtained with two or more micrometers symmetrically disposed round the circle. Other errors of graduation are eliminated as far as possible by repeating the observations on different parts of the circle.

Two methods by which horizontal angles may be measured with a theodolite are :

(1) The Direction Method.
(2) The Repetition Method (or Repeating Angles).

(1) The Direction Method.

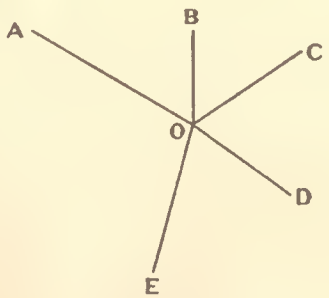


FIG. 23

If angles are required at the station O (Fig. 23) to the points A, B, C, D and E, the theodolite is set up and levelled over the ground mark at O, and the telescope and micrometers adjusted for focus and parallax. The procedure is then as follows :

(*a*) Select a station, say A, as a starting point, or *Reference Object* (R.O.). The point selected as R.O. should be a well-defined object at a good distance from the observer, though not so far that it is likely to be obscured by haze during the period of the observations, and should, if possible, be one of the main trigonometrical stations of the survey. When taking angles with a theodolite to *floating marks* from a shore station, it is essential to have a fixed R.O. and if no other trigonometrical point can be seen, some arbitrary point, such as a tree-trunk, or an artificial mark, such as a stake, must be selected for the purpose.

(*b*) See that the various slow-motion screws are roughly in the centre of their runs.

(*c*) Clamp the lower plate, and turn the upper plate until the horizontal circle reading is approximately 360° (say, within 1′) ; then re-clamp.

(*d*) Unclamp the lower plate and bring the cross-wires nearly onto A : re-clamp and bring the vertical wire onto A by means of the lower plate slow-motion screw.

(*e*) Unclamp the upper plate and rotate it through nearly 360° *clockwise*, stopping it just before the vertical wire reaches A, then re-clamping again.

(*f*) Bring the vertical wire onto A with the upper plate slow-motion screw. It is not essential that the horizontal wire should also be on A and it will generally be found more convenient to adjust the line of sight so that A appears slightly above or below the intersection of the cross-wires, all other stations being then intersected in a similar manner.

(*g*) Read and book the horizontal circle reading which should be about 360° 00′. If the instrument is fitted with two separate micrometers for the horizontal circle, both should be read and booked.

(*h*) Unclamp the upper plate and rotate it clockwise, stopping it just before the vertical wire reaches B, and then re-clamp.

(*i*) Bring the vertical wire onto B with the upper plate slow-motion screw and read and book the horizontal circle reading.

(*j*) Similarly for the stations C, D and E ; finally, intersect A again, continuing the clockwise motion of the theodolite throughout. The second circle reading of A should be the same as the first within ± 10″ (with the standard Tavistock model). If it differs by more than this amount, the instrument has presumably been disturbed during the round of angles, and the whole round should be repeated until satisfactory agreement is obtained. Assuming the instrument was in the F.R. (face right) position to start with, this completes one series of observations in the " Face right, Swing right " position.

(*k*) Transit the telescope, thereby changing to " face left " (F.L.), and repeat operations (*f*) to (*j*), starting and finishing with A, but observing the remaining stations in reverse order by turning the theodolite *anti-clockwise* throughout the series. The circle reading of A on this " face " will be approximately 180°. This completes one set of observations.

(*l*) Repeat the entire series of observations on F.L. and F.R., using a different " zero," *i.e.*, in operation (*c*) set the horizontal circle reading to 225° 1′ (say) so that the angles will be measured on different parts of the arc.

The number of repetitions will depend on the nature of the survey. In primary triangulation six or more zeros are used, but in hydrographic surveys the following system should generally give a sufficient standard of accuracy :

1st Zero	0° 0′	F.R., swing	R.
	180° 0′	F.L., ,,	L.
2nd Zero	225° 1′	F.L., ,,	L.
	45° 1′	F.R., ,,	R.

If the triangulation is likely to embrace a large area or if very long rays are involved, additional rounds of angles should be observed as follows :

3rd Zero	90° 2′	F.R., swing	R.
	270° 2′	F.L., ,,	L.
4th Zero	315° 3′	F.L., ,,	L.
	135° 3′	F.R. ,,	R.

Angles to secondary points and sounding marks need only be observed on one zero, but a complete set, *i.e.*, a series on *each* face, should always be obtained, *All* prominent objects, which may be of value as marks, should be included in this series.

The most convenient method of booking the angles is to place all the observations to one station together, as in the example overleaf :

EXAMPLE FOR RECORDING HORIZONTAL ANGLES

Survey..... Theodolite .....
Name of Station..... Height of theodolite above surface mark.....
Letter of Station ..... Height of beacon above surface mark .....
Observer ..... Weather .....
Date.....

Station.	Round.	Face/ Swing.	Hor. Circle Readings.	Reduced Angles.	Abstract of Angles.
			° ′ ″ ″	″	° ′ ″
A (R.O.)	1	R/R	360 00 10 } 12 14 }	00	360 00 00·0
	2	L/L	180 00 20 } 21·5 23 }	00	
	3	L/L	225 01 44 } 46 48 }	00	
	4	R/R	45 01 37 } 34·5 32 }	00	
B	1	R/R	59 11 41	29″	59 11 31·0
	2	L/L	54	32·5	
	3	L/L	284 13 16	30	
	4	R/R	07	32·5	
				4)124·0	
				31·0	
etc.	etc.	etc.	etc.	etc.	etc.

The second readings recorded on each face against A, the first station, are the readings at the end of the round. The means of the first and last readings of A are accepted. The reduced angles in column 5 are found in the case of B, C, D, etc., by subtracting the mean readings of A from the corresponding readings of the station.

(2) Repetition Method.

In this method the observations of each angle are completed separately. It is rather slower than the Direction Method, but should give a high standard of accuracy. The procedure is as follows :

(*a*) Set the horizontal circle reading to approximately 360° ; unclamp the lower plate, and bring the cross-wires nearly onto the selected station (station 1), and re-clamp.

(*b*) Intersect station 1 by using the lower slow-motion screw, and read and book the horizontal circle reading.

(*c*) Unclamp the upper plate and, swinging right, intersect the next station (station 2) by using the upper slow-motion screw. This constitutes one repetition. Read and book the horizontal circle reading ; this is done to prevent gross errors.

(*d*) Unclamp the *lower* plate and intersect the original station 1 again, using the *lower* slow motion screw. Note that the circle reading has not changed since the second object was intersected.

(*e*) Unclamp the upper plate and again intersect station 2, and so on, repeating the procedure six times in all. After the sixth repetition read and book the horizontal circle reading.

(*f*) Change face and intersect station 2 with the *lower* plate motion.

(*g*) Free the upper plate and intersect station 1, and repeat the process six times in a similar manner to that used before. After the sixth repetition read and book the horizontal circle reading which should be the same (within, say ± 10″) as the initial reading when the measurement was started.

The method of booking the readings is shown in the following example :

Stations observed.	Face L. or R.	Repetitions.	Hor. Circle Readings.	Arc passed over.	Means.
			° ′ ″	″	° ′ ″ ″
A—B	R	0	360 00 10		
		1	70 12 44		
		6	61 16 19	09	70 12 41·5 } 40·8
	L	6	360 00 18	01	40·2 }
etc.	etc.	etc.	etc. etc.	etc.	etc.

Six repetitions on each face should give sufficiently accurate results for use in hydrographic surveys, but if greater accuracy is required additional sets can be observed using different initial settings of the horizontal circle. If angles are being taken by the repetition method to a number of objects, such as A, B, C, D and E (Fig. 23), the fifth angle which "closes the horizon" should also be measured although in theory this angle can be deduced from the sum of the other four. It will generally be found in such cases that the angles in column 6 do not sum to exactly 360°, and they should be made to do so by equal adjustments.

Vertical Angles.

The vertical angle to an object is the angle measured in the vertical plane between that object and a horizontal line through the trunnion axis of the instrument. Vertical angles, *i.e.*, elevations and depressions, are required for determining the heights of various points in the survey. To measure vertical angles, make one observation on each face of the instrument to eliminate any unadjusted error in the line of vertical collimation and in the relative positions of the setting index, or line of micrometers, and the graduations of the arc. The construction and graduation of the circle in double-reading theodolites differ slightly from those fitted with two or more independent micrometers.

Procedure with Tavistock Theodolite.

(*a*) Set up and level the instrument carefully, and measure and book the height of the trunnion axis above the surface mark.

(*b*) Leave the upper plate clamp free, intersect the object carefully with the horizontal wire, using the vertical slow-motion, and make sure that the two ends of the bubble in the upper level, as seen in the prism reader, are absolutely coincident *when perfect intersection is made*, if necessary adjusting the bubble in coincidence with the altitude level setting screw.

It will probably be found convenient to make the intersection slightly to the right or left of the vertical wire. If this is done, the intersection should

be made on the opposite side and at the same distance from the vertical wire when face is changed.

(*c*) Read and book the vertical circle reading, *e.g.*, 41° 10′ 18″.....(1)

(*d*) Change face and repeat operation (*b*).

(*e*) Read and book the vertical circle reading, *e.g.*, 138° 49′ 20″.....(2)

This completes the measurement. If the adjustments were perfect, (1) and (2) should sum to 180°, but in this case the sum is 179° 59′ 38″. There is therefore an index error of 11″, *i.e.*, half the difference between the sum of the two circle readings and 180°, and as the sum is smaller than 180° in this example, the index error must be added to each reading. The corrected readings are therefore 41° 10′ 29″ and 138° 49′ 31″. As a reading of 90° on the circle of the Tavistock instrument represents the horizontal, the true elevation of the object is 48° 49′ 31″.

Procedure with Theodolite Fitted with Independent Micrometers.

The operations are the same as with a double-reading theodolite, *but* as the position of the bubble in the upper level cannot be directly seen by the observer at the eyepiece end of the telescope, it is necessary in operations (*b*) and (*d*) to centre the bubble *before* the observations, and since it cannot be relied on to remain perfectly steady, its position in the tube must be read immediately *after* each observation. If, as will often be the case, the bubble is *not* central after the observation, it is necessary to apply a correction to vertical angles corresponding to the displacement of the bubble. The value of this " bubble " correction can be found by noting the positions of the two ends of the bubble in the tube, the upper surface of which is usually graduated with a scale of equally spaced divisions. The angular movement of the bubble tube in the vertical plane corresponding to a movement by the bubble of 1 division along the scale is also usually marked on the tube, *e.g.*, 1 div. = 10″. This value, however, cannot be regarded as constant, since the size of the bubble varies with changes of temperature, and it should be checked from time to time in the following manner:

Call the scale reading of the object-glass end of the bubble O, and that of the eyepiece end E. Select an object with a well-defined horizontal edge and bring the bubble near one end of its run in the tube with the aid of the clip-screw (corresponding to the altitude level setting screw in the Tavistock instrument). Intersect the object with the vertical motion and read and book the circle reading and the position of one end, say, the O end of the bubble. With the clip screw bring the bubble nearly to the other end of its run and intersect the object again with the vertical motion. Read and book the circle reading and the O end of the bubble. Thus:

	Elevation.	O end of bubble.
1st Observation ..	10° 12′ 28″	16
2nd ,, ..	10 10 02	4
	2′ 26″	12

$\therefore$ Value of 1 division $= \frac{146''}{12''} = 12''{\cdot}2$

This operation should be repeated several times to get a good mean value.

The position of the centre of the bubble is the mean of the O and E readings; if the tube scale is divided outwards from the centre and the graduations towards the O end are regarded as positive and those towards the E end as negative, the position of the centre of the bubble is $\frac{O + E}{2}$, and the correction to the circle readings is $\frac{O + E}{2} \times$ the value of 1 division of the scale. If the O

reading is the greater, the correction is positive to elevations, negative to depressions, and *vice versa*. Example :

The value of 1 division on bubble tube = 12″

Face Right :		O E
Elevation (mean of C and D micrometers) =	21° 10′ 18″	+ 5 − 1
Correction = $\frac{+5-1}{2} \times 12''$ . . . . . =	+ 24	
Corrected elevation	21 10 42	. . . . . (1)
Face Left :		O E
Elevation (mean of C and D micrometers) =	21° 09′ 30″	+ 8 + 2
Correction = $\frac{+8+2}{2} \times 12''$ . . . . =	+ 1 00	
Corrected elevation	21 10 30	. . . . . (2)

∴ True elevation (*i.e.*, mean of (1) and (2))= 21° 10′ 36″

The above rule for the sign of the bubble correction is general whatever method is used for graduating the vertical arc, *provided* the circle readings are always reduced to actual angles of elevation or depression.

Notes on Observing Horizontal and Vertical Angles.

(1) *Centring of Theodolite.*—It is most important to centre the theodolite carefully over the ground mark. This is especially the case when observing short rays, as any error due to eccentricity varies inversely as the length of the ray. An eccentricity of 6 inches may produce an error of 17″ in a ray one mile in length.

(2) *Comfort of Observer.*—Accuracy of observation is greatly facilitated if the observer has unimpeded access to the theodolite on all sides and can get his eye easily to the telescope and micrometers, with his feet planted on firm ground. This should be borne in mind by the surveyor when he selects the positions of his trigonometrical stations.

(3) *Protection from Sun.*—Exposure to the hot sun of one side of the theodolite is likely to cause twisting of the instrument during use, and, if possible it should be protected from the sun's rays.

(4) *Clamps.*—In observing horizontal angles the vertical motion should either be very lightly clamped or unclamped altogether. Similarly in observing vertical angles the upper plate should be left free.

(5) *Times for Observing.*—The atmosphere usually appears clearer and, steadier in the morning and evening and these are generally the most suitable times for observing horizontal angles. At these times, however, refraction, which affects horizontal angles very little, is liable to be variable, and vertical angles should therefore be observed in the middle of the day.

False Stations

It frequently happens when angular measurements are to be made that the theodolite cannot be centred directly over the observing station ; this will be the case when objects such as church spires and chimneys are used as trigonometrical points of the triangulation. Sometimes, also, angles to other marks can only be secured by moving the theodolite a short distance to one side of the observing station. In such cases, the observed angles at the " False (or Satellite) Station " must be corrected. This can be done provided :

(*a*) The distances to the observed marks are known : a rough plot will usually give these with all required accuracy.

(*b*) The distance and theodolite reading of the true station, *i.e.*, of the centre of the observing station itself, are measured from the false station.

The method of correcting the observed angles is as follows :

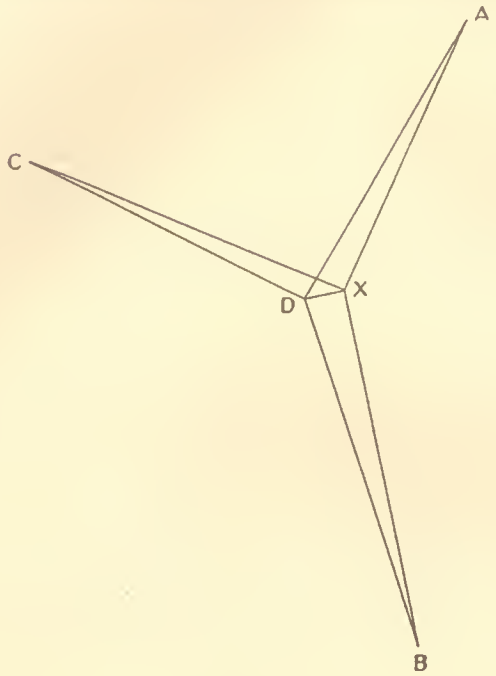


FIG. 24

In Fig. 24, D is a station from which angles are to be observed to A, B and C. Owing to the nature of the mark, the theodolite cannot be erected over the centre of the station but has to be placed at X, a short distance, DX, to one side. At X horizontal angles are measured between A, B and C and the centre of the true station D. Horizontal angular measurements are usually recorded as a series of directions or theodolite readings and, in finding the false station corrections, it is convenient to think of angles in this way. The angles ADB, BDC and CDA or, in other words, the directions DA, DB and DC are required, whereas the directions XA, XB and XC have been measured. To make them agree with DA, DB and DC it is evident that the directions XA, XB and XC must be corrected by the angles A, B and C respectively. The distances DA, DB and DC are measured from a rough plot and DX is measured on the ground. The required corrections are given by the formulae :

(1) $\text{Sin A} = \dfrac{\text{DX sin DXA}}{\text{DA}}$

(2) $\text{Sin B} = \dfrac{\text{DX sin DXB}}{\text{DB}}$

(3) $\text{Sin C} = \dfrac{\text{DX sin DXC}}{\text{DC}}$

The angles DXA, DXB and DXC are found by subtracting the theodolite reading of the true station D from those of A, B and C respectively, if necessary increasing any of the latter by 360°. The corrections can be calculated precisely by these formulae, taking care that the distances DA, DB and DC are expressed in the same unit of measurement as DX. When, as usually happens, DX is very small compared with DA, DB and DC, the corrections can normally be found with sufficient accuracy as follows :

If perpendiculars are drawn from X onto DA, DB and DC, their lengths represent the distance *subtended* by XD at A, B and C respectively. These

subtended distances can conveniently be taken out of a traverse table, using XD as the " distance " and the angles DXA, DXB and DXC as the " courses." The subtended distance is taken out under the heading " Diff. Lat." if the " course " lies between 045° and 135° or between 225° and 315°, and under the heading " Dep." if the " course " lies between 315° and 045° or between 135° and 225°. The corrections to the observed angles are then given by the following formula which can be worked out on a slide rule:

$$\text{Correction to angle (in secs.)} = \frac{\text{Distance subtended (in feet)} \times 34}{\text{Distance of object in sea miles (of 6,080 feet)}}$$

The signs of the angular corrections can be found from the following rule :

From the theodolite reading (increased by 360°, if necessary) it is desired to correct, subtract the reading of the true station : call the remainder θ. If θ is less than 180°, the correction is positive to the theodolite reading, and *vice versa*. Example :

In Fig. 24, the distance XD is 12·7 feet and the theodolite reading of D is 305°.

Obsd. Station.	Distance (in miles).	Theodolite Readings.	θ	Subtd. Dist. of XD (ft.).	Corrections.	Corrected Readings.
		° ′ ″	°		″	° ′ ″
A	9·00	360 00 00	55	10·40	+39·3	360 00 39·3
B	10·21	144 40 22	194⅔	3·21	−10·7	144 40 11·3
C	8·36	268 02 30	323	7·64	−31·1	268 01 58·9

It is customary to adjust the corrected reading of the reference object or zero mark (in this case A) to make it exactly 360°, and to do this a second correction, equal but of opposite sign to that of the zero mark reading, is applied to *all* the readings. In this example, then, a second correction of − 39″·3 would be applied to all corrected readings.

The problem can also be solved by the use of a nomogram or with a diagram for " Correcting False Stations."

CHAPTER IV

TRIANGULATION

Planning Triangulation.

The marks, by which the detail of a hydrographic survey is fixed, may be divided into three categories :

(1) The Stations of the Main Triangulation.

These are fixed in relation to each other by very careful angular measurements and their individual positions are precisely calculated. In order thait the quantity of observations and calculations may be kept within reasonable bounds and to avoid the accumulation of error, which is almost certa n to occur in chains of *small* figures, the number of stations forming the main triangulation system should be kept down to the minimum which will fulfil the essential requirement of furnishing a suitable framework for the survey.

(2) Secondary Stations.

These are required for providing an increased density of control which will often be necessary in some parts of the survey, *e.g.*, where the configuration of the coast is very involved. Their positions relative to the main framework will generally be calculated, but as the sides connecting them will be short and will not be used for extending the survey in any direction, a lower standard of accuracy in the observations connecting them may be accepted. Sextant angles may sometimes be used for fixing secondary points.

(3) Sounding Marks.

Under this heading may be included all other survey marks, natural and artificial, by which the soundings, coastline and topography are fixed. Their positions are not calculated trigonometrically but depend on the protraction on the plotting sheet of observed angles from the main and secondary stations or from positions dependent on these.

The density of the control provided by the triangulation will depend partly on the scale of the survey. The larger the scale the more sounding marks will be necessary to enable the surveyor to fix the detail in any part of the survey area. As these marks are fixed by the protraction of angles, sufficient main and secondary stations must be located within the limits imposed by the size of the plotting sheet and instruments to enable them to be plotted. In harbour or other large-scale surveys, therefore, the stations forming the main triangulation may be only a mile or less apart. In surveys of larger areas, the sounding marks will be relatively fewer and they can be plotted from the stations of the much larger framework which can be contained within the limits of the plotting sheet. The density of the control will also depend largely on the character of the coast. An irregular coastline will normally require a more complex system of triangulation than an open one, but when the irregularities are confined to small portions of the whole survey, the case can generally be met by small secondary systems of triangulation which should be in the form of connecting links between two parts of the main network though not used as integral parts of the latter.

In planning a triangulation scheme for the survey of a given area, the aim should be to provide a network of fixed points which will entirely enclose it. The network takes the form of a series of geometrical figures and the siting of the stations should be such that these figures are well-conditioned or, to put it in another way, that the " rays " connecting any station form a good " cut." Whenever possible, each main station should be connected by observation to

three or more other points in the system. In order to provide a check on the accuracy of the observations and plotting, every sounding mark must be fixed by at least three rays forming a good " cut " at it, and, to secure this, it is usually desirable to provide a framework which not only embraces the actual area of the survey but also extends some distance beyond its limits.

The most essential requirement in the selection of sites for main stations is that they should have as good an all-round field of view as possible. In consequence, hill summits, ridges, small isolated islands and conspicuous buildings, such as lighthouses and church towers, are usually the most suitable positions. Hill summits often require a great deal of clearing before they can be used, but time so spent will generally be well repaid in the increased value of the station itself. Marks low down near the shore may be easy to fix but they will seldom be useful for extending the triangulation.

When reliable maps are available, a general scheme of triangulation can often be evolved before a place is visited and this is a considerable saving of time although it invariably happens that some minor modifications of the scheme are found necessary when the ground is inspected. If reliable geodetic data can be obtained from local sources, the scheme of triangulation must be designed with a view to making connection with the local survey points. In unmapped districts it is essential to carry out a preliminary sketch survey before planning the triangulation.

In planning triangulation there is little to choose between a chain of quadrilaterals and a chain of polygons with central stations. The quadrilateral, if well conditioned, is a slightly " stronger " type of figure, *i.e.*, errors of observation have less effect on the accuracy of the final result, but the polygon, with five or more sides, covers a larger area for a corresponding number of stations and its adjustment and computation is somewhat simpler.

Triangulation

The framework, on which the survey depends, consists of a number of marked points on the earth's surface. These points must be fixed in relation to each other so that they can be plotted in corresponding positions on paper. If imaginary lines are drawn connecting the fixed points, we have a series of geometrical figures known as the " Triangulation." The relative positions of any two points must be determined by measuring the length and direction of the straight line joining them. Accurate measurement of distance on the earth is nearly always a slow and laborious process and it would be impracticable to measure directly all the distances between the adjacent points of a triangulation. If, however, the distance between any *two* adjacent points, *i.e.*, the length of *one* side, is measured and the angles of the various figures are observed, the remaining sides can be calculated by plane trigonometry. If the bearing or direction of one side is measured, that of any of the remainder may be found by subtraction or addition of the observed angles.

In Fig. 25, let ABC be any plane triangle.

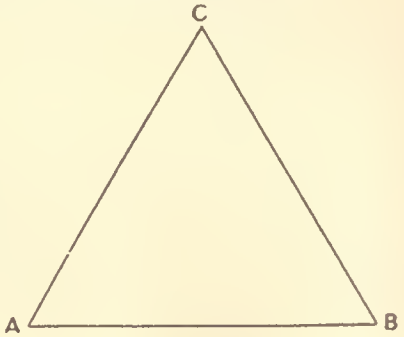


Fig. 25

Now, if the length of AB is known and the angles A, B and C are measured, AC and BC can be found from the equations :

$$AC = AB.\sin B.\operatorname{cosec} C$$
$$\text{and } BC = AB.\sin A.\operatorname{cosec} C$$

It may be noted that, as the angles of any plane triangle sum to 180°, it would be possible to calculate AC and BC if only *two* of the angles, say A and C, had been measured since the third angle B can be found from the equation, $B = 180° - (A + C)$.

Spherical Excess.—Triangles on the earth's surface are not, strictly speaking, "*plane*," but it can be shown mathematically that, if the observed angles of a triangle are adjusted so that they sum to 180°, the triangle may be treated as "plane" and the formulae of plane trigonometry can be used for finding the sides. The sum of the *true* angles exceeds 180° by an amount—the spherical excess—dependent on the area of the triangle, *vide* Table II. Whilst the plane angles can be used for computing the sides, the true spherical angles should be used for the transfer of azimuths from one point to another.

Fixing Stations.

The degree of accuracy, with which a point is fixed, depends to a great extent on the shape and character of the imaginary geometrical figures by which it is connected to other points of the triangulation. The simplest geometrical figure by which a point can be fixed in relation to others is the triangle.

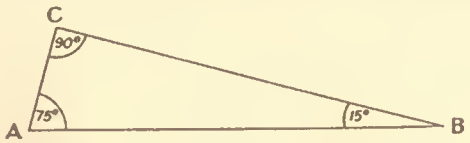


FIG. 26

Suppose, in Fig. 26, A, B and C are three points or stations of a triangulation and that the relative positions of two of them, and hence the length of one side, are known. The third station is to be fixed. Let the angles be :—A = 75°, B = 15°, and C = 90°. Now, if AB is the known side, this is a *well-conditioned* triangle for fixing C, as any errors in the angular measurements will have a minimum effect in displacing C from its true position ; the "receiving angle" at C, 90°, is as good as it can be. The triangle would also be well conditioned for fixing A, if BC was the known side, since the receiving angle at A is also large ; in other words the rays from A and B make a "good cut" at C, and those from B and C make a good cut at A. But if AC was the known side and it was required to fix B, the triangle is *ill conditioned* for *this* purpose since the receiving angle at B is only 15° and any errors of angular measurement must obviously have a *large* effect in displacing B from its true position.

In the example quoted it was supposed that angular measurements, or rays, from only two stations were available for fixing the third. Generally, there will be more observations than this.



FIG. 27

When there are three rays for fixing a point, as in Fig. 27, the most reliable position for it will be obtained, other things being equal, when the rays are inclined at angles of 60° or 120° to each other. Conditions in the field will rarely yield such ideal angles of intersection, but the siting of the trigonometrical stations should be arranged with a view to ensuring good receiving angles at all the points.

Examples of Main Triangulation.

The simplest form of triangulation is a series of triangles, each having a side common to its neighbour, see Fig. 28. In the figure, AB is a side of known length and direction. It is clear that if all the angles in the triangles ABC, BCD, CDE, etc. are measured, the lengths of all the sides, and hence the positions C, D, E, F and G relative to A and B, can be calculated (by plane trigonometry). For instance, AC and BC can be found from triangle ABC; then, BC being now known, BD and CD can be found from the triangle BCD, and so on. A triangulation of this form, however, is weak, as each station

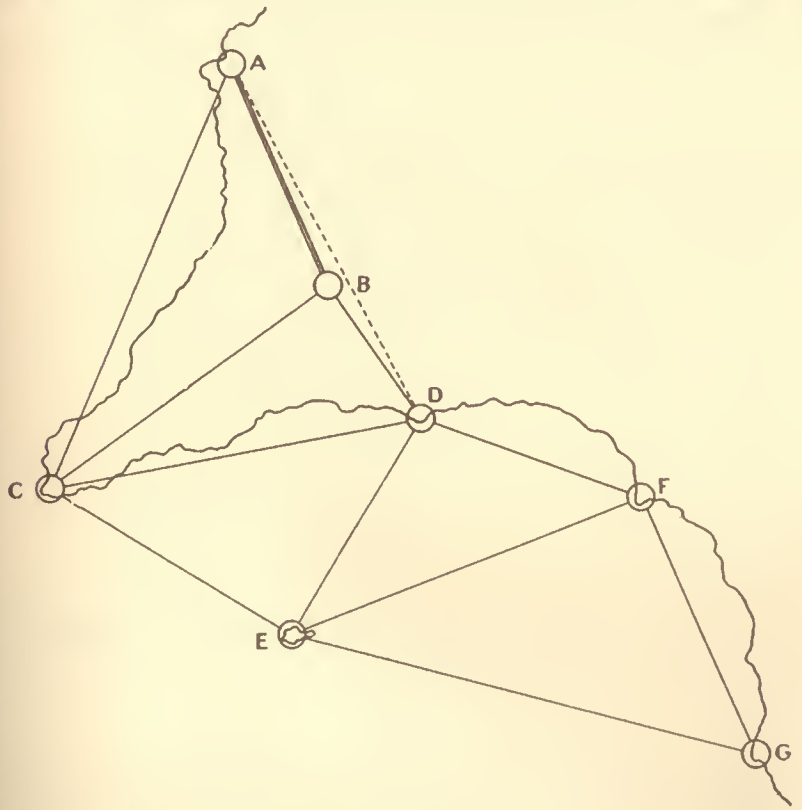


FIG. 28

to be fixed is dependent on a single triangle and has only two " rays " to it. No useful check on the accuracy of observation and computation is afforded, and errors of one triangle will be propagated through the remainder. If it had been possible to observe the ray AD (joined by a pecked line in the diagram) by measuring the angles A and D in the triangle ACD, the solution of this triangle would have provided a check for the length of the side CD and for the position of D, and as far as *this* point the triangulation would have been greatly strengthened. The extension of a triangulation by single triangles is sometimes unavoidable and in such cases special care is necessary in observing

the angles, but every effort should be made to site the marks so that continuous checks on the accuracy of the observations and computations are afforded. This will be attained if the figures composing the triangulation take the form of a series of quadrilaterals, each with a side common to its neighbour, or a series of polygons, each having a central station, and with two or more of its triangles common to adjacent figures. Often a triangulation includes both types of figure.

Figure 29 represents a triangulation system designed to provide a framework for a small coastal survey. The main stations only are shown. The positions of A and B were marked and known; hence the length and bearing of the side AB could be calculated. Further trigonometrical points for the survey were sited at C, D, E and F. Angles were observed at all stations, the observed rays being shown in the figure. The scheme may be described as consisting of three geometrical figures:

(1) A three-sided polygon BCD about a central station A, shown in a firm line.

(2) A three-sided polygon BCE about a central station A, shown in a pecked line.

(3) A quadrilateral ABFC, shown in a dotted line.

The triangle ABC is common to both polygons and the quadrilateral.

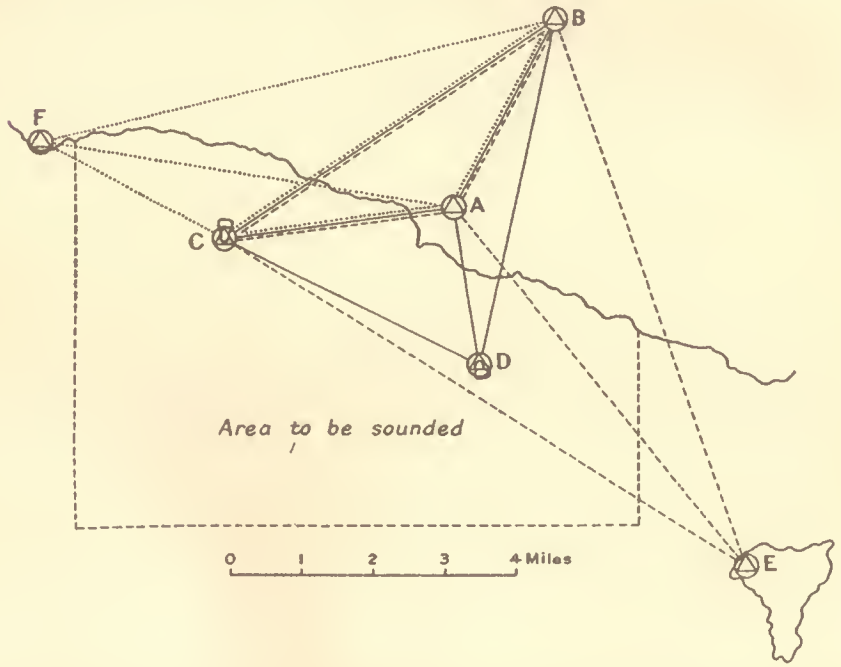


FIG. 29

The lengths of the sides AC and BC can be found by the sine formula from the triangle ABC, sides AD and BD from triangle ABD, and so on. Hence the positions of C, D, E and F relative to A and B can be calculated. The arrangement of the geometrical figures affords a check on the accuracy of the observations at each station. For instance, when the triangles ABC and ABD have been solved, CD can be calculated in the triangle ACD, using either the length AC or AD: both methods should give the same result, or results so nearly in agreement that their difference can be assigned to the small but inevitable errors of observation. Similarly, the observations connecting F can be checked by calculating the side AF independently from each of the triangles ABF and ACF, and those connecting E by calculating AE from the triangles ABE and ACE. In this example of triangulation it should be noted

that the receiving angles at E and F, in each case about 40°, are about the minimum value which should normally be accepted when there are no insuperable topographical difficulties in the selection of suitable triangulation points.

Fig. 30 shows a triangulation designed to provide a framework for the survey of a harbour and its approaches. The length of the side EF was deduced from a base measurement in its vicinity and its bearing was found from astronomical observations. The system may be considered as being comprised of three geometrical figures :

(1) A five-sided polygon EHIKJ about a central station F, shown in a firm line.

(2) A five-sided polygon EFKLM about a central station J, shown in a pecked line.

(3) A quadrilateral ENHF, shown in a dotted line.

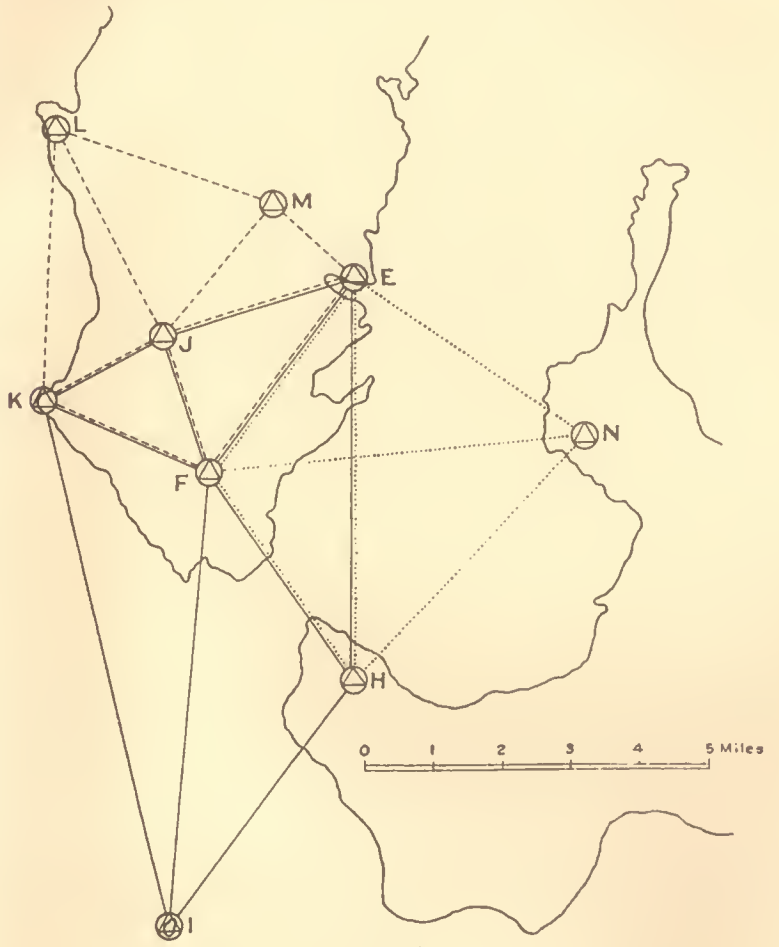


FIG. 30

The triangles EFJ and JFK are common to both polygons and the triangle EFH is common to the polygon EHIKJ and the quadrilateral ENHF. This example shows a triangulation of sound design and regular form. Every point is connected by observations to at least three other points in the system, thereby affording checks on the accuracy of the angular measurements and computations.

Fig. 31 illustrates part of a scheme of triangulation designed to provide a framework for surveys extending over about 50 miles of coastline and embracing

a number of off-lying islands on the West coast of the Malay peninsula. The positions of the points A, B and C were obtained from a land survey and hence the elements of the triangle ABC could be calculated and used for extending the triangulation to cover the hydrographic work. The various combinations of geometrical figures are shown in the diagram. Each figure has a side or sides common to adjacent figures, thereby ensuring, as far as possible, accuracy in the extension of the triangulation.

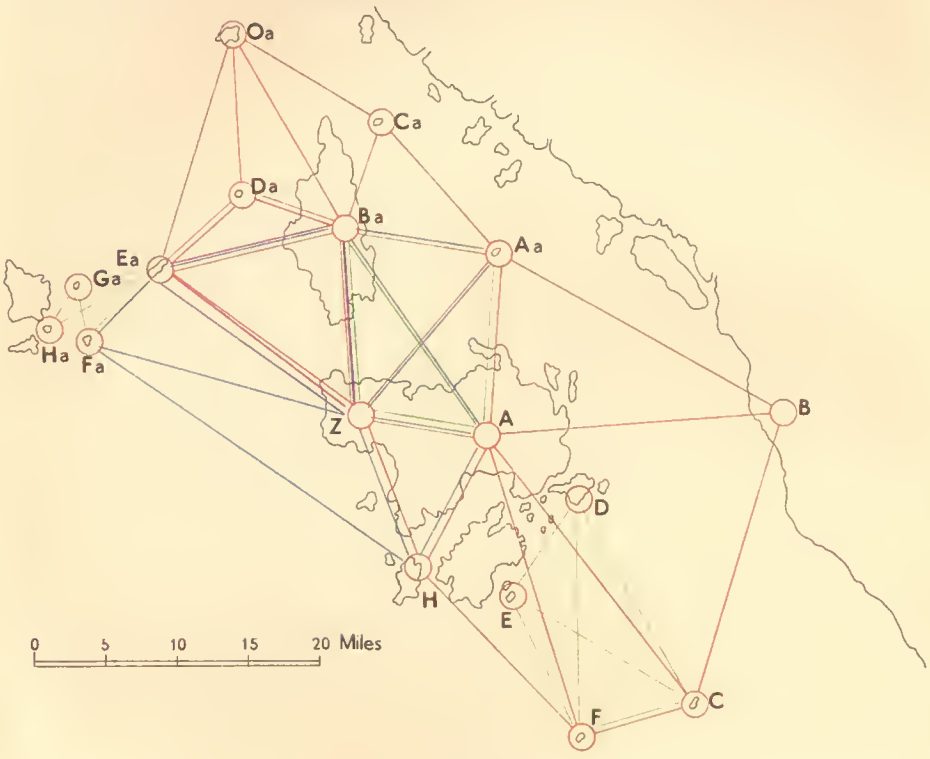


FIG. 31

BASE TRIANGULATION

The measurement of an accurate base line by means of steel tapes stretched over the ground (*q.v.*) is always a slow and laborious process. The preparation of a site, on which a base of, say 6,000 feet can be measured, may occupy several days, and it is usually uneconomical to attempt measurements much longer than this. Frequently the nature of the land features is an obstacle to a long base measurement and in the selection of the site it is impossible for the hydrographic surveyor to go very far outside the area of the survey owing to the amount of time which would be spent in connecting a distant base to the triangulation.

The triangulation of a harbour survey covers a comparatively small area, and it may be that the lengths of the sides are of the order of only ½ a mile to 1 or 2 miles. In such cases the base line may be long enough to be considered as an integral part of the main triangulation network. In surveys of larger areas, however, the sides of the main triangulation are much longer—they will usually be from 5 miles or so, up to 20 miles or more in length. In such cases the base line is too short to be included as one of the sides of the main triangulation, and it is necessary to extend the base by means of a subsidiary series of geometrical figures which will connect it to one or more sides of the main system. This subsidiary series is called the "Base Extension Triangulation." The stations or points forming this triangulation *must* be sited in such a way

that the geometrical figures consist of *well-conditioned* triangles in which all the angles can be observed and so that, if possible, two or more values can be obtained for the initial side of the main triangulation by using different means of approach in its calculation.

Examples of base extension triangulation :

(1) In Fig. 32, AB is a measured base and EF the initial side of the main triangulation. All rays shown in the figure are observed. The extension of the base is carried out by means of two quadrilateral figures, ACBD and ECFD, C and D being marks specially placed for the purpose of obtaining well-conditioned

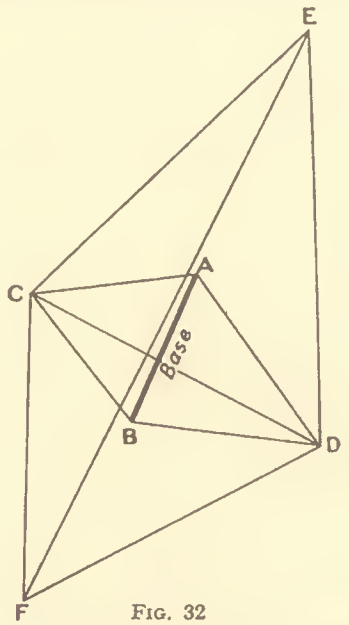


Fig. 32

triangles in the base extension system. This example, in which triangles on *each* side of the base are used for the extension, represents the ideal arrangement of the stations for the purpose in view.

The length of CD can be found from the quadrilateral ACBD and then EF from the quadrilateral CFDE. The adjustment of the angular measurements and the method of computation are discussed later.

(2)

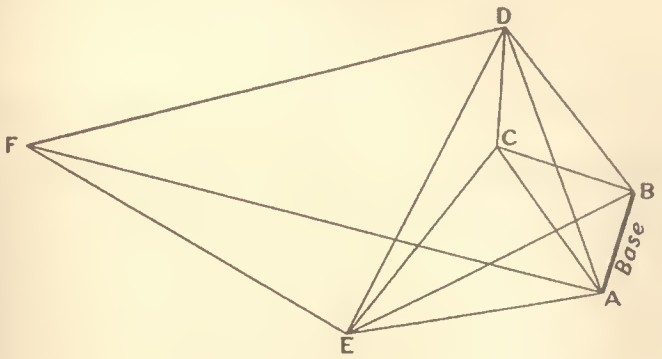


Fig. 33

Fig. 33 illustrates another good type of Base Extension Triangulation. AB is the measured base and AF a side of the main triangulation. All the

stations are intervisible and angles are observed at each to all other points. The method of computing the length of AF in this case would be as follows:

The sides AC and BC being calculated from the triangle ABC, AE and BD can be found from the triangles ACE and BCD respectively. Two values of DE can now be obtained from the quadrilateral ABDE, and, accepting the mean, AF can be found from the quadrilateral ADFE. It will be seen that this method of computation confines the working to well-conditioned figures and that errors of observation in the first triangles are to some extent counteracted by meaning the results obtained from different angular measurements. The observed angles require certain adjustments before the computation is done, and the method of finding and applying these are given below.

Calculation and Adjustment of Triangulation

Adjustment of Observations.

Before angular measurements are used for the calculation of distances and bearings in a triangulation net, they must be adjusted to satisfy certain geometric relations which exist among different measured quantities. For example, the three angles of a triangle on the earth's surface should, after correction for the small quantity known as spherical excess, sum to 180°, a relation which is used to check the accuracy of the observations. In practice it is found that, even when using instruments capable of being read with a high degree of precision, *e.g.*, to single seconds of arc, the three observed angles of a triangle will not, except accidentally, sum to exactly 180°, and they must be adjusted to do so before they are used in computation. The object of adjustment is to make the triangulation network symmetrically correct and geometrically consistent ; the values of the sides in the adjusted figures must bear a certain relation to the angles so that, once the adjustment has been made, no further correction is necessary whatever part of the network is used in conjunction with adjoining figures ; any discrepancies then found must be regarded as due to errors of computation, not to those of observation.

The triangulations of hydrographic surveys may generally be considered to come in the category of *Minor* Triangulation, in which the accepted principle is that all observations must be adjusted to self-consistency, but that *any* distribution of error is tolerable provided that the maximum correction to any angle is not too great. In primary triangulation the adjustment of the observations is a laborious process designed to give the most probable final results by the application of the mathematical principle of " Least Squares." In the triangulations of most hydrographic surveys, no practical purpose would be served by any lengthy and elaborate process of adjustment, and the methods to be described, though of a comparatively simple type, are adequate for the purpose. The results are, nevertheless, geometrically consistent, although they will not as a rule exactly represent the most probable values of the measurements.

The adjustments of the observations consist of :

(1) The Station Adjustment, *i.e.*, the correction of the angles at each station so that they satisfy the geometric conditions existing among angles measured at a fixed point.

(2) The Figure Adjustment, *i.e.*, the correction of the angles of each figure so that it is made geometrically consistent. It should be noted that every triangulation network can be arbitrarily divided into a series of *simple* figures, namely, the polygon with or without a central station, the quadrilateral and the triangle, and each figure can be adjusted in turn. If often happens that one or more of the diagonals across a polygon with central station may have been observed, but, though such rays sometimes afford a valuable check on the other observations, they cannot, as a rule, be conveniently used in the process of adjustment without greatly complicating it.

(3) The Chain Adjustment.—When a chain or network of triangulation starting from one known position is " closed " onto another known position,

the sides and angles of the intermediate figures require adjustment to bring them into harmony with the distance and bearing between the two accepted positions. Similarly, if a triangulation network is carried from one measured base to another, an adjustment of the intervening figures will generally be necessary to make the angles and sides consistent with the two measured bases.

Station Adjustment.

The angles at a point must sum to 360°. When all the observations at a station have been made at the same time by the *direction* method (*q.v.* p. 48), the angles are simply the differences in the observed directions and they will naturally sum to 360° so that no station adjustment is necessary. When the angles at a station have been observed independently or by the *repetition* method, they will probably not sum to exactly 360°. If it is assumed that they have all been equally well observed, they must be adjusted to sum to 360° by distributing any excess or deficiency equally among them. If the angles have *not* been equally well observed, the excess or deficiency should be distributed in proportion to the estimated accuracy of each observation.

Figure Adjustment.

Triangle.—The simplest case of figure adjustment is that of the single triangle. Before the angles are used in the computation of the sides, etc., they must be made to sum to exactly 180° by distributing any excess or deficiency among them. Normally the distribution is made in equal proportions among the angles, but, if there is reason to suppose that some of the angles have been observed with greater accuracy than others, the corrections may be distributed in proportion to the estimated accuracy of each observed angle. As a rule triangles form parts of more complicated figures, and the adjustment of their angles is carried out as a part of the adjustment of the whole figure. When only two angles of a triangle, say the angles A and B of a triangle ABC, have been observed, the third angle, C, is assumed to equal 180° − (A + B).

Quadrilateral.—Single triangles need only an " angle " adjustment to make them geometrically consistent. The quadrilateral, however, requires in addition a " side " adjustment as will now be explained.

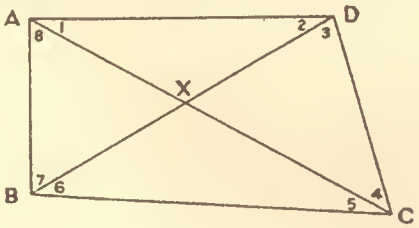


FIG. 34

In Fig. 34, ABCD is a quadrilateral in which the eight exterior angles have been observed. By the laws of geometry,

(i) The eight exterior angles should sum to 360° : correction of the angles to satisfy this equation is the " quadrilateral " adjustment.

(ii) The angles of each of the four triangles composing the quadrilateral should sum to 180° : correction of the angles to satisfy this equation is the " triangle " adjustment.

(iii) The angles should have such values that, if any one side of the quadrilateral is known, it is possible to calculate any other side by different " routes " and obtain identical values for it ; correction of the angles to satisfy this condition is the " Side " adjustment.

The necessity of satisfying equations (i) and (ii) is obvious from first geometrical principles. At first sight it might be thought that the figure would

then be geometrically consistent, but the necessity of a third equation may be demonstrated as follows :

Suppose in the quadrilateral ABCD that the length of CD is required, the side AB being known and equations (i) and (ii) having been satisfied. The calculation may be made in two ways, viz :

(1) Through triangles ABC and BCD.
or (2) ,, ,, ABD ,, ACD.

The results will not agree (except accidentally) because equations (i) and (ii) have not definitely fixed the values of the angles. For instance, it is possible to decrease angle 2 by any definite amount provided angle 1 is increased by the same amount. The triangles ABD and ACD will still close to 180°, but the value of CD calculated through them will be altered. It is evident that a third equation is necessary to complete the adjustment of the figure.

If AB, as before, is a known side, the formulae for calculating CD may be either :

(i) $CD = AD \sin 1 \operatorname{cosec} 4$
$= AB \sin 1 \operatorname{cosec} 4 \sin 7 \operatorname{cosec} 2 \ldots\ldots\ldots (1)$

or (ii) $CD = BC \sin 6 \operatorname{cosec} 3$
$= AB \sin 6 \operatorname{cosec} 3 \sin 8 \operatorname{cosec} 5 \ldots\ldots\ldots (2)$

If the same value is to be obtained for CD by either equation, the angles must be so adjusted that the expressions (1) and (2) are equal, *i.e.* :

$AB \sin 1 \operatorname{cosec} 4 \sin 7 \operatorname{cosec} 2 = AB \sin 6 \operatorname{cosec} 3 \sin 8 \operatorname{cosec} 5.$

Cancelling AB from both sides, this may be re-written as :

$\sin 1 \sin 3 \sin 5 \sin 7 = \sin 2 \sin 4 \sin 6 \sin 8,$

or, using logarithms,

$\text{Log} \sin 1 + \text{Log} \sin 3 + \ldots\ldots = \text{Log} \sin 2 + \text{Log} \sin 4 + \ldots$

The most convenient method of applying the " triangle " adjustment is by correcting the angles to bring them into agreement with a further geometric relation in the quadrilateral which is that the sum of the angles of each interior triangle and its opposite must be equal. Since the opposite angles at X, the intersection of the diagonals are equal, it follows that the sums of the alternate angles must also be equal, *i.e.*, in this figure,

Angles $1 + 2$ = angles $5 + 6$
,, $3 + 4$ = ,, $7 + 8$

The " triangle " adjustment is made by correcting the angles to satisfy these two equations—an inspection of the figure will show that it also has the effect of closing the four triangles ABC, ABD, ACD and BCD and that the quadrilateral adjustment need not be disturbed.

Example illustrating the adjustment of the angles of a quadrilateral :

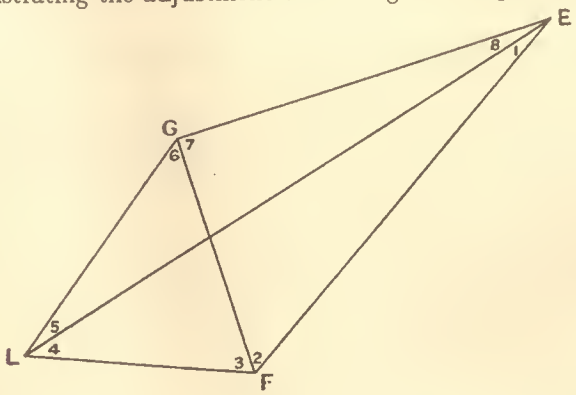


FIG. 35

EFLG is a quadrilateral in which the eight exterior angles have been observed. The procedure is as follows:

(1) Tabulate the observed angles of the four component triangles and sum them, in order that the triangular mis-closures may be seen.

Triangle.	Angles.	Observed Angles.	Triangle.	Angles.	Observed Angles.
		° ′ ″			° ′ ″
EFG	1 + 8	32 37 13·0	EFL	1	17 23 14·0
	2	57 49 31·9		2 + 3	125 58 41·5
	7	89 33 15·9		4	36 38 19·5
		180 00 00·8			180 00 15·0
EGL	8	15 13 59·0	FGL	3	68 09 09·6
	6 + 7	142 21 28·2		6	52 48 12·3
	5	22 24 31·0		4 + 5	59 02 50·5
		179 59 58·2			180 00 12·4

(2) Prepare a form as illustrated on page 68.

(3) In column 2, tabulate the eight observed angles, putting the odd-numbered angles in the upper half of the form and the even-numbered angles in the lower half.

(4) Sum the observed angles. The excess or deficiency from 360° is the amount required for the quadrilateral correction. This may be distributed equally among the eight angles, but an inspection of angles of the four triangles composing the quadrilateral will sometimes indicate that certain angles are more likely to be in error than others. In this example, the triangles EFL and FGL have large closing errors in comparison with EFG and EGL. As 3 and 4 are the only angles to appear in both the former triangles and not in the latter, it is reasonable to suppose that one or both of *these* angles must be chiefly responsible for the mis-closures. In this instance, therefore, most of the quadrilateral adjustment is put into these two angles. The quadrilateral adjustment is the 1st Correction and is tabulated in column 3.

(5) Applying these corrections, tabulate the angles under the heading "Angles, 1st Adjustment" in column 4.

(6) It is now necessary to make the angles (1 + 2) equal to angles (5 + 6) and angles (3 + 4) equal to angles (7 + 8). These corrections are calculated in the lower part of the form, the quantity for each angle being a quarter of the difference between each of the pairs. This is called the Triangle Adjustment, as it has the effect of closing all the triangles, and is tabulated under the heading "2nd Correction" in column 5.

(7) Applying these corrections, tabulate the angles under the heading "Angles, 2nd Adjustment" in column 6.

(8) Tabulate the Log Sines of the angles in column 7, at the same time noting the "Diffs. for 1 sec." in column 8. These differences are given at the foot of each column in Shortrede's Tables. IF AN ANGLE IS OVER 90°, ITS "Diff. for 1 sec." MUST BE PREFIXED BY A MINUS SIGN.

(9) Sum the Log Sines of each set of angles and find the difference between the sums: call this quantity D.

(10) Sum algebraically the "Diffs. for 1 sec." for all the angles: call this quantity S (d).

(11) Divide D by S (d): the resulting quantity "e" is the correction in seconds to be applied to each of the angles; in this example the sum of the Log Sines of the even-numbered angles is less than that of the odd-numbered angles; "e" is therefore additive to the former and subtractive from the latter angles.

Form 1.

ADJUSTMENT OF QUADRILATERAL

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
Angle.	Observed Angles.	1st Correction.	Angles. (1st Adjustment.)	2nd Correction.	Angles. (2nd Adjustment.)	Log Sines.	Diff. for 1″.	Angles (finally adjusted).	Log Sines.
	° ′ ″	″	° ′ ″	″	° ′ ″			° ′ ″	
1	17 23 14·0	− 0·6	17 23 13·4	− 0·6	17 23 12·8	9·475 4132	67·2	17 23 10·9	9·475 4004
3	68 09 09·6	− 5·0	68 09 04·6	− 1·3	68 09 03·3	9·967 6263	8·4	68 09 01·4	9·967 6246
5	22 24 31·0	− 0·5	22 24 30·5	+ 0·6	22 24 31·1	9·581 1640	51·1	22 24 29·2	9·581 1543
7	89 33 15·9	− 0·5	89 33 15·4	+ 1·3	89 33 16·7	9·999 9869	0·2	89 33 14·8	9·999 9869
						S_1 = 9·024 1904			1662
2	57 49 31·9	− 0·6	57 49 31·3	− 0·6	57 49 30·7	9·927 5897	13·2	57 49 32·6	9·927 5922
4	36 38 19·5	− 5·0	36 38 14·5	− 1·3	36 38 13·2	9·775 7875	28·3	36 38 15·1	9·775 7929
6	52 48 12·3	− 0·5	52 48 11·8	+ 0·6	52 48 12·4	9·901 2219	16·0	52 48 14·3	9·901 2250
8	15 13 59·0	− 0·5	15 13 58·5	+ 1·3	15 13 59·8	9·419 5421	77·4	15 14 01·7	9·419 5568
	360 00 13·2		360 00 00·0		360 00 00·0	S_2 = 9·024 1412 S_1 = 9·024 1904	S(*d*)261·8	360 00 00·0	1669
						D 492			

		°	′	″
Angles (1st Adjustment)	1 + 2 =	75	12	44·7
,, ,,	5 + 6 =	75	12	42·3
	Diff.			2·4
	½ ,,			± 0·6
Angles 1st adjustment)	3 + 4 =	104	47	19·1
,, ,,	7 + 8 =	104	47	13·9
	Diff.			5·2
	½ ,,			± 1·3

$$e = \frac{D}{S(d)} = \frac{492}{261{\cdot}8} = \pm\ 1''{\cdot}9$$

(12) Applying the correction "*e*," tabulate the finally adjusted angles in column 9 and their Log sines in column 10. Sum the two sets of Log Sines which should now be equal, subject to a small discrepancy (in the last figure) which cannot be avoided unless the computation of the angles is taken to hundredths of a second; this provides a check on the working.

The adjustment of the figure is now complete, *i.e.*, the angles are geometrically consistent. The adjusted triangles may now be put down and the sides of the quadrilateral calculated. Assuming EF to be a side of known length; the following shows a convenient lay-out for the computation:

Triangle.	Station.	Adjusted Angles.	Sides.
		° ′ ″	
EFG	E	32 37 12·6	L. sin .. 9·731 6430
	G	89 33 14·8	L. cosec 0·000 0131
			log EF .. 4·649 2669
	F	57 49 32·6	L. sin .. 9·927 5922
		180 00 00	log FG .. 4·380 9230
			log EG .. 4·576 8722
EFL	E	17 23 10·9	L. sin .. 9·475 4004
	L	36 38 15·1	L. cosec 0·224 2071
			log EF .. 4·649 2669
	F	125 58 34·0	L. sin .. 9·908 0891
		180 00 00	log FL .. 4·348 8744
			log EL .. 4·781 5631
EGL	E	15 14 01·7	L. sin .. 9·419 5568
	L	22 24 29·2	L. cosec 0·418 8457
			log EG .. 4·576 8722
	G	142 21 29·1	L. sin .. 9·785 8454
		180 00 00	log GL .. 4·415 2747
			log EL .. 4·781 5633
FGL	F	68 09 01·4	L. sin .. 9·967 6246
	L	59 02 44·3	L. cosec 0·066 7267
			log FG .. 4·380 9230
	G	52 48 14·3	L. sin .. 9·901 2250
		180 00 00	log GL .. 4·415 2743
			log FL .. 4·348 8747

When two results are obtained for a side, that found from the best-conditioned triangle should be accepted. As with the Log Sine equation, discrepancies occur in the last figure of the logarithms unless angles are taken to hundredths of a second.

Quadrilateral with one Triangle already Adjusted.

It sometimes happens that it is necessary to adjust to geometrical consistency a quadrilateral in which three of the stations have already been fixed in a previously adjusted figure. In such cases the adjustment must be effected without disturbing the values of the angles in this triangle, and a slightly different method must be used. Example (Fig. 36):

FLGM is a quadrilateral in which the triangle FGL has already been adjusted:

(1) Prepare a form, as shown on pp. 70 and 71, and in column 2 tabulate the observed angles of the four component triangles.

Form 2.

ADJUSTMENT OF QUADRILATERAL WITH ONE TRIANGLE ALREADY ADJUSTED

1. Triangle and Angles.	2. Observed Angles.	3. 1st Correction.	4. Angles. 1st Adjustment.)	5. Double Angles.	6. 2nd Correction.	7. Angles. (2nd Adjustment.)	8. 3rd Correction.	9. Angles. (3rd Adjustment.)
	° ′ ″	″	° ′ ″	° ′ ″	″		″	° ′ ″
2	52 48 12·3	(+ 2·0)*	52 48 14·3*		—	Previously Adjusted.	Previously Adjusted.	Previously Adjusted.
FGL 1 + 8	59 02 50·5	(— 6·2)*	59 02 44·3*	59 02 44·3	—			
7	68 09 09·6	(— 8·2)*	68 09 01·4*		—			
	180 00 12·4		180 00 00·0					
						° ′ ″	″	° ′ ″
6	59 22 15·7		59 22 15·7		+ 2·5	59 22 18·2	—	59 22 18·2
FGM 4 + 5	35 22 24·8		35 22 24·8	35 22 24·8	+ 2·4	35 22 27·2	—	35 22 27·2
3	85 15 12·1		85 15 12·1		+ 2·5	85 15 14·6	—	85 15 14·6
	179 59 52·6		179 59 52·6			180 00 00·0		180 00 00·0
5	17 51 02·8		17 51 02·8		+ 1·2	17 51 04·0	+ 5·9	17 51 09·9
FLM 6 + 7	127 31 25·3	— 8·2	127 31 17·1	127 31 17·1	+ 2·5	127 31 19·6	—	127 31 19·6
8	34 37 27·7	— 3·1	34 37 24·6			34 37 24·6	+ 5·9	34 37 30·5
	179 59 55·8		179 59 44·5			179 59 48·2		180 00 00·0
1	24 25 22·8	— 3·1	24 25 19·7			24 25 19·7	— 5·9	24 25 13·8
GLM 2 + 3	138 03 24·4	+ 2·0	138 03 26·4	138 03 26·4	+ 2·5	138 03 28·9	—	138 03 28·9
4	17 31 22·0		17 31 22·0		+ 1·2	17 31 23·2	— 5·9	17 31 17·3
	180 00 09·2		180 00 08·1	359 59 52·6		180 00 11·8		180 00 00·0

* From previous adjustment.

10.	11.	12.	13.	14.	15.
No. of Angle.	Angles. (3rd Adjustment.)	Log Sines.	Diff. for 1″.	Angles. (finally adjusted).	Log Sines.
	° ′ ″			° ′ ″	
1	24 25 13·8	9·616 4023	46·4	24 25 15·7	9·616 4110
3	85 15 14·6	9·998 5084	1·7	85 15 16·5	9·998 5087
5	17 51 09·9	9·486 5321	(×2) 130·7	17 51 13·7	9·486 5570
7	68 09 01·4*	9·967 6246	—	68 09 01·4	9·967 6246
		S_1 = 9·069 0674			1013
2	52 48 14·3*	9·901 2250	—	52 48 14·3	9·901 2250
4	17 31 17·3	9·478 6577	(×2) 133·4	17 31 13·5	9·478 6323
6	59 22 18·2	9·934 7461	12·5	59 22 16·3	9·934 7438
8	34 37 30·5	9·754 5049	30·5	34 37 28·6	9·754 4991
		S_2 = 9·069 1337	$S(d)$ = 355·2		1002
		S_1 = 9·069 0674			
		Diff. D = 663	$e = \frac{D}{S(d)} = \frac{663}{355{\cdot}2} = \pm 1''{\cdot}9$		

* Previously adjusted.

(2) Apply the corrections already found for the angles of the adjusted triangle FGL where these appear in the other triangles of the figure. *i.e.*, The corrections to 2 and 7 must be applied to the double angles (2 + 3) and (6 + 7) respectively, while half the correction to the double angle (1 + 8) must be applied to each of the single angles 1 and 8.

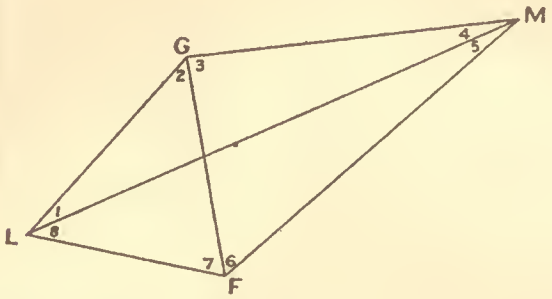


FIG. 36

(3) Place these corrections in column 3 and, applying them, tabulate the angles again in column 4 under the heading "Angles, 1st Adjustment."

(4) It is now necessary to find the Quadrilateral Correction; therefore tabulate the double angles in column 5 and sum them. The correction, tabulated in column 6, is found by dividing the excess or deficiency from 360° equally between the angles of the triangle FGM, *i.e.*, one-third to each of the angles 3 and 6, and one-sixth, to each of the angles 4 and 5. The double angles (2 + 3) and (6 + 7) obviously receive the same corrections as angles 3 and 6 respectively.

(5) Applying the 2nd correction, tabulate the angles in column 7. It will be seen that the 2nd correction, in addition to closing the quadrilateral, has also closed the triangle FGM.

(6) Any closing error in the two remaining triangles will be equal, but of opposite sign, and must be divided equally between the two single angles of each triangle and tabulated in column 8 under the heading "3rd Correction."

(7) Applying the 3rd Correction, tabulate the angles in column 9 and make the final adjustment by the sine rule in the usual way except that, owing to the fact that angles Nos. 2 and 7 must not be altered, the application of a constant correction to the remaining six angles is inadmissible, as it would disturb the closing of the triangles FLM and GLM.

This difficulty is surmounted by putting a double correction, *i.e.*, $2e$, into the two angles, 4 and 5 (opposite to the previously adjusted triangle), which has the result of leaving the closing of the figure undisturbed by the final adjustment. It should be noted that in finding "e," the figures for angles 4 and 5 in column 13 under the heading "Diff. for 1 Sec." are obtained by multiplying the tabulated differences by 2.

Polygon with a Central Station.

The adjustment of the observed angles of a polygon is similar in general principles to that of the quadrilateral. It consists of two parts:

(1) **Angle Adjustment.**—The angles of each component triangle must be adjusted to sum to 180° and the angles at the central station must be adjusted

to sum to 360° less one-third of the spherical excess of the whole figure. (The spherical excess need not be taken into account in *small* figures.)

(2) **Side Adjustment.**—The angles must be further adjusted so that they are made geometrically consistent. At first sight it might be thought that this would be achieved by making the angle adjustment alone, but an example will show that this is not so. In a polygon ABEFH about the central Z (Fig. 37) the length of side AZ is known and the triangles have been closed by the " angle " adjustments.

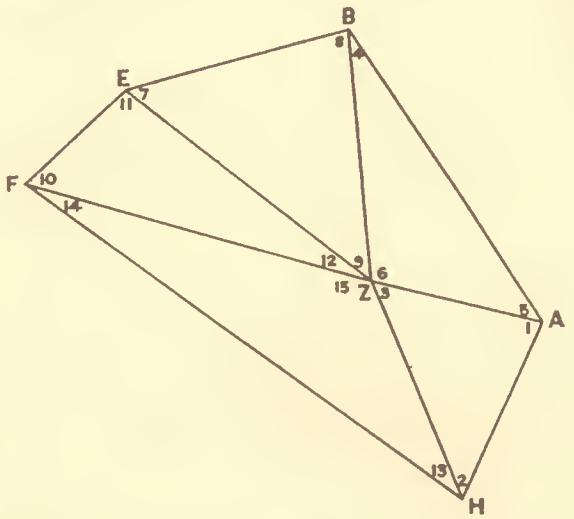


FIG. 37

Now, if the length of EF is required, it may be calculated :

(i) Through the triangles ABZ, BEZ and EFZ, or

(ii) ,, ,, ,, AHZ, HFZ and EFZ.

The two results would only agree accidentally as the angle adjustments have not definitely fixed the values of the angles which must be used in the computation. For instance, any definite quantity may be subtracted from angle 5 if an equal quantity is added to angle 4 without disturbing the angular closure of the triangle ABZ or the polygon in any way, but obviously a different result would be obtained for the length of EF. If the figure is geometrically consistent, calculations by any " route " should give the same result.

If, as before, AZ is a known side and EF is required, the equations would be as follows :

(i) EF = EZ sin 12 cosec 10
= BZ sin 8 cosec 7 sin 12 cosec 10
= AZ sin 5 cosec 4 sin 8 cosec 7 sin 12 cosec 10

or

(ii) EF = FZ sin 12 cosec 11
= HZ sin 13 cosec 14 sin 12 cosec 11
= AZ sin 1 cosec 2 sin 13 cosec 14 sin 12 cosec 11

If the two equations are to give identical results, it is clear that the following must be true :

AZ sin 5 cosec 4 sin 8 cosec 7 sin 12 cosec 10 = AZ sin 1 cosec 2 sin 13 cosec 14 sin 12 cosec 11.

Cancelling identical terms and transposing, we have :

Sin 1 sin 4 sin 7 sin 10 sin 13 = Sin 2 sin 5 sin 8 sin 11 sin 14.

Or, using logarithms,

Log sin 1 + Log sin 4 + ... = Log sin 2 + Log sin 5 + ...

The side adjustment consists in correcting the angles to fulfil this equation.

Example of adjusting the observed angles of a polygon ABEFH (Fig. 37) :

In this polygon suppose the triangles ABZ and AHZ form part of a previously adjusted figure ; their angles are therefore fixed so that all the necessary correction must be put into the angles of the three remaining triangles. The area of this figure is about 280 square nautical miles and its spherical excess (very nearly equal to 1″ per 60 square nautical miles of area) is 4″·8. The *plane* adjusted angles at the central station must therefore sum to $360° - \frac{1}{3}(4''{\cdot}8) = 359°\ 59'\ 58''{\cdot}4$.

Proceed as follows :

(1) Prepare form 3 in two parts, as illustrated on pp. 75 and 76.

(2) In column 2, tabulate the observed angles of the five triangles : it is convenient to keep them in the same sequence when writing them down, *e.g.*, forward angle, back angle, central angle. When any parts of the figure have been previously adjusted (as is the case with triangles AHZ and ABZ in this example), tabulate the adjusted angles.

(3) In the example it is assumed that all the angles have been equally well observed and have equal " weighting values," so that the first adjustment is distributed equally among the angles of each triangle. If, however, there is reason to suppose that the observations are not all of equal " weight," an additional column of " weighting values " should be inserted between columns 1 and 2.

(4) Sum the angles of each triangle and in column 3 tabulate the corrections (in proportion to weighting values) necessary to make them sum to 180°.

(5) Deduce the central angles at Z and tabulate them in column 4. Sum the angles in column 4 : the excess or deficiency from 360° (less one-third the spherical excess of the whole figure) must be distributed equally among the central angles. In this example angles 3 and 6 are already fixed, and the deficiency must be divided among the other three central angles. Tabulate the corrections in column 5.

(6) The adjustment of the central angles disturbs the closures effected by the 1st Correction, therefore re-adjust the exterior angles so that the triangles again sum to 180°, tabulating the corrections also in column 5. As the latter corrections will always be comparatively small, these re-adjustments may be distributed equally without taking into consideration the weighting values, if such exist.

(7) Applying the 2nd Correction, tabulate the angles in column 6 under the heading " Angles, 2nd Adjustment." The central angles receive no further adjustment in the subsequent working and their Log Sines can be tabulated in column 7 for future use.

(8) In column 9 (on the lower part of the form) tabulate all the exterior angles of the polygon taking them from column 6 and placing the forward angles, 1, 4, 7, 10 and 13 in the upper half of the column and the back angles 2, 5, 8, 11 and 14 in the lower half.

(9) Tabulate the Log Sines of these angles in column 10, at the same time noting in column 11 the " Diff. for 1 sec." for each angle which is not already fixed by adjustment in a previous figure. When an angle is over 90° (as No. 11 in the example) its " Diff. for 1 sec." must be prefixed by a minus sign.

(10) Sum the Log Sines of each set of angles, calling the sums S_1 and S_2, and find the difference D between S_1 and S_2.

(11) Sum algebraically the " Diffs. for 1 sec." and call this S(d).

(12) The final adjustment " e " is found from the equation :

$$e = \frac{D}{S\,(d)}$$

" e " being the number of seconds to be added to each of the angles 1, 4, 7, 10 and 13 and subtracted from each of the angles 2, 5, 8, 11 and 14 ; in the example S_1 is smaller than S_2, and " e " is consequently additive to the forward angles.

(13) Applying the quantity " e " to each of the un-fixed angles, tabulate the finally adjusted angles in column 12 and their Log Sines in colum 13. It will be seen that the two sets of Log Sines now sum equally, subject to a discrepancy in the last figure of the logarithms which cannot be avoided unless calculations are taken to hundredths of a second.

The adjustment of the figure is now complete, *i.e.*, the angles are geometrically consistent.

Form 3.

ADJUSTMENT OF POLYGON WITH CENTRAL STATION

1.	2.	3.	4.	5.	6.	7.
Angle.	Observed Angles.	1st Correction.	Deduced Central Angles.	2nd Correction.	Angles. (2nd Adjustment.)	Log Sines.
	° ′ ″	″	° ′ ″	″		
1	77 46 08·4*			—	Previously	
2	47 11 55·2*			—	Adjusted.	
3	55 01 56·4*		55 01 56·4	—		9·913 5360
	180 00 00·0					
4	28 11 38·0*			—	Previously	
5	43 54 02·1*			—	Adjusted.	
6	107 54 19·9*		107 54 19·9	—		9·978 4384
	180 00 00·0				° ′ ″	
7	51 39 33·1	+ 1·9		− 0·6	51 39 34·4	
8	82 36 15·8	+ 2·0		− 0·6	82 36 17·2	
9	45 44 05·3	+ 1·9	45 44 07·2	+ 1·2	45 44 08·4	9·854 9902
	179 59 54·2				180 00 00·0	
10	61 09 03·6	− 0·6		− 0·6	61 09 02·4	
11	96 10 21·8	− 0·6		− 0·6	96 10 20·6	
12	22 40 36·4	− 0·6	22 40 35·8	+ 1·2	22 40 37·0	9·586 0636
	180 00 01·8				180 00 00·0	
13	31 51 19·2	− 3·3		− 0·6	31 51 15·3	
14	19 29 51·8	− 3·2		− 0·6	19 29 48·0	
15	128 38 58·8	− 3·3	128 38 55·5	+ 1·2	128 38 56·7	9·892 6431
	180 00 09·8	Sum	359 59 54·8		180 00 00·0	
Central angles should sum to			359 59 58·4			
		Diff.	3·6			

* Previously adjusted.

8. Angle.	9. Angles. (2nd Adjustment.)	10. Log Sines.	11. Diff. for 1″.	12. Angles. (finally adjusted.)	13. Log Sines.
	° ′ ″			° ′ ″	
1	77 46 08·4*	9·990 0285	—	77 46 08·4	9·990 0285
4	28 11 38·0*	9·674 3621	—	28 11 38·0	9·674 3621
7	51 39 34·4	9·894 5037	16·7	51 39 34·9	9·894 5044
10	61 09 02·4	9·942 4504	11·6	61 09 02·9	9·942 4509
13	31 51 15·3	9·722 4367	33·9	31 51 15·8	9·722 4384
		S_1 = 9·223 7814			7843
2	47 11 55·2*	9·865 5268	—	47 11 55·2	9·865 5268
5	43 54 02·1*	9·840 9896	—	43 54 02·1	9·840 9896
8	82 36 17·2	9·996 3724	2·7	82 36 16·7	9·996 3722
11	96 10 20·6	9·997 4750	−2·3	96 10 20·1	9·997 4752
14	19 29 48·0	9·523 4239	59·5	19 29 47·5	9·523 4210
		S_2 = 9·223 7877	S (*d*) = 122·1		7848
		S_1 = 9·223 7814			
		D 63			

$$\text{“}e\text{”} = \frac{D}{S\,(d)} = \frac{63}{122{\cdot}1} = \pm 0''{\cdot}5$$

* Previously adjusted.

Polygon without a Central Station.

A type of figure, which is occasionally met with in triangulation, is the polygon without a central station, illustrated in Fig. 38.

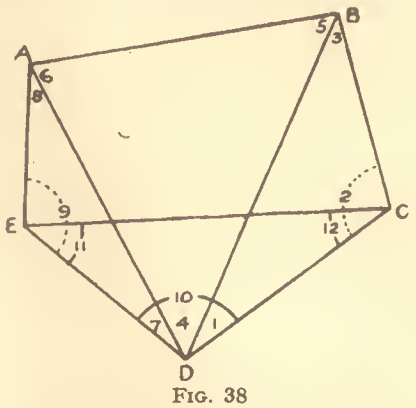


FIG. 38

The angles of this figure are adjusted by methods similar to those employed for a polygon *with* a central station, but the “ angle ” adjustments to close the figure must be effected in a slightly different manner. Using Form 3 as before the procedure is as follows :

(1) Select the station which is connected by the largest number of rays (or, if all stations are interconnected, the station at the most obtuse-angled corner of the figure)—in this case, D—and regard it as the “ central ” station. Number the angles of each of the triangles in sequence, taking particular care that the sequence is maintained in the last triangle, whose “ central ” angle is the sum of the other “ central ” angles (in this case, the triangle DEC).

(2) Sum the angles of each triangle and, in col. 3, tabulate the corrections necessary to make them add up to 180°.

(3) Tabulate the " central " angles at D in col. 4 but, instead of making them sum to 360°, the sum of the part angles (1, 4, and 7) must be made to equal the whole angle (10) by applying equal corrections (though of different signs) to each " central " angle (1, 4, 7, and 10) ; as this will upset the closure effected by the first correction, the other numbered angles must be readjusted by 2nd corrections so that the triangles again close to 180°. Tabulate all these corrections in col. 5.

(4) This completes the " angle " adjustments and the " side " adjustment is effected by making the product of the sines of the forward angles 2, 5, 8 and 11 equal to that of the back angles 3, 6, 9 and 12. It will be seen that, as with an ordinary polygon, the " central " angles at D are not brought into the " side " adjustment.

CHAIN ADJUSTMENT.

Adjustment to a whole series of figures, *i.e.*, a chain of triangulation, may be necessary in the following cases :

(i) When the triangulation extends between two measured bases.

(ii) When the triangulation extends between two known or accepted positions.

(iii) When the triangulation extends between two astronomically determined positions.

1. Adjustment between Measured Bases.

When a chain of triangulation, starting from a side of known length and bearing, extends over a great distance and consists of a large number of figures, it must be anticipated that some error in scale and bearing will accumulate, and it is advisable to measure a new or check base and make further astronomical observations for bearing in order to prevent the errors assuming large proportions. In such cases, the intermediate figures constituting the chain of triangulation between the two measured sides should be readjusted to secure consistency with the measurements. A rigorous method of chain adjustment is a long and complicated process, but a simple method which will be described is based only on the principle that adjustment to meet discrepancies may be distributed in any manner we please provided the corrections to angles and sides are not unreasonably large.

AB, Fig. 39, is a side of known length and bearing and a chain triangulation, dependent on this side for scale and bearing, is extended to the side GH. (Normally there would probably be more intermediate figures than are shown in the diagram.)

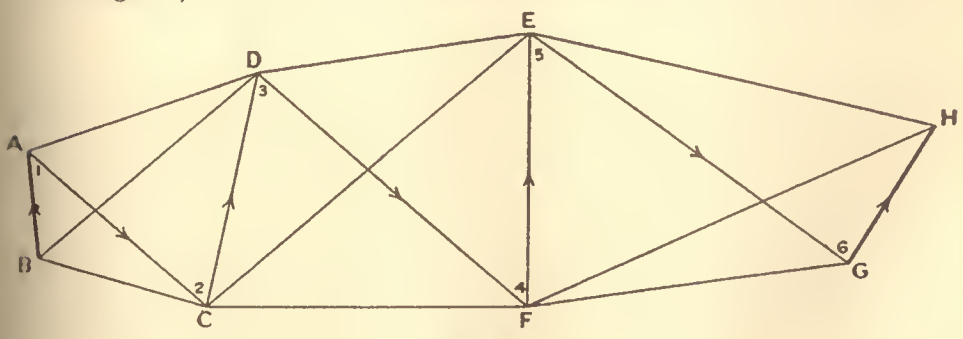


FIG. 39

The length and bearing of GH are calculated through the triangulation and are also measured to provide a check on the work. A certain discrepancy is

found between the observed and calculated values of GH and the problem is to readjust the figures between AB and GH to bring the calculated and observed values into agreement. It is assumed that the measured values of AB and GH are correct, *i.e.*, that the discrepancies are entirely due to errors of angular measurement in the intermediate figures. The procedure is as follows :

(1) Let the true (*i.e.*, measured) length and bearing of GH be x and a respectively, and its length and bearing as calculated through the triangulation, be $x + \Delta x$ and $a + \Delta a$.

(2) Starting with the initial side AB select one of the diagonals and a side common to adjacent figures in each quadrilateral finishing with the final side GH. The sides so chosen are marked with arrows in the figure.

(3) The adjustments to compensate for the discrepancies Δx and Δa may be distributed as follows :

(i) *In Length* :

Correction to AB = 0

$$\text{,, ,, AC} = -\Delta x \frac{\text{AC}}{\text{AC} + \text{CD} + \text{DF} + \text{FE} + \text{EG} + \text{GH}}$$

$$\text{Correction to CD} = -\Delta x \frac{\text{AC} + \text{CD}}{\text{AC} + \text{CD} + \text{DF} + \text{FE} + \text{EG} + \text{GH}}$$

etc., etc.

(ii) *In Bearing* :

Correction to AB = 0

,, ,, AC $= -\frac{1}{3}\Delta a$.

,, ,, CD $= -\frac{1}{3}\Delta a$.

etc., etc.

The numbered angles in the readjusted figures can now be deduced from the corrected bearings of the sides which contain them and, using these angles and the corrected lengths of AC, CD, etc., the remaining angles and sides of the figures can be re-computed.

2. Adjustment between two Known Positions.

When a chain of triangulation extends from one *known or accepted position* to another, the simplest method of adjusting the figures to secure agreement with the distance and bearing between the two known positions is by local adjustments to the positions of all the intermediate stations in the chain. The method by which these adjustments can be made is described under the Adjustment of a Traverse (Chapter VI, pp. 145–6). When they have been applied, the angles and sides of the figures can be recomputed.

3. Adjustment between Astronomically-determined Positions.

With modern instruments and careful observations, very little error should accumulate in a chain of triangulation, extending over a distance of, say 200 miles. The accuracy of the angular measurements can be assessed from a knowledge of the geometrical conditions which they should fulfil and cumulative errors of scale and bearing should be largely eliminated by check bases and bearings during the progress of the survey. If astronomical observations for geographical positions are made at two stations near the opposite ends of a chain of triangulation, it must be anticipated that there will be some discrepancy in the distance and bearing between them as derived (1) from the observed differences of latitude and longitude and (2) through the triangulation. Observations for geographical position with portable instruments can only be relied on to give an *apparent* accuracy of about $\pm$ 100 feet in latitude and about $\pm$ 200 feet in longitude ; actually, they may be in error to a much greater extent than this since unsuspected deflection of the plumbline from the

vertical may exist at one or both places of observation. Consequently, if the triangulation is of regular form and sound design, it is probable that *most*, if not *all*, the discrepancy may fairly be assigned to errors in the astronomically determined positions and *these* should be adjusted to conform with the results of the triangulation.

When it is necessary to resort to triangulations of a less accurate character, *e.g.*, those involving the use of floating beacons, the cumulative error in a long chain may be considerable and it might be unreasonable to assign the entire discrepancy to the astronomical observations. No hard-and-fast rule can be laid down as to what proportion of the discrepancy can fairly be allotted to each, for the surveyor must be guided by the character of the triangulation and the estimated accuracy of the astronomical observations. When the proportions have been decided, the figures of the triangulation can be readjusted to harmonise with the " corrected " geographical positions in the same manner as that outlined in paragraph 2.

Irregular Triangulation

Use of Ship and Floating Marks.

The majority of present-day hydrographic surveys are of a detailed character and are based on triangulations of orthodox form. In sketch surveys, where it may be assumed that time is the important factor, less regular methods of triangulation may be accepted, and even in detailed surveys, when the physical obstacles to regular triangulation are great, resort may be made to various artifices which will give sufficiently accurate results without an undue expenditure of time and labour. Such artifices may include the fixation of some

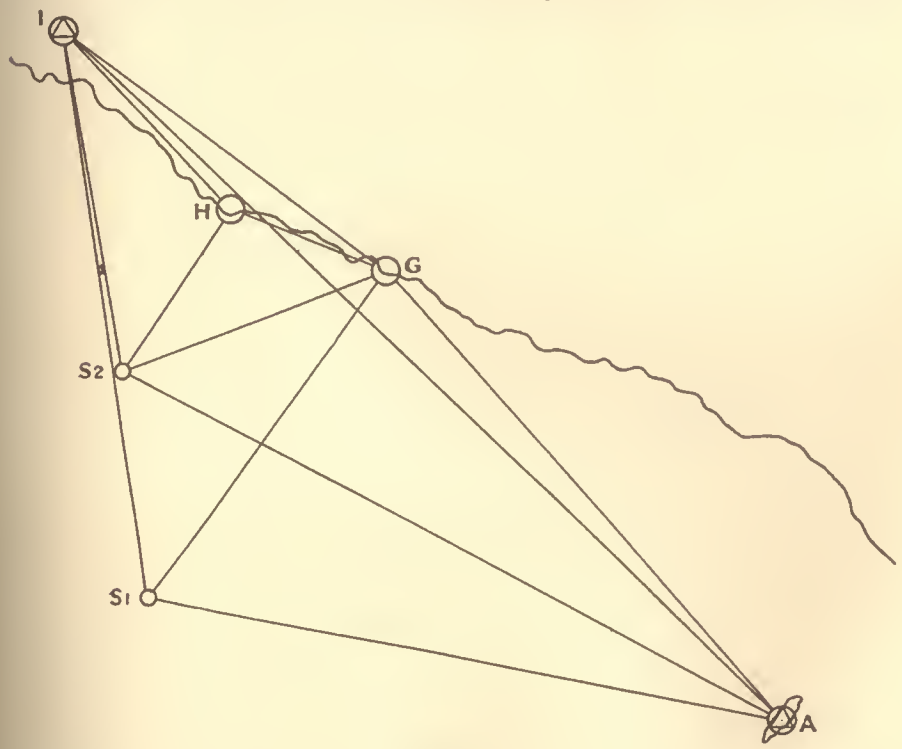


Fig. 40

of the stations solely by resection, *i.e.*, by angles observed *at* them, or by means of traverses. Often the ship or boats at anchor, or moored beacons,

can be used as temporary marks which will enable connection to be made between shore stations by well-conditioned triangles.

For example :

In Fig. 40, A and I are two stations whose positions have been previously determined ; the length and direction of the side AI are therefore known. For the purposes of a survey off the coast further trigonometrical points are required, and two positions at G and H are selected and marked on the shore. A, I, G and H are all intervisible, with the exception of the " ray " A to H, and it is not possible to carry the triangulation through on land. Now, it is clear from the shape of the triangles AIG and IGH that they are quite unsuitable for fixing G and H relative to A and I, for a very small error in the observed angles would beget large errors of position if these triangles were used. Two positions S_1 and S_2 offshore are therefore selected and the ship anchored at each in turn. When the ship is at S_1, simultaneous angles are taken (at a given signal) to the foremast from A, G and I. Two well-conditioned triangles AGS_1 and IGS_1 are now available for fixing G, for the angles at A, G and I have been measured and those at S_1 can be deduced, and the sides AS_1 and IS_1 can be computed from the triangle AIS_1. The position of G is found from both triangles and a partial check on the accuracy of the observations is thereby afforded.

A similar procedure is adopted with the ship anchored at S_2, simultaneous angles being observed from A, I, G and H. The position of H can now be computed from either of the triangles GHS_2 or IHS_2 in which the angles have been measured or can be deduced and the sides GS_2 and IS_2 can be found from the triangles AGS_2 and AIS_2.

The observations to S_1 and S_2 should be repeated two or three times and the results of each set worked out independently. Where shorter distances are involved beacons or boats may be used in a similar manner.

" Ship Stations " may sometimes be used for fixing stations which are too far apart to be visible from each other, as in Fig. 41.

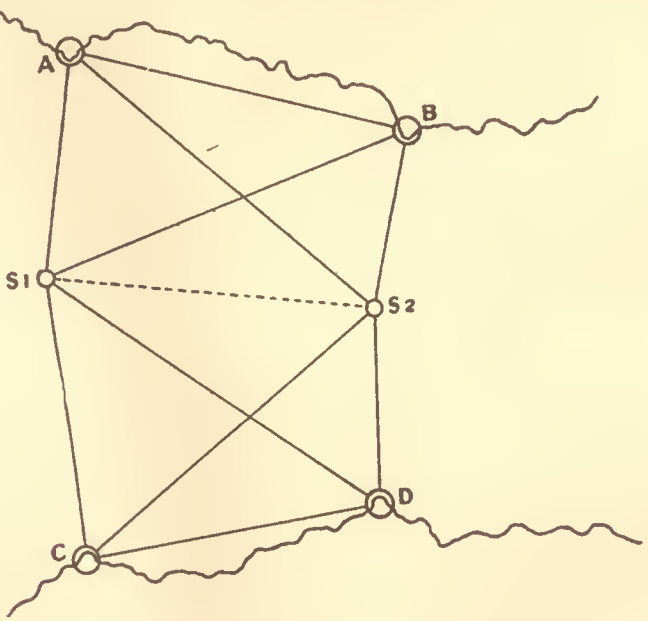


FIG. 41

Fig. 41 represents a channel about 15 miles wide separating two low-lying stretches of land. On the north side two intervisible stations A and B have

been fixed. It is required to fix two intervisible points C and D on the south shore, but the distance across the channel is too great to permit direct observations to be made. The ship is anchored at S_1 and simultaneous angles are taken to the foremast from the four stations: she then proceeds to S_2 where a similar programme is carried out. These observations will suffice to connect C and D with A and B, for, the side AB being known, S_1S_2 can be found from the quadrilateral ABS_1S_2, and the sides and unknown angles of the quadrilateral CDS_1S_2 can now be computed by the method applicable to solving the problem of the "Inaccessible Base," *vide* Chapter II, pp. 22–5.

Floating Beacons in Main Triangulation

In hydrographic surveys a satisfactory triangulation may often be carried out with the aid of floating beacons moored offshore. Such expedients may be resorted to when more regular methods are impossible or would involve an expenditure of time and labour out of all proportion to the value of the results. Straight low-lying stretches of the coast, with dense bush reaching down to the high-water line and no hill features or off-lying islets, present an almost insuperable obstacle to regular methods of triangulation, but the use of moored beacons enables a survey to be made with all necessary accuracy. Under favourable conditions and with proper safeguards in the taking of the angular measurements, very little error should accumulate and the shore stations may be fixed with a precision not far short of that obtainable in a regular triangulation. Example:

A survey is required off a low-lying coast between B and F, Fig. 42, but owing to the nature of the land features no regular shore triangulation can be made. The points A and B have been fixed by a previous survey and the side AB may therefore be used for extending the triangulation. The procedure is as follows:

Five points, B, C, D, E and F, are selected on the coast in such positions that triangulation marks can be erected at them and that each is visible from adjacent marks. Four beacons, **a**, **b**, **c** and **d**, are moored at a distance of about 5 miles offshore, each beacon being approximately abreast a shore station.

Stations A, B and C are manned by observers with theodolites and the ship proceeds to the vicinity of beacon **a**. At a given signal from the ship, the three observers take angles to **a**, *i.e.*, the angles BA**a**, AB**a**, a**B**C and a**C**B are measured. The length and bearing of AB being known, the angles at **a** at the moment of observation are deduced by closing the triangles AB**a** and BC**a**, and the length and bearing of BC can be calculated. Hence the position of C is found.

Stations B, C and D are next manned and the ship proceeds to the vicinity of beacon **b** to which simultaneous angles are obtained as before. The length and bearing of CD can now be calculated from the triangles BC**b** and CD**b**, and hence the position of D is fixed. A similar procedure enables E and F to be fixed from the triangles CD**c** and DE**c** and from DE**d** and EF**d** respectively.

The observations to each beacon should be repeated two or three times and the results of each set worked out *independently* and the values of the sides meaned. If this method of using moored beacons in the triangulation is adopted, any movement of the beacons previous or subsequent to the actual observations, has no effect on the accuracy of the work. The angles at the beacons themselves may be observed by sextant and will provide a check against gross errors of observation at the shore stations, but they should not be used in the computation.

If intervisibility cannot be secured between two adjacent shore stations, angles at the beacons must be observed and used in the triangulation, but in such cases a considerable loss in accuracy must be anticipated since the triangulation will be partly dependent on sextant angles and it will probably be impossible to maintain the principle of simultaneous observation which is essential for accurate results.

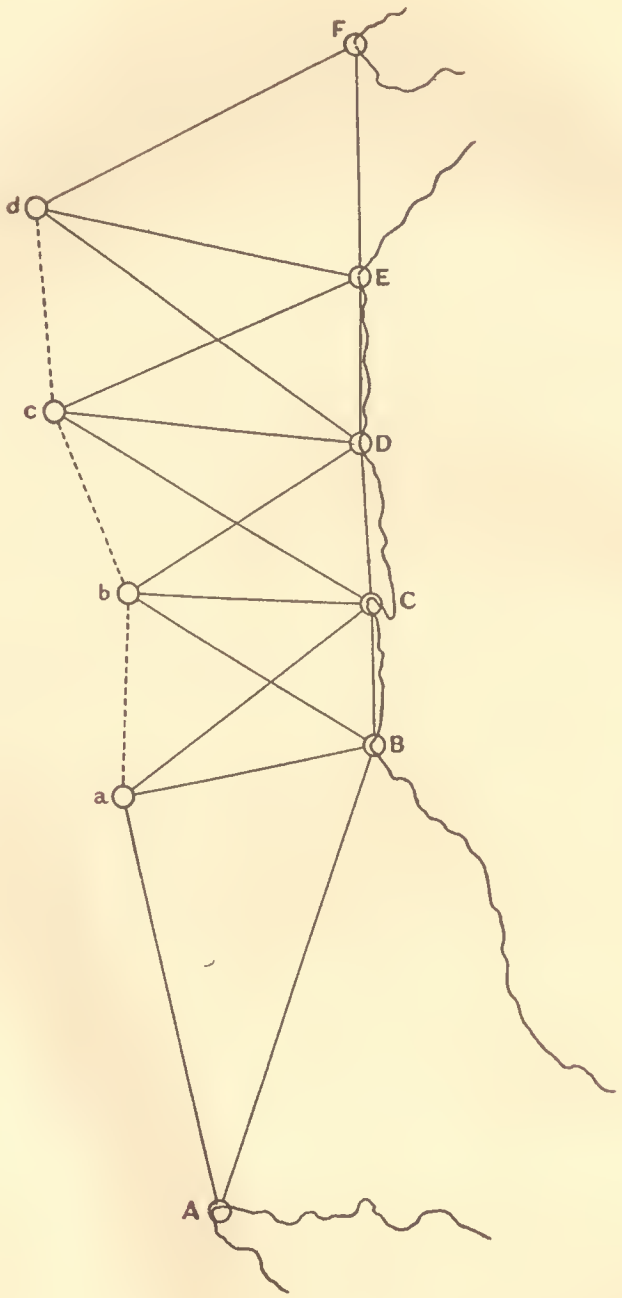


FIG. 42

FLOATING TRIANGULATION

Many hydrographic surveys are made out of sight of land or at such distances from the land that shore marks cannot be directly used for locating the soundings, either because such marks are not suitably disposed to afford good "fixes" or because they lie outside the limits of a plotting sheet. Also it often occurs that a part only of a hydrographical work can be fixed by direct angles

to shore stations whilst other means must be devised to locate the remainder.

When shore stations cannot be made to serve as a framework for the survey a floating triangulation can be carried out by means of anchored or moored beacons, if the depth of water permits. Such triangulations may be divided into:

(i) Those which are connected to and extended from the stations of a shore triangulation, when the scale, bearing and geographical position will usually depend on observations and measurements taken on shore.

(ii) Those of a fully floating type in which the scale, bearing and geographical position depend on observations and measurements taken at sea.

Beacons are anchored in such positions that they provide a framework for the survey and their arrangement should be such that the sides connecting them form a series of geometrical figures of which the most suitable types are quadrilaterals, with sides common to adjacent figures, and polygons with central stations and with one or more of their component triangles common to adjacent figures. In shore triangulations the shape of the figures is determined to a large extent by the physical characteristics of the land and it is often necessary to use figures which are comparatively ill-conditioned. In floating triangulations, however, it is generally possible to dispose the beacons in such a manner that the geometrical figures approximate to the ideal shapes, *i.e.*, the quadrilateral can be nearly a square, and the central-station polygon can consist of six nearly-equilateral triangles. As far as possible extension of the network by *single* triangles should be avoided.

The angles in a floating triangulation are measured by sextant, those at each beacon being observed from the ship (or boats) hove to in close proximity and corrections being applied, as necessary, for the "false station." The accuracy of this type of triangulation cannot be very great because:

(i) The beacons have a certain amount of movement round their anchors, even if moored, and it is impossible to make all the observations simultaneously. This is the principle source of error.

(ii) The angles are measured from a platform which cannot be kept absolutely stationary.

(iii) The type of sextant generally used is graduated only to minutes of arc and can be read by estimation to the nearest half minute. (The use of observing sextants, which can be read to 10 seconds of arc, rarely gives any increase in accuracy as both telescopes and mirrors are ill adapted to this sort of work.)

Although subject to considerable errors from these causes, a floating triangulation should nevertheless be of such accuracy that there is *no plottable error* in the resulting survey, on the scale which is being used. When beacons are anchored with single anchors only, triangular misclosures up to 4′ of arc can be accepted; but if moored with two anchors, it should be possible to keep triangular misclosures within a maximum of 2½′ of arc. The question whether beacons should be anchored or moored depends chiefly on the scale of the survey. On scales of 3 inches to the mile (about 1/25,000) and larger, the swing of a beacon round a single anchor may produce plottable errors; beacons should therefore be moored, except possibly in shallow water where little cable is necessary. On smaller scales beacons with single anchors should serve unless they occupy some exceptionally important or key position in the scheme of triangulation.

To minimise accumulation of error in a floating triangulation, attention must be directed to the following points:

(*a*) The figures forming the triangulation system should be planned with a view to affording a check on the accuracy of the angular measurements.

(*b*) No opportunity should be lost of checking the accuracy of the work by means of "rays" to or from distant mountain peaks or other land objects.

(*c*) The sides forming the triangulation should be as long as possible, consistent with the conditions of visibility which may be expected. Additional beacons may be required for fixing the soundings, but these should be regarded as sounding marks only and should not be considered as an integral part of the triangulation.

(*d*) When the triangulation has been extended through a long chain of figures to a distance of, say, 40 miles, a new " base " should be measured by taut wire and a new azimuth determined by astronomical observations. The subsequent extension of the work should depend on these new measurements.

The adjustment of a floating triangulation and the computation of the sides, etc., is carried out in a similar manner to that used in ordinary land triangulation. When the network consists of a series of interlacing polygons, the adjustments which are required to make the figures geometrically consistent

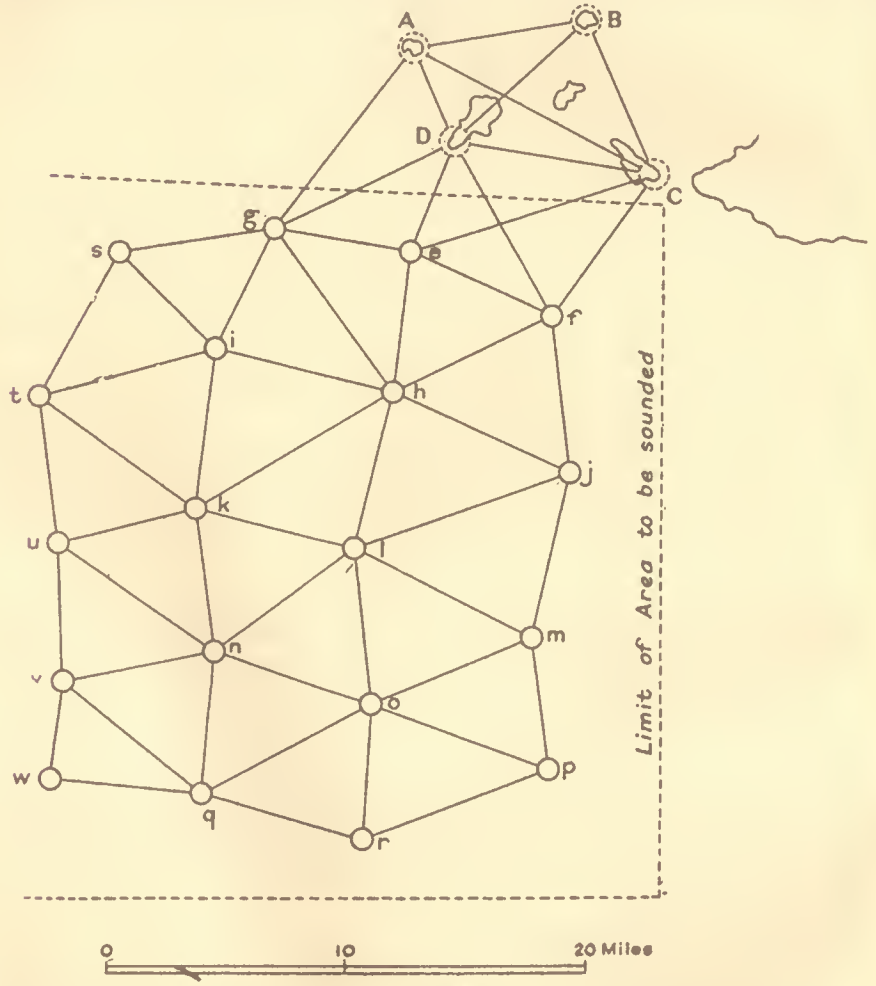


FIG. 43

may assume rather large proportions and for *this* reason, in a floating triangulation, a network of quadrilaterals is generally preferable although it is not quite so economical in beacons in relation to the area covered.

Examples :

In Fig. 43 a floating triangulation has been extended from three shore stations A, C and D, previously fixed in a former triangulation.

The beacons, **e, f, g,** etc., are disposed so that a series of interlacing polygons is formed. Their average distance apart is about 6 miles and they provide a framework sufficient to cover an area of approximately 500 square miles. Their positions are dependent on those of the shore stations.

In Fig. 44 a floating triangulation has been connected to three shore stations A, B and C.

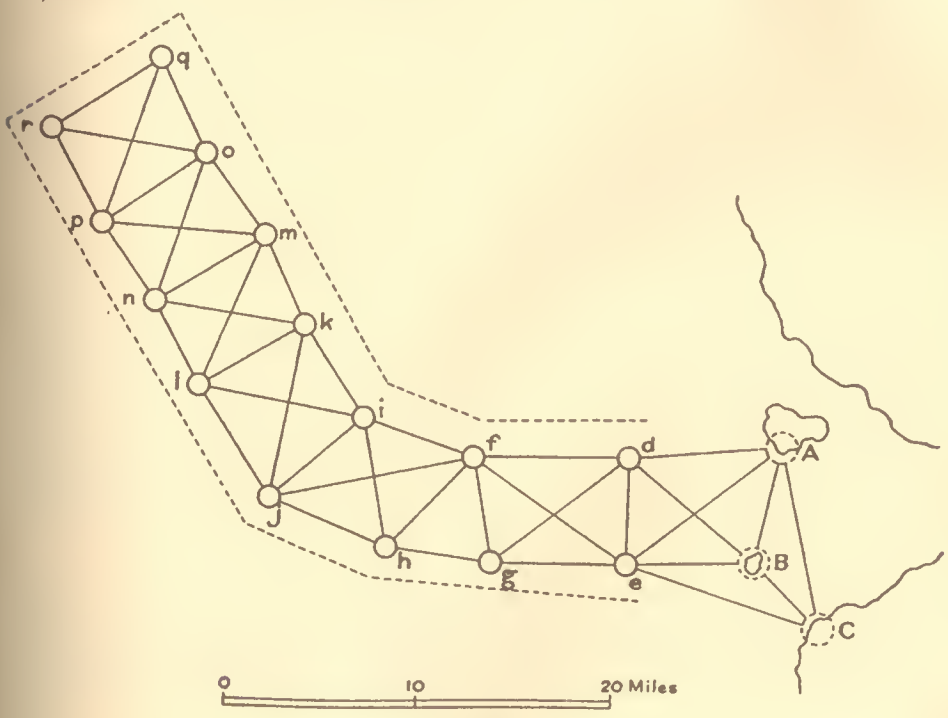


FIG. 44

This system of figures consists mainly of quadrilaterals, beacons being moored at **d, e, f,** etc. The geographical position of C, and the azimuth of the side AC, are known. Conditions are unsuitable for a base measurement ashore, and the length of the side " **ef** " is found by taut wire. When the triangulation has been carried as far as the quadrilateral **klmn**, the side **mn** is measured by taut wire and its azimuth determined by astronomical observations. The results of these measurements should then be used as a basis for the further extension of the triangulation to the north-westward. The diagonals of the quadrilaterals are from 8 to 10 miles in length, *i.e.*, about the maximum distance that can be observed. The system provides a framework sufficient to cover an area of about 400 square miles, but some additional beacons (not shown in the figure) may be needed as sounding marks.

An alternative method of control of a floating triangulation is by taut-wire traverses measured along lines of floating beacons laid in transit.

The azimuth of any convenient leg of the traverse is observed from a position ashore, or from the ship hove to, on the extension of the line of the leg, by the methods described in Chapter XVI. Angles between successive legs of the traverse are observed by sextant from the ship and corrected for false station.

The traverse may either start from a beacon fixed by shore marks and close

on another beacon similarly fixed, or may start from a beacon astronomically fixed and close on the same beacon. Adjustment for errors of closure, as described in Chapter VI, "Adjustment of a Traverse," is applied, the closing errors obtained indicating whether the accuracy desired for the scale in use has been achieved or whether additional measurements and observations are required.

Fig. 44A illustrates the method.

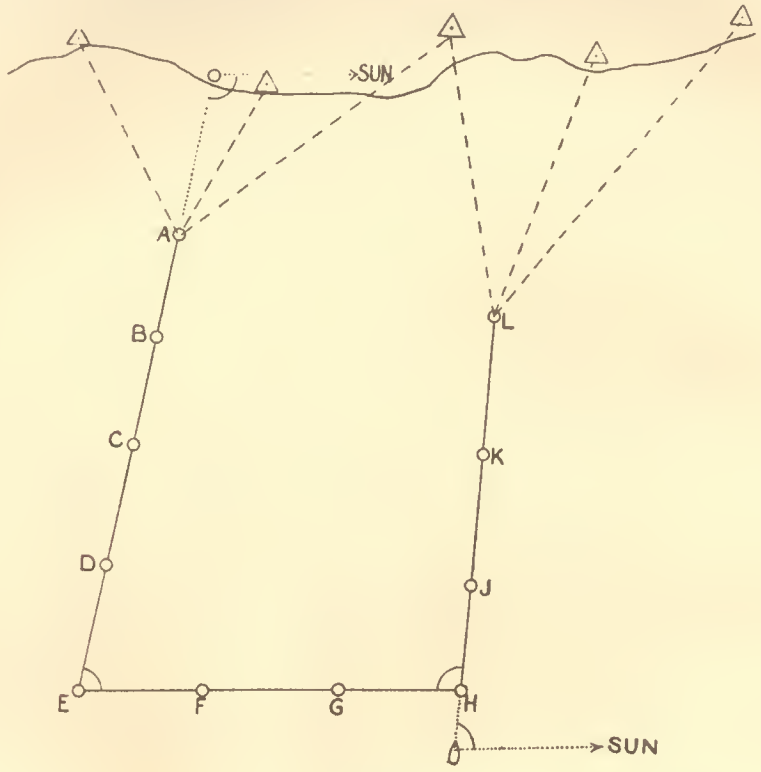


FIG. 44A

Beacons C, D and E are laid on the transit of A and B; G and H on the transit of E and F, and K and L on the transit of H and J.

Distances AE, EH and HL are measured with taut-wire and the angles DEF and GHJ observed.

A and L are fixed from shore marks and the azimuth of AB and hence AE, or HJ and hence HL is observed from the shore or from the ship.

In order that the beacons may be laid in transit their distance apart must be regulated so that alternate beacons are intervisible, also, if it is desired to observe the azimuth from a shore station, the two inshore beacons of the leg must be laid close enough together for both to be visible from the shore.

The advantages of this method compared with triangulation are its quickness, that fewer beacons are required to cover the same area, and that unreliable beacons usually disclose themselves at once by drifting off transit.

It should be noted that if a leg whose azimuth is to be observed lies in an east-west direction it may be necessary to lay an additional beacon to act as a referring object so as to make the observed angle between it and the sun as near 90° as possible; in such a case the angle between this beacon and the leg of the traverse whose azimuth is required must be observed simultaneously with the azimuth of the sun.

Errors of Triangulation

It is useful to have some method of assessing the accuracy of any series of observations such as that composing a triangulation. Thereby the work of different observers can be compared and an estimate be formed of the relative merits of various independent measurements.

The error of an observation is the difference between the true and measured values of a quantity. Errors may be divided into three classes :

(1) **Mistakes ;** *e.g.*, reading off a micrometer wrongly. Such errors should be noticed and rectified in the field by arranging for proper safeguards in the mode of observation.

(2) **Constant or Regular Errors** which include those due to refraction, aberration, etc., special peculiarities or faults in the instrument, and personal idiosyncrasies of the observer.

(3) **Irregular or Accidental Errors,** such as those arising from the imperfection of the eye in measuring small spaces, changes in the instrument produced by anomalous variation of temperature, and which are in general beyond the control of the observer.

It is only the third class of errors, *i.e.*, those of an irregular type, which can be considered mathematically in assessing the degree of precision of a series of observations.

If a number of measurements are made of the same quantity, the most probable value of that quantity is the arithmetical mean of all the measurements. A residual is the result of subtracting the mean from a measured value, and it is a characteristic of the arithmetical mean that it makes the sum of the squares of the residuals a minimum.

In an indefinitely large series of observations, the *probable error* is that error than which there are as many errors greater as there are smaller.

The probable error (P.E.) of the arithmetical mean of a series of observations

$$= 0{\cdot}67.\sqrt{\frac{\Sigma r^2}{n(n-1)}}$$

where n is the number of observations, and Σr^2 is the sum of the squares of the residuals. Example :

A series of measurements of an angle give the following results (seconds only given) :

$''$	r	r^2
20	4·3	18
26	1·7	3
29	4·7	22
22	2·3	5
22	2·3	5
27	2·7	7
6)146		$\Sigma r^2 =$ 60

Mean $= 24''{\cdot}3$

$$\therefore \quad \text{P.E.} = 0{\cdot}67\sqrt{\frac{60}{6 \times 5}} = 0''{\cdot}9$$

The figure ($0''{\cdot}9$) provides a basis for determining the degree of precision of these measurements in comparison with any other similar series. It should be clearly understood that this is the only purpose of calculating the probable error ; such a small number of observations can provide little guide to the *possible* error of the mean since there is no certainty that all constant or systematic sources of error have been eliminated. Little, if *any*, value can be attached to the " probable error " of a very small number of measurements

(*e.g.* a base measured two or three times with a steel tape over the ground) owing to the number of sources of error which may occur in such work.

Probable Errors of a Triangulation.—If the P.E. of each angle of a triangle is p, the P.E. of the sum of the three angles is $p.\sqrt{3}$; conversely, if the triangular error is e, the P.E. of an angle is $\frac{e}{\sqrt{3}}$

In n triangles the P.E. of an angle is $0{\cdot}67\sqrt{\frac{\Sigma e^2}{3n}}$, since the probable value of a triangular error is $0{\cdot}67\sqrt{\frac{\Sigma e^2}{n}}$.

Example :
The triangular errors in a chain of eight triangles are as follows :

e "	e^2
0·5	0
16·7	279
9·0	81
7·5	56
3·0	9
8·0	64
13·0	169
2·0	4
	$\Sigma e^2 = 662$

P.E. of a triangle $= 0{\cdot}67\sqrt{\frac{662}{8}} = 6''{\cdot}1$

and P.E. of an angle $= 0{\cdot}67\sqrt{\frac{662}{24}} = 3''{\cdot}5$

Latitude, Longitude and Reverse Azimuth

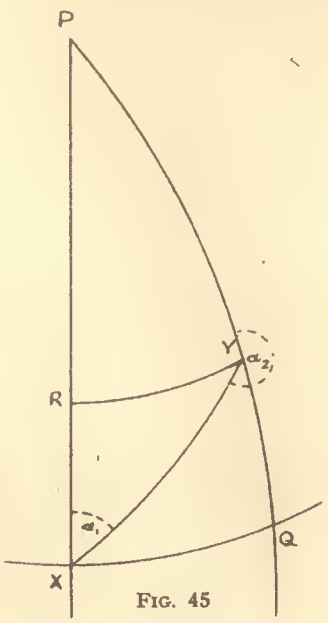


Fig. 45

The geographical co-ordinates (Latitude and Longitude) of one station in any triangulation must be known in order that the position of the survey as a whole can be correctly located on the earth's surface. The latitude and longitude of the other main stations should be calculated as they may be required for the extension of the survey or for other purposes.

X and Y (Fig. 45) are two main stations and XY one of the sides of a triangulation: the length of XY is therefore known. The latitude and longitude, ϕ_A ω_A, of X, and the true bearing or azimuth, α_1, of Y from X are known: the latitude and longitude, ϕ_B ω_B, of Y and the reverse azimuth, α_2, of X from Y are required.

PRX and PYQ are meridians through X and Y respectively, meeting at the pole P. RY and XQ are parallels of latitude through Y and X respectively. The azimuth, α_1, of Y from X is the angle PXY between the meridian PRX and the arc of a great circle, XY. The reverse azimuth is the angle α_2

The fundamental formulæ are as follows :—

$$\tan \tfrac{1}{2}\Delta\alpha = \tan \tfrac{1}{2}\Delta\omega \,.\, \sin (\phi_A + \tfrac{1}{2}\Delta\phi) \sec \tfrac{1}{2}\Delta\phi \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\Delta\phi = \frac{s}{\rho \,.\, \sin 1''} \cos (\alpha_1 \pm \tfrac{1}{2}\Delta\alpha) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\Delta\omega = \frac{s}{\nu \,.\, \sin 1''} \frac{\sin (\alpha_1 \pm \tfrac{1}{2}\Delta\alpha)}{\cos (\phi_A + \tfrac{1}{2}\Delta\phi)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where $\Delta\phi$, $\Delta\omega$ and $\Delta\alpha$ are the required differences in latitude, longitude and azimuth, $\Delta\alpha$ being regarded as $(\alpha_2 \pm 180°) - \alpha_1$.

s is the true distance between the stations.

ρ is the radius of curvature to the meridian at the mid-latitude.

ν is the radius of curvature at right-angles to the meridian at the mid-latitude.

Log $1/\rho \sin 1''$ and log $1/\nu \sin 1''$ are tabulated under the headings log P and log Q in Tables V, Va and Vb. The values of these terms depend on the " Figure of the Earth " which is adopted in the survey and must be taken out of the corresponding table. The figures most likely to be met with are Clarke's 1858 and 1866 (which differ very little), Airy's, which is used for the Ordnance Survey of Great Britain, and Everest's, which is used for the Survey of India and the Federated Malay States. Clarke's 1880 figure is also used in several colonial surveys.

In using the formulæ the terms ϕ_A and ω_A are regarded as **positive** when the initial station is north of the Equator and east of Greenwich, and ***vice versa***. Azimuths and reverse azimuths should always be measured from 0° to 360° clockwise from north.

The signs of $\Delta\phi$, $\Delta\omega$ and $\Delta\alpha$ may be found from the following rules :—

(1) $\Delta\phi$ is positive when $(\alpha_1 \pm \frac{1}{2}\Delta\alpha)$ lies between 270° and 090°

(2) $\Delta\phi$ is negative ,, ,, lies between 090° and 270°

(3) $\Delta\omega$ is positive ,, ,, lies between 0° and 180°

(4) $\Delta\omega$ is negative ,, ,, lies between 180° and 360°

(5) In N. latitude, $\Delta\alpha$ is $\frac{\text{positive}}{\text{negative}}$ when α_1 lies between $\frac{0° \text{ and } 180°}{180° \text{ and } 360°}$

(6) In S. latitude, $\Delta\alpha$ is $\frac{\text{negative}}{\text{positive}}$ when α_1 lies between $\frac{0° \text{ and } 180°}{180° \text{ and } 360°}$

It should be noted that the azimuth and reverse azimuth between two points on the earth's surface do not differ by exactly 180° owing to the convergence ($\Delta\alpha$) of the meridians.

The angles $\Delta\omega$ and $\Delta\alpha$ are in general very small, so that some difficulty may be experienced in taking out the logarithms of their tangents from ordinary tables. This difficulty may be overcome by adding the log $\frac{\tan \text{arc}}{\text{arc}''}$ to the log of the angle reduced to seconds. Log $\frac{\tan \text{arc}}{\text{arc}''}$ is tabulated on the last page of Shortrede's Tables.

In computing by these formulæ it is first necessary to find approximate values, $\Delta\phi_A$ and $\Delta\omega_A$, of $\Delta\phi$ and $\Delta\omega$ to use in equation (1). If a large scale plot is available, they may be read off from it with sufficient accuracy ; usually, however, it is necessary to make an approximate calculation of these terms.

Azimuth, Reverse Azimuth and Distance.

When the geographical co-ordinates of two stations are known the reverse

process of calculating azimuths and distance is often required. No preliminary approximation is required. The formulæ are :

$$\tan \tfrac{1}{2}\Delta\alpha = \tan \tfrac{1}{2}\Delta\omega \sin (\phi_A + \tfrac{1}{2}\Delta\phi) \sec \tfrac{1}{2}\Delta\phi \quad \ldots\ldots(1)$$

$$\tan (\alpha_1 + \tfrac{1}{2}\Delta\alpha) = \frac{\nu\Delta\omega}{\rho\Delta\phi} \cos (\phi_A + \tfrac{1}{2}\Delta\phi) \quad \ldots\ldots(2)$$

$$\left.\begin{aligned} s &= \Delta\phi\, \rho \sin 1'' \sec (\alpha_1 \pm \tfrac{1}{2}\Delta\alpha) \\ \text{or,} \quad s &= \Delta\omega\, \nu \sin 1'' \cos (\phi_A + \tfrac{1}{2}\Delta\phi) \operatorname{cosec} (\alpha_1 \pm \tfrac{1}{2}\Delta\alpha) \end{aligned}\right\} \quad \ldots\ldots(3)$$

Log $\frac{\nu}{\rho}$ is log $\frac{P}{O}$ from Table V (Va or Vb), the values of log P and log O being taken out, as before, for the mid-latitude.

In the approximate part of the calculation, log P and log O may be taken out with ϕ_A as argument. In the examples given the constants for the Everest " figure " have been used, *vide* Table Va.

Form for Calculating Latitudes, Longitudes and Reverse Azimuths.

Data	Initial Station A		Lad.	
	Lat. of A (ϕ_A)	+	6° 12′	57″·820
	Long. of A (ϕ_A)	+	99° 45′	02″·960
	Final Station B		Pin.	
	Azimuth of B from A (α_1)		305° 07′	45″·84
	log AB (s) in feet		5·230 8859	
Approx. Calculation.	log P (for ϕ_A)		$\bar{3}$·996 64	
	L. cos α_1		9·759 99	
	log s		5·230 89	
	log $\Delta\phi_a$ (secs.)		2·987 52	
	$\Delta\phi_a$	+		971″·7
		= +	16′	11″·7
	$\frac{1}{2}\Delta\phi_a$	+	8′	05″·8
	ϕ_A	+	6° 12′	57″·8
	$\phi_A + \frac{1}{2}\Delta\phi_a$ (ϕ_{am})	+	6° 21′	03″·6

Approx. Calculation.	log O (for ϕ_{am})	$\bar{3}$·993 79	
	L. sin α_1	9·912 68	
	L. sec ϕ_{am}	0·002 67	
	log s	5·230 89	
	log $\Delta\omega_a$ (secs.)	3·140 03	
	log 0·5	$\bar{1}$·698 97	
	log $\frac{\text{tan arc}}{\text{arc}''}$	4·685 58	
	L. sin ϕ_{am}	9·043 83	
	L sec $\frac{1}{2}\Delta\phi_a$	0·000 00	
	L. tan $\frac{1}{2}\Delta\alpha$	6·568 41	
	log $\frac{\text{tan arc}}{\text{arc}''}$	4·685 58	
	log $\frac{1}{2}\Delta\alpha$	1·882 83	
	$\frac{1}{2}\Delta\alpha$	−	76″·35
		= − 01′	16″·35
	α_1	305° 07′	45″·84
	$\alpha_1 \pm \frac{1}{2}\Delta\alpha$	305° 06′	29″·49

log P (for ϕ_{am})			$\bar{3}\cdot996\ 6421$
L. cos $(a_1 \pm \frac{1}{2}\Delta a)$			$9\cdot759\ 7602$
log s			$5\cdot230\ 8859$
log $\Delta\phi$ (secs.)			$2\cdot987\ 2882$
$\Delta\phi$	+		$971''\cdot154$
	=	+	$16'\ 11''\cdot154$
ϕ_A	+	+	$6°\ 12'\ 57''\cdot820$
ϕ_B	+	+	$6°\ 29'\ 08''\cdot974$

log O (for ϕ_m)			$\bar{3}\cdot993\ 7850$
L. sin $(a_1 \pm \frac{1}{2}\Delta a)$			$9\cdot912\ 7892$
L. sec ϕ_m			$0\cdot002\ 6735$
log s			$5\cdot230\ 8859$
log $\Delta\omega$ (secs.)			$3\cdot140\ 1336$
$\Delta\omega$	−		$1380''\cdot809$
	=	−	$23'\ 00''\cdot809$
ω_A	+	+	$99°\ 45'\ 02''\cdot960$
ω_B	+	+	$99°\ 22'\ 02''\cdot151$

$a_1 \pm 180°$			$125°\ 07'\ 45''\cdot84$
Δa	−		$02'\ 32''\cdot70$
a_2			$125°\ 05'\ 13''\cdot14$

Form for Calculating Distances and Azimuths.

Initial Station A			Cone
Final Station B			Kweh
Data ϕ_A	+		$6°\ 24'\ 01''\cdot968$
Data ϕ_B	+	+	$6°\ 35'\ 07''\cdot715$
$\Delta\phi$	−		$11'\ 05''\cdot747$
$\frac{1}{2}\Delta\phi$	−		$5'\ 32''\cdot873$
Mid. Lat. (ϕ_m)	+		$6°\ 29'\ 34''\cdot841$

Data ω_A	+		$99°\ 40'\ 20''\cdot905$
Data ω_B	+	+	$99°\ 50'\ 15''\cdot741$
$\Delta\omega$	−		$9'\ 54''\cdot836$
	=	−	$594''\cdot836$
$\frac{1}{2}\Delta\omega$	−		$297''\cdot418$
log $\frac{1}{2}\Delta\omega$ (secs.)			$2\cdot473\ 367$
log $\frac{\text{tan arc}}{\text{arc}''}$			$4\cdot685\ 575$
L. sin ϕ_m			$9\cdot053\ 393$
L. sec $\frac{1}{2}\Delta\phi$			$0\cdot000\ 001$
L. tan $\frac{1}{2}\Delta a$			$6\cdot212\ 336$
log $\frac{\text{tan arc}}{\text{arc}''}$			$4\cdot685\ 575$
log $\frac{1}{2}\Delta a$ (secs.)			$1\cdot526\ 761$
$\frac{1}{2}\Delta a$			$33''\cdot63$

	log P (for ϕ_m)	$\bar{3}\cdot996\ 6397$
	colog O (for ϕ_m)	$2\cdot006\ 2158$
	log $\Delta\omega$ (secs.)	$2\cdot774\ 3973$
	colog $\Delta\phi$ (secs.)	$\bar{3}\cdot176\ 6908$
	L. cos ϕ_m	$9\cdot997\ 2053$
	L. tan $(a_1 \pm \frac{1}{2}\Delta a)$	$9\cdot951\ 1489$
	$a_1 \pm \frac{1}{2}\Delta a$	041° 47′ 03″·67
	$\frac{1}{2}\Delta a$	33″·63
	a_1 (Azimuth of B from A)	041° 46′ 30″·04
	a_2 (Azimuth of A from B)	221° 47′ 37″·30
	L. sec. $(a_1 \pm \frac{1}{2}\Delta a)$	$0\cdot127\ 4603$
	colog P (for ϕ_m)	$2\cdot003\ 3603$
	log $\Delta\phi$ (secs.)	$2\cdot823\ 3092$
	log s (AB)	$4\cdot954\ 1298$
∴	s ... ... =	89 976·6 feet

Machine computation of Azimuths and Distance from Geographical Co-ordinates. The following formulæ are suitable :

$$\text{Tan } \tfrac{1}{2}(a_2 + a_1) = \frac{\cos \frac{1}{2}\Delta\phi}{\sin \phi_m} \text{ Cot } \tfrac{1}{2}\Delta\omega \quad \ldots\ldots(1)$$

$$\text{Tan } \tfrac{1}{2}(a_2 - a_1) = \frac{\sin \frac{1}{2}\Delta\phi}{\cos \phi_m} \cot \tfrac{1}{2}\Delta\omega \, \frac{\rho}{\nu} \quad \ldots\ldots(2)$$

$$\nu_A \sin a_1 \sec \phi_B = \nu_B \sin a_2 \sec \phi_A = \nu_{90} \sin \Delta\omega \operatorname{cosec} s \text{ (at radius r)} \quad \ldots(3)$$

Equation (1) is Dalby's Theorem in a suitable form ; (2) is an approximation, although a very close one ; in (3), the first two quantities are the characteristic equation of the geodesic line and are absolute in their identity, while the third part is an approximation on similar lines.

$\frac{\rho}{\nu}$ and r are for the mid-latitude (ϕ_m) of the line s.

ν_{90} (*i.e.*, the normal at the pole) $= \frac{a^2}{b}$, and is a constant for any given spheroid :

For Clarke's 1858 spheroid it is 2099 7705 ft.
,, Everest's 1st ,, ,, ,, 2099 2721 ft.
,, Airy's ,, ,, ,, 2099 3850 ft.

Caution. As the above formulæ are based on Napier's Analogies, both a_1 and a_2 are on the poleward side of s, ∴ the azimuth in the 3rd or 4th quadrant, either a_1 or a_2, must be subtracted from 360° for it to be rendered in the conventional form.

Example of computing Azimuths and Distance by machine.

ϕ_A +6° 24′ 01″·968		ω_A +99° 40′ 20″·905			
ϕ_B +6° 35′ 07″·715		ω_B +99° 50′ 15″·741			
$\Delta\phi$	11′ 05″·747	$\Delta\omega$	9′ 54″·836	ν_A	20923790
$\frac{1}{2}\Delta\phi$	5′ 32″·873	$\frac{1}{2}\Delta\omega$	4′ 57″·418	sin a_1	0·6620809
ϕ_m	6° 29′ 34″·841	sin $\frac{1}{2}\Delta\phi$	0·00161381	sec ϕ_B	1·0066419
		sec ϕ_m	1·00645575	Const.	14032184
cos $\frac{1}{2}\Delta\phi$	0·99999870	cot $\frac{1}{2}\Delta\omega$	**693·51774**	ν_{90}	20992721
cosec ϕ_m	8·8431380	$\frac{\rho}{\nu}$ (for ϕ_m)	0·9934466		
cot $\frac{1}{2}\Delta\omega$	693·51774			Const/ν_{90}	0·66843091
				cosec $\Delta\omega$	346·760
tan $\frac{1}{2}(a_2+a_1)$	6132·8651	tan $\frac{1}{2}(a_2-a_1)$	1·11905133		
				cosec s	231·785
$\frac{1}{2}(a_2+a_1)$	89° 59′ 26″·367	$\frac{1}{2}(a_2-a_1)$	48° 12′ 56″·101		
		$\frac{1}{2}(a_2+a_1)$	89° 59′ 26″·367	s″	889″·901
				r sin 1″ (for ϕ_m)	101 ·1086ft.
		a_1	N 41° 46′ 30″·266 E		
		a_2	N138° 12′ 22″·468 W	s	89 976 ·6 ft.

ν_A is obtained by multiplying ν_A sin 1″ in Table III (IIIA or IIIB) by cosec 1″ (206 264·81).

In this example the errors in the results obtained do not exceed 0·1 in. in distance (*i.e.*, less than 1:10 000 000), and 0″·003 in azimuth ; and this may be taken to be the general accuracy of the above formulæ.

For computing Geographical co-ordinates from Bearing and Distance, the formulæ are :

$$\text{Tan } \tfrac{1}{2}(\alpha_2+\Delta\omega) = -\frac{\sin \tfrac{1}{2}(90-\phi_A-s)}{\sin \tfrac{1}{2}(90-\phi_A+s)} \cot \tfrac{1}{2}\alpha_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$\text{Tan } \tfrac{1}{2}(\alpha_2-\Delta\omega) = -\frac{\cos \tfrac{1}{2}(90-\phi_A-s)}{\cos \tfrac{1}{2}(90-\phi_A+s)} \cot \tfrac{1}{2}\alpha_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$\phi_B-\phi_A\ (=\Delta\phi) = -\frac{s \sin \tfrac{1}{2}(\alpha_1+\alpha_2)}{\rho \sin 1'' \text{ (for } \phi_A)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

where the distance s is expressed in arc with radius ν_A.

As in the preceding case, these formulæ are based on Napier's Analogies, with the omission of minute angular quantities, negligible on all but long sides (of 200 000 ft. or more).

Example of computing Geographical Co-ordinates and Reverse Azimuth by machine.

s 170 171·1 ft.	ϕ_A + 6° 12′ 57″·820 ω_A +99° 45′ 02″·960	α_1 305° 07′ 45″·840
$\nu_A \sin 1''$ 101·4412	90—ϕ_A 83° 47′ 02″·180	$\frac{1}{2}\alpha_1$ 152° 33′ 52″·920
s″ 1677″·534	—cos ½ (90—ϕ_A—s) —0·74711427	
= 27′ 57″·534	sec ½ (90—ϕ_A+s) 1·34828364	½ α_2 62° 32′ 36″·559
(90—ϕ_A—s) 83° 19′ 04″·646	cot ½α_1 —1·92628991	½ ($\alpha_1+\alpha_2$) 215° 06′ 29″·479
(90—ϕ_A+s) 84° 14′ 59″·714	tan ½ ($\alpha_2-\Delta\omega$) +1·94039411	—sin ½ ($\alpha_1+\alpha_2$) +0·57512217
		s 170171·1 ft.
½(90—ϕ_A—s) 41° 39′ 32″·323 ½(90—ϕ_A+s) 42° 07′ 29″·857	½ ($\alpha_2-\Delta\omega$) 62° 44′ 06″·963 ½ ($\alpha_2+\Delta\omega$) 62° 21′ 06″·155	$\Delta\phi$ +97869·172 ft.
—sin ½ (90—ϕ_A—s) —0·66469562	α_2 125° 05′ 13″·118	$\rho_A \sin 1''$ 100·7757 ft.
cosec ½(90—ϕ_A+s) 1·49086890		$\Delta\phi''$ + 971″·158
cot ½ α_1 —1·92628991	$\Delta\omega$ — 0° 23′ 00″·808	= +0° 16′ 11″·158
tan ½($\alpha_2+\Delta\omega$) +1·90890327	ω_A +99° 45′ 02″·960	ϕ_A +6° 12′ 57″·820
½($\alpha_2+\Delta\omega$) 62° 21′ 06″·155	ω_B +99° 22′ 02″·152	ϕ_B +6° 29′ 08″·978

These results show small differences from those derived from the logarithmic formulæ ; they are, however, in exact agreement with results obtained from precise computation. Over long distances (200 000 ft. or so) the formulæ given for machine computation will give slightly more accurate results than those given for logarithmic computation.

Rectangular Co-ordinates

The position of a point in a plane may be determined by two measurements. If OX and OY (Fig. 46) are any two fixed straight lines at right angles in a plane, the position of any point P, in that plane, is fixed if the distances OM and MP (x and y), measured along OX and parallel to OY, are known. These lengths are the Cartesian Rectangular Co-ordinates of the point, the name being derived from Descartes, the inventor of the system.

OM is called the abscissa and MP the ordinate of P. The lines OX and OY are called the axes, and O is the " origin."

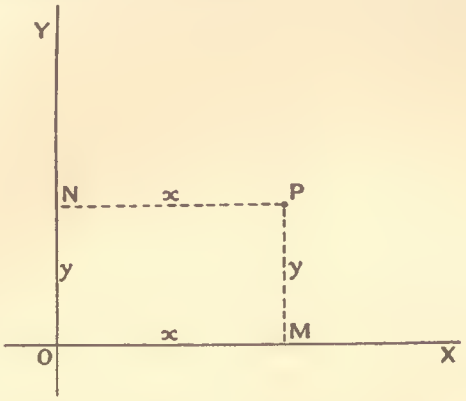


Fig. 46

Rectangular co-ordinates provide a convenient and simple means of recording the positions of the trigonometrical points in a hydrographic survey, since small areas of the earth's surface may be regarded as " plane " without introducing appreciable errors. The calculation of rectangular co-ordinates is less laborious than that of geographical co-ordinates (latitude and longitude), and they provide data which may be used in a simple manner for plotting the framework of the survey.

When the method is applied to a triangulation, it is usual to select one of the trigonometrical points as the initial station, or " Point of Origin," O, and OY—the y-axis—becomes the true meridian through that station, and OX—the x-axis—is a line drawn at right angles to the meridian through the station. The x-axis on the earth is consequently a great circle at right angles to the meridian through O ; it is *not* a parallel of latitude which is a small circle on the earth. By convention, the x co-ordinates are regarded as positive when a station lies to the east of the y-axis and negative when it lies to the west, and the y co-ordinates are regarded as positive when the station lies to the north of the x-axis and negative when it lies to the south.

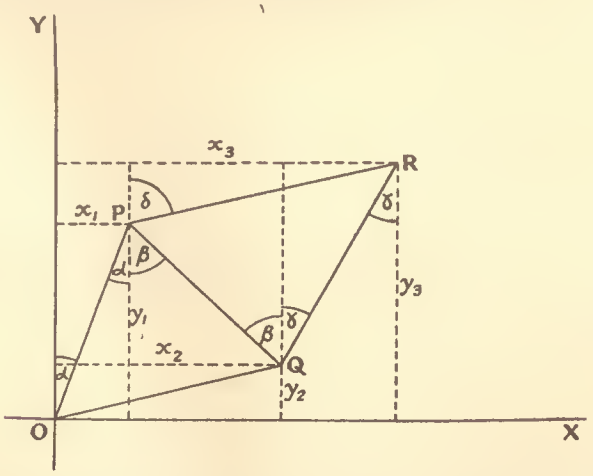


Fig. 47

Before co-ordinates can be calculated from the triangulation, it is necessary to know the true bearing or azimuth of one of the stations from that selected

as the Point of Origin. An arbitrary azimuth could be used but the co-ordinates would have to be subsequently changed when the true azimuth was measured. The method of calculating the rectangular co-ordinates will now be explained.

O, P, Q and R (Fig. 47) are four trigonometrical points of a survey and O is selected as the point of origin for the calculation of rectangular co-ordinates. The azimuth (a) of the side OP is known. The angles of the triangulation are adjusted and the lengths of all the sides are calculated by plane trigonometry by the methods already explained. Then it is clear from the figure that the co-ordinates of P ($x_1 y_1$) are :

$$x_1 = \text{OP} \sin a$$
$$y_1 = \text{OP} \cos a$$

And the co-ordinates of Q ($x_2 y_2$) are :

$$x_2 = \text{OQ} \sin (a + \text{POQ})$$
$$y_2 = \text{OQ} \cos (a + \text{POQ})$$

The co-ordinates of Q may also be obtained through P, for, if lines parallel to the axes are drawn through P, it will be seen that the co-ordinates of Q may be found from the equations :

$$x_2 = x_1 + \text{PQ} \sin \beta$$
$$y_2 = y_1 + \text{PQ} \cos \beta$$

Where $\beta = \text{OPQ} - a$.

Attention must be paid to the signs of the various terms ; as Q lies to the East and South of P, PQ sin β will be positive, and PQ cos β will be negative.

The co-ordinates of R($x_3 y_3$) cannot be found directly from O but may be calculated either through P or Q, as follows :

$$x_3 = x_1 + \text{PR} \sin \delta$$
$$y_3 = y_1 + \text{PR} \cos \delta$$

Where $\delta = 180° - (\text{RPQ} + \beta)$

Or,

$$x_3 = x_2 + \text{QR} \sin \gamma$$
$$y_3 = y_2 + \text{QR} \cos \gamma$$

Where $\gamma = \text{PQR} - \beta$

The co-ordinates are usually measured in feet. It must be clearly understood that the lines drawn through P, Q and R parallel to the axis OY are *not* true meridians, and the angles β, γ and δ do not represent true bearings : they are usually called " false " or " co-ordinate " bearings and they differ from true bearings by an amount equal to the " convergency of the meridians."

When the rectangular co-ordinates are given, the reverse process of calculating the sides and angles is equally simple. Supposing the sides and angles of triangle PQR are required and the co-ordinates of P, Q and R are given as $x_1 y_1$, $x_2 y_2$, and $x_3 y_3$, respectively. Then the angles can be found as follows :

$$\tan \beta = \frac{x_2 - x_1}{y_1 - y_2}$$
$$\tan \gamma = \frac{x_3 - x_2}{y_3 - y_2}$$
$$\tan \delta = \frac{x_3 - x_1}{y_3 - y_1}$$

Angles P, Q and R can be deduced from the values of β, γ and δ. The sides may then be found from the equations :

$$\text{PQ} = (x_2 - x_1) \operatorname{cosec} \beta = (y_1 - y_2) \sec \beta$$
$$\text{PR} = (x_3 - x_1) \operatorname{cosec} \delta = (y_3 - y_1) \sec \delta$$
$$\text{QR} = (x_3 - x_2) \operatorname{cosec} \gamma = (y_3 - y_2) \sec \gamma.$$

Rectangular Spheroidal Co-ordinates.—The methods described above made use of *plane* angles no account being taken of the curvature of the earth. In surveys of small extent this practice is quite in order, since errors will be negligible, but, when a survey extends over large areas, and particularly when the distance east or west of the Point of Origin is great, the use of " plane " co-ordinates is inadmissible.

It is clear that the position of a point on the earth's surface can be precisely defined, with reference to another point, by the intercept of a great circle drawn through it at right angles to the meridian through the point of reference, and the distance along that meridian between the point of reference and the point of intersection of the great circle. Rectangular spheroidal co-ordinates define positions in this manner.

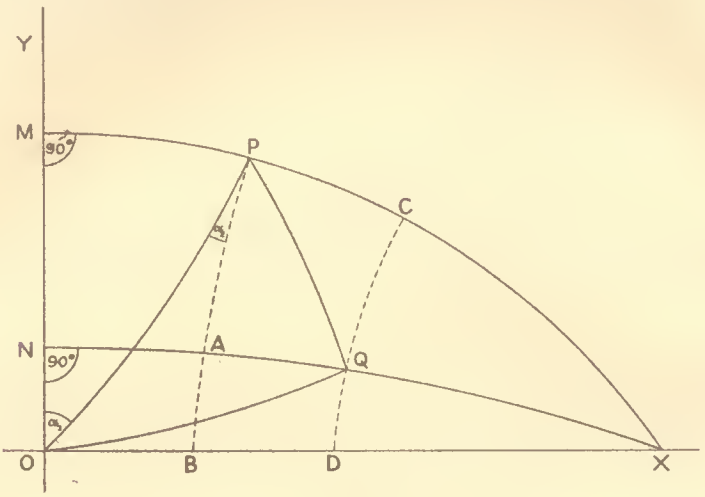


FIG. 48

O, P and Q (Fig. 48) are three trigonometrical stations of a survey, O is selected as the point of origin ; OX and OY are therefore the axes of the co-ordinates. Through P and Q draw the great circles MPCX and NAQX at right angles to OY ; these will meet OX at the point X, a pole on the equator 90° from O. Draw the small circles PAB and CQD through P and Q respectively and parallel to OY.

Then, the co-ordinates of P are :

$$x = \text{MP and } y = \text{OM}$$

And the co-ordinates of Q are :

$$x = \text{NQ and } y = \text{ON}$$

It will also be seen that MP=NA=OB and MC=NQ=OD. Now, all points on MPCX have the same y co-ordinate, *i.e.*, the length OM, and all those on NAQX also have the same y co-ordinate, *i.e.*, the length ON. When the distance of stations from the y-axis is great, their y co-ordinates exaggerate their true distance North or South of the x-axis, and the distance between two such stations, obtained directly from the differences of their co-ordinates, is not a true distance : it may be called a " false " or " co-ordinate " distance.

The adjustment and calculation of a triangulation give plane angles and the true lengths of sides. When rectangular spheroidal co-ordinates have to be computed, or when the sides and angles have to be found from these co-ordinates, it is necessary :

(i) To take into account the curvature of the earth.

(ii) To modify the equations used for " plane " co-ordinates by including second order terms.

The equations necessary for finding spheroidal co-ordinates from bearing and distance, and *vice versa*, are as follows :

(1) $a_m = \frac{1}{2}(a_1+a_2)$

(2) $\text{Log tan } a_m = \log\frac{x^1-x}{y^1-y}+p-q-r$

(3) $\log(a_1-a_2)'' = \bar{1}0{\cdot}3714+\log(x^1+x)+\log(y^1-y)$

(4) $\log(x^1-x) = \log(s \cdot \sin a_m)+r$

(5) $\log (y^1-y) = \log (s \cdot \cos \alpha_m)+p-q$
(6) $\log s = \log (x^1-x)+\log \operatorname{cosec} \alpha_m - r$
$= \log (y^1-y)+\log \sec \alpha_m - p+q$

in which, α_1 is the " bearing " of the second station from the first.
,, ,, α_2 is the reverse " bearing."
,, ,, xy, x^1y^1 are the co-ordinates of the two stations expressed in feet.
,, ,, s is the true distance between the two stations.
,, ,, $(\overline{10}\cdot3714)$ is a constant log : it is not quite correct for all latitudes, but may be used without appreciable error.
,, ,, p, q and r are quantities which can be taken from Table VI and applied as corrections to the logarithmic terms.

The " bearing " α_1 is not a true bearing except when the initial station is the point of origin. *e.g.*, In Fig. 48, if the co-ordinates of P are being calculated from the side OP, α_1 = POM, and is in *this* case a true bearing ; and α_2 = OPB.

If the co-ordinates of Q are being calculated from the side PQ, α_1 = QPB ; and α_2 = CQP.

(Should any doubt arise, a glance at formula (3), above, will show that the sign of the expression $(\alpha_1 - \alpha_2)$ is determined by the signs of $(x_1 + x)$ and $(y_1 - y)$; when both these expressions have similar signs $(\alpha_1 - \alpha_2)$ is positive, *i.e.*, $\alpha_1 > \alpha_2 + 180°$; when they have different signs $(\alpha_1 - \alpha_2)$ is negative, *i.e.*, $\alpha_1 < \alpha_2 + 180°$.)

To calculate rectangular spheroidal co-ordinates :

In Fig. 48, the co-ordinates of P and Q are required and the elements of the triangle OPQ found from the triangulation are as follows :

		Plane Angles.	Spherical angles.	
Angle	O	39° 10′ 18″·1	19″·0	Area of triangle 170 sq. nautical miles.
,,	P	57° 56′ 22″·5	23″·5	
,,	Q	82° 53′ 19″·4	20″·4	
			Spherical excess=2″·9.	
log	OP	5·208 4217		
,,	OQ	5·139 9094		
,,	PQ	5·012 2495		

The true bearing, or azimuth, OP=035° 02′ 10″·8.

The approximate co-ordinates of P and Q, which must first be obtained from a rough plot or calculation, are :—

P : x=+ 92770 ft. : y=+132310 ft.
Q : x=+132800 ,, : y=+ 37560 ,,

When the co-ordinates of P have been found, those of Q may be calculated from the side PQ as a check on those obtained from the side OQ. The computation is shown below. It should be noted that in finding the bearing α, for use in columns 3 and 4, the spherical angles of the triangle are used. The reverse process of finding the sides and angles from the co-ordinates is rather shorter as p, q and r may be looked out at once without a preliminary calculation to find the approximate co-ordinates.

Form for Computation of Rectangular Spheroidal Co-ordinates.

1. Stations.	2. O — P	3. O — Q	4. P — Q
x^1	+ 92770	+ 132800	+ 132800
x	0	0	+ 92770
x^1-x	+ 92770	+ 132800	+ 40030
x^1+x	+ 92770	+ 132800	+ 225570
y^1	+ 132310	+ 37560	+ 37560
y	0	0	+ 132310
y^1-y	+ 132310	+ 37560	— 94750

1. Stations.	2. O — P	3. O — Q	4. P — Q
p (for (x^1+x))	11	22	63
q (for $)x^1-x)$)	4	7	1
r (for (y^1-y))	14	2	7
Const. log	$\overline{10}\cdot3714$	$\overline{10}\cdot3714$	$\overline{10}\cdot3714$
log (x^1+x)	+ 4·9674	+ 5·1232	+ 5·3533
log (y^1-y)	+ 5·1216	+ 4·5747	− 4·9766
log (a_1-a_2)	+ 0·4604	+ 0·0693	− 0·7013
a_1-a_2	+ 2″·89	+ 1″·17	− 5″·03
$\frac{1}{2}(a_1-a_2)$	+ 1″·44	+ 0″·58	− 2″·51
a_1	035° 02′ 10″·8	074° 12′ 29″·8	157° 05′ 44″·41
a_2	215° 02′ 07″·91	254° 12′ 28″·63	337° 05′ 49″·44
a_m	035° 02′ 09″·35	074° 12′ 29″·21	157° 05′ 46″·92
log s...	5·208 4217	5·139 9094	5·012 2495
L. sin a_m	9·758 9799	9·983 2908	9·590 1532
+ r	14	2	7
log (x^1-x)	4·967 4030	5·123 2004	4·602 4034
log s	5·208 4217	5·139 9094	5·012 2495
L. cos a_m	9·913 1737	9·434 7987	9·964 3354
$+p-q$	7	15	62
log (y^1-y)	5·121 5961	4·574 7096	4·976 5911
x^1-x	+ 92769·0	+ 132800·7	+ 40031·7
x	0	0	+ 92769·0
x^1	+ 92769·0	+ 132800·7	+ 132800·7
y^1-y	+ 132311·0	+ 37558·6	− 94752·6
y	0	0	+ 132311·0
y^1	+ 132311·0	+ 37558·6	+ 37558·4

When the stations, whose co-ordinates are calculated in this manner, are plotted, great circles at right angles to the y-axis are treated as straight lines parallel to the x-axis. There is, therefore, an error in projection in a north and south direction increasing as the square of the distance at which the plotted point lies from the y-axis. At a distance of 30 sea miles the error amounts to about 1/26,550 ; at 60 sea miles it is 1/6,637. The existence of such errors in plotting does not invalidate the use of this method since *every* method of plotting terrestrial features on a plane is necessarily burdened with *some* systematic errors. Rectangular spheroidal co-ordinates represent actual distances on the earth's surface ; for the purposes of plotting they are regarded as " plane " co-ordinates and the resulting projection is known as the Cassini-Soldner. In other projections, such as the Transverse Mercator, the " rectangular " co-ordinates, used for plotting, are calculated by formulæ which differ slightly from those given here, and they do not represent actual distances on the earth.

The formulæ applicable to the calculation of *plane* rectangular co-ordinates may be used when the distance east or west of the Point of Origin does not exceed 100,000 ft. If the distance exceeds this, the formulæ for rectangular *spheroidal* co-ordinates should be used.

Computation of Rectangular Spheroidal Co-ordinates from Bearing and Distance by machine.

The formulæ in this case are :

$$x = s \sin \alpha - \frac{s^3}{6r^2} \sin \alpha \cos^2 \alpha + \ldots\ldots\ldots\ldots$$

$$y = s \cos \alpha + \frac{s^3}{3r^2} \sin^2 \alpha \cos \alpha + \ldots\ldots\ldots\ldots$$

Here r is the mean radius of the earth at the mid-latitude of the side s, but in all normal circumstances the second term in each case is so small that a mean value (at latitude 45°) will suffice. This mean value of $\frac{1}{3r^2}$ can be taken as $0{\cdot}76 \times 10^{-15}$, and to obviate any difficulty in using it in this form we divide s by 10^5 (or move the decimal point five places to the *left*) and take the cube of the result. Thus, for s=161,592·7 ft. we have $(1{\cdot}616)^3 = 4{\cdot}22$, so that

$$\frac{s^3}{3r^2} = 4{\cdot}22 \times 0{\cdot}76$$

An example of computation (by machine) follows :

s= 161592·7
α= 35° 02′ 10″·8

s 161592·7		s 161592·7	
sin α + 0·57409578		cos α + 0·81878816	
$\Delta_1 x$ = +92769·69 ft.		$\Delta_1 y$ = +132310·19 ft.	
s^3 4·22	sin α+0·574	s^3 4·22	$\sin^3 \alpha$ 0·329
Const. −0·38	$\cos^3 \alpha$ 0·670	Const. +0·76	cos α +0·819
$\Delta_2 x$ = −0·62 ft.		$\Delta_2 y$ = +0·86 ft.	
x = +92769·07 ft.		y = +132311·05 ft.	

TRANSFORMATION OF RECTANGULAR INTO GEOGRAPHICAL CO-ORDINATES, AND THE REVERSE PROCESS.—It is often necessary to transform the rectangular co-ordinates of trigonometrical points into geographical co-ordinates, and *vice versa*.

In Fig. 49, diagrams (i) to (iv), O is the point of origin of the rectangular co-ordinates and Q another point in the triangulation. P is the Pole, PO and PQ are meridians through O and Q respectively, and QM is a parallel of latitude through Q. QN is the arc of a great circle, drawn through Q to cut PO at right angles. Then, by definition, the rectangular co-ordinates of Q, *xy*, are the lengths QN and ON.

To Transform Rectangular into Geographical Co-ordinates.

$\phi_A \omega_A$ and $\phi_B \omega_B$ are the latitude and longitude of O and Q respectively, and ϕ_N the latitude of N. If $\phi_A \omega_A$ are known, and $\phi_B \omega_B$ are to be calculated ; it is required to find :

$\Delta\phi$ = OM, the difference of latitude between O and Q.
$\Delta\omega$ = OPQ, the difference of longitude between O and Q.
Let MN = η : then $\Delta\phi$=OM=ON$\pm\eta$.
,, ρ = radius of curvature in the meridian.
,, ν = radius of curvature at right angles to the meridian.

The fundamental formulæ are :

$$ON = \frac{y}{\rho \sin 1''}$$

$$\eta = \phi_N - \phi_B = \frac{x^2}{2 \rho \nu \sin 1''} \tan \phi_N ;$$ the values of ρ and ν being taken out with ϕ_N as argument.

$\Delta\omega = \pm\omega_A \pm\omega_B = \frac{x}{\nu \sin 1''} \sec(\phi_N - 2/3\,\eta)$; the value of ν being taken out with ϕ_N as argument.

Second terms are omitted from these formulæ, but they are sufficiently accurate for all normal requirements.

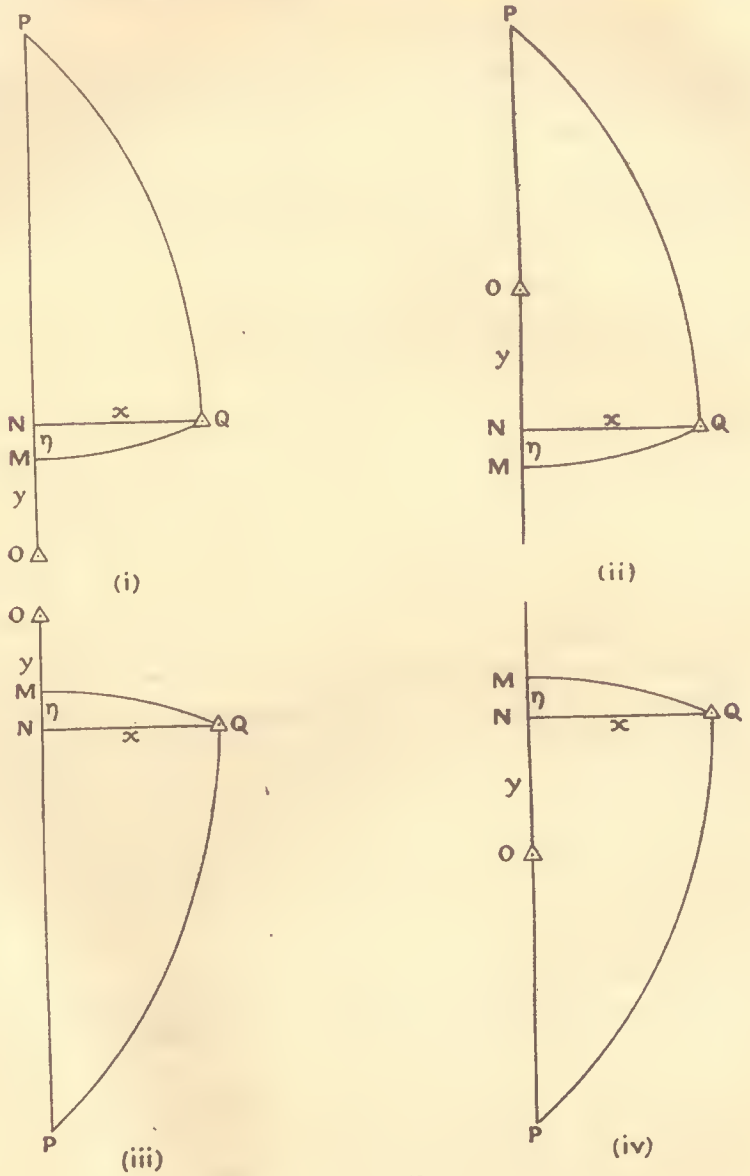


FIG. 49

Before an accurate value of $\Delta\phi$ can be determined, an approximate value, $\Delta\phi_a$ must be found from the formula :

$\Delta\phi_a = \frac{y}{\rho \sin 1''} = y\,P$, where $P = \frac{1}{\rho \sin 1''}$, and log P is tabulated in Tables V, Va and Vb : ϕ_A may be used as argument.

$\Delta\phi_a$, applied to ϕ_A, gives ϕ_N, and an approximate value for the latitude of Q.

Then $\Delta\phi = \text{ON} \pm \eta$

$= y\,\text{P} \pm x^2 \dfrac{\tan \phi_N}{2\,\rho\,\nu \sin 1''}$

the values of $\log \dfrac{\tan \phi}{2\,\rho\,\nu \sin 1''}$ are tabulated in Tables V, Va and Vb for every 10′ of latitude. In this equation, P is taken out with $(\phi_A + \frac{1}{2}\Delta\phi_a)$ as argument, and $\log \dfrac{\tan \phi}{2\,\rho\,\nu \sin 1''}$ is taken out with ϕ_N as argument.

$\Delta\phi$ applied to ϕ_A gives ϕ_B.

Then $\Delta\omega = x \sec (\phi_B + 1/3\,\eta)\,.\,\text{O}$; where $\text{O} = \dfrac{1}{\nu \sin 1''}$ and log O is tabulated in Tables V, Va, Vb, and is found with ϕ_B as argument.

$\Delta\omega$, applied to ω_A, gives ω_B.

Sign Conventions.—The normal sign conventions for the rectangular and geographical co-ordinates are used, *i.e.*, x and y are positive to the east and north of the axes, negative to the west and south; latitude is positive north of the Equator, longitude is positive east of the meridian of Greenwich, and *vice versa.*

It will be seen from Fig. 49, diagrams (i) to (iv), that the sign of η is different from that of y, if Q is nearer the pole than O, but the same as that of y, if Q is further from the pole. This gives the following convention for the sign of η in the formula for $\Delta\phi$:

η is $\dfrac{-}{+}$ according as Q is $\dfrac{\text{N}}{\text{S}}$ of the equator.

It will also be seen from the figures that, in the expression $(\phi_B + 1/3\,\eta)$, the arithmetical, not the algebraical, sum is required.

The full expression for η is:

$$\eta = \frac{x^2 \tan \phi_N}{2\,\rho\,\nu \sin 1''} - \frac{x^4 \tan \phi_N\,(1 + 3\tan^2 \phi_N)}{24\,\rho^2\,\nu^2 \sin 1''}$$

The second term becomes appreciable, *i.e.*, 0″·0005 in precise working, when log x (feet) has the following values:—

Lat. 30°, log x=5·44; lat. 40°, log x=5·39; lat. 50°, log x=5·31; lat. 60°, log x=5·20.

The values of log P, log O and $\log \dfrac{\tan \phi}{2\,\rho\,\nu \sin 1''}$ will depend on the "Figure of the Earth" which is adopted in the survey. The tables of these expressions appropriate to the adopted "figure" must be used. The formulæ here given are applicable to *plane* rectangular co-ordinates, confined to short distances, and to rectangular spheroidal co-ordinates calculated for the Cassini projection: they cannot be used for rectangular co-ordinates which have been calculated for the Transverse Mercator or other orthomorphic projections.

An example of transforming rectangular spheroidal co-ordinates into geographical co-ordinates is given below. Constants for Everest's figure (Table Va) are used.

Example of Transformation of Rectangular into Geographical Co-ordinates.

Data	Origin, A: Lat. (ϕ_A)	+	5° 57′ 54″·530	
	Long. (ω_A)	+	100° 38′ 15″·340	
	Point required, B: x	−	321934·2 feet	
	y	+	91298·8 ,,	
log y			4·960 4651	
log P (for ϕ_A)			$\overline{3}$·996 65	
log $\Delta\phi_a$ (secs.)			2·957 11	
$\Delta\phi_a$	+			906″
	= +		15′	06″
$\frac{1}{2}\Delta\phi_a$	+		7′	33″

ϕ_A ... ... +	5°	57′ 55″
$\Delta\phi_a$ ... ... +		15′ 06″
ϕ_N ... ... +	6°	13′ 01″
ϕ_A ... ... +	5°	57′ 55″
$\frac{1}{2}\Delta\phi_a$ ... ... +		7′ 33°
$\phi_A+\frac{1}{2}\Delta\phi_a$ (ϕ_{am}) ... ... +	6°	05′ 28″
log y		4·960 4651
log P (for ϕ_{am})		$\bar{3}$·996 6463
log y . P		2·957 1114
y . P ... ... +		905″·965
= +		15′ 05″·965
log x		5·507 7671
2 log x		11·015 53
log $\frac{\tan\phi}{2\rho\,.\,\nu\,.\,\sin 1''}$ (for ϕ_N)		$\bar{11}$·412 11
log η		0·427 64
η ... ... —		02″·677
y . P ... ... +		15′ 05″·965
$\Delta\phi$ ... ... +		15′ 03″·288
ϕ_A ... ... +	5°	57′ 54″·530
ϕ_B ... ... +	6°	12′ 57″·818
$\phi_B+1/3\,\eta$ ... ... +	6°	12′ 58″·71
log x		5·507 7671
L. sec ($\phi_B+1/3\,\eta$)		0·002 5611
log O (for ϕ_B)		$\bar{3}$·993 7857
log $\Delta\omega$ (secs.)		3·504 1139
$\Delta\omega$ ... ... —		3192″·375
= —		53′ 12″·375
ω_A ... ... +	100°	38′ 15″·340
ω_B ... ... +	99°	45′ 02″·965

To Transform Geographical into Rectangular Co-ordinates.

The fundamental formulæ are :

$x=\Delta\omega\ \nu \sin 1'' \cos(\phi_B+1/3\,\eta)$, where ν is taken out with ϕ_B as argument.

$y=(\phi_N-\phi_A)\,\rho \sin 1''$, where ρ is taken out with $\frac{\phi_N+\phi_A}{2}$ as argument.

Before an accurate value of x can be determined, an approximate value, x_a, must be found from the formula

$x_a = \Delta\omega\ \nu \sin 1'' \cos\phi_B$

$= \frac{\Delta\omega \cos\phi_B}{O}$, where O is found from Tables V, Va or Vb, with ϕ_B as argument.

Then, $\eta = \frac{(x_a)^2}{2\rho\,\nu \sin 1''}\tan\phi_B$, where $\frac{\tan\phi}{2\rho\,\nu \sin 1''}$ is found from Tables V, Va or Vb, with ϕ_B as argument.

Then, η, applied to ϕ_B, gives ϕ_N, since $\phi_N=\phi_B+\eta$; and

$x = \frac{\Delta\omega \cos(\phi_B+1/3\,\eta)}{O}$, where O is found with ϕ_B as argument.

$y = \frac{\phi_N-\phi_A}{P}$, where P is found with $\frac{\phi_N+\phi_A}{2}$ as argument.

The sign conventions are the same as those given for the reverse process, except that in transforming geographical into rectangular co-ordinates, ϕ_N will always be arithmetically greater than ϕ_B. This gives the following sign convention for η:

η is $\frac{+}{-}$ according as B is $\frac{N}{S}$ of the equator.

An example of transforming geographical into rectangular co-ordinates is given below. The constants for Airy's "Figure" are used for finding the values of log P, log O and log $\frac{\tan \phi}{2 \rho \nu \sin 1''}$, (Table Vb).

Example of Transformation of Geographical into Rectangular Co-ordinates.

Origin, A Lat. (ϕ_A)	+ 51° 16′ 10″·380
Long. (ω_A)	+ 0° 39′ 55″·564
Point required, B Lat. (ϕ_B)	+ 51° 36′ 05″·195
Long. (ω_B)	+ 0° 53′ 37″·820
$\omega_B - \omega_A$	+ 13′ 42″·256
$\therefore \Delta\omega$ ... ... =	+ 822″·256
log $\Delta\omega$	2·915 0071
L. cos ϕ_B	9·793 18
colog O (for ϕ_B)	2·007 11
log x_B	4·715 30
2 log x_B	9·430 60
log $\frac{\tan \phi_B}{2 \rho \nu \sin 1''}$ (for ϕ_B)	$\bar{1}$·472 43
log η	$\bar{1}$·903 03
η	+ 0″·800
ϕ_B	+ 51° 36′ 05″·195
ϕ_N	+ 51° 36′ 05″·995
$\phi_B + 1/3\ \eta$	+ 51° 36′ 05″·46
log $\Delta\omega$ (secs.)	2·915 0071
L. cos ($\phi_B + 1/3\ \eta$)	9·793 1805
colog O (for ϕ_B)	2·007 1051
log x	4·715 2927
x	+ 51915·0 feet
ϕ_N	+ 51° 36′ 05″·995
ϕ_A	+ 51° 16′ 10″·380
$\phi_N - \phi_A$	+ 19′ 55″·615
$\therefore \Delta\phi_B$ ... ... =	+ 1195″·615
ϕ_N	+ 51° 36′ 05″·995
ϕ_A	+ 51° 16′ 10″·380
$\phi_N + \phi_A$	+ 102° 52′ 16″·375
$\frac{\phi_N + \phi_A}{2}$	+ 51° 26′ 08″·187
log $\Delta\phi_B$	3·077 5913
colog P for $\left(\frac{\phi_N + \phi_A}{2}\right)$	2·005 9691
log y	5·083 5604
y	+ 121216·1 feet

Transformation of Co-ordinates by machine.

For machine working of Rectangular into Geographical co-ordinates, the following formulæ may be used :

$$\text{Tan } \Delta\omega = \tan x \sec \phi_N$$
$$\eta = x \sin \tfrac{1}{2}\Delta\omega \sin \phi_N$$

The first expression may be regarded as absolute ; while the error of the second will not exceed 0·01 feet in latitude 53° with x co-ordinates up to 30 miles ; this error decreases somewhat in lower latitudes. A worked example follows :

ϕ_A	+ 5° 57′ 54″·530
ω_A	+100° 38′ 15″·340
y	91298·8 ft.
ρ sin 1″ (for ϕ_A)	100·8 ft.
$\Delta\phi_a$	906″
$\frac{1}{2}\Delta\phi_a$	7′ 33″
ϕ_{am}	6° 05′ 28″
y	91298·8 ft.
ρ sin 1″ (for ϕ_{am})	100·7752 ft.
y P	905″·965
	= 0° 15′ 05″·965
ϕ_A	5° 57′ 54″·530
ϕ_N	6° 13′ 00″·495

x	−321 934·2 ft.
y	+91 298·8 ft.
x	−321934·2 ft.
ν sin 1″ (for ϕ_N)	101·4412 ft.
x (secs)	−3173″·604
	= −0° 52′ 53″·604
tan x	−0·01538728
sec ϕ_N	+1·00591554
tan Δ ω	−0·01547831
Δω	−0° 53′ 12″·375
½ Δω	−0° 26′ 36″·2
Δω	−0° 53′ 12″·375
ω_A	+100° 38′ 15″·340
ω_B	+99° 45′ 02″·965

x	−321934·2 ft.
sin ½ Δω	0·007738
sin ϕ_N	+0·10829
η	269·8 ft.
y	91298·8 ft.
$\Delta\phi$ ($y-\eta$)	+91029·0 ft.
ρ sin 1″ (for ϕ_{am})	100·7752
$\Delta\phi$	903″·288
	= +0° 15′ 03″·288
ϕ_A	+5° 57′ 54″·530
ϕ_B	+6° 12′ 57″·818

For machine working of the reverse process, Geographical into Rectangular co-ordinates, the appropriate formulæ are :

$$\text{Sin } x = \sin \Delta\omega \cos \phi_B$$
$$\eta = x \tan \tfrac{1}{2}\Delta\omega \sin \phi_B$$

The accuracy of these equations corresponds to that of their counterparts above. A worked example follows :

ϕ_A	+51° 16′ 10″·380
ω_A	+ 0° 39′ 55″·564
Δω	+0° 13′ 42″·256
sin Δω	+0·00398640
cos ϕ_B	+0·62112804
sin x	+0·00247606
x (secs)	+510″·725
ν sin 1″ (for ϕ_B)	101·6494 ft.
x	+51 914·9 ft.

ϕ_B	+51° 36′ 05″·195
ω_B	+ 0° 53′ 37″·820
½ Δω	+0° 06′ 51″·128
x	51914·9 ft.
tan ½ Δω	+0·001993
sin ϕ_B	+0·783709
η	81·1 ft.

$\Delta\phi$	0° 19′ 54″·815
½ $\Delta\phi$	0° 09′ 57″·4
ϕ_m	51° 26′ 08″
$\Delta\phi$	1194″·815
ρ sin 1″ (for ϕ_m)	101·3838 ft.
$\Delta\phi$	+121 134·9 ft.
$+\eta$	81·1 ft.
y	+121 216·0 ft.

Transferring of Rectangular Co-ordinates from One Point of Origin to Another.—It is often necessary to transfer rectangular co-ordinates from one Point of Origin to another. If, for instance, a survey is being made in the Thames Estuary, use may be made of Ordnance Survey points on the Kent and Essex coasts ; the co-ordinates of the former may be based on a Point of Origin in Kent, those of the latter on a Point of Origin in Essex. For the purposes of the survey it is convenient to transfer them all to a common point of origin. Again, the Point of Origin may be so distant from the scene of the survey that the rectangular co-ordinates of the stations are expressed by inconveniently large figures, in which case a more central point of origin

may be selected and the co-ordinates can be transferred to this new point of reference.

It is clear that the transference cannot be effected by direct addition and subtraction since, owing to the convergency of the meridians through the two points of origin, the axes on which the co-ordinates are based are not parallel. The transference must be made by :

(i) Calculating the *geographical* co-ordinates of the required points from their rectangular co-ordinates ;

(ii) Calculating their new rectangular co-ordinates on the new point of origin from these geographical co-ordinates.

The methods of doing (i) and (ii) have already been explained.

Intersection

The trigonometrical points of a triangulation are normally fixed by rays observed both to and from them, and the observations are adjusted in the manner already described. It sometimes happens, however, that it is necessary to fix accurately a point which is either inaccessible or is of such character that angles cannot be observed at it. Such points are said to be " intersected " ; preferably their use as a part of the main triangulation system should be avoided, but it occasionally happens that there is no alternative to this.

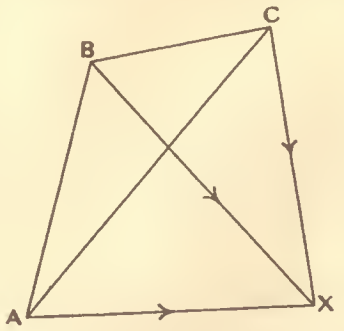


Fig. 50

In Fig. 50, X is an inaccessible point which it is required to fix ; rays to it are observed from A, B and C, three trigonometrical points of which the positions are known. The receiving angles at X, *i.e.*, AXB, CXB and AXC, can be deduced from the observations by closing the three triangles in each of which two angles have been measured. The lengths of sides AB, BC and AC are known from the triangulation, and the position of X (*e.g.*, by rectangular co-ordinates) can, therefore, be determined independently from each of the three triangles which connect it to A, B and C. A mean of the three results may generally be adopted, but if one of the receiving angles is small, say, less than 25°, the final position should be obtained by " weighting " each result in proportion to the values of the sines of the receiving angles.

When more than three rays are available, it is a matter of some difficulty to weight the results properly and it is preferable to solve the problem by the semi-graphic method.

Semi-Graphic Intersection.—The semi-graphic method of intersection is useful when there are a considerable number of rays to the station which is to be fixed. The object of this method is to provide a large scale diagram showing the paths of the observed rays in the immediate vicinity of the object which is being intersected. The picture so formed enables the most probable position of the intersected object to be seen almost at a glance and also makes it obvious which rays, if any, should be rejected owing to errors of observation.

Example :

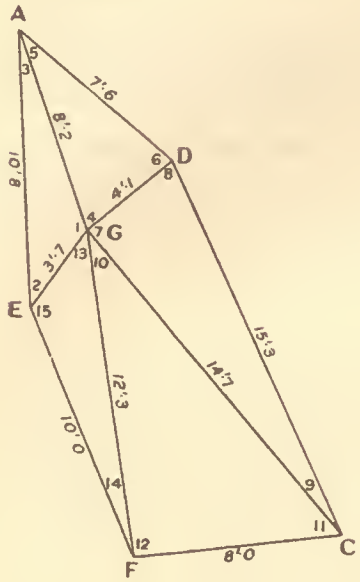


FIG. 51

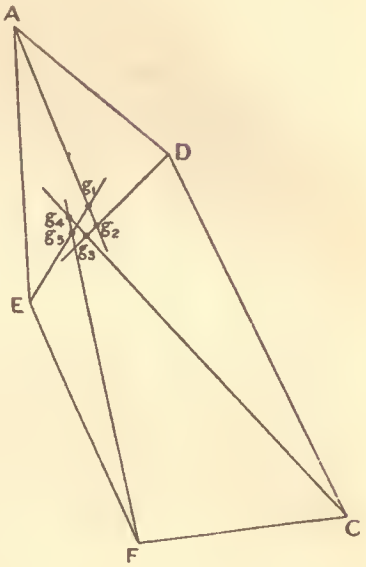


FIG. 52

In Fig. 51, A, D, C, F and E are five points of an adjusted triangulation. The angles and the lengths of the sides of this figure are therefore known, and it is a condition for the applicability of the method that these sides and angles must be mutually consistent. It is required to fix the station G to which angles have been observed from A, D, C, F and E. The rays AG, DG, etc., should all meet at one point, but owing to errors of observation they will not in general do so precisely.

In practice, the state of affairs will be as exemplified and exaggerated in Fig. 52, where the intersection of the rays from A and E is at g_1, that of the rays from A and D at g_2, and so on.

The problem is to find a position for G which will best accord with the various observations, and the method is as follows : The triangle AEg_1 can be solved, since the side AE and the angles at A and E are known : so also the triangles ADg_2, DCg_3, CFg_4 and FEg_5. The side Ag_1 is found from the triangle AEg_1 and the side Ag_2 from the triangles ADg_2 ; the differences of these lengths is g_1g_2. The differences g_2g_3, g_3g_4, g_4g_5 and g_5g_1 can be found in a similar manner. Knowing these differences, a large-scale figure can be drawn representing the rays in the immediate vicinity of G, as will be clear by a numerical example.

The adjusted figure ADCFE has the following dimensions :—

Fixed Spherical Angles.				Fixed Sides.			
EAD	45°	58′	48″·7	Log AE in ft.		4·816	2849
ADC	157	26	22·7	,, AD ,,		4·666	2812
DCF	71	52	03·4	,, DC ,,		4·969	2497
CFE	105	59	59·0	,, CF ,,		4·685	7492
FEA	158	42	48·3	,, FE ,,		4·784	1030
Sum ..	540	00	02·1				

The first step is to make a " station " adjustment at each observing station ; for instance, the observed angles 9 and 11 will not generally sum exactly to the fixed angle DCF, and they must be made to do so by arbitrary adjustments, *e.g.*, equal corrections to each angle. It is not essential that both angles at

each observing station shall have been observed ; if only one has been observed the other is inferred and adjustment is unnecessary. In the example, angles 2, 3 and 6 are not observed.

The observed angles and station adjustments are as follows :

Observed Spherical Angles.				Adjustments.	Working Angles.		
	°	′	″	″	°	′	″
3.	16	43	21·9 (inferred)	—	16	43	21·9
5.	29	15	26·8	—	29	15	26·8
	45	58	48·7				
6.	83	36	47·7 (inferred)	—	83	36	47·7
8.	73	49	35·0	—	73	49	35·0
	157	26	22·7				
9.	15	18	39·0	— 5·3	15	18	33·7
11.	56	33	35·0	— 5·3	56	33	29·7
	71	52	14·0				03·4
			03·4				
Diff.			10·6				
12.	90	37	32·0	+ 2·0	90	37	34·0
14.	15	22	23·0	+ 2·0	15	22	25·0
	105	59	55·0				59·0
			59·0				
Diff.			4·0				
15.	119	26	41·4	—	119	26	41·4
2.	39	16	06·9 (inferred)	—	39	16	06·9
	158	42	48·3				

The spherical excess of each triangle is now found from a rough plot by which the areas of the triangles can be estimated. Spherical excess is practically equal to 1″ of arc for every 60 square nautical miles, *see* Table II. In this example the values (to the nearest tenth of a second) are :

Triangle ..	AEG	ADG	CDG	CGF	FEG
Spherical excess	0″·2	0″·3	0″·5	0″·8	0″·3

The sum of these values must be the same as the spherical excess of the whole figure, *i.e.*, 2″·1. The central angles 1, 4, 7, etc., can be deduced from the exterior " working " angles 3, 5, 6, etc., and the unknown sides are calculated as follows :

Spherical Angles.				Excess.	Plane Angles.	Log Sides.		Sides (feet).	
	°	′	″	″	″				
2.	39	16	06·9	0·1	06·8	L. sin	9·801 3736		
1.	124	00	31·4	0·1	31·3	L. cosec	0·081 4702		
						log AE	4·816 2849		
3.	16	43	21·9		21·9	L. sin	9·459 0015		
						log Ag_1	4·699 1287	Ag_1	50018·3
						log Eg_1	4·356 7566	Eg_1	22738·2
5.	29	15	26·8	0·1	26·7	L. sin	9·689 0727		
4.	67	07	45·8	0·1	45·7	L. cosec	0·035 5590		
						log AD	4·666 2812		
6.	83	36	47·7	0·1	47·6	L. sin	9·997 2962		
						log Dg_2	4·390 9129	Dg_2	24598·7
						log Ag_2	4·699 1364	Ag_2	50019·2

Spherical Angles.				Excess.	Plane Angles.	Log Sides.		Sides (feet).	
	°	′	″	″	″				
8.	73	49	35·0	0·2	34·8	L. sin	9·982 4620		
7.	90	51	51·8	0·2	51·6	L. cosec	0·000 0494		
						log DC	4·969 2497		
9.	15	18	33·7	0·1	33·6	L. sin	9·421 6535		
						log Cg_3	4·951 7611	Cg_3	89487·2
						log Dg_3	4·390 9526	Dg_3	24601·0
11.	56	33	29·7	0·3	29·4	L. sin	9·921 3981		
10.	32	48	57·1	0·2	56·9	L. cosec	0·266 0487		
						log CF	4·685 7492		
12.	90	37	34·0	0·3	33·7	L. sin	9·999 9741		
						log Fg_4	4·873 1960	Fg_4	74678·6
						log Cg_4	4·951 7720	Cg_4	89489·5
14.	15	22	25·0	0·1	24·9	L. sin	9·423 4287		
13.	45	10	53·9	0·1	53·8	L. cosec	0·149 1427		
						log FE	4·784 1030		
15.	119	26	41·4	0·1	41·3	L. sin	9·939 9332		
						log Eg_5	4·356 6744	Eg_5	22733·9
						log Fg_5	4·873 1789	Fg_5	74675·6

The differences between the lengths Ag_1 and Ag_2, Dg_2 and Dg_3, etc., are as follows :

Sides ..	Ag_2-Ag_1	Dg_3-Dg_2	Cg_4-Cg_3	Fg_4-Fg_5	Eg_1-Eg_5
Diffs. (ft.)	0·9	2·3	2·3	3·0	4·3

A large-scale diagram (Fig. 53) can now be drawn on a convenient scale, say, ½ an inch to 1 foot. Draw a line as in Fig. 53, representing the observation

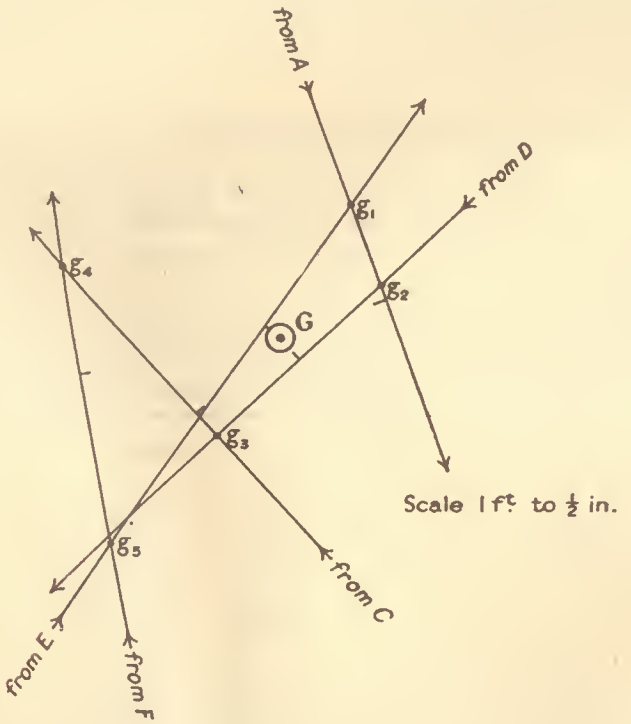


FIG. 53

from A and for convenience having approximately the same bearing as that

observation, and on it mark two points g_1 and g_2 0·45 in. (representing 0·9 ft.) apart. Since Ag_1 is shorter than Ag_2, g_1 will be the nearer point to A. Through g_2 draw a line from D, making the angle Dg_2A equal to 67° 08′ and through g_1 draw a line from E making the angle Eg_1A equal to 124° 01′. Now Dg_2 is shorter than Dg_3 by 2·3 ft., and so g_3 can be marked on the line from D at a point 1·15 ins. from g_2. Similarly Eg_5 is shorter than Eg_1 by 4·3 ft. and so g_5 can be marked on the line from E at a point 2·15 ins. from g_1 in the direction of E. From g_3 and g_5 draw the lines from C and F respectively, making the angle Cg_3D equal to 90° 52′, and Fg_5E equal to 45° 11′. These two lines meet at g_4 and, if the figure has been correctly drawn, the distance g_3g_4 should be 1·15 ins., *i.e.*, equivalent to 2·3 ft., the difference between Cg_3 and Cg_4, and g_5g_4 should be 1·50 ins., *i.e.* equivalent to 3·0 ft., the difference between Fg_5 and Fg_4. The diagram now represents a picture of the observations in the vicinity of G. The final position may be interpolated by eye, having regard to the relative lengths of the observed rays.

Assuming that the position adopted is that shown in Fig. 53, the corrections to the angles and sides, which are required to adjust the observations to conformity with this position, may be found as follows :

(i) *Angles.*—Measure the perpendiculars from G to each of the rays.

In the example the measurements are :

To ray from :	A.	D.	C.	F.	E.
Perp. (ins.)	0·57	0·16	0·59	1·06	0·10
Corresponding to (ft.) ..	1·1	0·3	1·2	2·1	0·2

Then, correction in secs. $= \dfrac{\text{Dist. subtended in ft.} \times 34}{\text{Dist. of object in naut. miles}}$

These may be worked out on a slide rule : the results are :

Correction to ray from A $= \pm 4''{\cdot}9$
,, ,, ,, D $= \pm 2''{\cdot}5$
,, ,, ,, C $= \pm 2''{\cdot}8$
,, ,, ,, F $= \pm 5''{\cdot}8$
,, ,, ,, E $= \pm 1''{\cdot}8$

It is obvious from the figure which way the rays must be swung, and hence which of the exterior angles receive the + and − corrections respectively.

(ii) *Sides.*—Draw perpendiculars from G to each of the rays : for the sake of clearness, only the roots of the perpendiculars are shown in the figure. The corrections to the sides can then be measured off and converted into feet as follows :

Correction to $Ag_1 = +0{\cdot}51$ ins. *i.e.* $+1{\cdot}0$ ft.
,, $Ag_2 = +0{\cdot}06$,, *i.e.* $+0{\cdot}1$,,
,, $Dg_2 = +0{\cdot}56$,, *i.e.* $+1{\cdot}1$,,
,, $Dg_3 = -0{\cdot}59$,, *i.e.* $-1{\cdot}2$,,
,, $Cg_3 = +0{\cdot}15$,, *i.e.* $+0{\cdot}3$,,
,, $Cg_4 = -1{\cdot}02$,, *i.e.* $-2{\cdot}0$,,
,, $Fg_4 = -0{\cdot}58$,, *i.e.* $-1{\cdot}2$,,
,, $Fg_5 = +0{\cdot}90$,, *i.e.* $+1{\cdot}8$,,
,, $Eg_5 = +1{\cdot}40$,, *i.e.* $+2{\cdot}8$,,
,, $Eg_1 = -0{\cdot}77$,, *i.e.* $-1{\cdot}5$,,

The final angles and sides may now be re-tabulated, the adjusted central angles being deduced from the corrected exterior angles. Two results are obtained for the length of each side and these should agree with each other and with the corrections found from the plot, subject to small discrepancies which cannot be avoided unless the computations are taken to hundredths of a second. If the spherical excess of the whole figure does not exceed 2″, it may be neglected and plane angles can be used throughout the working.

Adjusted Angles and Sides

Spherical Angles.				Excess.	Plane Angles.	Log Sides.		Sides.
	°	′	″	″	″			
2.	39	16	08·7	0·1	08·6	L. sin	9·801 3782	
1.	124	00	34·5	0·1	34·4	L. cosec	0·081 4747	
						log AE	4·816 2849	
3.	16	43	17·0		17·0	L. sin	9·458 9671	
						log AG	4·699 1378	50019·3
						log EG	4·356 7267	22736·7
5.	29	15	31·7	0·1	31·6	L. sin	9·689 0911	
4.	67	07	43·4	0·1	43·3	L. cosec	0·035 5611	
						log AD	4·666 2812	
6.	83	36	45·2	0·1	45·1	L. sin	9·997 2956	
						log DG	4·390 9334	24599·9
						log AG	4·699 1379	50019·3
8.	73	49	37·5	0·2	37·3	L. sin	9·982 4636	
7.	90	51	52·1	0·2	51·9	L. cosec	0·000 0494	
						log DC	4·969 2497	
9.	15	18	30·9	0·1	30·8	L. sin	9·421 6321	
						log CG	4·951 7627	89487·6
						log DG	4·390 9312	24599·8
11.	56	33	32·5	0·3	32·2	L. sin	9·921 4020	
10.	32	49	00·1	0·2	59·9	L. cosec	0·266 0389	
						log CF	4·685 7492	
12.	90	37	28·2	0·3	27·9	L. sin	9·999 9742	
						log FG	4·873 1901	74677·6
						log CG	4·951 7623	89487·5
14.	15	22	30·8	0·1	30·7	L. sin	9·423 4732	
13.	45	10	49·9	0·1	49·8	L. cosec	0·149 1511	
						log FE	4·784 1030	
15.	119	26	39·6	0·1	39·5	L. sin	9·939 9353	
						log EG	4·356 7273	22736·7
						log FG	4·873 1894	74677·4

Semi-Graphic Intersection by Co-ordinate Method.—When the co-ordinates of the stations in a triangulation have been calculated, the position of an intersected point can, under certain conditions, be determined in a way which is rather simpler and shorter than that previously described. The method however, must *not* be used with rectangular spheroidal co-ordinates if the distances of the stations east or west of the Origin are great, since the Cassini projection is not orthomorphic. It may be applied within the limits which are applicable to the use of *plane* rectangular co-ordinates, but may be used under all conditions if the co-ordinates are calculated for an orthomorphic projection. A convenient orthomorphic projection to which Cassini co-ordinates may be converted is the Transverse Mercator, whose Northings are the same as Cassini Northings. Table IV gives the formulæ for the conversion of Cassini to Transverse Mercator Eastings and *vice versa*, and also the values of Log $1/6\ \nu^2$ and $1/24\ \nu^4$. In cases where the limits of use of plane co-ordinates are to be exceeded, Cassini co-ordinates should first be converted to Transverse Mercator ; and the co-ordinates arrived at by semi-graphic methods on this projection can then be re-converted to Cassini for record purposes. The purpose of the method is to provide a large-scale diagram of the observed rays in the immediate vicinity of the unfixed point ; thereby its position may be accurately interpolated on the picture.

The approximate co-ordinates of the point must first be found either from one of the triangles connecting it or from a large-scale plot if such is available. A rectangular grid on some convenient scale, say, $\frac{1}{2}$ in. to 1 ft., is plotted on squared paper, the vertical and horizontal sides of the grid being regarded as parallel to the y and x axes of the system respectively. Co-ordinates are assigned to the sides which will ensure that the unfixed point is located within

the grid. For instance, if the approximate co-ordinates of a station were

$x = -4550$ ft.
$y = +2100$,,

a suitable grid would be one enclosing an area 10 ft. by 10 ft. (5 ins. × 5 ins. on the paper) with the vertical sides having co-ordinates of $x = -4545$ and $x = -4555$ respectively, and the horizontal sides $y = +2105$ and $y = +2095$ respectively†. The problem is then to find the points at which the observed rays cut the sides of the grid so that they may be drawn on the diagram. It is sufficient to calculate the position of the point on the *nearer* side of the grid at which each ray cuts it ; the path of the ray across the grid can then be laid down by protraction of the bearing. Example :

A, B, C are three main stations in a triangulation, and X an intersected point of which the co-ordinates are required (Fig. 54).

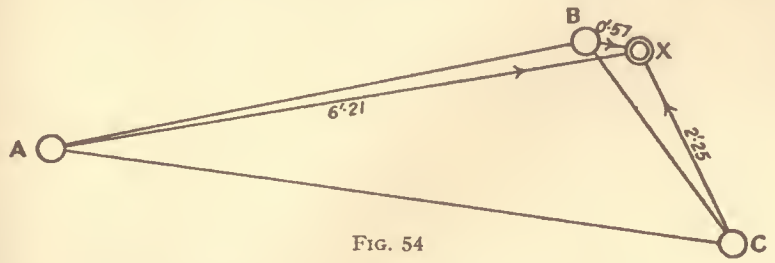


FIG. 54

The rectangular co-ordinates of A, B and C are :

A : $x = -36132{\cdot}4$ ft., $y = +3661{\cdot}0$ ft.
B : $x = -1962{\cdot}9$,, $y = +8855{\cdot}9$,,
C : $x = +6610{\cdot}7$,, $y = -4594{\cdot}5$,,

The approximate lengths in miles of the sides AX, BX and CX are shown in the figure. "Co-ordinate" bearings connecting the main stations are as follows :

A − B = 081° 21′ 19″
B − C = 147° 29′ 08″
C − B = 327° 29′ 08″

The approximate co-ordinates of X are $x = +1427$ ft., $y = +8{,}156$ ft. Co-ordinates are, therefore, selected for the grid sides as follows :

E. side, $x = +1433$: N. side, $y = +8162$.
W. ,, $x = +1423$: S. ,, $y = +8152$.

The further computation is as follows :

Observing Station.	A		B		C	
Bearing	AB	081 21 19	BC	147 29 08	CB	327 29 08
Observed angle	BAX	1 48 51	CBX	45 49 48	BCX	10 23 29
Sum or Diff. = Bearing	AX	083 10 10	BX	101 39 20	CX	337 52 37
Observing Station ; x or y*	x	−36132·4	x	−1962·9	y	− 4594·5
$\frac{E}{W}$ or $\frac{N}{S}$ side ; x or y ...	x_w	+ 1423·0	x_w	+1423·0	y_s	+ 8152·0
Δx or Δy	Δx	37555·4	Δx	3385·9	Δy	12746·5
log Δx or Δy		4·574 6724		3·529 6741		4·105 3910
Bg. ; L. cot or tan*	cot	9·078 3977	cot	9·314 4597(−)	tan	9·609 0891(−)
Sum		3·653 0701		2·844 1338		3·714 4801
Antilog		+ 4498·5		− 698·4		− 5181·8
Observing Station ; y or x	y	+ 3661·0	y	+ 8855·9	x	+ 6610·7
Intersection on grid, x or y	y	+ 8159·5	y	+ 8157·5	x	+ 1428·9

* Use x and L. cot when bearing is between 045° and 135°, or 225° and 315°
Use y and L. tan ,, ,, 135° ,, 225°, ,, 315° ,, 045°

†Note.—If preferred simple x and y axes drawn through the approximate position can be used instead of a containing rectangle.

This computation provides sufficient data for drawing the large scale figure, since the ray from A cuts the nearer (W.) side of the grid at the point $y = +8159{\cdot}5$ on a bearing of 083° 10′, that from B cuts the W. side at $y = +8157{\cdot}5$ on a bearing of 101° 39′ and that from C cuts the S. side at $x = +1428{\cdot}9$ on a bearing of 337° 53′ as shown on Fig. 55.

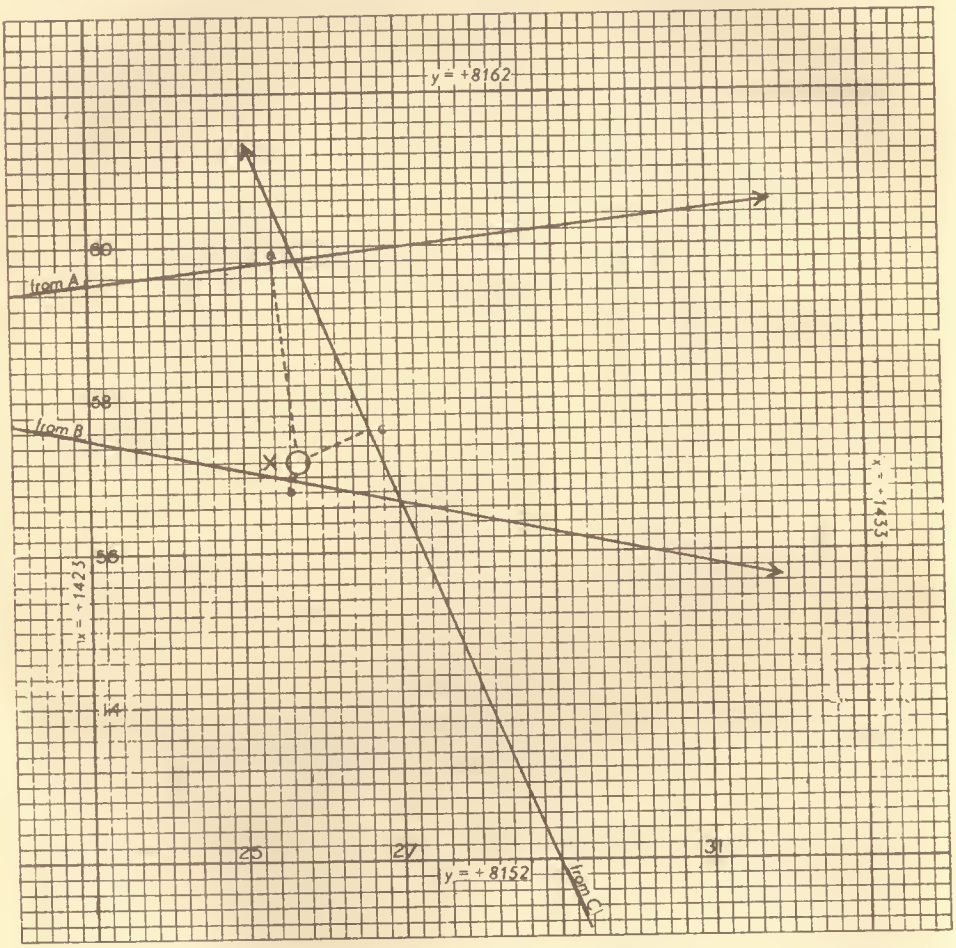


FIG. 55

The position of X is determined by the rule that the lengths of the perpendiculars from X to each of the rays should be proportional to the lengths of these rays, *i.e.*, $Xa : Xb : Xc = 6{\cdot}21 : 0{\cdot}57 : 2{\cdot}25$. The position of the point can usually be interpolated with sufficient accuracy by eye; alternatively a mathematical solution can be found without much difficulty.

From the diagram, the co-ordinates of X are:

$$x = +1425{\cdot}7 \text{ ft.}$$
$$y = +8157{\cdot}2 \text{ ,,}$$

When there are more than three rays to the intersected station, it will not generally be possible to find a position from which the perpendiculars are all proportional to the lengths of the rays; the position selected should be that which most nearly fulfils this relation.

Resection

"Resection" describes the fixing of positions *from* which angles have been observed to points of known position in the survey. This procedure may be necessary when the known points are inaccessible or when a new point has to be accurately established after the completion of the main triangulation. Examples :

1. *Fixing a point by angles taken from it to two known points, with the help of an azimuth.*

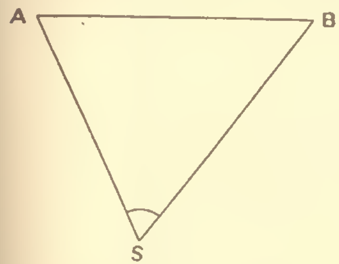


Fig. 56

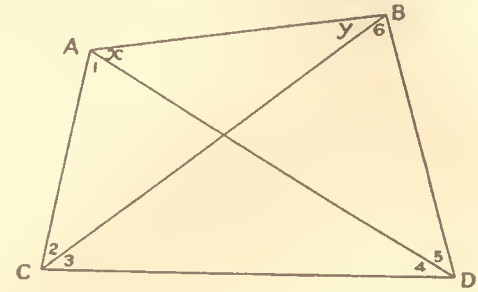


Fig. 57

A and B are two points, of which the latitude, longitude, azimuth (or true bearing) AB and reverse azimuth BA are known. S is the observer's position which it is required to fix.

The angle ASB is observed, and the azimuth SA or SB. Approximate values for the latitude and longitude of S must first be found ; this can be done with sufficient accuracy from a plotting sheet.

The reverse azimuths AS and BS can be computed, for Azimuth AS $= 180°$ $+$ Azimuth SA $+ \Delta\alpha$, and $\Delta\alpha = \Delta\omega \sin \phi_m \sec \frac{1}{2} \Delta\phi$.

Where

$\Delta\alpha$ = convergency in seconds between the meridians at A and S.

$\Delta\omega$ = difference of longitude between A and S.

ϕ_m = mean latitude of A and S.

$\Delta\phi$ = difference of latitude between A and S.

Assuming that azimuths are measured from 0° to 360° clockwise from north, the sign of $\Delta\alpha$ can be found from the rule :

In *N*. Latitude, $\Delta\alpha$ is $\dfrac{\text{positive}}{\text{negative}}$ when α is between $\dfrac{0° \text{ and } 180°}{180° \text{ and } 360°}$.

In *S*. Latitude, $\Delta\alpha$ is $\dfrac{\text{negative}}{\text{positive}}$ when α is between $\dfrac{0° \text{ and } 180°}{180° \text{ and } 360°}$.

The triangle ABS can now be solved since the side AB is known and the angles SAB and SBA can be deduced from the azimuths AB and AS, BA and BS.

2. *Fixing the position of two points by angles taken from them to two previously fixed points.*

In Fig. 57, A and B are two known points in a triangulation. It is required to fix C and D, from which angles have been observed to A and B. The angles 2, 3, 4 and 5 are measured, 1 and 6 can therefore be deduced. x and y are the unknown angles.

This problem is the same as that of the "Inaccessible Base," a numerical example of which is given in Chapter II, pp. 23–4.

The necessary formulæ are :

$$x + y = 3 + 4 \quad \ldots\ldots(1)$$

$$\tan \frac{y - x}{2} = \tan \frac{(3 + 4)}{2} \cdot \frac{1 - P}{1 + P} \quad \ldots\ldots(2)$$

$$\text{Where } P = \frac{\sin 1 \sin 3 \sin 5.}{\sin 2 \sin 4 \sin 6.}$$

From (1) and (2) x and y are found, and the triangles can then be solved.

3. *Fixing the position of a point by angles taken from it to three previously fixed points.*

In this, there are three cases, as shown in Fig. 58.

A, B and C are three fixed points in the triangulation, and S is the observer's position which it is required to fix. The angles α and β are observed. The problem is the same as that which is solved mechanically, but less accurately, by the station pointer, and, as with that instrument, the problem is insoluble

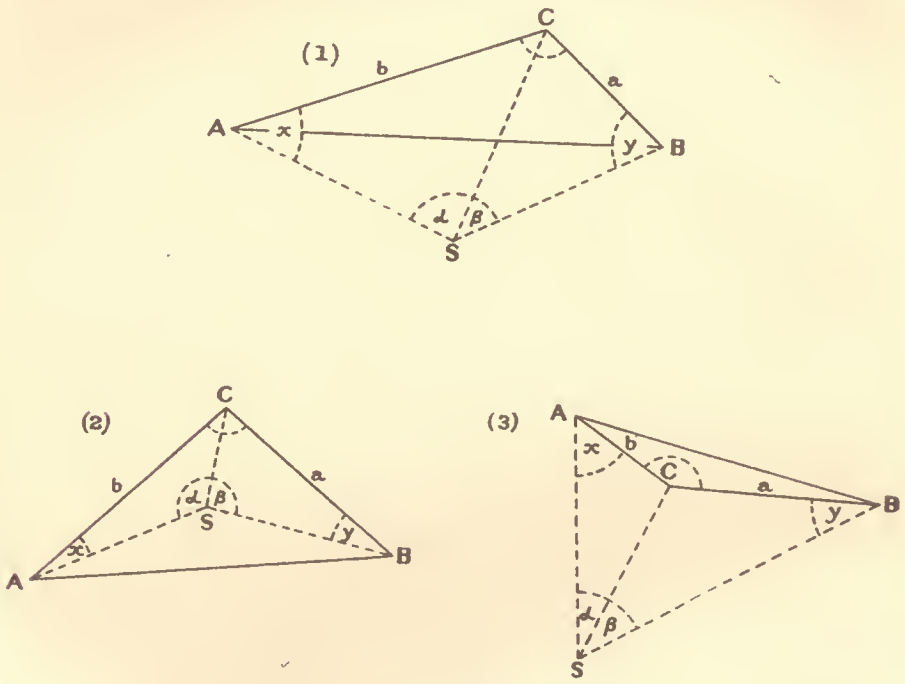


FIG. 58

if S lies on the circumference of the circle through A, B and C. This will occur when angles ACB and ASB sum to 180°.

If the angles CAS and CBS be denoted by x and y, and the angle ACB by C; then, in cases (1) and (2),

$$x + y = 360° - (\alpha + \beta + C) \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

and, in case (3)

$$x + y = C - (\alpha + \beta) \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

From the figure:

$$\sin x = \frac{CS \sin \alpha}{b}$$

$$\text{and } \sin y = \frac{CS \sin \beta}{a}$$

$$\text{Therefore, } \frac{\sin x}{\sin y} = \frac{a \sin \alpha}{b \sin \beta}$$

$$\text{Let } \tan \phi = \frac{\sin x}{\sin y}$$

$$\text{Now, } \tan(\phi - 45°) = \frac{\tan\phi - \tan 45°}{1 + \tan\phi \tan 45°}$$

$$= \frac{\tan\phi - 1}{\tan\phi + 1}$$

$$= \frac{\frac{\sin x}{\sin y} - 1}{\frac{\sin x}{\sin y} + 1}$$

$$= \frac{\sin x - \sin y}{\sin x + \sin y}$$

$$= \frac{2\cos\frac{x+y}{2}\sin\frac{x-y}{2}}{2\sin\frac{x+y}{2}\cos\frac{x-y}{2}}$$

$$= \frac{\tan\frac{x-y}{2}}{\tan\frac{x+y}{2}}$$

Therefore, $\tan\frac{x-y}{2} = \tan\frac{x+y}{2}\tan(\phi - 45°)$ .....(2)

From (1) and (2) angles x and y can be found, and the figure solved. If there is uncertainty as to whether x or y is the larger angle, compute the terms $\frac{\sin\alpha}{b}$ and $\frac{\sin\beta}{a}$; if $\frac{\sin\alpha}{b}$ is greater than $\frac{\sin\beta}{a}$, x must be the larger angle and *vice versa*.

Example :

Data :—log a	4·543 4645	} From triangulation.
log b	4·424 3334	
Angle C	124° 47′ 03″	
Obsd. angle α	7° 38′ 00″	
,, ,, β	74° 13′ 15″ .	

Computation :

Log b	4·424 3334	Log a	4·543 4645	
L. sin β	9·983 3181	L. sin α	9·123 3061	
Log (b sin β)	4·407 6515	Log (a sin α)	3·666 7706	
Log (a sin α)	3·666 7706			
L. tan ϕ	10·740 8809			
ϕ	79° 42′ 26″	$\phi - 45°$	34° 42′ 26″	L. tan 9·840 4946
Angle C	124° 47′ 03″			
$\alpha + \beta$	81° 51′ 15″			
$x + y$	42° 55′ 48″	$\frac{x+y}{2}$	21° 27′ 54″	L. tan 9·594 6190
		$\frac{y-x}{2}$	15° 14′ 04″	L. tan 9·435 1136
		$\therefore\ y =$	36° 41′ 58″	
		and $x =$	6° 13′ 50″	

The angles ACS and BCS can now be deduced from the triangles ACS and BCS, and the unknown sides AS, BS and CS found from these triangles by the sine rule.

The computation gives *no* check on the accuracy of the observed angles, and this method of fixing positions is greatly strengthened if an azimuth is observed from S to one of the known points ; alternatively, if angles have been observed to four previously fixed points the necessary check can be obtained by recomputing, using a different combination of points (*e.g.*, first compute resection using points A, B and C, and check by computing resection using points B, C and D).

3(*a*). *Resection by " Collins Point " Method.*—As an alternative to that given in paragraph 3, the " Collins Point " method may be used for calculating the resection ; it is, perhaps, easier to understand and compute when the rectangular co-ordinates of the trigonometrial points have been calculated. Also, if the angle to a fourth point has been measured, it is a simple matter to check the other angles and the computation by comparison with this observation. The method should not be used with rectangular *spheroidal* (Cassini) co-ordinates if the distances east or west of the Point of Origin are great, but should be confined to cases where *plane* rectangular co-ordinates may be safely employed, or to rectangular co-ordinates calculated for orthomorphic projections, such as the Transverse Mercator.

An auxiliary point is introduced and the co-ordinates of this must first be computed. The geometric construction, by which this point is located, must be considered in the two general cases :

(i) When the point to be fixed lies *inside* the triangle formed by the three trigonometrical points to which angles have been observed.

(ii) When the point lies *outside* the triangle so formed.

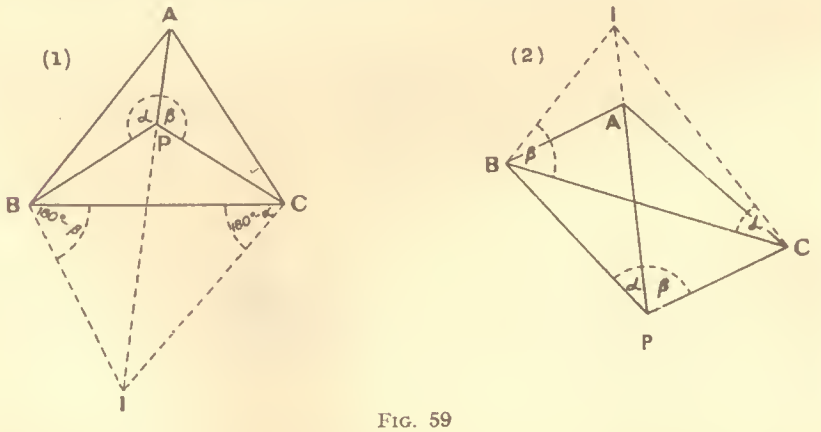


FIG. 59

Case (i). *See* Fig. 59 (1).

P is a point whose position is required and A, B and C three points whose positions are known. The angles APB and APC (α and β), have been observed.

Join AB, AC, BC, AP, BP and CP.

From C draw a straight line CI, making with CB an angle equal to $180° - \alpha$, and meeting AP produced in I. Join BI.

Then, angle BCI = angle BPI = $180° - \alpha$.

Since angles BCI and BPI lie on the same chord BI, the points B, P, C and I must lie on the circumference of a circle.

Therefore, angle CBI = angle CPI = $180° - \beta$ since CBI and CPI lie on the same chord CI.

Case (ii). *See* Fig. 59 (2).

The construction and proof are the same as for case (i), but the angle laid off at C should be equal to α, and not $180° - \alpha$, and the angle at B will be equal to β.

In both cases P and I lie on opposite sides of BC. If P lies on the circumference of a circle passing through A, B and C, the problem is insoluble as the position of I would coincide with A.

Computation.—The computation is the same in either case, and is as follows :

(1) The co-ordinates of B and C being known, the length and bearing of BC can be calculated.

(2) In the triangle BCI, the side BC and the angles B and C are known : this triangle can therefore be solved and the co-ordinates of I found.

(3) The co-ordinates of A and I being known, the bearing I—A can be found, and by construction this is the same as the bearing I—P.

(4) In the triangle CPI, angle I can be found from the difference of the bearings I—P and I—C, angle P = $180° - \beta$ (case (i)) or β (case (ii)), and the side CI has been found in (2). This triangle can therefore be solved and the co-ordinates of P calculated.

(5) A check on the computations can be made by finding from the co-ordinates the bearings P—A and P—B and comparing their difference with the observed angle α.

(6) The working affords no check on the accuracy of the observed angles, but if a fourth point has been observed, a check may be provided by a comparison of the observed angle with that obtained by the differences of bearings derived from the co-ordinates.

A fairly accurate diagram should always be drawn to guard against gross errors.

Example :

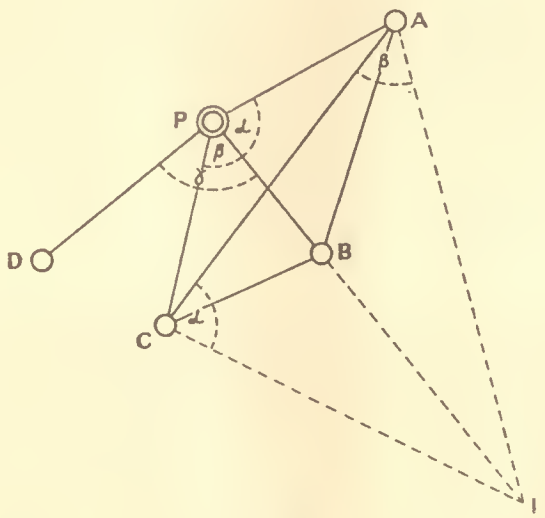


FIG. 60

In Fig. 60, P is a point which it is required to fix, and angles have been observed to the four trigonometrical points, A, B, C and D, of which the rectangular co-ordinates are known. I is the " auxiliary " point.

Resection—Collins Point Method

Data.

Points observed to			Observed Angles.		Deduced Angles.
Point.	x (feet).	y (feet).	α	80° 33′ 17″	ACI 80° 33′ 17″ } by con-
A	+41433·5	+64171·0	β	51° 52′ 08″	CAI 51° 52′ 08″ } struction
B	+26769·2	+15756·6	γ	37° 36′ 42″	AIC 47° 34′ 35″ deduced
C	− 5809·1	+ 49·3			
D	−31615·4	+14358·9			

1. *Bearings A—C, A—I, I—A ; Length AC.*

x_a	+41433·5
x_c	− 5809·1
Δx	47242·6
y_a	+64171·0
y_c	+ 49·3
Δy	64121·7
log Δx ..	4·674 3338
log Δy ..	4·807 0051
L. tan A—C ..	9·867 3287
A—C ..	216° 22′ 53″
∠ CAI (β) ..	51° 52′ 08″
A—I	164° 30′ 45″
I—A	344° 30′ 45″
log Δy ..	4·807 0051
L. cos A—C ..	9·905 8426
log AC ..	4·901 1625

2. *Co-ordinates of* I.

log AC ..	4·901 1625
L. sin ∠ ACI	9·994 0719
L. cosec ∠ AIC	10·131 8393
log AI ..	5·027 0737
L. sin A—I ..	9·426 5570
L. cos A—I ..	9·983 9368
log Δx ..	4·453 6307
log Δy ..	5·011 0105
Δx	+28420·4
x_a	+41433·5
x_i	+69853·9
Δy	−102567·7
y_a	+ 64171·0
y_i	− 38396·7

3. *Bearings* I—P, P—I ; *Angle* PAI.

x_i	+69853·9
x_b	+26769·2
Δx	43084·7
y_i	−38396·7
y_b	+15756·6
Δy	54153·3
log Δx ..	4·634 3231
log Δy ..	4·733 6249
L. tan I—B	9·900 6982
I—B ..	321° 29′ 38″
I—P ..	321° 29′ 38″
P—I ..	141° 29′ 38″
I—P ..	321° 29′ 38″
I—A ..	344° 30′ 45″
∠ AIP ..	23° 01′ 07″
∠ API (α) ..	80° 33′ 17″
Sum	103° 34′ 24″
∴ ∠ PAI ..	76° 25′ 36″

4. *Co-ordinates of P.*

log AI ..	5·027 0737
L. sin ∠ PAI	9·987 6977
L. cosec ∠ API	10·005 9281
log IP ..	5·020 6995
L. sin I—P	9·794 2078
L. cos I—P	9·893 5075
log Δx ..	4·814 9073
log Δy ..	4·914 2070
Δx	−65299·1
x_i	+69853·9
x_p	+ 4554·8
Δy	+82074·3
y_i	−38396·7
y_p	+43677·6

5. *Check on Computations, (From* C.)

x_c	− 5809·1
x_p	+ 4554·8
Δx	10363·9
y_c	+ 49·3
y_p	+43677·6
Δy	43628·3
log Δx ..	4·015 5232
log Δy ..	4·639 7683
L. tan P—C ..	9·375 7549
P—C ..	193° 21′ 47″
P—B ..	141° 29′ 38″
∠β (Compd.)	51° 52′ 09″
∠β (Obsd.) ..	51° 52′ 08″

6. *Check on Data. (From* D.)

x_d	−31615·4
x_p	+ 4554·8
Δx	36170·2
y_d	+14358·9
y_p	+43677·6
Δy	29318·7
log Δx ..	4·558 3509
log Δy ..	4·467 1447
L. tan P—D	10·091 2062
P—D ..	230° 58′ 21″
P—C ..	193° 21′ 47″
∠γ (Compd.)	37° 36′ 34″
∠γ (obsd.)	37° 36′ 42″
Diff. ..	8″

4. *Resection by Semi-graphic Methods.*—When angles to more than three trigonometrical points of known position have been observed from a point which it is required to fix, a semi-graphic method of resection is sometimes found useful. In this method the approximate rectangular co-ordinates of the point must first be found by calculation or from a plot, and a bearing from it to one of the known points must be assumed. By applying the observed angles, assumed, or " false," bearings are obtained to each of the other points. In a similar manner to that used in semi-graphic intersection (*q.v.*, pp. 110–12), a rectangular grid is drawn on a suitable scale and co-ordinates are assigned to its sides in anticipation that they will enclose the true position of the unfixed point. The points, at which the reverse false bearings cut the sides of the grid, are calculated as in the intersection problem, and a large scale picture of the rays in the vicinity of the unfixed point is provided. Two difficulties, which may be found with this method, are :

(1) Unless the approximate position of the point and the assumed bearings are fairly accurate, say, within 5 ft. and 1′ of arc respectively, the picture formed by the rays will be unduly large—this is not evidence of inaccurate observation—and they will not cut the sides of the grid *within* the rectangle. Under these circumstances, it will probably be necessary to carry out a recomputation, using more accurate assumptions for position and bearing.

(2) The selection of the most probable position for the unknown point is not such a simple matter as in the intersection problem, for the " rays " in this case are not fixed, and the position must be interpolated after " swinging " each ray in the same direction, clockwise or anti-clockwise, through equal angles until the figure formed is as small as possible.

Shooting up Marks from the Ship and Boats

The marks in a survey either form part of the triangulation or are fixed by one or more of the normal methods already described ; but it is often necessary to resort to less regular means to fix sounding marks and other minor points. In the case of main and secondary stations, the degree of accuracy in fixing is usually *far* in excess of that actually required to plot them correctly on the scale in use, but in the case of the minor points the only essential criterion is that the observations must be capable of defining their positions without *plottable* error. The configuration of the coastline and the character of the land features often obstruct direct observations from the main and secondary stations to the sounding marks, and in such cases the expedient of shooting them up from the ship or boats can be usefully employed. A short distance off shore it may be possible to obtain good fixes by resection from the main and secondary stations and to observe angles from these resected positions to the sounding marks.

Example :

In Fig. 61, A, B, C and D are four main stations of the triangulation. For fixing the coastline and inshore soundings, the marks a, b and c are erected, but, owing to the shape of the coastline and the high trees along it, they cannot be seen from any of the main stations. A short distance out from the coast, however, the four main stations are visible and a boat therefore anchors successively in the positions marked 1, 2, 3, etc. Each position is fixed by observing sextant angles to A, B, C and D and angles are measured to all visible sounding marks. The positions of 1, 2, 3, etc., are selected having regard to :

(1) The necessity of having a good fix by resection from the main stations.

(2) The necessity of obtaining enough observations to make well-conditioned " cuts " at a, b and c.

The points, 1, 2, 3, etc., are plotted by station pointer on the plotting sheet

and the shots to the sounding marks are protracted in the usual manner. The three or more observations to each mark should, of course, meet in a point and thereby prove the accuracy of the observations and plotting.

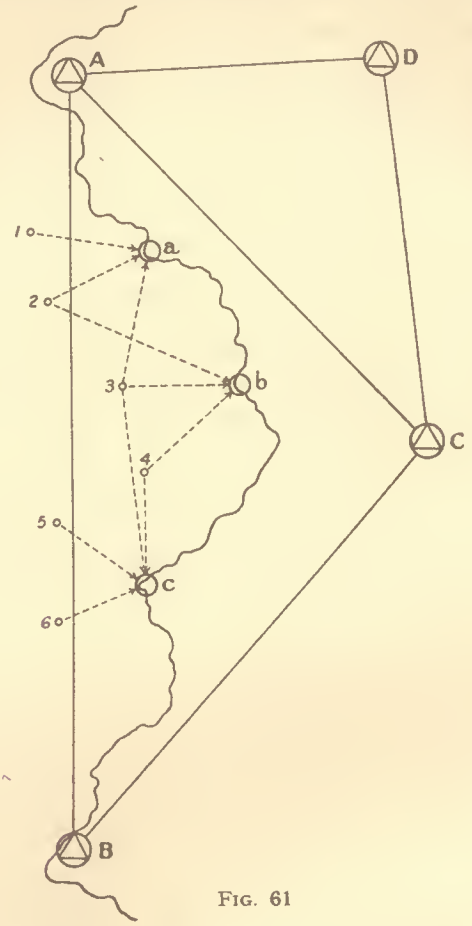


FIG. 61

When the ship, instead of a boat, can be used for shooting up marks, time is often saved as a sufficient number of observers can be stationed on the bridge to obtain all the required angles at each point simultaneously, and this can be done under way. It will, nevertheless, be found advisable to stop the ship, or at least reduce to slow speed for each set of observations.

As all the observations are, of necessity, made by sextant, any errors in the observed angles due to the objects observed being at different heights must be allowed for. *Vide* Chapter XVII, Page 395, and Table XIX.

TRIANGULATION DATA

The principal record of a hydrographic survey is the Fair Chart (*vide* Chapter XVIII), but comprehensive details of the triangulation must also be furnished in order that the accuracy of the survey may be appraised and so that in after years surveyors may be able to use the original triangulation data as a basis for new work, thereby providing a rigid connection between the old and the new and eliminating waste of time in the repetition of previous measurements and observations.

The following information should be embodied in returns of Triangulation Data :

(1) *Diagram of the Triangulation* : drawn on tracing cloth on any convenient scale. The rough coastline should be indicated and the positions and letters of all main and secondary stations with all observed rays should be shown. By the use of different colours, the grouping of the various figures in the process of adjustment should be made clear. (See Fig. 31.)

(2) *Rectangular Co-ordinates* : of all main and secondary stations.

(3) *Description of Triangulation* : indicating methods employed in the measurements of bases and angles and the extent to which data from previous surveys has been employed. The figure of the earth used in the computations must be stated.

(4) *Observed and Adjusted Angles* : with a list of the various figures in the order of their adjustment.

(5) *Lengths of all Sides.*

(6) *Base Measurements* : full details with methods employed and corrections applied, together with a diagram and details of the base extension triangulation.

(7) *Geographical Co-ordinates* : of all recoverable stations.

(8) *Probable Errors* : of base measurements, angular measurements and astronomical observations.

(9) *Descriptions of Stations* : full details being given to assist recovery in the future (*vide* Chapter V).

CHAPTER V

SURVEYING MARKS

Selection of Stations

The general principles, on which the positions of the trigonometrical stations should be selected, have been discussed in the previous pages. The actual site of a station must be decided after consideration of the following points:

(1) The position should preferably have an all-round view as it may subsequently be wanted for extending the triangulation in other directions, and it must be visible from other trigonometrical stations with which it is to be connected by angular measurements; it should also see as many sounding marks as possible.

(2) It must be possible to erect a theodolite at, or in close proximity to, the station in such a way that angles can be observed in reasonable comfort with the instrument firmly set up.

(3) The station should be sited to afford the best obtainable background from the point of view of the observers at other marks. A background of sky is probably the ideal, but when this cannot be arranged, the station should be as much in the open as possible so that it is well illuminated by the sun.

(4) It should usually be possible to mark the position in a permanent manner and stations should preferably be placed where their mark-stones are unlikely to be disturbed; if solid rock is available, this is perhaps the most satisfactory position.

(5) The labour involved in clearing the site should be the minimum compatible with the foregoing requirements.

(6) Church spires, lighthouses, chimneys, beacons and other marks of an artificial nature may often be used as trigometrical stations in hydrographic surveys. The advantages of using these " ready-made " marks are great, but the surveyor must guard against possible inaccuracies due to difficulty in measuring the exact distance from the theodolite to the centre of the mark or to want of perpendicularity in the marks themselves, etc.

Marking and Description of Stations.—Station marks in hydrographic surveys usually consist of two parts:

(i) A temporary mark consisting of a beacon or luminous signal.

(ii) A permanent mark consisting of a buried mark, stone or tile, or, if the station is on solid rock or masonry, a cut surface mark.

Beacons.—The best type of beacon for most purposes is of the quadripod form, which is substantial, easy to erect and a symmetrical mark for intersection. It can be constructed with four hop poles lashed together near their tops and the feet splayed apart so that the cross at the lashing is plumbed over the ground mark. The upper part of the beacon may be covered with calico, canvas or wooden slats, painted black or white to contrast with the nature of the background as seen by the observers at other stations. The lower edge of the " solid " portion of the mark should be not less than 6 feet from the ground,

so that the theodolite may be centred over the ground mark and observations taken without interference. The size of the beacon depends on the distance from which it is to be observed ; marks of this type with 18-ft. legs, can be intersected under good conditions from a distance of 20 miles. Rather a large quantity of material is required for a quadripod beacon and it is often necessary to use pole marks with flags. These are not so satisfactory as it is difficult to guy them rigidly enough to remain perfectly upright for any length of time ; at long distances the pole cannot be seen and there is often an element of uncertainty as to which way the flag is blowing out. Many errors in triangulation are due to eccentricity in the marks themselves ; those parts of the marks which are intersected from the distant stations are only too frequently out of the vertical from the true position of the station. Wire should be used in preference to rope for the guys and the importance of erecting the marks so that they are rigid and perpendicular cannot be over-emphasised.

In thickly-wooded country high marks will often save a great deal of clearing and may, indeed, be essential if a regular triangulation is to be carried out. To observe angles from such marks, some form of tower is necessary. A large quantity of material is required and two or three days will probably be needed for assembly and erection. Towers of heights of 80 ft. and more have been successfully used on many occasions. Satisfactory observations can only be made from them in calm weather or very light winds.

For short rays, *e.g.*, those involved in the base extension triangulation, marks consisting of thin poles with small topmarks should be used. When distances of only a few hundred yards are involved, a large nail may make a good mark for intersection.

Luminous Signals.—These consist of heliostats by day and lamps at night. A 6-volt electric lamp with a small reflector can be seen at a distance of two or three miles and is often useful for night observations. For longer shots, an Aldis signalling lamp may be used ; it is powerful enough to be seen at distances of more than 20 miles. Luminous signals, provided they are not too large, are more satisfactory objects to intersect than any type of opaque day signal.

Mark Stones and Surface Marks.—Whenever possible, the main stations of a triangulation should be marked in a permanent manner, so that the exact positions can be subsequently recovered. In this connection the word " permanent " must not be taken too literally, but the aim should be to provide some positive means of identification which will last for at least 20 years without attention. In hydrographic surveys it is sometimes necessary to site main stations on coral reefs, sandbanks and other similar positions which may be covered at high water. In such cases it may be a waste of time to attempt to provide a permanent mark at a station, but the surveyor should not on that account omit to mark it in such a manner that its position can be re-identified at any time during the actual progress of the survey, if the necessity arises. Stations may be permanently marked by one of the following means :

(i) **Buried Mark-stones or Tiles.**—These can be easily constructed on board and may be in the form of a concrete block about 9 ins. square and 3 to 4 ins. thick with a brass disc embedded in its centre. The disc should be marked, as shown in Fig. 62, with the ship's name, a reference number and a date. The dot in the centre of the triangle indicates the exact position of the trigonometrical station.

Mark stones should be buried at a depth of at least a foot. In many land surveys a *surface* mark stone is used in addition to the buried one over which it is plumbed. The surface mark in these cases may be

assumed to indicate the exact position of the station if it shows no sign of having been disturbed.



FIG. 62

When a tripod or quadripod type of beacon is used, it should be erected *first* and the buried and surface mark stones then plumbed below its centre. When a pole mark is used, the buried mark stone should be placed in position *before* the pole is erected.

(ii) **Concrete Pillars.**—These can be made on board and should be about 2 ft. long and 9 ins. square. They should be let into the earth to a depth of a foot or more. As they are liable to disturbance, it is advisable to bury a mark below them.

(iii) **Iron Bolt Staves.**—These are not very satisfactory as, besides the likelihood of interference, they cannot be easily inscribed in a distinctive manner. On coral reefs and similar places, however, they may be the only form of permanent mark which can be used.

(iv) **Surface Marks.**—When the station is on solid rock or on a permanent building, it should be marked by a triangle cut on the surface of the rock or material of the building with, if possible, the addition of the ship's name and a reference number. When the surface is irregular, a brass disc or a piece of piping set in concrete should be cemented to the rock.

Descriptions of Stations.—Much excellent work has been impaired by the failure of surveyors to mark their triangulation stations in a permanent manner and to provide adequate " descriptions " by which the positions can be subsequently recovered. The recording of angles and measurements, by which the spot can be re-identified should be the *first* consideration when the site of a station has been selected, for it frequently happens that meddlesome persons tamper with or remove marks even during the actual progress of a survey. In such cases much time is wasted in re-establishing the station if no measurements for that purpose have been made. The provision of an adequate " description " often calls for the exercise of considerable ingenuity on the part of the surveyor. His aim should be to furnish a sufficient number of bearings and measurements to stable points in the immediate neighbourhood to enable the position to be recovered equally well (as far as can then be foreseen) from any one of several different combinations of measurements. Angles should also be observed to more distant objects which are prominent

and well defined, as these will often enable the position to be recovered with sufficient accuracy for the mark stone to be found by digging over a small area. Recovery of stations is greatly assisted if a few reference marks can be established in its immediate vicinity. These marks may take the form of drill holes in rock, arrows cut on trees, posts, etc. Sometimes a sketch, showing the relative positions of some prominent features of the landscape as seen from the station, is a useful addition to the " description." When objects such as flagstaffs are used as trigonometrical stations, it is generally unsafe to regard them as " permanent," and therefore needing no " description " ; they are certain to be renewed from time to time, and it is quite possible that they will not be re-erected in the same position. Whenever possible, therefore, measurements as well as a written description should be given.

Examples of Descriptions of Stations :

(1) On Tagrin Point, 10 yards from H.W.L. on flat grassy ground with sandy soil. The station is marked by a concrete block (11 ins. by 11 ins.) buried 1 ft. below the surface : a brass disc in the centre is engraved " H.M.S. ——, 4/1936." Reference marks ▲ are cut on the drying rocks about 5 yards from their inshore limit—see diagram. The station is 2 ft. above M.H.W.S. (Fig. 63.)

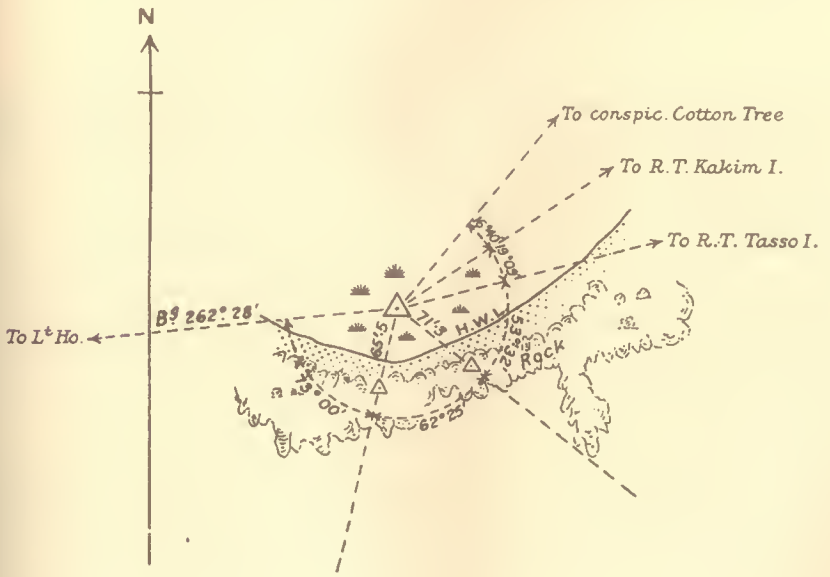


FIG. 63

(2) On the top of Belham Church Tower. The station is marked by a triangle ▲ cut in the lead roof. Measurements are shown in the diagram. (Fig. 64.)

MARKS FOR FIXING THE DETAIL OF THE SURVEY

1. Use of Conspicuous Natural Marks.—The main stations of the triangulation form a framework for the survey, but a number of additional marks are usually needed to enable the detail to be fixed by angles and distances measured to or from them. Many of these additional marks may have to be in the form of temporary beacons erected by the surveyor, but it often happens that the area contains a number of objects which may be used as " natural " marks, thereby enabling him to dispense with artificial ones to a considerable extent,

or even entirely. The surveyor should be on the look-out to make the fullest possible use of such natural marks as, by so doing, there is not only a great saving of time, labour and material, but there is also a reasonable certainty that they will not be removed or tampered with during the progress of the

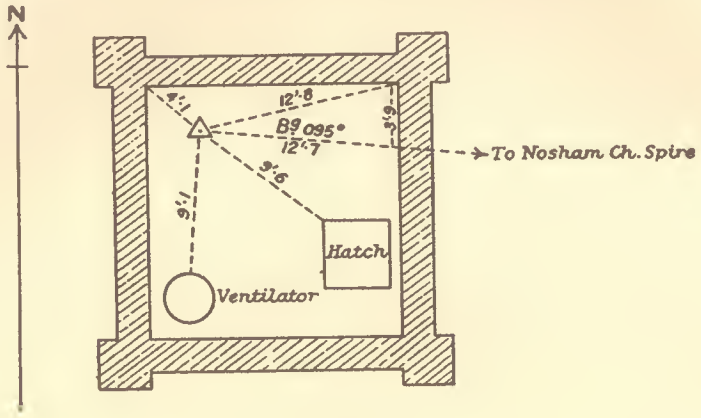


FIG. 64

survey. Natural marks may include peaks, clumps of trees, single trees, isolated or conspicuous patches of rock, buildings of any description and, in fact, any objects that are well defined and easily recognisable from the different directions in which it may be necessary to see them.

The suitability of natural marks for the purpose in view must be carefully considered with regard to the scale of the survey. For instance, on small scales, *e.g.*, 1/50,000, a clump of trees 100 yards in diameter or a whole house could be regarded as sufficiently well-defined points, for the clump would be represented on the chart by a circle less than 1/10th of an inch in diameter and its centre could be estimated with sufficient accuracy by eye, whilst even a large house would appear as little more than a dot. On large scales, however, such objects would be unsuitable; it would be necessary to select some particular tree in the clump as the mark, or some special point, *e.g.*, one of the eaves, on the house.

In general, natural marks are not satisfactory for use as *main* stations; there are usually difficulties in observing angles at them and they are unlikely to be sufficiently well defined for accurate intersection. Exceptions are symmetrical objects such as lighthouses, church spires and chimneys which, besides being well defined and symmetrical, are usually conspicuous. Tree stumps have often been used as main stations, but this practice should be avoided, at all events in the Tropics, where they are soon destroyed by white ants.

2. Surveying Marks on Shore.—Artificial marks in a survey may take a variety of forms, depending on the precise purposes for which they are required and the material available for their construction. Such marks may be divided into the following categories:

(*a*) **Towers.**—These are essential when the observations at a station have to be made at a considerable height above the level of the ground. They should consist of two independent but concentric structures, the inner one supporting the theodolite and the outer one carrying a platform to support the observer. Any scaffolding material will suffice for a small tower, but high ones, *e.g.*, 40 ft. or more, require a great quantity of wood or steel and all the parts must be specially made and fitted for the purpose. Both inner and outer towers must

be strongly (and independently) guyed if they are to withstand anything more than a light breeze. For these reasons and on the score of expense, the construction of high towers can only be justified in exceptional cases, *e.g.*, in flat densely-wooded country where the same results cannot be achieved so economically by the ordinary method of clearing the forest or by other means.

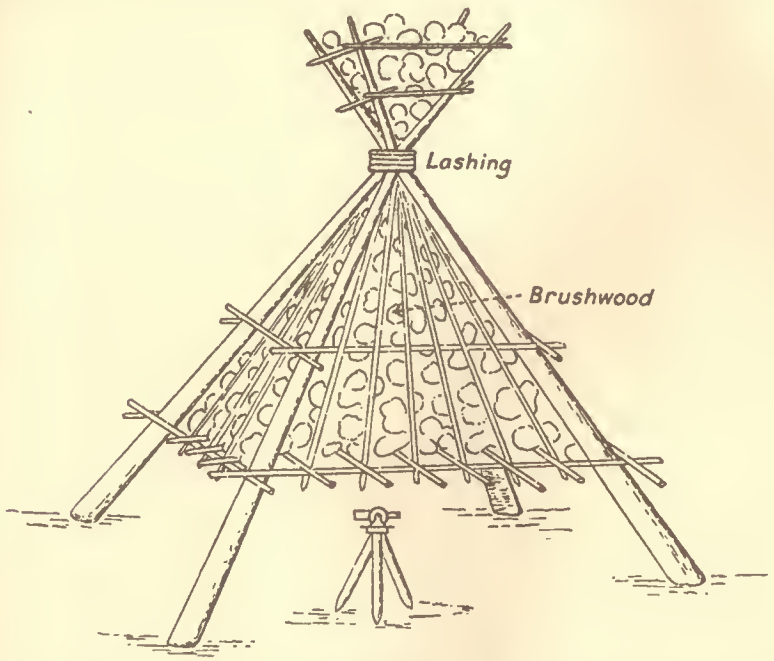


FIG. 65

(*b*) **Quadripods and Tripods.**—These are easily constructed and offer an admirable target for a distant observer. The quadripod is preferable, it is firmer than the tripod, and its symmetrical appearance from all directions is also an advantage.

The quadripod (Fig. 65) may be built of four small tree-trunks, about 20 ft. long, or of hop-poles if the former are not available. A floor and side pieces of saplings and light branches can be lashed on and the inside is then filled with brushwood : alternatively, the upper part may be covered with canvas, calico, or wooden slats painted black or white as may be necessary to make the mark distinctive against its background. If the heels of the spars can be buried a foot or two in the ground, guys will probably be unnecessary except in very exposed positions.

(*c*) **Pole Marks.**—In various forms these are largely used in hydrographic surveys. They may consist of barlings or bamboos (or a combination of these) for high marks carrying large flags, or of hop-poles or boathook staves carrying smaller flags which will suffice when the marks need only be seen up to distances of three or four miles. Pole marks require less material than quadripods or tripods and are more quickly erected, but they must be strongly guyed. It is difficult to ensure that high marks of this type are perfectly upright and rope guys stretch so much that they will not long remain so. For this reason it is better to use wire guys at main stations.

With the resources normally available, pole marks up to a maximum of 70 or 80 feet in height may be erected without much difficulty. Fig. 66 shows a mark of this type and the following notes on the material required and the method of erection may be of use :

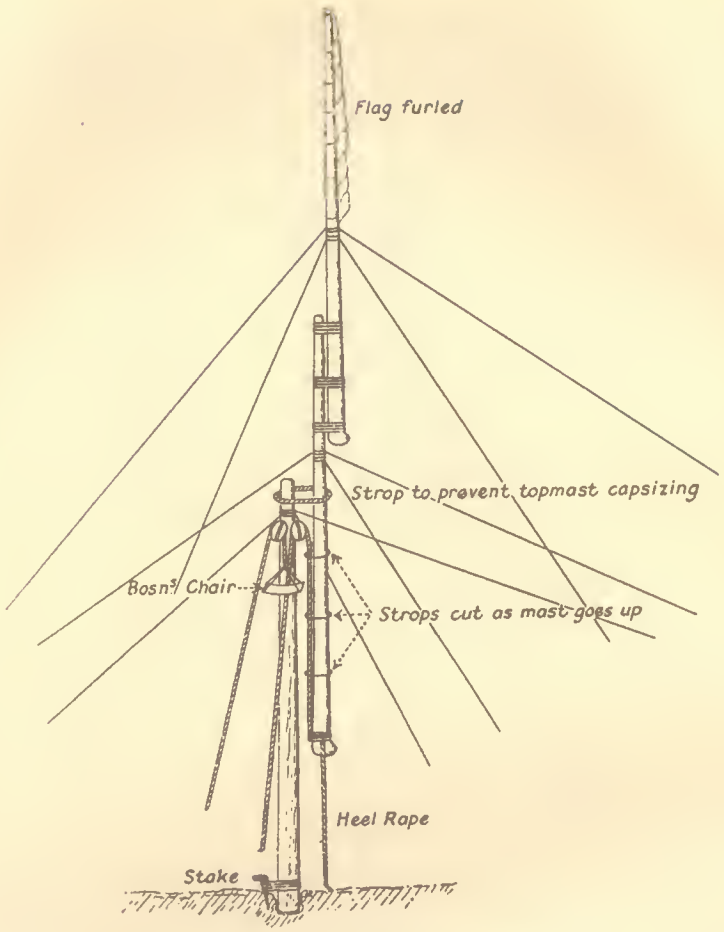


FIG. 66

The gear needed will vary slightly with the conditions, but will generally be as follows :

One barling and two bamboos, each doubling 10 ft.
Four lowermast guys ; $\frac{1}{2}$-in. or $\frac{3}{4}$-in. F.S. Wire tailed, each 10 fms.
Four topmast guys ; $1\frac{1}{2}$-in. rope, each 12 fms.
Four top-gallant mast guys ; 4-yarn spun yarn, each 15 fms.
Two 5-in. tail blocks.
Gantline ; 10 fms. of $1\frac{1}{2}$-in. rope, and boatswain's chair.
Mast rope ; 25 fms. of 2-in. rope.
Heel rope ; 10 fms. of $1\frac{1}{2}$-in. rope.
Two large balls of spun yarn for lashings and smiting line.
Maul.
Flag ; 16 ft. by 10 ft.
Five iron stakes (9, if ground is soft).

The stakes for the guys and heel should be driven in first ; a prismatic compass is useful for placing the former correctly. The tail blocks having been bent on to the lowermast head, the gantline and mast rope are rove and the heel is loosely lashed to the centre stake. The lowermast may then be hoisted and securely guyed in position. The topmast and top-gallant mast are then lashed together on the ground and the eight guys cut to the necessary lengths and bent on. The flag should be furled and a smiting line fitted so that

it can be broken out *after* hoisting. The heel of the topmast is then temporarily lashed (loosely) to the heel of the lowermast, and the mast rope bent on and secured along the topmast by stops at intervals nearly to its head. The topmast and top-gallant mast can now be raised together to an upright position by hauling on the mast rope, the eight guys being tended to steady it. A man sent up in the boatswain's chair should then pass a loose strop round the head of the lowermast and the adjacent portion of the topmast ; the latter can then be hoisted, the guys and heel rope being tended and eased off as it goes up and the strops cut as they come level with the tail block. When high enough, the topmast is secured to the lowermast, the guys are belayed and the flag broken out.

(*d*) **Cairns.**—Where there are boulders and stones, cairns form very satisfactory marks and are easily constructed. If possible, the foot of a pole mark should be surrounded by a cairn which will help to support it and make it easier to distinguish. Except perhaps when there is a sky background, cairns should be whitewashed.

(*e*) **Whitewash.**—Whitewash is invaluable to the surveyor. Its components, lime, whiting and sea water, are cheap and always obtainable and it may be used on cairns, tree trunks, angles of houses, canvas, etc. Whitewashed marks show well under most conditions. A further advantage in the use of whitewash is that it cannot disappear by being blown over nor can it be removed by meddlesome persons.

3. Marks on Drying Sandbanks.—The seaward extension of triangulation over large shallow estuaries is often a matter of some difficulty, but sometimes may be facilitated by the erection of marks on drying banks of sand or mud. As the time available for putting them up is limited, a pole mark, such as a bamboo, is usually the most feasible. The heel of the mark should have a heavy sinker attached and four wire guys to 1-cwt. anchors, buried in the sand or mud, should be fitted. If tidal streams are strong, such marks cannot be expected to last more than a few days ; it is impossible to prevent a deep scour forming round the heel and the mark will capsize before long, but it should by then have served its purpose in enabling the triangulation to be carried farther seaward or across the estuary more accurately than would have been possible with floating marks.

4. Marks below Low Water.—It is sometimes convenient to erect fixed marks in the sea. The most useful type is the tripod, formed of three hop-poles or barlings. The former can be laid in depths up to 8 ft., the latter in depths up to 15 ft. or so at low water. To erect a barling tripod, a motor boat or cutter should be used and the procedure is as follows :

Select an even portion of the bottom with a depths of 10 to 15 ft. at low water ; anchor the boat there about an hour before that time. The method of rigging the tripod in the boat is shown in Fig. 67.

$1\frac{3}{4}$-cwt. sinkers are attached as heel weights and one leg is pushed out over each quarter and one over the bow. Stop the legs on each quarter and in the bow and lash the cross ; a temporary stop will also be necessary to hold this down in the boat.

At a given signal cut all the stops and at the same time haul away on the anchor chain and light aft the foremost leg. The heels of the two after legs fall to the bottom and are splayed apart by the beam of the boat. Continue to haul on the anchor chain and, when the heel of the foremost leg comes aft, drop it over the quarter or stern. A swifter should then be passed round and secured to the legs at the water line to prevent them splaying any farther apart. Attach four wire guys to the cross and lay out four 1-cwt. anchors. A pole mark with heel weight and flag can then be lowered over the side under the cross and secured to it, and lashings should be passed at the water line from the pole to the legs to prevent capsizing.

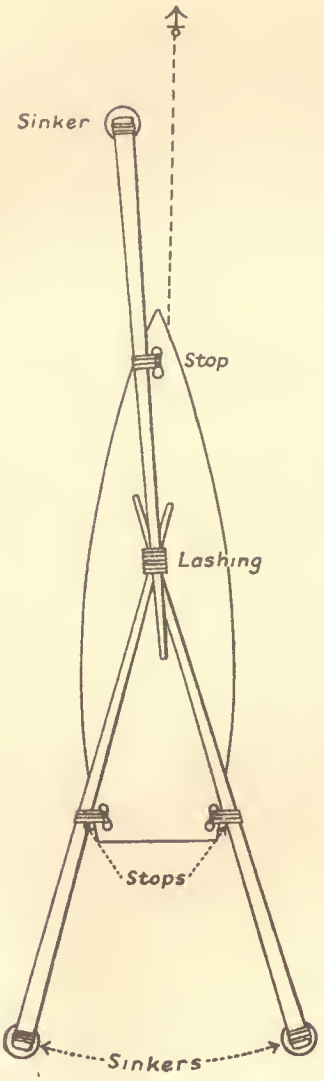


FIG. 67

5. Mark Boats.—A whaler, moored head and stern, with a bamboo carrying a large flag lashed to the mast, forms a useful temporary mark for sounding.

6. Floating Beacons.—When floating marks of a more permanent nature than the above are required " beacons " are used. Three types in common use are :

(*a*) Egeria Beacon.

(*b*) Ormonde Beacon.

(*c*) Dan Buoy.

Other types of beacon may be improvised with rum casks, fishermen's pellets, etc.

(*a*) **Egeria Beacon.**

Figs. 68 and 69 show the details of the Egeria type beacon. It consists of two 40-gall. boom defence barrels clamped together between two pieces of wood which are pierced to take a 35-ft. beacon pole. Split pins prevent the

pole slipping up and down through the pieces of wood. A bamboo (about 35 ft. long) carrying a 16-ft. by 10-ft. flag is lashed to the upper part of the pole and the foot of the pole is pierced so that a large D-shackle can be fitted to take

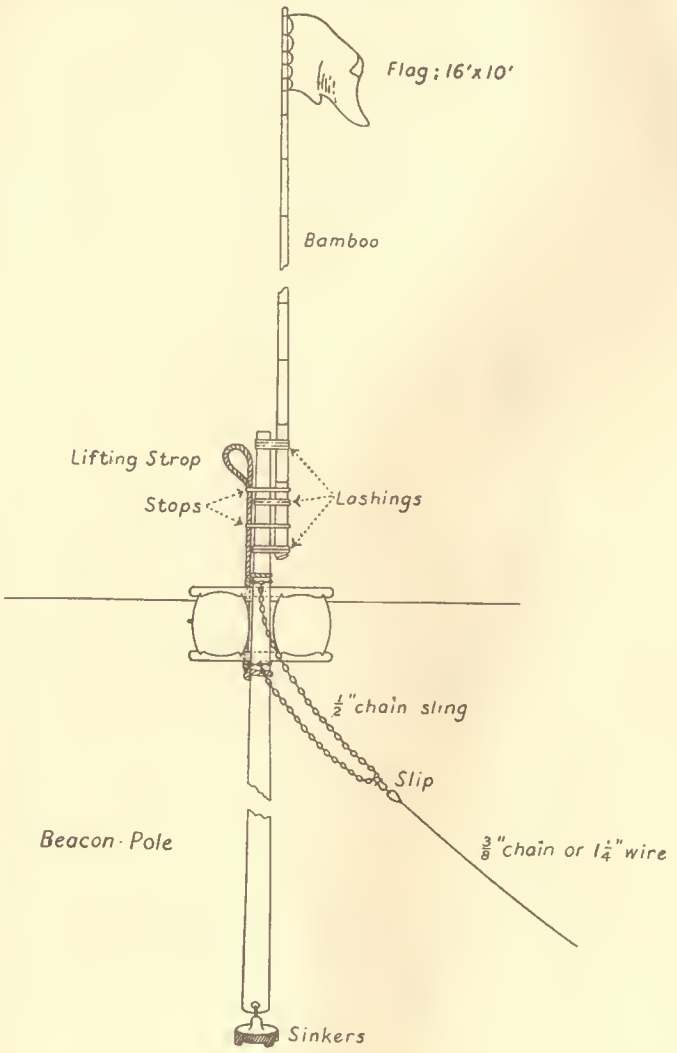


FIG. 68

heel weights aggregating 2½ to 3 cwt. This type of beacon is usually anchored with a 4-cwt. anchor and, up to depths of 20 fathoms or so, ⅜-in. chain. In deeper water, 1¼-in. or 1-in. wire (being lighter) is used with a few fathoms of chain tailing at the anchor. The upper end of the mooring chain or wire is fitted with a long link in which a small Senhouse slip engages, the latter being shackled to a ½-in. chain sling secured to the beacon spar above and below the drums. A lifting strop, with a large soft eye in each end, is secured below the drums (and sling), half-hitched above them and stopped up nearly to the head of the beacon pole.

The beacon is rigged in a horizontal position on deck and hoisted out by means of a whip on the fore derrick, which should be topped up sufficiently for the keel weights to clear the deck when the beacon is hoisted close up in a vertical

position. The anchor is taken forward and slung outboard on a rope's end and the mooring secured to the beacon when it has been lowered over the side; this can be done whilst the heel is still clear of the water. The beacon whip is fitted with a tripping hook or equivalent slipping arrangement.

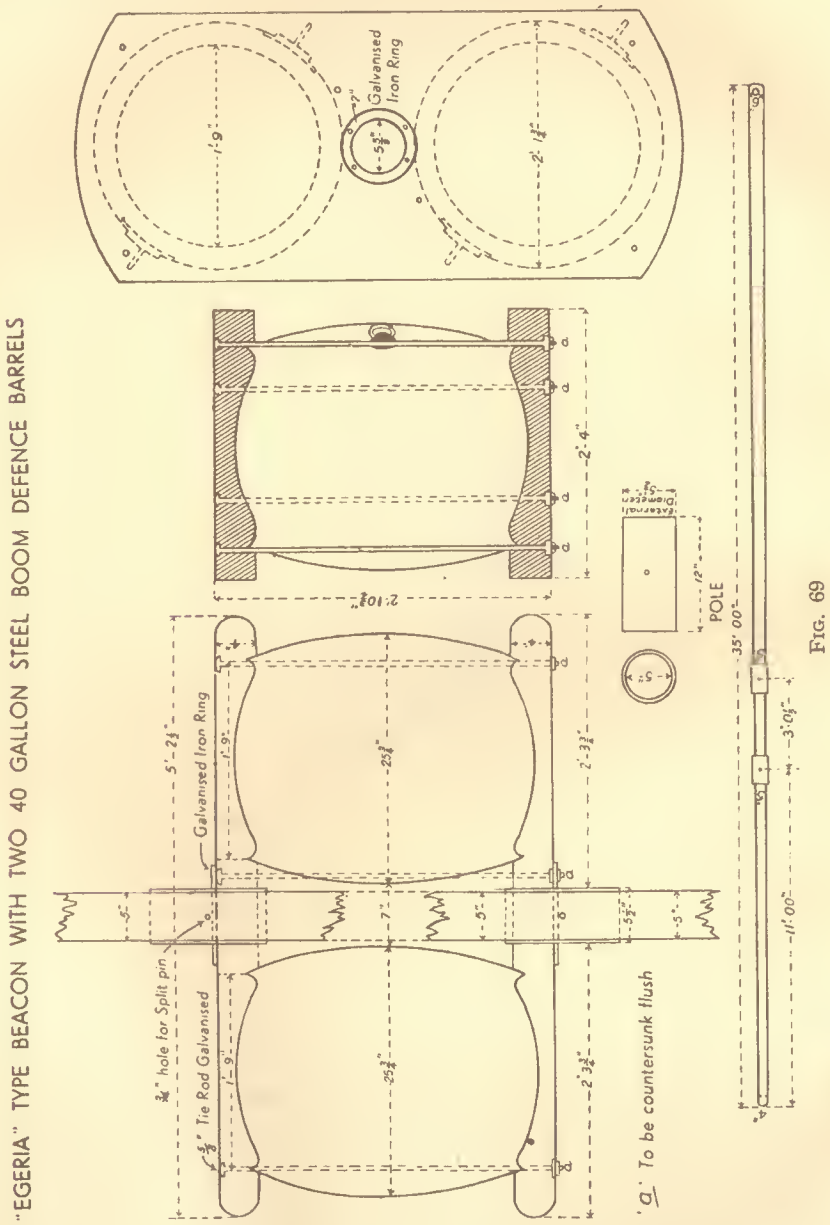


FIG. 69

On arrival at the required position the beacon is lowered till the drums are just clear of the water. Then, with a little stern way on, the anchor is cut loose and the beacon is slipped. The flag should be broken out with a smiting line as the beacon draws clear of the ship.

When mooring a beacon with two anchors, the second anchor is usually

rigged about midships, and is let go as the strain comes on when the beacon draws ahead. This ensures a taut moor.

To weigh a beacon, plumb the derrick head over the side with the hook of the whip a few feet clear of the water and have a man ready at it in a boatswain's chair. Run a weighing rope from the capstan through the foremost fairlead and aft to a point abreast the derrick. Bring the ship up to the beacon head to wind or tide (whichever is the stronger) and hook the whip onto the lifting strop. See the man over the side clear and hoist the beacon till the inboard end of the moorings can be reached from the deck. Then shackle on the weighing rope below the eye of the mooring wire and knock off the slip. The beacon may then be hoisted in and lowered on deck, and the moorings weighed at leisure. Care must be taken to prevent too much strain being put on the derrick before the slip is knocked off.

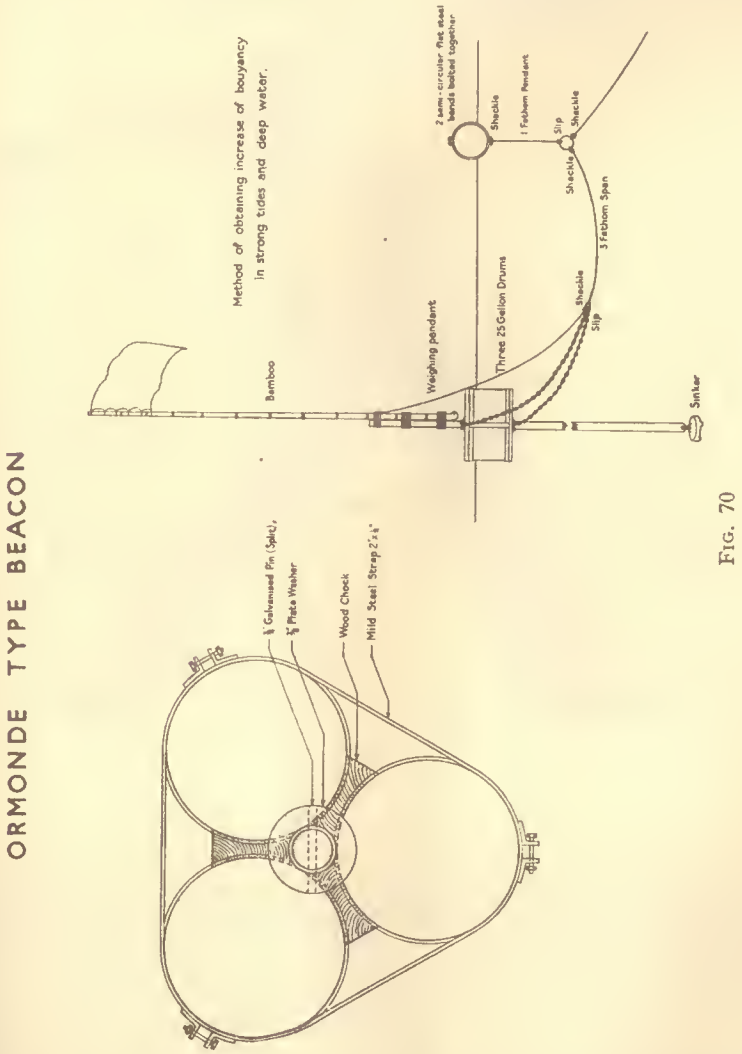


FIG. 70

When two anchors are used, the weighing rope can frequently be shackled onto both moorings as the anchors usually drag together when they are hove in. If they will not do this, a second weighing rope should be rigged and the anchors weighed independently.

An alternative method of weighing (preferred by many) can be gathered from Fig. 70. The beacon is rigged with a weighing pendant, shackled to the upper end of the mooring wire, and with a large eye stopped to the top of the beacon pole. When weighing, the weighing rope (see above) is secured to the weighing pendant and the mooring hove into the hawse pipe *before* the derrick is hooked onto the beacon lifting strop and thus takes the strain of the moorings before any can come on the derrick.

In bad holding ground and when strong currents exist, it has sometimes been found necessary to back up the 4-cwt. anchors with $1\frac{3}{4}$-cwt. sinkers.

(*b*) **Ormonde Beacon.**

Fig. 70 shows the details of the Ormonde type beacon. This is made up of three 25-gall. oil drums and has less buoyancy than the Egeria type. The moorings and anchors of the two types are similar.

(*c*) **Dan Buoy.**

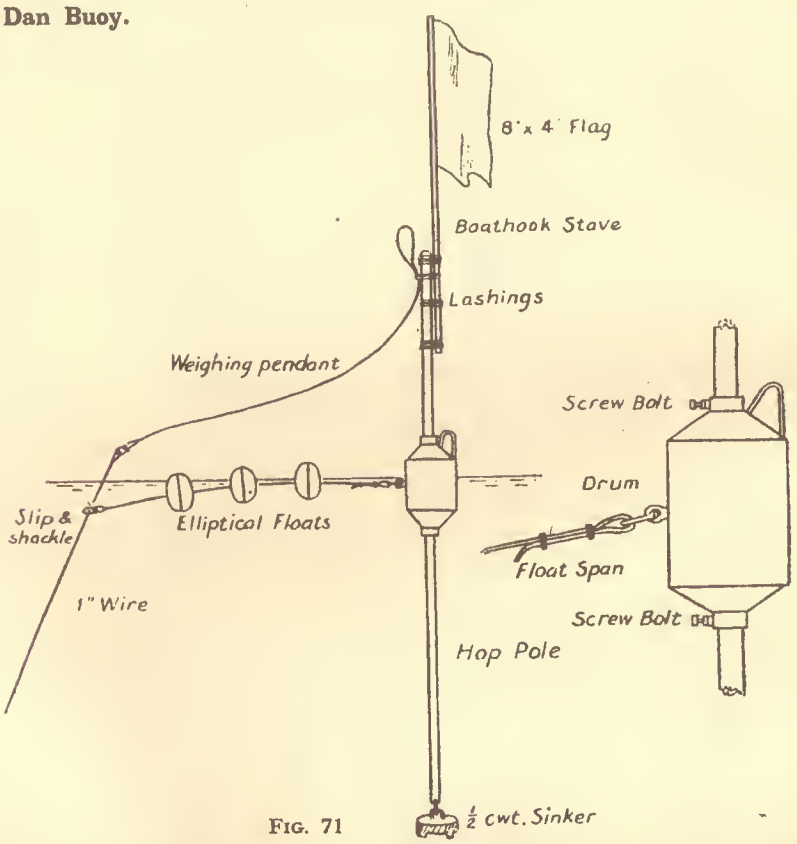


FIG. 71

This is a light type, consisting of a cylindrical drum with a hop-pole passing through it which is prevented from slipping by screw bolts at the top and bottom of the drum. A boat-hook stave carrying an 8-ft. by 4-ft. flag is lashed to the pole, and a $\frac{1}{2}$-cwt. sinker is used as a heel weight. The mooring wire can be shackled to a lug on the side of the drum, but is more usually rove through a shackle at one end of a short span on which are threaded three or more elliptical floats, as shown in Fig. 71. It is convenient to fit a slip in the moorings to ensure quick detachment from the buoy. A dan buoy is light enough to be laid and weighed from a boat, and a 1-cwt. anchor will usually hold it.

Extra buoyancy to support the weight of the moorings in deep water, or in strong currents, can be given by the methods shown in Figs. 70 and 71.

Galton's Sun Signal or Heliostat.—This instrument is supplied for use when it is necessary to take observations to marks which are very distant or which, owing to haze, bad background, etc., cannot be distinctly seen. With its aid



PLATE G

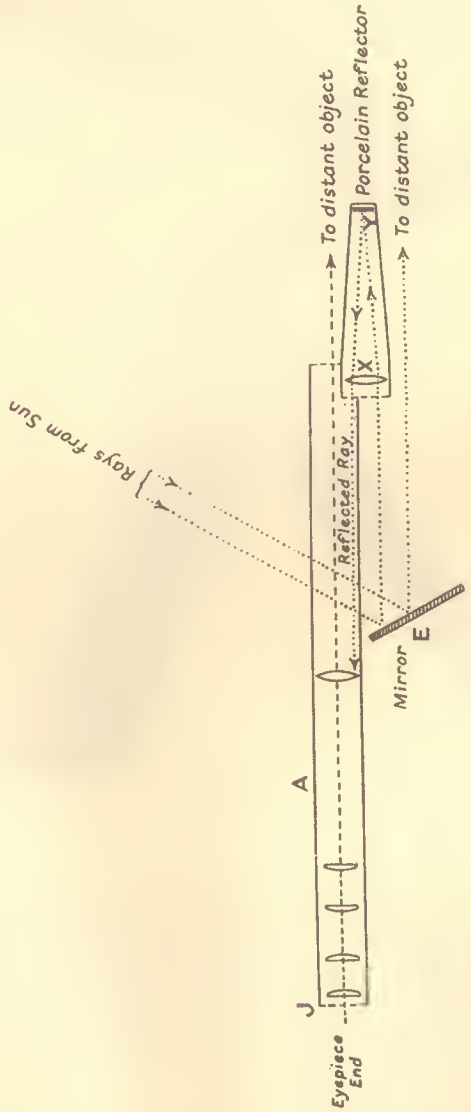
the sun's rays are reflected from a small mirror centred over one mark to an observer at another, and observations over distances of more than 40 miles can be made under good conditions. The flash from the mirror makes an admirable mark to intersect with the diaphragm webs of a telescope, and, *whenever* there is doubt as to the surveyor's ability to see a distant mark clearly, it is strongly recommended that the heliostat should be used for daylight observations. Many errors of theodolite observations are due to faulty intersection of marks or to want of perpendicularity in the marks themselves, and these should be eliminated by the use of the heliostat.

The instrument is fitted on a light wooden tripod and its general form is seen in plate G.

It consists primarily of a telescope A, which is mounted so that it can be moved in azimuth and elevation, and a plane mirror E, about 4 ins. by 2½ ins., which is attached to the telescope barrel but can be turned in relation to the

telescope by means of the rack and pinion D. A tubular extension, attached rigidly to the telescope, carries a lens at its wider end and a small porcelain reflector at its narrower end. The telescope, mirror and extension tube can be rotated about the axis of the latter by means of the rack and pinion B. By this method of mounting, the reflected beam of the sun can be directed to any desired point other than in a direction exactly away from the sun. The tubular extension and its fittings are provided to assist the observer in directing the reflected beam in a line parallel to the axis of the telescope so that when the telescope is aligned on a mark the reflected beam can also be directed on to that mark.

The optical arrangement is diagrammatically shown in Fig. 72.



Heliostat: Diagrammatic Sketch of Optical Arrangements
(Not to Scale)
FIG. 72

When the telescope has been aligned on a distant mark, the mirror E is adjusted in two dimensions by means of the racks and pinions B and D (this does not affect the alignment of the telescope) until a small portion of the

reflected beam enters the extension tube and after reflection from Y emerges parallel to its direction on reflection from E. The object end of the telescope partly overlaps the extension tube and an image of the sun is seen in the field of view. By adjustment of the mirror this image can be made to cover the distant mark and, if the instrument is in adjustment, the reflected beam will then be directed on to that mark. It is evident that the axis of the lens X should be parallel to that of the telescope and the reflector Y should be perpendicular to this axis. Two " open " sights F and G are fitted on the extension tube to assist in bringing the reflected beam into an approximately correct alignment. A movable shutter H on the extension tube allows the brightness of the sun's image, as seen in the telescope, to be adjusted to suit the prevailing conditions.

The reflected beam forms a cone with an angle of about $\frac{1}{2}°$; the adjustment of the instrument must therefore be correct to within about $\frac{1}{4}°$ for the distant station to see the flash. Owing to the apparent movement of the sun, it is necessary to make frequent small adjustments in the azimuth and elevation of the mirror so that the reflected beam can be kept continuously on the distant mark. The quantity of light transmitted by the mirror will depend on the angle between the sun and the distant mark and is, of course, greatest when this angle is smallest. When the angle is 90°, the effective area of the mirror is reduced by about one-quarter; when the angle is 140°, the effective area is reduced by nearly three-quarters and it is unlikely that under these conditions the flash would be seen very far. It is well to remember this when organising a programme involving heliostat work.

In using the heliostat, care should be taken that when set up, the mirror is exactly over the station which is to be observed or, if this cannot be arranged, that it is on the direct line between the station and the distant mark. After alignment on the distant mark, the telescope is rigidly clamped by means of the clamps C and L. The preliminary adjustment of the mirror, which is effected by moving it until the shadow of the back sight F falls on the foresight G, is simplified by holding a sheet of white paper behind the foresight and noting where the shadow falls on it. It is then an easy matter to adjust the mirror so that the shadow is made to fall on the centre of the foresight.

To Test the Adjustment of a Heliostat.—When the image of the sun, seen in the field of the telescope, covers the distant mark the reflected beam should also be directed on the mark. It is advisable to check this alignment from time to time. To do this, set up a field board on edge at a distance of about 70

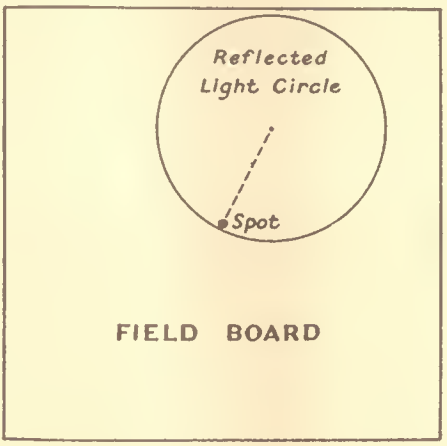


FIG. 73

yards and mark on it a black spot, as a target. If the telescope is then directed on the target and the sun's image, as seen in the field of view, adjusted so that

its centre coincides with the spot, the reflected beam from the mirror should throw a circle of light (about 18 ins. in diameter) on the board and the centre of this should also be coincident with the spot. Strictly speaking, an allowance should be made for the distance between the centre of the mirror and the axis of the telescope, about 3 ins., but in practice this may be neglected. If there is coincidence, the instrument is in adjustment. If not, nothing can be done to remedy it in the field, but it is a simple matter to make an allowance for the error if it is not too large. Supposing, for instance, the spot on the board is found to lie near the edge of the light circle " at 7 o'clock," as in Fig. 73.

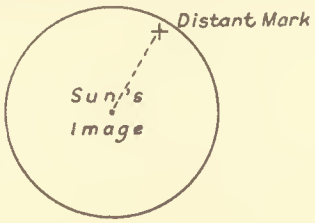


FIG. 74

A diagram can then be drawn, as in Fig. 74, with a circle representing the sun's image and the distant mark near its edge " *at* 1.0 *o'clock* " : this diagram should be pasted in the lid of the instrument box. Then if the instrument is adjusted in use so that the distant mark is seen near the edge of the sun's image at 1.0 o'clock, the mark will be in the centre of the reflected beam.

Improvised Heliostat.—If no other is available, it is a fairly simple matter to improvise an effective form of heliostat by mounting a small mirror on the end of a metal or wooden arm which can be moved in azimuth and elevation. A sighting vane is fitted at the opposite end of the arm and a small hole drilled in the centre of the mirror to serve as a back sight. The mounting of the mirror must be such that it will enable it to be moved in two dimensions relatively to the arm. If the sights are now aligned on a distant station and the reflected beam brought on to the foresight and kept there by adjustment of the mirror, the beam should also be directed on the distant station. Heliostats of this type, however, are not so easy to align accurately as the telescopic variety described.

CHAPTER VI

TRAVERSES

A traverse consists of a connected series of straight lines (on the earth's surface) the lengths and bearings of which are measured. By these measurements the relative positions of various points and detail in the survey can be found. In some countries traverse takes the place of minor triangulation to some extent, and, although not so frequently used in this manner in hydrographic surveys, it may occasionally be employed for fixing trigonometrical points which cannot easily be connected by triangulation in the ordinary way, and it is often used for fixing subsidiary marks from which the detail of the survey can be plotted. Each straight section of a traverse is termed a " leg," and the junctions of the legs are called " turning points."

Traverse involves angular and linear measurements, and the principal methods, by which these are effected in hydrographic surveys, are as follows :

Angular Measurements.—These are needed to determine the bearing of each leg and may be made by :

(*a*) Theodolite, or level fitted with a horizontal circle.

(*b*) Sextant.

(*c*) Compass.

(*a*) For accurate traverse work a theodolite must be used to measure the angles at each turning point. An azimuth of the first leg having been observed from the initial station, either astronomically or by reference to one of the other fixed points of the triangulation, the theodolite is set up over each turning point in succession and the angle between the forward and backward leg is measured. The azimuth of the second and succeeding legs can then be deduced from the turning point angles. As the legs may often be very short, it is important that the theodolite should be centred carefully over the ground mark at each turning point, and that the marks, which are being intersected, are as fine as the magnification of the telescope will allow. For instance, for legs up to 200 ft. in length a nail is large enough ; at longer distances, up to 1,000 ft., a small wooden peg can be used. Angles should be measured at least once on each " face," with further repetitions if a high standard of accuracy is necessary. In long traverses a check on the angular measurements may be provided by observing astronomical azimuths at intervals as the work proceeds.

(*b*) The use of the sextant for the angular measurements should be confined to short traverses, such as those extending from one sounding mark to the next. In such traverses the only essential requirement in the measurements is that there should be no *plottable* error on the scale in use ; the positions of turning points, from which the detail is fixed, are plotted by the protraction of the observed bearing and length of each leg, and their rectangular co-ordinates are not usually required.

(*c*) The compass gives the magnetic bearings in a direct form. Both forward and backward bearings of each leg should be observed, and a mean accepted. With careful observations very good results are obtainable from a compass traverse.

Linear Measurements.—These are needed to determine the length of each leg and, when accurate results are required, must be corrected for slope. They may be made by :

(*a*) Steel tape, or rangefinder, giving direct results.
(*b*) Tachymetry.
(*c*) Subtense methods.

(*a*) The steel tape affords the most accurate method which is normally available to the hydrographic surveyor. The measurements are carried out in a similar manner to that used in base measurement by this means, but, as a lower standard of accuracy can usually be accepted, less rigorous methods may be adopted and the length of the tape may be regarded as that calculated for the mean temperature during use. Traverse measurement with steel tapes will not often be practicable or necessary in hydrographic surveys since the operation is slow and can only be undertaken when the ground is reasonably even.

Portable range-finders may be employed in short traverses if no very high standard of accuracy is required, but their use is not recommended if other methods can be applied.

(*b*) The linear measurements may be made by tachymetric methods. Most theodolites and levels are now fitted with telescopes in which two (or sometimes four) horizontal wires at equal distances from the axis are fixed in the diaphragm or, in the case of a glass diaphragm, are scribed on the glass. These wires or lines are called " stadia " lines. A typical glass diaphragm, as seen through the eyepiece, has the appearance shown in Fig. 75.

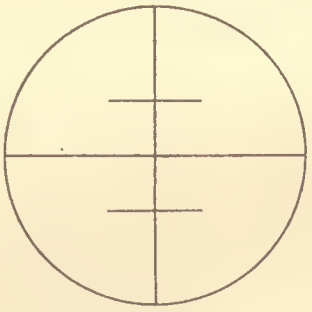


FIG. 75

If a graduated staff is held vertically and its divisions intersected by the cross-wires of the telescope, its distance from the theodolite (or level) can be found from the difference of the readings of the upper and lower stadia lines. The angle subtended by these lines is constant at a point (on the axis of the telescope) which depends on the instrument employed. When the telescope is fitted with only an object glass and an eyepiece, this point is at a distance $(f+c)$ in front of the vertical axis of the instrument, where f=focal length of the telescope (at solar focus), and c=distance from the centre of the object glass to the vertical axis.

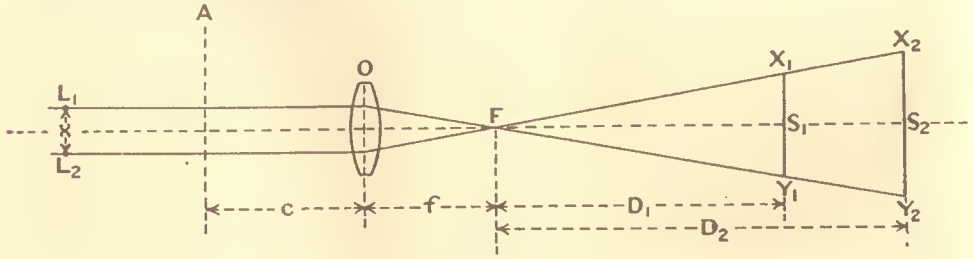


FIG. 76

In Fig. 76, L_1L_2 are the upper and lower stadia lines on the telescope diaphragm at a distance x apart, A is the vertical axis of the instrument, O is the

object glass at a distance c from A, and S_1S_2 are two positions of the graduated staff. Light falling on L_1 and L_2 will be brought to a focus at F, where OF ($=f$) is the focal length of the object glass, and, continuing, would cut S_1 at X_1Y_1 and S_2 at X_2Y_2. The distance of S_1 from A is D_1+f+c, and the distance of S_2 is D_2+f+c.

If the dimensions of x and f are so chosen that $\frac{x}{f}=\frac{1}{100}$, then $\frac{X_1Y_1}{D_1}=\frac{1}{100}$, and $\frac{X_2Y_2}{D_2}=\frac{1}{100}$.

The number of divisions, represented by the lengths X_1Y_1 and X_2Y_2 can be read off, and then :—

At S_1 the distance of the staff from the vertical axis $=100\times X_1Y_1+(f+c)$.

At S_2, the distance of the staff from the vertical axis $=100\times X_2Y_2+(f+c)$.

The value of the correction $(f+c)$ should be found by testing the instrument over known distances, such as 100, 200 and 300 ft. $\frac{f}{x}$ is a constant multiplier at all distances.

In some instruments an anallatic lens is fitted between the object glass and the diaphragm, and no correction $(f+c)$ is then required. The optical arrangement is shown in Fig. 77.

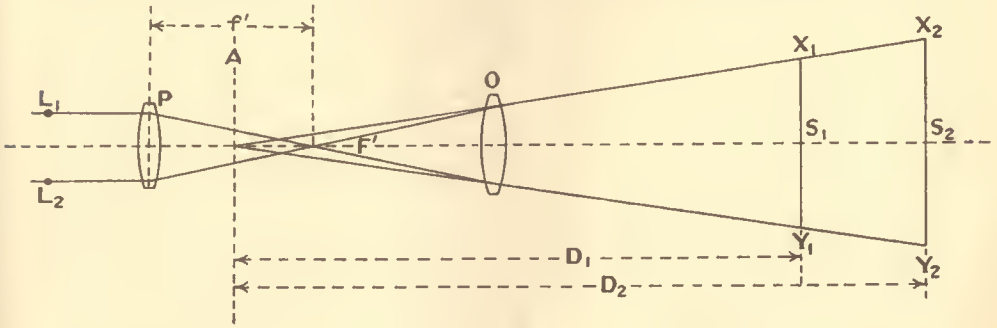


FIG. 77

L_1L_2 are the upper and lower stadia lines on the diaphragm, A is the vertical axis, O is the object glass, P is the anallatic lens, and S_1, S_2 are two positions of the graduated staff. If two rays intercept distances X_1Y_1 and X_2Y_2 on the staff at S_1 and S_2 respectively, in the absence of an object glass they would meet on the axis at A, but owing to the existence of that lens they are deflected and meet at F', and, continuing their paths, fall on the anallatic lens P. If this lens is made with a focal length PF' ($=f$), they will emerge parallel and fall on the diaphragm at L_1 and L_2 where they may be examined in the eyepiece in the usual way.

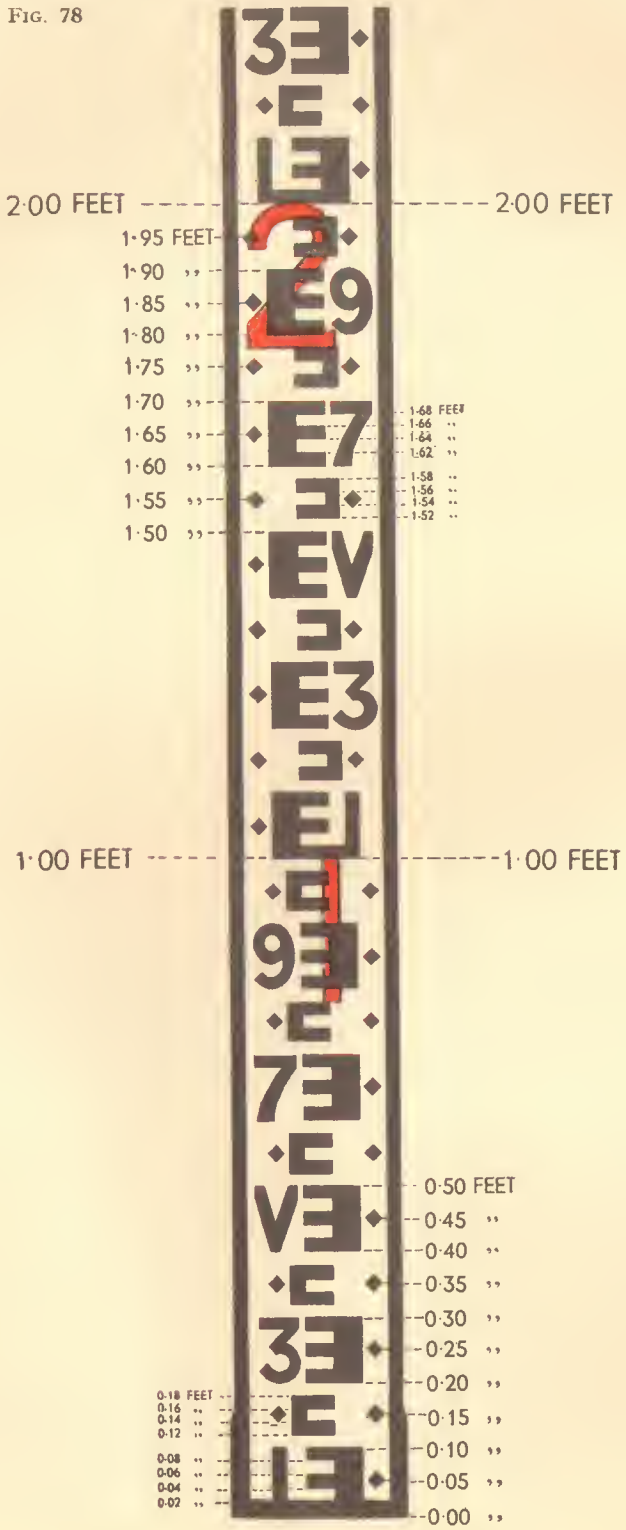
If the distance D_1 is 100 times the length X_1Y_1 and the upper and lower stadia lines on the diaphragm are arranged to cut the rays as shown, the distance of the staff from the axis of the instrument is equal to $100\times X_1Y_1$. Similarly, if the staff is at D_2, its distance is $100\times X_2Y_2$.

Tachymeter measurements in traverses should be limited to distances of about 800 ft. as a maximum. At longer distances it is rarely possible to read the staff accurately unless the light is particularly good. Errors of 1/500 should not be exceeded under good conditions.

A typical example, showing the style of graduation of a levelling staff, is shown in Fig. 78. This Admiralty pattern staff is made in three sections and can be extended to a length of 15 ft. It is fitted with a circular bubble level and is intended for use in a vertical position only.

ADMIRALTY PATTERN

FIG. 78



Correction of Tachymeter Measurements for Slope.

Small differences of level, up to an equivalent of 3° of elevation or depression, may be neglected as the corrections required to reduce the measurements to their horizontal components are insignificant. When larger elevations or depressions are involved, a correction is required which will :

(1) Reduce the measurement to its horizontal component.

(2) Adjust the tachymeter reading to what it would be if the staff was held perpendicular to the line of sight.

In Fig. 79, the observing instrument is at O and the staff is held vertically at S. OL is the horizontal through O so OLS is a right angle. OE is the line of sight, *i.e.*, the collimation line of the telescope, and angle EOL (=e) is the angle of elevation. The distance OL is required.

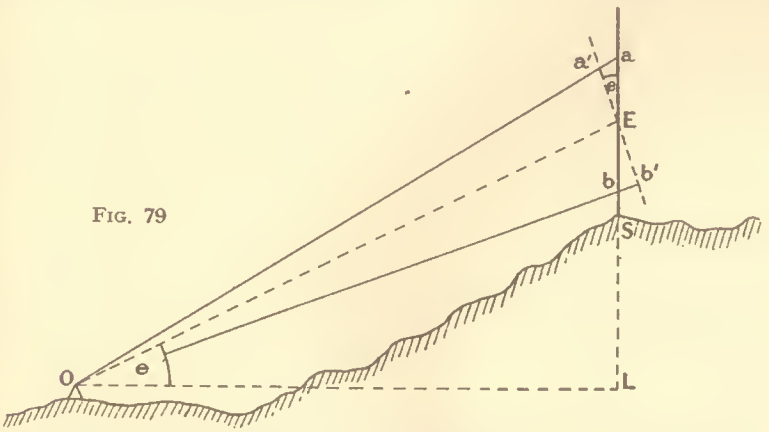


FIG. 79

The distance OE is observed and the readings intersected on the staff are a and b, but it is clear that the readings required are a' and b' where (a' Eb') is drawn perpendicular to OE. Therefore $a'-b'=(a-b)\cos e$.

If k is the constant multiplier of the instrument, $OE=k\,(a-b)\cos e+(f+c)$.

$$OL=OE\cos e.$$
$$=k\,(a-b)\cos^2 e+(f+c)\cos e.$$

Or, if an anallatic lens is fitted,

$$OL=k\,(a-b)\cos^2 e.$$

(NOTE.—Corrections for slope are given in tabular form in Table X A.)

(*c*) The linear measurements may be made by the subtense method, which has been explained under the heading " Base Measurement," in Chap. II (p. 17). Some theodolites are fitted with an eyepiece micrometer, which enables the small angles to be measured with rapidity and accuracy, and very good results can be obtained.

For short traverses, such as are used for fixing small portions of the coastline and other topographical detail, a sextant and 10-ft. pole are frequently used.

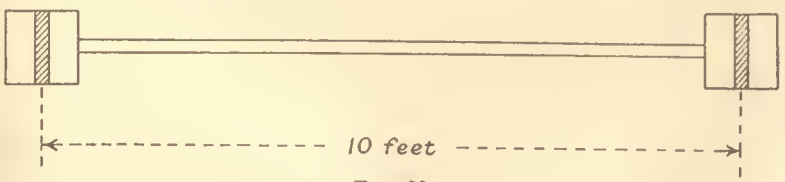


FIG. 80

A convenient form of the 10-ft. pole can be constructed with two rectangular wooden frames, each about 18 ins. square and covered with canvas, fitted by means of sockets at the back to the ends of a light pole (Fig. 80). Pins

passed through the sockets and pole keep the frames fixed. The canvas on each frame is painted white, with a black stripe about 2½ ins. wide and perpendicular to the axis of the pole. The sockets and pins are so arranged that, when the frames are mounted, the distance between the centres of the black stripes is exactly 10 ft. (This is a convenient length for transport.)

The pole is held at right angles to the line of sight and the angle subtended by the stripes is measured with a sextant by bringing them into coincidence. A table of distances, corresponding to the angle subtended by the length of the pole in use, is carried in the field for reference on the spot : *vide* Table XV.

A sighting vane may be fitted to the centre of the pole to enable it to be held at right angles to the line of sight, or, alternatively, the pole may be swayed backwards and forwards a few degrees in such a manner that it must at some point be at right angles to the line of sight ; the maximum angle subtended by the stripes is the angle required.

The maximum distance, at which good results can be obtained by this method, when using a 10-ft. pole, is not more than 250 to 300 yds., and measurements should be limited accordingly. Under poor light conditions measurements should be shorter. Longer poles may be used but are a doubtful advantage unless the subtended angles are measured by more accurate means than is possible with a sextant. When long " shots " are unavoidable, it is better to measure a small distance on the ground at right angles to the line of sight and observe the subtended angle with a theodolite.

Even with careful observations, errors of 1/100 must be anticipated when using a sextant and 10-ft. pole, and, in view of this low standard of accuracy, this method should be confined to very short traverses, such as those between adjacent sounding marks, and to cases where the nature of the ground precludes the use of fixed instruments. Tachymeter methods are always preferable on large scales.

Traverse Measurement : Procedure, Computation and Adjustment.

The type of traverse most commonly required in hydrographic surveys is the *closed* traverse, *i.e.*, that which begins and ends at the same point, or which extends between two points whose positions have already been determined.

Suppose it is required to fix some secondary points for the survey of a narrow strait which runs between A and B, Fig. 81, A and B are two intervisible stations which have already been fixed by triangulation ; the length and bearing of the side AB are therefore known. The procedure is as follows :

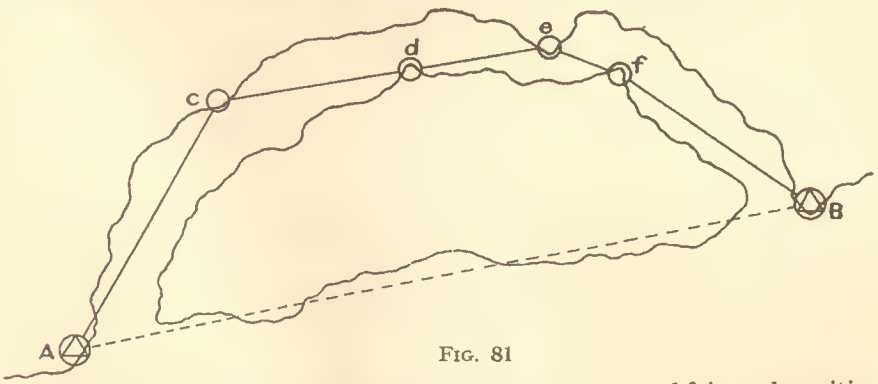


Fig. 81

(1) Select and mark intermediate points, such as c, d, e and f, in such positions that each is visible from adjacent marks on either side of it.

(2) Set up the theodolite at A, and, using B as the reference object, observe the angle BAc and measure Ac.

(3) Proceed to c, and observe the angle Acd and measure cd. Similarly at d, e and f, and at B observe the angle ABf.

Computation and Adjustment.

Traverses met with in hydrographic surveying are generally over such short distances that plane co-ordinates can be used. The work of computation is a summation of the terms s.sin α and s.cos α where s is the distance traversed on each leg and α is the co-ordinate bearing of the leg. The work is much simplified by the use of a calculating machine.

Errors of measurement which combine to result in a misclosure of the traverse are of two kinds, angular and linear. Those due to angular measurement may be either positive or negative and will not normally be cumulative, whereas linear errors, owing to the nature of the ground or lack of accuracy in a tape, are more likely to be so.

The total error is, therefore, one of scale rather than angle and will be proportional to the traversed distance from the starting point. It should be noted that it is not proportional to the co-ordinate distance and that an attempt to adjust from this would correct for scale but not for the " swing " of the traverse.

The observed angles are therefore used in the adjustment and the co-ordinates of each station in the traverse are amended in the ratio of the distance measured. If required, the adjusted co-ordinate bearings of the legs may be deduced from the corrected values of the co-ordinates.

An example of the adjustment of a traverse appears on the following page. If the angles are uniformly measured *clockwise* at each station, the bearings of the legs are more easily calculated ; merely adding the clockwise observed angles to the previous forward bearing and deducting 180°.

From the final entry in columns 10 and 11 it is seen that the closing error of the x co-ordinate is 1·61 ft. or ·4147 ft. per 1,000 ft. measured, and to the y co-ordinate is 7·25 ft., or 1.8633 ft. per 1,000 ft. measured. Multiplying ·4147 by 1·221, 2·037, . . . etc., from column 5 gives the corrections to the x co-ordinates in column 12. Similarly multiplying 1·8633 by 1·221, 2·037, . . . etc., from column 5 gives the corrections to the y co-ordinates in column 13.

Columns 14 and 15 show the final co-ordinate values.

It often happens that the traverse crosses and re-crosses the line joining the two terminal points, as in Fig. 82.

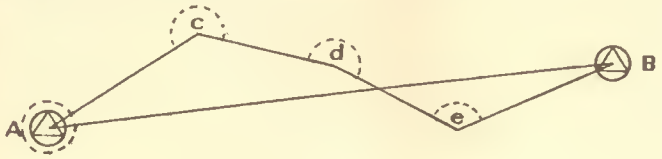


Fig. 82

The procedure is exactly the same as in the previous case. The angles are measured clockwise uniformly at each station. As before the bearings of the legs are calculated by adding the clockwise observed angles to the previous forward bearings and deducting 180°.

When a traverse is carried round a complete circuit and " closed " on the starting point, it is clear that the co-ordinates of this point, as deduced through the traverse, should be zero. Any error in this respect is dealt with in precisely the same manner as in the numerical example already given.

Open or hanging traverses, *i.e.* those which do not close on a previously fixed point, are rarely used in hydrographic work. They can only be checked by astronomical observations for geographical position and azimuth, in which case the traverse may be adjusted by a similar process to that in the numerical example just given, differences of latitude and departure replacing differences of rectangular co-ordinates.

EXAMPLE OF TRAVERSE COMPUTATION

DATA: Initial Station, A: Rectangular co-ordinates, x = 0 y = 0
Final Station, B: Rectangular co-ordinates, x = + 3050·6 y = + 674·6

Co-ordinate bearing A — B = 077° 31′ 50″

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Station	Included Angle	"α" Forward Bearing	"s" Forward Distance	Total Distance	sin α	cos α	s sin α	s cos α	x′	y′	Corrn. to x′	Corrn. to y′	x (10 ± 12)	y (11 ± 13)
	° ′	° ′ ″	feet	feet			feet	feet	feet	feet	feet	feet	feet	feet
A (data)		077 31 50	(3124·3)		+0·97641	+0·21592	+3050·6	+ 674·6			(0·4147 per 1000)	(1·8633 per 1000)		
A	311 20	028 51 50	1221·0	0·0	+0·48273	+0·87577	+ 589·41	+1069·32	+ 0·00	+ 0·00	+0·00	+0·00	+ 0·0	+ 0·0
c	232 02	080 53 50	816·5	1221·0	+0·98741	+0·15821	+ 806·23	+ 129·16	+ 589·41	+1069·32	+0·51	+2·28	+ 589·9	+1071·6
d	177 47	078 40 50	572·5	2037·5	+0·98055	+0·19628	+ 561·37	+ 112·37	+1395·64	+1198·48	+0·85	+3·81	+1396·5	+1202·3
e	213 30	112 10 50	321·0	2610·0	+0·92600	−0·37753	+ 297·25	— 121·19	+1957·01	+1310·85	+1·08	+4·87	+1958·1	+1315·7
f	191 08	123 18 50	951·0	2931·0	+0·83567	−0·54923	+ 794·73	— 522·31	+2254·26	+1189·66	+1·22	+5·47	+2255·5	+1195·1
B	314 18	257 36 50		3882·0					+3048·99	+ 667·35	+1·61	+7·25	+3050·6	+ 674·6
B (data)		257 31 50							+3050·6	+ 674·6				
								diff.	1·61	7·25				
						Error per 1000 ft. = diff. ÷ 3·882			0·4147	1·8633				

NOTES: Column 2—The "included angle" at a station is measured clockwise from previous station.
3—"Forward Bearing" = previous forward bearing + included angle — 180°.
5—"Total Distance" is the distance from A measured along the traverse, *i.e.* the accumulative sums of column 4.
10—x′ of a station is the algebraical sum of the entries in columns 8 and 10 for the preceding station.
11—y′ of a station is the algebraical sum of the entries in columns 9 and 11 for the preceding station.

CHAPTER VII

HEIGHTS AND LEVELLING

The heights of all the more prominent land features in the vicinity of the coast must be determined in order that they may be recorded on the chart. These heights are of value to the mariner in enabling him to form a general idea of the character of the land features and to obtain his distance off-shore by means of vertical angles, and they are essential to the surveyor to enable him to draw in the contours of the ground.

On Admiralty charts all such heights are referred to the level of Mean High Water Springs. This datum is chosen for the benefit of the mariner so that he will not be led into underestimating his distance off-shore, as found from vertical angles, if he neglects to apply tidal corrections. From the surveyor's point of view the datum is not so satisfactory since the level of mean high water springs may vary several feet within the confines of a single chart and contours obtained from land survey maps using mean sea level as datum cannot, strictly speaking, be transferred, as they stand, to a chart. Fortunately, heights and contours on the chart need not always be of the standard of accuracy required in land surveys.

The height of *low water* features, *i.e.*, those which show only at certain states of the tide, are referred to sounding datum which is approximately the level of mean *low* water springs. Such heights can usually be obtained by direct measurement from the water level and a reference to the tide-pole readings at the time, or by sounding over the features in the ordinary way.

The methods generally used by hydrographic surveyors for determining differences of height are as follows :—

(1) Vertical angles with a theodolite from stations of known height on shore ;
(2) Elevations with a sextant measured above a sea or shore horizon from a ship or boat at anchor ;
(3) Aneroid barometer ;
(4) Spirit levelling.

In methods (1) and (2) the horizontal distance between the observer and the object must be calculated from the triangulation or taken from the plotting sheet.

(1) Theodolite Heights

The measurement of vertical angles with the theodolite has been discussed in previous pages. Before such angles can be used for calculating differences of height, they must be corrected for :—

(i) Curvature.
(ii) Refraction.

(i) **Curvature.**

Fig. 83 shows a vertical section through O, the centre of the earth. A and B are two objects on the surface *at the same level.* AP, a tangent to the earth's surface at A and therefore the horizontal at A, is drawn to meet OB produced at P. The vertical angle to B, observed at A, is the angle PAB, a depression, and, the effect of refraction being for the moment neglected, the difference between the apparent and true levels of B is represented by BP = x. If AP = s and the radius of the earth (OA or OB) = r, then

$$(r + x)^2 = r^2 + s^2 \quad \text{or,} \quad x\,(2r + x) = s^2$$

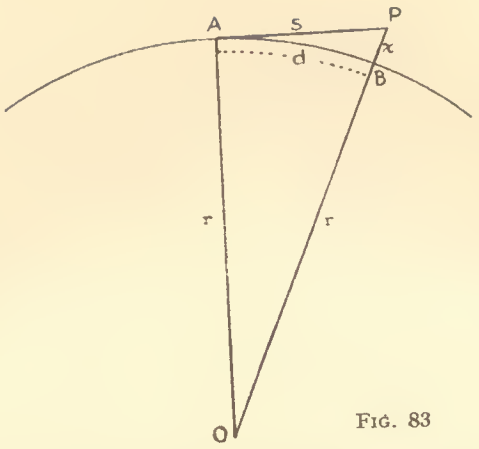


FIG. 83

Since the distance AB is always very short in comparison with r, 2r can be substituted for $2r + x$ and d^2 for s^2 without appreciable error, where d is the length of the arc of the earth's surface between A and B.

The equation then becomes :

$$2r\,x = d^2, \quad \text{or} \quad x = \frac{d^2}{2r}$$

The correction represented by x is always additive to angles of elevation and subtractive to those of depression.

(ii) **Refraction.**

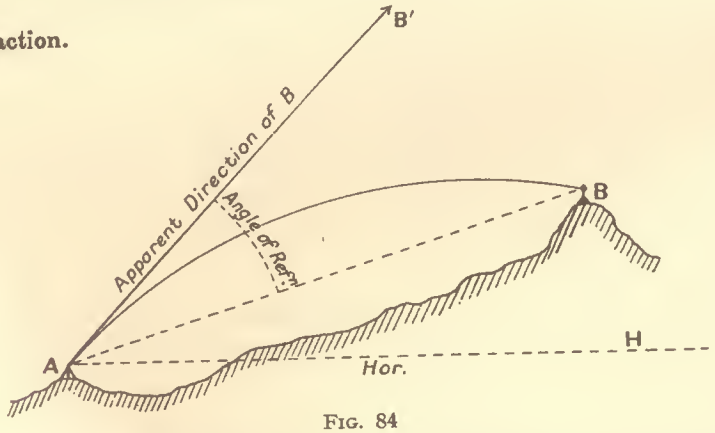


FIG. 84

The ray of light from a distant object follows a curved path to the observer, making the object appear, as a rule, higher than it really is. In Fig. 84, A is an observer and B a distant object. The ray from B follows a curved path as shown in the figure so that to the observer at A it appears that B lies in the direction B′ where AB′ is drawn tangential to the curve at A. If AH is the horizontal plane through A, then the true vertical angle to B is BAH, but the observed angle is B′AH. The difference, angle B′AB, may be called the angle of refraction, and this varies a small amount with the place, season and time of day. The *co-efficient of refraction* k is expressed by the quantity r r sin $1''/d$, where r is the angle of refraction in seconds, r is the radius of the earth at that place, and d is the distance between the observer and the distant object. The average value of k is about 0·07.

The correction for terrestrial refraction, y, may be obtained approximately from the formula $y = \frac{k\ d^2}{r}$.

Combined Correction for Curvature and Refraction.

Combining the two corrections x and y, the total correction, K, in seconds to vertical angles $= \frac{d^2}{r} \cdot \frac{(1 - 2k)}{2}$. Table VII is calculated from this formula, using a value of 0·07 for k. As x is always larger than y, K is additive to elevations and subtractive from depressions.

When the distance, d, between the observer and the distant object is more conveniently found in terms of nautical miles, the value of K in seconds may be taken as $\frac{100d}{4}$ with sufficient accuracy for most purposes.

If a vertical angle has been observed at only one of the stations (*e.g.*, vertical angles to intersected points), the correction for curvature and refraction is conveniently applied as follows :

Call angles of depression D, those of elevation E. The value of K, the combined correction, is found from table VII.

Then, if S = K − D or K + E,

Δh = tan S × distance between the stations.

Reciprocal Observations.

By observing reciprocal vertical angles between two stations, the corrections for curvature and refraction can be avoided, while the mean refraction can, if necessary, be calculated.

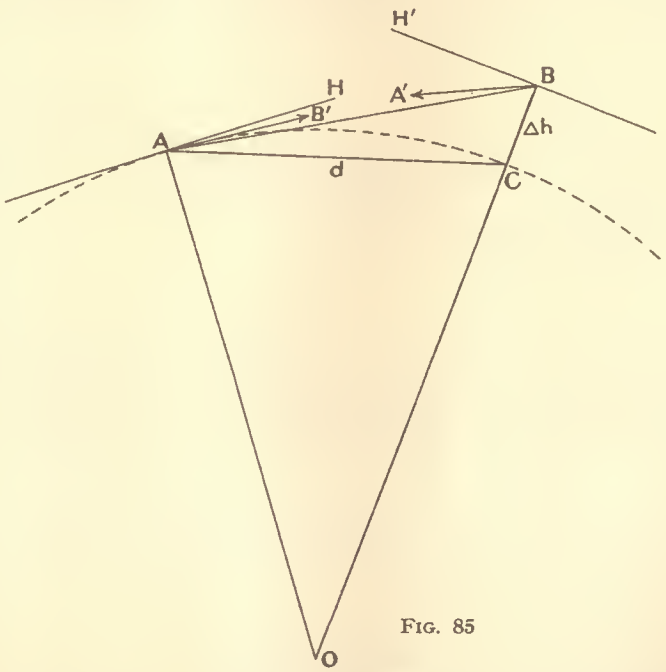


FIG. 85

Fig. 85 represents a vertical section through O, the centre of the earth ; A and B are two stations from which reciprocal vertical angles are measured. Join OB and draw AC to meet it at C so that OA=OC. Then the difference of height, Δh, between A and B is BC. AH and BH′ are the horizontals through A and B respectively. Owing to refraction B appears from A to be in the direction AB′ and A appears from B to be in the direction BA′. In this example both observed angles are depressions ; that at A, D*a*, is HAB′ and that at B, D*b*, is H′ BA′. It is assumed that refraction is equal at A and B, *i.e.*, angle B′ AB = angle A′ BA : call the value of these angles θ.

Then angle BAC $= 90° - Da - \theta -$ CAO .....(i)

Angle CAO = angle ACO (by construction).

$= BAC + ABC$

$= BAC + (90 - Db - \theta)$.....(ii)

Combining (i) and (ii),

$$2\,BAC = Db - Da \quad \text{or,} \quad BAC = \frac{Db - Da}{2}$$

Since the distance between A and B is very small in comparison with the radius of the earth, the chord AC may be taken as equal to its arc d and ACB may be regarded as a right angle. The difference of height, Δh, is then given by :

$$\Delta h = d \tan BAC = d \tan \frac{Db - Da}{2}$$

If one angle is an elevation and the other a depression, the formula becomes :

$$\Delta h = d \tan \frac{Ea + Db}{2}, \quad \text{or} \quad \Delta h = d \tan \frac{Da + Eb}{2}$$

Correction for Heights of Beacons and Instruments.

The required heights of stations are usually those of the ground level or surface marks at the stations. It is, therefore, obviously necessary to correct the results for the heights of beacons and instruments.

Supposing vertical angles are observed from A (Fig. 86) to B and C.

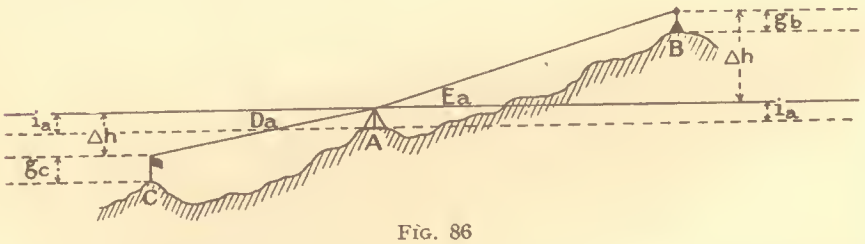


FIG. 86

The height of the instrument at A is i_a and that of the marks at B and C, g_b and g_c respectively. The difference of height found by calculation is Δh, and it will be seen that in the case of B, where the vertical angle is one of elevation, the true difference of height is $\Delta h + i_a - g_b$, and in the case of C, where the angle is one of depression, the true difference of height is $\Delta h - i_a + g_c$.

Generally in reciprocal observations, if i_a, i_b are the heights of the instruments at two observing stations, and g_a, g_b, the heights of those points on the beacons to which observations have been taken, the correction to be applied to the difference of height (already calculated) is

$$+ \tfrac{1}{2}(g_a - g_b + i_a - i_b).$$

Examples of calculating Trigonometrical Heights.

Differences of height can be conveniently calculated in the manner shown in the following form. The meanings of the various symbols are as follows :

D_a and E_a denote the vertical angle of depression or elevation at A.

D_b' and E_b denote the vertical angle of depression or elevation at B.

i_a and g_a denote heights of instrument and beacon respectively above ground level at A.

i_b and g_b denote corresponding measurements at B.

K denotes the combined correction for curvature and refraction.

Three examples are given :

(1) With the height of A known and reciprocal observations having been made between A and B.

(2) With the height of A known and a vertical angle from A to B.

(3) With the height of B known and a vertical angle from A to B.

	1.	2.	3.
Station A	Bang.	Bang.	Line.
Height of A above M.H.W.S.	51·5	51·5	
i_a	4·0	4·0	5·3
g_a	27·4		
Station B	Ham.	Zed.	Hold.
Height of B above M.H.W.S.			472·2
i_b	4·8		
g_b	15·0	63·4	27·5
Distance AB in feet (d)	36580	22610	58770
I (a)—	° ′ ″		
$+ E_a$ or $- D_a$	−0 01 08		
$+ D_b$ or $- E_b$	+0 04 12		
	+0 03 04		
Divide algebraic sum by 2			
∴ S (±)	+0 01 32		
I (b)—		° ′ ″	° ′ ″
$+ E_a$ or $- D_a$		(E_a) +3 15 05	(D_a) −1 00 12
K		+ 1 36	+ 4 10
∴ S (=K − D or K + E) ±		+3 16 41	−0 56 02
II—			
log tan S	$\overline{4}$·64936	$\overline{2}$·75797	$\overline{2}$·21221
log d	4·56324	4·35430	4·76916
log Δh	$\overline{1}$·21260	$\overline{3}$·11227	$\overline{2}$·98137
∴ Δh (±)	+16·3	+ 1295·0	+ 958·0
Correction for instruments and beacons	+ 5·8	− 59·4	+ 22·2
Known height of A or B	A 51·5	A 51·5	B 472·2
Required height of B or A	B 73·6	B 1287·1	A 1452·4

Note.—If reciprocal angles have been observed, use space I (a) : otherwise use I (b).

(2) Heights by Sextant Elevations

Sextant angles from a ship or boat at anchor are often used for determining the approximate heights of the land features. The results are considerably less accurate than those obtainable with a theodolite but are sufficiently good for intersected points, such as hill summits, which are not usually very definite in outline. In wooded country the actual ground level is of little interest to the mariner, for when determining distance by means of vertical angles, these must be observed, in such cases, to the tops of the trees and it will be sufficiently accurate in a coastal survey to record their heights (above mean high water springs) to the nearest 10 ft. Points, the heights of which are determined by sextant angles, should not be used as reference levels for finding the heights of other points in the survey.

Two general cases arise :

(a) When the sextant angles are measured above a *sea* horizon.

(b) When the sextant angles are measured above a *shore* horizon.

(*a*) Heights from Vertical Angles to a Sea Horizon.

Figure 87 represents a vertical section through O, the centre of the earth. E is the observer's eye at a height x feet above the water level and B is a hill summit, the height of which is to be determined. EH is the horizontal through E, and EL is drawn tangential to the water surface.

Then the angle measured by the observer is the elevation BEL and the angle HEL is the " Dip " of the sea horizon which will depend on the height of eye of the observer. Values of the angle of " Dip " for various heights of eye are given in table VIII.

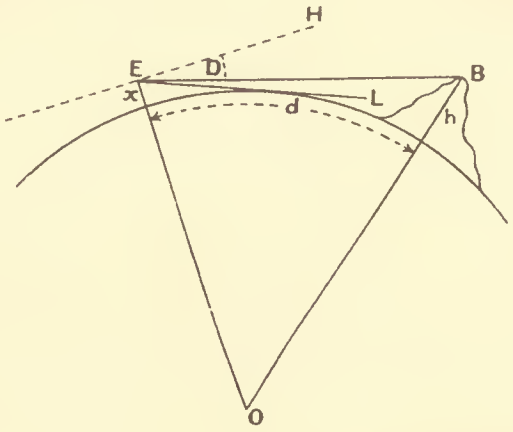


FIG. 87

If a theodolite could be used at E, the angle HEB (D) would be measured, and in dealing with sextant elevations the first process is to convert the sextant angle into what it would have been if a theodolite had been used. The calculation then follows the lines of that used in finding ordinary trigonometrical differences of height. Example :

Data :

Height of eye = 30 feet.
Height of tide = 5 feet : level of M.H.W.S. = 20 feet.
Distance EB (d) = 9·40 nautical miles (of 6,080 feet).
Observed sextant elevation of B = 1° 10′ 30″

Computation :

		1°	10′	30″
Observed sextant angle (corrected for index error) ..		1	05	07
Dip	—		5	23
True elevation (E)		1	05	07
Correction for curvature and refraction $\frac{(100d)}{4}$	+		3	55
Angle " S "		1	09	02
log tan S		$\bar{2}$·30274		
log d		0·97313		
log 6080 (feet in 1 nautical mile)		3·78390		
log Δh		3·05977		
Δh	+	1148 feet		
Correction for height of eye	+	30 ,,		
Correction for height of tide	−	15 ,,		
Height of B		1163 feet		

(*b*) **Heights from Vertical Angles to a Shore Horizon.**

Fig. 88 represents a vertical section through O, the centre of the earth. E is the observer's eye and B a distant hill summit. A vertical plane through E and B cuts the coastline at L and the observer measures the angle BEL. EH is the horizontal through E. In this case the " Dip " is the angle HEL which will vary with the observer's height of eye and the distance EL of the shore horizon. The value of the angle of dip is given by the formula :

$$\text{Tan dip} = \frac{\text{height of eye}}{\text{distance of shore horizon}}$$

Values of dip for various heights of eye and distances are given in Table IX. Using the appropriate value of dip, the computation is the same as with observations to a sea horizon.

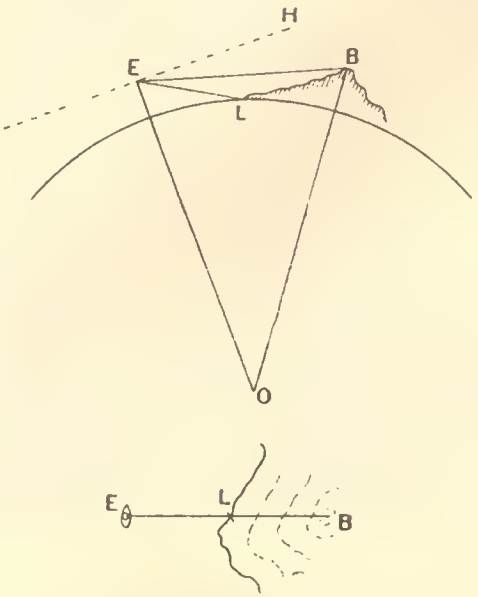


Fig. 88

Notes on Sextant Elevations.

(i) Abnormal refraction cannot be allowed for and may be responsible for erroneous results.

(ii) When the shore horizon is close, observe from as low a position as possible. When the distance of the horizon is less than half a mile, the method should not be used.

(iii) Errors due to observing the shoulder of a hill instead of its summit must be guarded against by locating the observing stations in such positions that the required summits are not accidentally obscured.

(iv) For long rays observing sextants should be used and angles should be measured " on " and " off " the arc.

(3) Barometer Heights.

The aneroid barometer provides a convenient means of determining differences of height with fair precision. Changes of height are accompanied by differences of atmospheric pressure amounting to approximately 1 inch per 1,000 feet, and a portable aneroid is usually graduated with a scale of feet which can be read with an accuracy of about $\pm$ 5 ft. It is clear, however, that,

before the aneroid readings can be accepted, allowance must be made for any changes of atmospheric pressure common to the district during the time the instrument is in use. Furthermore, the readings of all barometers are affected by changes of air temperature (this applies as much to aneroids which are stated to be " compensated " as to other types of barometer) and, to obtain the best results, readings should be taken in inches or millibars and corrected for temperature (*vide* Close's Text Book of Topographical Surveying, Chapter IX). The temperature corrections, however, are not very large and for the hydrographic surveyor's purpose may normally be neglected.

To use the aneroid, proceed first to some place of known height—the hydrographic surveyor can generally use sea level—and note the reading and time. Carry out the same procedure at those places whose heights are required and return finally to some other place of known height for a final comparison. Assuming the operation starts and finishes at sea level, a curve should be drawn showing the correct aneroid readings at sea level, allowing for the changes in pressure which have occurred during the day. These changes can be deduced from the readings of the mercurial barometer or barograph on board ship, for it may be assumed that pressure changes near the coast will be the same as those experienced on board ship. The required differences of height can be obtained by subtraction of the curve readings at the times of observation.

Aneroids should always be read in the same position, *i.e.*, horizontal or vertical ; they should be tapped gently and given a short period to settle down before being read.

(4) Levelling.

Levelling is one of the means adopted for finding differences of height during a survey and is carried out with an instrument called a " level." These differences are directly obtained by readings of a horizontal line of sight directed on graduated (" levelling ") staves held vertically over the stations and at intermediate points. In land surveys levelling occupies a position of great importance, but in hydrographic surveys it is normally confined to the determination of the difference in level between sounding datum and adjacent bench-marks.

Description of Level.

The essential feature of a level is a telescope mounted in such a manner that ts line of collimation may be made horizontal. Plate H shows a modern type of instrument.

The level is mounted on tripod legs, three footscrews B engaging in slots on the tripod head and the attachment being made by means of the screw A which passes through the head and engages in a flexible tribrach attached to the footscrews.

Rough levelling is effected by means of the three footscrews and a circular spirit level C under the eyepiece end of the telescope.

The telescope is of × 25 magnification and is internal focusing. It is fitted with cross-wires and stadia wires : when used for measuring distances by tachymeter methods there is no " constant " correction to be applied.

The main spirit level D is fitted in a metal guard which is rigidly attached to the telescope barrel. Adjusting screws E (not visible in the plate) enable the level tube to be tilted through a small angle and brought into correct adjustment with the telescope. The position of the bubble in its tube is seen from the eye end of the telescope by means of a prism reader and the eyepiece G. A movable reflector F throws light into the tube to illuminate the bubble. Both ends of the bubble are seen in the eyepiece and the instrument is " levelled " by tilting the telescope by means of the fine-setting screw H until they coincide. The appearance of the bubble in the reader is shown in Fig. 89.

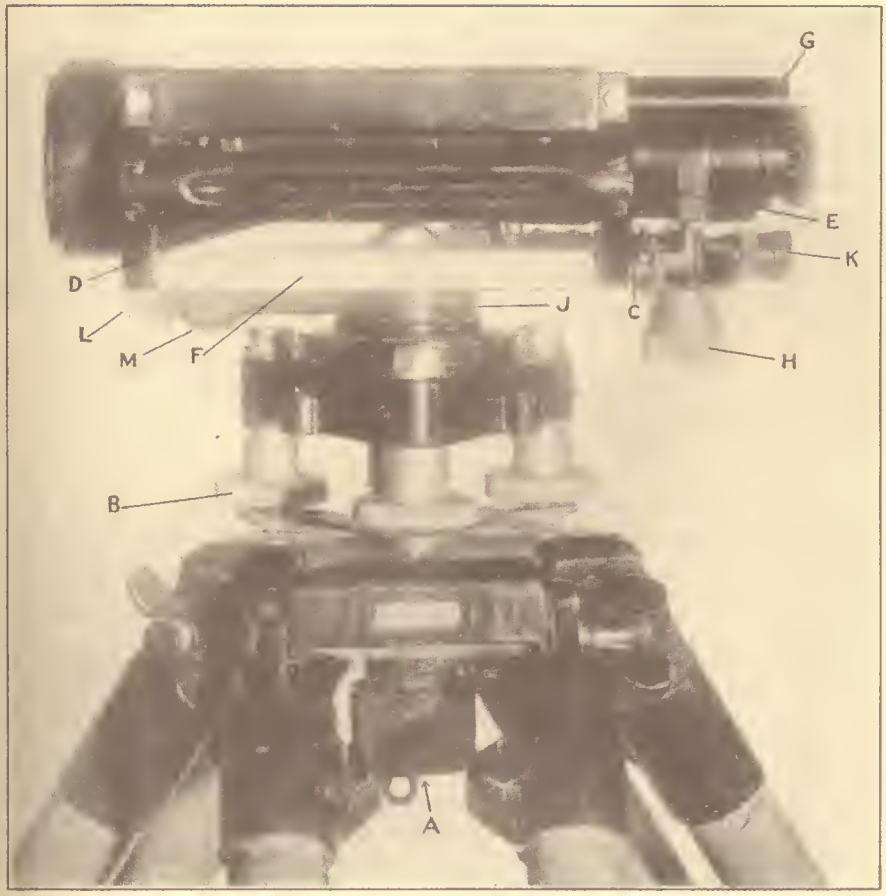


PLATE H

A horizontal circle is fitted under the cover J and can be read by the eyepiece K to an accuracy of 2′ of arc. The instrument can therefore be conveniently used for coastlining and other topographical purposes. The clamp L and slow-motion screw M enable the telescope to be set on any required bearing. The horizontal circle is illuminated by a reflector which cannot be seen in the plate.

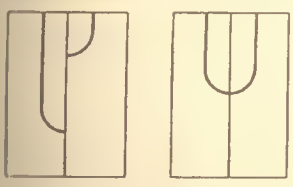


FIG. 89

Other modern types of level are similar in principle though they may differ in design; in some a ball and socket arrangement is fitted for rough levelling; in some the position of the main bubble is read by means of a mirror reflector mounted on the tube.

Setting up and Adjustment of Level.

(*a*) To set up the level, push the legs firmly into the ground so that the tripod head is roughly level.

(*b*) Secure the upper part of the instrument to the tripod head and by means of the footscrews bring the bubble in the circular level central.

(*c*) Having placed a levelling staff over the first station from which the line of levelling is to be carried, intersect the staff and focus the telescope for distinct vision and elimination of parallax (see adjustment of theodolite telescope (p. 34)).

(*d*) Complete the levelling by bringing the two ends of the bubble of the main

level into coincidence with the fine setting screw. Read and book the reading of the horizontal cross-wire on the staff.

(*e*) Operation (*d*) will have levelled the telescope for that observation *only*. When the staff is moved to a new position and the level is turned in azimuth to intersect it again, the ends of the bubble in the main spirit level must *again* be brought into coincidence before the staff is read.

Procedure when levelling.

Example. It is required to determine the difference in height between the zero of a tide pole and a fixed bench-mark on a house.

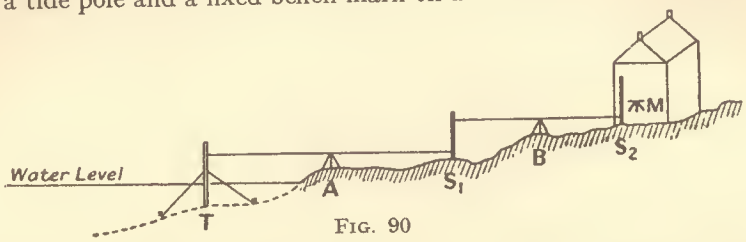


FIG. 90

In Fig. 90, T represents the tide pole and M the bench-mark. (Note that bench-marks are usually cut in the form of a broad arrow with a horizontal line (the mark) at its head.)

(*a*) Set up the instrument at A and direct the telescope on the tide pole. Make the final levelling adjustment ((*d*) above), then read and book the height on the pole cut by the horizontal wire.

(*b*) Send an assistant to hold the levelling staff at S_1, in such a position that $AT = AS_1$ approximately. Rotate the telescope and direct it on the staff, and after re-levelling, read and book the height on the staff cut by the horizontal wire.

(*c*) Take the level to a position B, so that $BM = BS_1$ approximately. Set up and adjust, and read the height on the staff at S_1 (which must be turned round to face the level) again.

(*d*) Send the staff to S_2 to be held alongside M, and after re-adjusting the telescope, observe and book the reading on the staff.

(*e*) Before removing the staff note the height on it corresponding to the horizontal line of the bench-mark.

(*f*) Compute the difference in height as follows, noting that a line of levels consists of a series of *back* and *forward* readings, and that the difference in height between the beginning and end of the line is the difference between the sums of the back readings and the sums of the forward readings.

Station.	Back reading.	Fore reading.
	ft.	ft.
A	17·64 (tide pole)	1·02
B	12·17	2·10 (by bench-mark)
Sums	+ 29·81	— 3·12
	— 3·12	
Difference of height between zero of tide pole and zero of staff at S_2	+ 26·69	
Height of M above zero of staff at S_2 ...	+ 3·95	
∴ Therefore zero of tide pole is	30·64 ft. below bench-mark M	

It should be noted that the accuracy of the levelling depends largely on the staff holder, who must pay particular attention to the following points:

(i) The staff must be held truly vertical. In most patterns of staff a small spirit level is fitted to assist in this.

(ii) When rotating the staff from its position for a forward reading to that for the following back reading, great care must be taken that no vertical displacement occurs. On soft or steeply sloping ground a peg should be driven in for the staff to rest on.

Errors of Levelling and their Correction.

The most important source of error in levelling, and the only one which need be considered here, is collimation error. If the bubble is brought to the centre of its run (by making the ends as seen in the prism reader coincident) the line of collimation, which may be defined for our purposes as the line joining the centre of the object glass to the intersection of the cross-wires, should be horizontal. To test whether this is the case, proceed as follows :

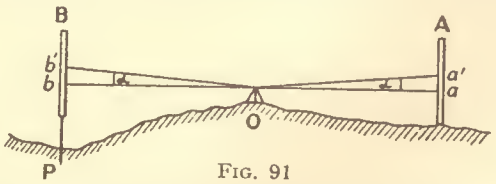


FIG. 91

Select a piece of fairly level ground and have two levelling staves, A and B (Fig. 91) held vertically about 100 yards apart. The ground at B being lower than at A, put B on a peg P driven into the ground. Set up the level at O exactly midway between A and B.

Bring the bubble to the centre of its run, and intersect and read staff A. Rotate the level and, again bringing the bubble to the centre of its run, intersect staff B. Drive in P until the reading of B is the same as that of A. A and B must then be at the same level whether the instrument is in adjustment or not. For, supposing *ab* is the horizontal and that the line of collimation (when the bubble is central) makes an angle α with the horizontal, then, since the line of collimation is fixed with reference to the horizontal when the bubble is central, the staff readings will be a' and b', and $aa' = bb'$.

Now set up the instrument as in Fig. 92 on the alignment of A and B.

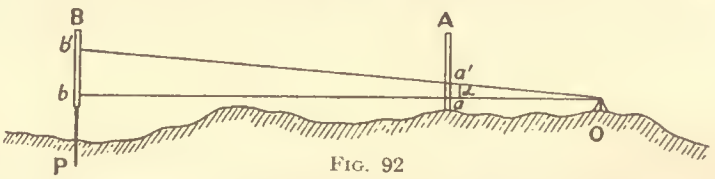


FIG. 92

Bring the bubble central and intersect the staves. If the instrument is in perfect adjustment, the readings a and b will be the same since the staves are at the same level. If, however, the readings are a' and b' which are not equal, a' b' being inclined at an angle α to *ab* the horizontal, the telescope should be tilted by means of the fine setting screw until the two readings are the same, and the bubble should then be centred in its tube by the adjusting screws fitted for the purpose.

An objection to this method of adjustment is that is is necessary to alter the focus of the telescope in the second part of the operation as the staves are not equidistant from the instrument. This may alter the line of collimation, but with modern internally-focusing telescopes the displacement should be practically negligible.

When levelling the forward and back shots should always be made (as nearly as possible) equal in length ; by so doing, errors of collimation are automatically cancelled and no corrections for curvature or refraction are necessary.

Note.—A theodolite can be used for levelling, but is much less handy than the level.

CHAPTER VIII

PLOTTING AND GRADUATION

Triangulation provides the surveyor with a series of points fixed in relation to each other. From these points the detail of the survey is located. When the triangulation has been completed and the observations have been adjusted, the next step is to plot the positions of the various trigonometrical points on paper.

The Plotting Sheet.

The following descriptions of paper are supplied for use as plotting sheets :

(1) Paper, Drawing, Antiquarian, H.P. (53 ins. × 31 ins.) mounted on linen.

(2) Paper, Drawing, Double Elephant, H.P. (40 ins. × 26¾ ins.), mounted on linen.

(3) Paper, Drawing, continuous Roll, 6 ft. × 6 yds., mounted on linen.

No. (1) or (2) should be used if large enough to embrace the survey. When larger sheets are required, they must be cut off No. (3).

Surveying ships are provided with a plotting table which is solidly constructed and carefully planed to ensure a flat and even surface. These tables are usually about 6 ft. square and this, therefore, is the size of the largest sheet which can be plotted conveniently.

The paper for the plotting sheet should be stretched out flat and well weighted down on the table several days before it is actually required for use. This gives the paper a chance of " settling down " after being unrolled and should ensure that it does not change shape to any material extent whilst in actual use. If possible, the plotting sheet should be kept flat during the whole course of the survey and until it is finished with.

The accuracy of the plot is a matter of great importance and every care should be taken of the plotting sheet. Anything in the way of rough handling is certain to distort the paper and consequently impair the results of the survey.

Scale.

The scale of the chart has been discussed in Chapter II (p. 8). The methods employed for plotting the main stations of the survey involve the use of a rectangular grid which must be of such size as to enclose, if possible, the whole area which is to be surveyed. In surveys of great extent it will, of course, usually be necessary to have a series of rectangular grids, each overlapping the next by a small amount. The vertical sides of the grid are parallel to the co-ordinate meridian employed in the survey. The size of the grid (on the paper) and the amount of paper required for the plotting sheet are determined by the area and scale of the survey. If, for instance, the limits of the survey can be contained in a grid whose sides run North-South and East-West and measure 100,000 ft. × 200,000 ft. on the earth, and if the scale of the survey is to be 1/50,000, it is clear that this will be represented by a grid on the paper measuring 2 ft. × 4 ft. and, to allow a suitable margin, a piece of paper 30 ins. × 54 ins. will be required as a plotting sheet.

The rectangular co-ordinates of the main stations forming the framework of the survey are calculated from the triangulation and these provide a ready means of determining what will be suitable dimensions for the grid. Assuming, as will usually be the case, that the limits of the survey are indicated on a small scale chart, the approximate positions of some of the main stations can be

also plotted on the chart and measurements made from them to the sides of a suitable grid. Example :

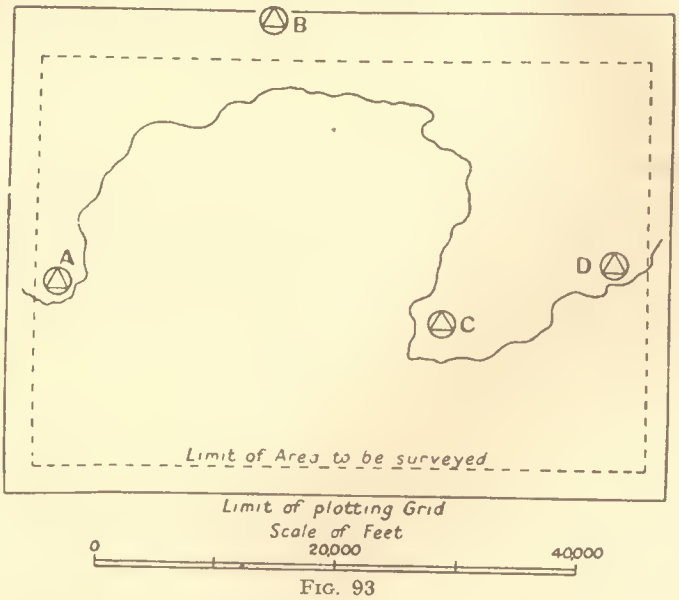


FIG. 93

In Fig. 93, A, B, C and D represent four main stations of the survey plotted on a chart approximately in their correct positions. The limits of the area which is to be surveyed are shown by pecked lines. A rectangular grid, shown with continuous lines, is drawn on the chart to enclose the survey area with a little to spare. Its north side is taken up sufficiently far to enclose the main station B (it is not absolutely necessary for the grid to enclose all the main stations—they can be plotted if some of them lie slightly outside its limits—but it is more convenient to make it do so, if possible). If, for example, the natural scale of the survey be 1/12,500 and the approximate co-ordinates of the stations are :

	x (E. or W.) feet	y (N. or S.) feet
A	0	0
B	+ 18000	+ 21500
C	+ 32000	− 3500
D	+ 46000	+ 1500

Then, by measurement on a rough plot, suitable co-ordinates for the sides of the grid can be selected as follows :

North side .. .. .. $y = +22500$ feet
South ,, ... $y' = -18000$,,
East ,, ... $x = +50500$,,
West ,, ... $x' = -4000$,,

Hence, $y \sim y' = 40{,}500$ ft. and $x \sim x' = 54{,}500$ ft., and since the scale of the survey is 1/12,500, the length of the grid sides on the paper will be :

North and south sides : $\frac{54500}{12500}$ feet $= 52{\cdot}320$ ins.

East and west sides : $\frac{40500}{12500}$ feet $= 38{\cdot}880$ ins.

Allowing a suitable margin, a piece of paper measuring about 58 ins. × 45 ins. will be required for the plotting sheet.

Projections

Before plotting, the various "projections," by means of which the curved surface of the sphere can be represented on a flat sheet of paper, must be considered.

A projection is defined as being any systematic method of representing parallels and meridians, which on the earth's surface are curved, on a flat sheet. By definition therefore, a projection need not be strictly geometrical and it is only in a few cases that it is so.

Representation of Shape.

Since the surface of the earth is curved and is to be represented on a flat sheet, it is not possible to preserve the shape of large areas on the representation. But if at any point the scale along the meridian and parallel through that point is the same (not necessarily correct) and if the meridian and parallel are at right angles to each other, then very small areas on the representation will be the same shape as the corresponding small areas on the earth. A projection which has this property is called *orthomorphic.*

The essential feature of an orthomorphic projection may be defined, therefore, in either of two ways (*a*) at any point the scale is the same in all directions or (*b*) the angles in any small figure on the earth are preserved unaltered on the representation.

It is proposed to deal only with those projections which are in common use for charts and topographical maps. In the general description of these the earth is assumed to be spherical. When considering projections from the standpoint of the cartographer or in surveying computations, the earth must be treated as a spheroid.

There are two broad divisions which may be made; those which can be derived from a cone and those which are classed as "conventional." In the first of these classes are the conical, zenithal and cylindrical projections. Among the conventional projections is Mercator's, and Cassini's is sometimes referred to as a conventional projection though, as will be seen, it can be derived from a simple conical projection.

Conical Projections.

In all common examples of these the meridians are represented by straight lines converging to a point, the vertex and the parallels are circles concentric with this vertex.

The angles between the meridians are a certain fraction n of the angles which the terrestrial meridians make with one another and from Fig. 94 it is clear that n must lie between zero, which will produce a cylindrical projection, and unity which will result in a zenithal projection. (This fraction n is called the constant of the cone.)

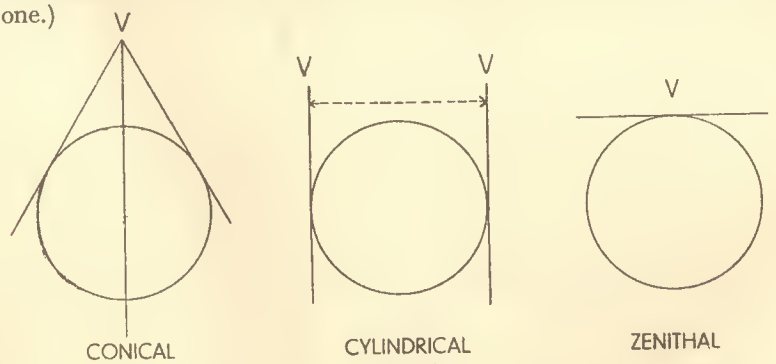


Fig. 94

Only the line of contact of the cone with the sphere will be true to scale. In the case of a cone whose axis is the polar axis of the sphere this true to scale parallel is known as a Standard Parallel.

Conical Projection with one Standard Parallel.

The construction shown in Figs. 95 and 96 illustrates the derivation of this projection. The standard parallel is at Latitude $\phi°$ and R is the radius of the sphere.

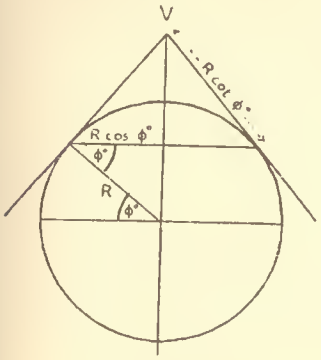


FIG. 95

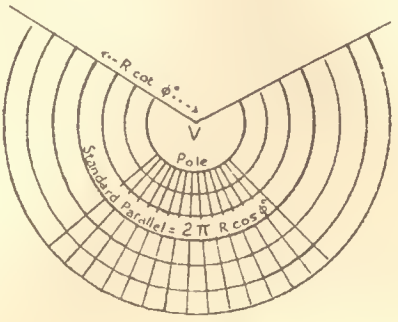


FIG. 96

From Fig. 95 it is seen that the radius of the standard parallel is R cos $\phi°$ and its circumference is 2π R cos $\phi°$. An arc of a circle is therefore drawn, see Fig. 96, whose length is 2π R cos $\phi°$ and whose radius is R cot $\phi°$. If the arc is divided into 36 equal parts and such divisions are joined to the centre of the arc, the lines so formed will represent meridians at intervals of 10°. Concentric circles are drawn at distances equal to their distances, reduced to scale, on the sphere. These will be the parallels of the projection ; the pole is represented by an arc of a circle.

It will be seen that since the distances between the parallels has been made equal, the projection is not geometrical. If it were so, the distances between parallels would increase on either side of the standard parallel as in Fig. 97.

This projection should not, therefore, be used in high latitudes unless the latitude range of the area covered is small and that the area in the immediate vicinity of the pole is not shown.

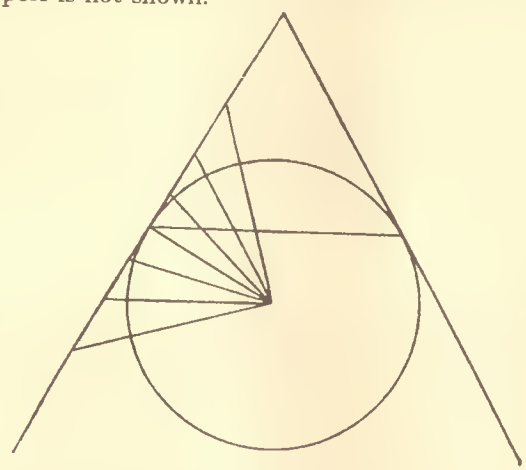


FIG. 97

Referring to Fig. 96, the angles subtended at V, the vertex, by the arc of the standard parallel will be given by,

$$\frac{\text{Angle at vertex}}{360°} = \frac{2\pi \text{ R cos } \phi}{2\pi \text{ R cot } \phi} = \sin \phi$$

In other words n, the constant of the cone, is sin ϕ.

Considering the properties of this projection, it is seen that the scale is correct everywhere along the standard parallel and along each meridian. It can be shown that the scale along any parallel other than the standard parallel is too great and that this error increases with the distance away from this standard parallel; therefore the projection is not orthomorphic since the scale is not equal in all directions at any point.

This increasing error of scale along the parallels as the standard parallel is left can be much reduced by making two standard parallels instead of one.

Conical Projection with Two Standard Parallels.

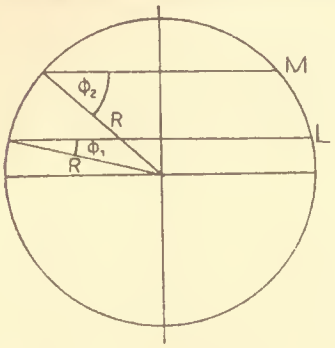


FIG. 98

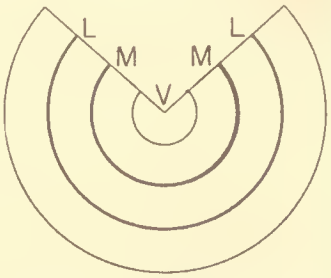


FIG. 99

This is a projection which represents two parallels at their true lengths, separated by their true distance, as concentric arcs round the vertex from which the meridians radiate. Its construction is illustrated in Figs. 98 and 99, in which the selected parallels are those drawn through L and M in latitudes $\phi_1°$ and $\phi_2°$.

From Fig. 98 it can be seen that the lengths of the parallels through L and M are $2\pi R\cos\phi_1$ and $2\pi R\cos\phi_2$ .....(1)

and the distance between them, the arc ML =

$$\frac{2\pi R(\phi_2-\phi_1)}{360} = R\frac{\pi}{180}(\phi_2-\phi_1) \quad \text{.....(2)}$$

Refer now to Fig. 99, showing the developed cone; to calculate n, the constant of the cone, it is necessary to find the angle LVL and the radius VL.

$$n = \frac{LVL}{360} = \frac{\text{arc LL}}{2\pi VL} = \frac{2\pi R\cos\phi_1}{2\pi VL} = \frac{R\cos\phi_1}{VL} \quad \text{.....(3)}$$

Now $\frac{VL}{VM} = \frac{\text{arc LL}}{\text{arc MM}}$ and $\frac{VL}{ML} = \frac{VL}{VL-VM}$ so, $VL = ML\,\frac{LL}{LL-MM}$

Substituting (1) and (2), $VL = \dfrac{R\frac{\pi}{180}(\phi_2-\phi_1)\,2\pi R\cos\phi_1}{2\pi R\cos\phi_1 - 2\pi R\cos\phi_2}$

$$= \frac{R\cos\phi_1\,\frac{\pi}{180}(\phi_2-\phi_1)}{\cos\phi_1-\cos\phi_2}$$

From (3), $n = \dfrac{R\cos\phi_1}{VL}$ and therefore, $= \dfrac{\cos\phi_1-\cos\phi_2}{\frac{\pi}{180}(\phi_2-\phi_1)}$

The scale of this projection is correct along the two selected parallels and along all meridians. On any parallel between the two standard ones the scale will be too small and outside them too large but the error does not increase as

rapidly as it does in the case of the conical projection with one standard parallel.

The same considerations as before will show that this projection is not orthomorphic, but it is more nearly so since the scale errors are smaller.

There are many possible variations of this form of projection which are obtained by (*a*) adjustments of the scale along the meridians, (*b*) by the selection of the position of the two parallels. Common examples are (i) scale along the meridian equal to scale along parallels throughout gives an orthomorphic projection (usually known, after its author, as Lambert's Conical Orthomorphic projection) ; (ii) scale along the meridian inversely proportional to scale along the parallels throughout gives an equal area projection (this is usually known as Albers' Equal Area Projection). For other forms of this basic projection see Hydrographic Department Technical Publication No. 1, "Notes on the Minimum Error Conical Projections."

Polyconic Projection.

This is a widely used variant of the conical projection. If the sphere is supposed to have its surface divided into a large number of conical surfaces

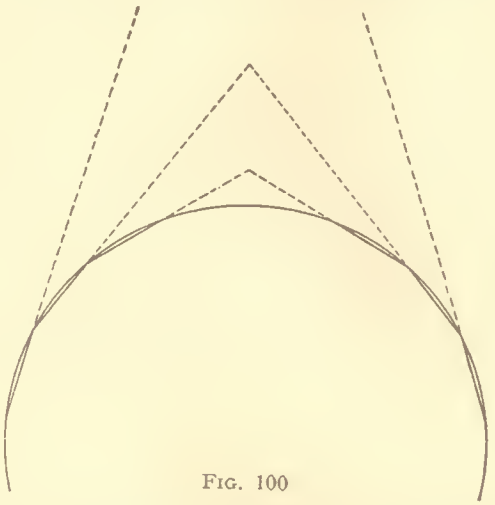


Fig. 100

as in Fig. 100 so that the difference between the figure so formed and the true sphere is very small, each strip of the surface can be developed as a narrow band, the radius of whose curvature is R cot latitude.

The bands if laid flat will appear as in Fig. 101.



Fig. 101

The projection offers an advantage that is not possessed by a simple conical projection in that whilst the simple conical is suitable for areas having a limited extent in latitude and a large extent in longitude, the polyconic is suitable for areas having a limited extent in longitude and a large extent in latitude.

Conical Orthomorphic Projection.

All conical projections have their meridians and parallels at right angles to each other and if they can be modified so that the scale is the same in both these directions, the projection will be orthomorphic.

It can be shown that if r_1, the radius upon the projection of the co-latitude χ of a parallel, is computed from the formula,

$$r_1 = m\ (\tan \tfrac{1}{2}\chi)^n$$

the projection is orthomorphic. In this formula m is an operator to obtain the desired scale and n is the constant of the cone.

Zenithal Projections.

If n, the constant of the cone, is increased in value to unity, the cone may be imagined as becoming flatter at the vertex until it has degenerated into a plane. If the cone was normal to the polar axis of the sphere, the plane will be tangential to the sphere at the pole.

These projections form an important class, their principal feature being that azimuths on the sphere from the point of contact of the plane are preserved on the plane.

Included among the zenithal projections are nearly all those which can be termed geometrical projections in the strict sense of the word, being made by projecting a portion of the sphere onto a plane by straight rays proceeding from a point, the point of projection.

The following table shows the projections obtained by projecting from points on the diameter (or diameter extended) of the sphere, see Fig. 102.

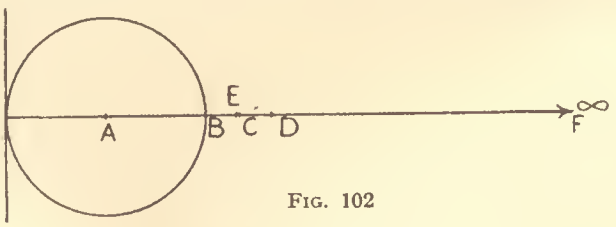


FIG. 102

Point of projection.	Distance from centre of sphere.	Projection obtained.
A. ..	At centre of sphere.	Gnomonic.
B. ..	At opposite end of diameter to tangent plane.	Stereographic.
C. ..	Radius × 1·37.	Sir Henry James'.
D. ..	Radius × 1·71.	La Hire's.
E. ..	Radius × 1·3676	Clarke's Minimum Error Perspective.
F. ..	At infinity.	Orthographic.

Gnomonic Projection.

If θ is the angular distance of any point on the sphere from the point which is to be the centre of the tangent plane, then the linear distance of the projected point from the centre of the tangent plane is given by,

$$s = R \tan \theta$$

It is evident that as the distance from the centre of the tangent plane increases, the scale along the radii becomes rapidly greater as θ approaches 90°. This can be seen from the fact that $\tan 90° = \infty$, or, geometrically, that when $\theta = 90°$ the projecting ray will be parallel to the plane that it is required to cut.

It follows, therefore, that since all distances, areas and shapes are badly represented on the Gnomonic projection it is unsuitable for small scale maps but it has certain advantages when special charts are considered.

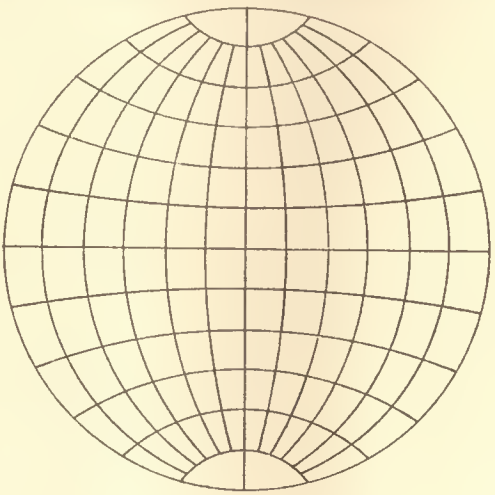
Since the plane of any Great Circle passes through the centre of the sphere, the rays which project this circle will lie in one plane and the projection of the Great Circle will be a straight line. This property is made use of on special charts to facilitate the laying off of Great Circle courses and the plotting of radio bearings.

Cylindrical Projections.

These, as has already been stated, result from decreasing the value of the constant of a cone until it becomes zero. As the constant of a cone is given by $\sin \phi$, if $\sin \phi$ is zero, the standard parallel will be the equator.

Cassini-Soldner Projection.

If a cylindrical projection is drawn so that distances are truly represented to scale along the meridians and along the equator, a network of square rectangles will result which is known as a "projection plate carrée." If this projection is rotated through 90° so that the equator becomes a central meridian the resulting projection is the Cassini-Soldner.



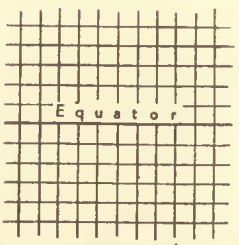
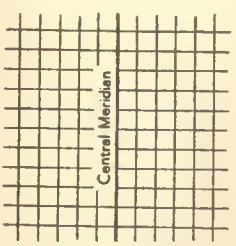


PLATE CARRÉE



CASSINI-SOLDNER

FIG. 103

Fig. 103 shows diagrammatically the parallels and meridians on the sphere and as they will appear on the projections. It should be compared with Fig. 106. The advantage of these projections is that positions may be plotted on them by their rectangular co-ordinates. Neither of them is orthomorphic since in the "plate carrée" scale is true along the equator and in a north-south direction only, whilst in the Cassini scale is true along the central meridian and in an east-west direction only. In neither projection is scale true in any direction at any one point.

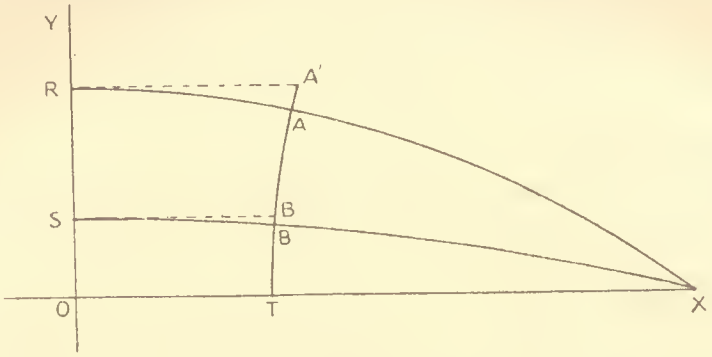


FIG. 104

Fig. 104 illustrates the Cassini-Soldner projection with OX and OY representing the x and y axes of a co-ordinate system whose point of origin is O.

X is a pole 90° from the y axis. A and B are two trigonometrical points having the same x co-ordinate; RAX and SBX are great circles drawn at right angles to OY through A and B respectively, and meeting at the pole X. Since A and B have the same x co-ordinate, AR = BS, and so a small circle TBA can be drawn parallel to OY passing through A and B. RA′ and SB′ are drawn perpendicular to OY to meet the small circle TBAA′ at A′ and B′ respectively.

In the explanation of the methods used for calculating rectangular spheroidal co-ordinates (p. 95) it was shown that the co-ordinates of A are defined by RA and RO, but that for the purposes of plotting they are treated as "plane" co-ordinates, so that the small circle TBAA′ becomes a straight line parallel the y axis, and A is plotted at A′, with co-ordinates RA′ and TA′, and B is plotted at B′, with co-ordinates SB′ and TB′.

Now RA′ = RA, and SB′ = SB, so that the x co-ordinates truly define the distances of A and B from the y axis, but the y co-ordinates exaggerate their true distances from the x axis. It is clear from the figure that the error of the plot from this cause will be very small in the vicinity of the y axis; even at a distance of 60 sea miles east or west (in latitude 52°) it only amounts to 1 : 6,637 in a north and south direction, a quantity which cannot be measured on the paper. So, provided distances east or west of the origin are not too great, the errors of plotting by this method are negligible, and this justifies the use of this projection which, like every other, is *necessarily* burdened with errors of some sort.

Meridians (which are also great circles), other than that through the origin, are not represented by straight lines on this projection. Within a reasonable distance, however, east or west of the origin (say, 100 miles) the deviation of the true curves is so small that they can be plotted as straight lines without measurable error.

Mercator's Projection.

Let it be imagined that a very narrow strip of the surface of a sphere, extending north and south from the pole to the equator and limited on the east and west by two meridians, is laid flat. In fact this will be impossible unless

the width of the strip is infinitely small, but it will be nearly possible if the meridians are very close together.

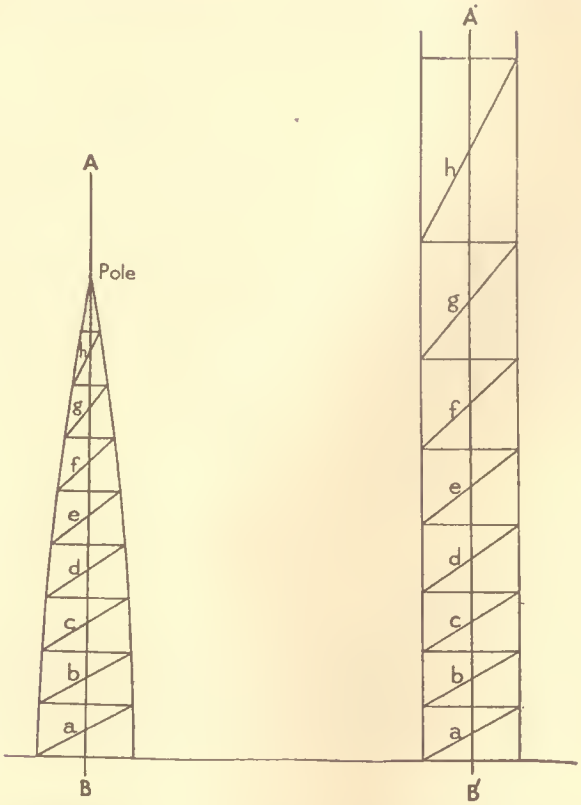


FIG. 105a FIG. 105b

If the strip is divided equally along the meridians into units of equal latitude and if the units have their diagonals drawn as at a, b, c, d, e, f, g, and h of Fig. 105a, these diagonals may be regarded as Rhumb lines cutting the centre line of the strip AB.

If the meridians are now drawn parallel to each other as in Fig. 105b, and the rhumb lines are drawn to cut A'B' at the same angle as they cut AB in Fig. 105a, it can be seen that the units must be progressively widened in a north-south direction towards the pole.

This is the basis of Mercator's projection. The equator is divided equally to obtain meridians which are drawn at right angles to it and parallel to each other. The parallels are at right angles to the meridians and have their distance apart increased in the ratio,

$$\frac{\text{radius of equator}}{\text{radius of latitude circle}}$$

i.e. Chart length of 1′ Lat. = Chart length of 1′ Long. × sec. Lat.

The projection is orthomorphic and Rhumb lines, since the meridians and parallels are at right angles to each other, are represented by straight lines.

For convenience in constructing a graticule on the Mercator projection, tables of meridional parts are included in various nautical tables, *e.g.*, Tables for the Construction of Charts, 1932. These tables give the relative distance from the equator of each minute of latitude in terms of a minute of longitude.

Suppose, for instance, it is desired to construct a Mercator graticule to cover

a 1° square between latitudes 52° and 53° N. and longitudes 15° and 16° W. on a scale of ½ in. to 1′ of latitude at mid-lat.

From Table III : for lat. 52° 30′ ; 1′ of lat. ($\rho \sin 1'' \times 60$) = 6084·864 ft.
and 1′ of long. ($\nu \sin 1'' \cos 52° 30' \times 60$) = 3713·609 ft.

1′ of lat. at mid-lat. = ½ in. so 1′ of long. $= \left(\frac{1}{2} \times \frac{3713 \cdot 609}{6084 \cdot 864}\right)$ ins. = 0·30515 ins.

From Table of Meridional Parts : Mer. parts for 53° 00′ = 3745·05
Mer. parts for 52° 00′ = 3646·74
Difference = 98·31

So the degree of longitude will measure (60 × 0·30515) ins. = 18·309 ins. and the degree of latitude will measure (98·31 × 0·30515) ins. = 29·999 ins.

A rectangle is drawn with dimensions of 29.999 ins. × 18·309 ins. to represent the area. The upper and lower sides can at once be divided into equal divisions, say, at every 3·052 ins. representing 10′ of longitude. The division of the vertical sides is made by further reference to the table of meridional parts.

For instance : Mer. parts for 52° 20′ = 3679·26
Mer. parts for 52° 00′ = 3646·74
Difference = 32·52

The position of the 52° 20′ mark will be, therefore, 32·52 × 0·30515 = 9·923 ins. above the lower edge of the rectangle ; and similarly for any other divisions of latitude.

Transverse Mercator Projection.

This projection, which is also known as the Gauss Conformal Projection, is constructed by rotating the Mercator through 90°. Figs. 106 (a) and (b) show the sphere and the resulting graticule on the Mercator projection : Figs. 106 (c) and (d) show the same thing when the graduation on the sphere is turned through 90° while the sphere itself remains fixed.

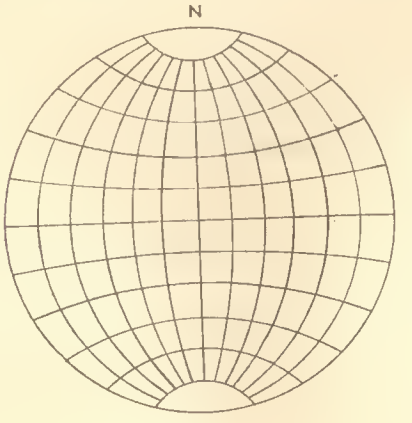


Fig. 106a

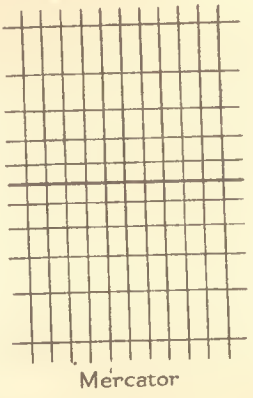


Fig. 106b

The North and South Poles are here at the extremities of the Great Circle that was formerly the equator and this Great Circle is the only meridian remaining : it becomes the Central Meridian of the projection. The former meridians now become Great Circles running East and West and appear on the projection as equidistant horizontal lines. The former parallels become small circles parallel to the Central Meridian and appear on the projection as vertical straight lines set at increasing distances from the Central Meridian just as the real parallels increase in distance from the equator in the normal Mercator projection.

It will be seen that, whereas in the ordinary Mercator projection the scale increases to the north and south of the equator, in the Transverse Mercator the scale increases to the east and west of the Central Meridian : but on both projections the scale at any one point is the same in all directions.

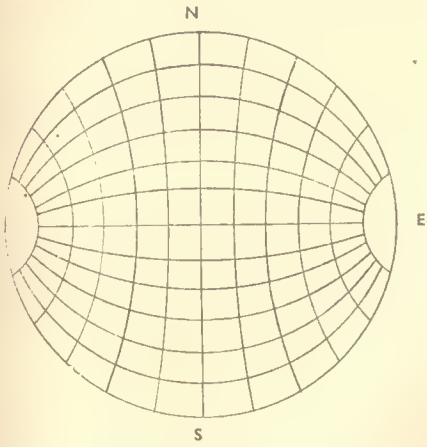


Fig. 106c

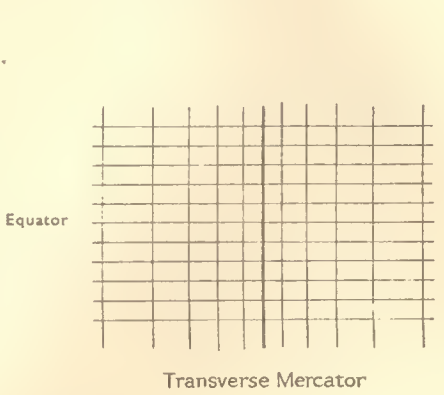


Fig. 106d

The Transverse Mercator projection provides a rectangular framework into which the real meridians and parallels must be plotted : the parallels will be curves that are concave towards the poles and the meridians will be concave towards the central meridian, but since the projection is orthomorphic all meridians and parallels will cut at right angles.

This projection is clearly best suited to an area that has a North to South length considerably greater than its East to West breadth.

Plotting Instruments

The principal instruments required for plotting are :

(*a*) **Scales.**—These are of metal and supplied in various sizes, from 24 ins. to about 60 ins. in length. They should be used in their boxes and should never be taken out of them even for cleaning purposes. Great care should be taken not to damage the surfaces of scales with scratches of beam compass points, and no abrasive material should be used for cleaning them.

(*b*) **Straight-Edges.**—These are made of nickel-plated or rustless steel and are supplied in various sizes up to 72 ins. in length. One edge is chamfered and it is this edge which is used for ruling lines. Provided straight-edges are carefully treated, they should retain their initial accuracy which can be guaranteed by the methods used in their construction. The straightness of the chamfered edge should, nevertheless, be tested from time to time by ruling a fine line (with a pricker) and reversing the straight-edge when examination of the coincidence of the edge with the line by means of a reading-glass will show whether it is perfect. As with scales, no abrasive materials should be used for cleaning purposes.

(*c*) **Beam Compasses.**—These are obtainable in various sizes to measure distances from about 3 ins. to 60 ins. No beam compass is absolutely rigid and in making long measurements care should be taken to hold and support the beam in the same manner when using it on the scale as when sweeping an arc or setting it between two points on the plotting sheet.

(*d*) **Chord Dividers and Spring Bows.**—These are needed for all short measurements of less than about 3 ins.

(*e*) **Protractors.**—Circular, semi-circular and rectangular protractors in various sizes can be used for protracting angles when *short* rays only are involved.

(*f*) **Pricker.**—A pricker should be used for laying off the lines for plotting the grid and all the more important points on the plotting sheet. Finer and more accurate lines can be drawn with a pricker than with a pencil and the plotting sheet is kept cleaner.

(*g*) **Magnifying Glasses.**—These are required for examining the scale when setting measurements on beam compasses, etc., and for inspecting the lines on the plotting sheet.

(*h*) **Weights.**—A good supply of canvas, leather or baize-covered weights is essential for keeping the plotting sheet flat and for weighting down straight edges before drawing lines.

Plotting of Main Triangulation

The plotting of the main stations of a survey is usually effected by means of their rectangular co-ordinates. The selection of a suitable rectangular grid to contain the main stations and the survey area has already been discussed. The mechanical construction of a grid is best explained by a practical example.

(*a*) **Construction of Grid.**

The first process is to find the plotting lengths of the vertical and horizontal sides and the diagonals, and the diagonal bearing as measured from the vertical sides.

Assuming that the scale of the survey is 1/12,500 and the co-ordinates selected for the grid sides are:

North side, $y = +12500$ ft.: East side, $x = +42500$ ft.
South ,, $y' = -18000$ ft.: West ,, $x' = -4000$ ft.

Then, $x \sim x' = 46500$	log	4·667 4530
$y \sim y' = 30500$	log	4·484 2998
$\therefore$ *Diag. bearing* $= 56° 44' 19''$:	*L. tan* ..	10·183 1532
Log $x \sim x'$	.. .. ..	4·667 4530
L. sin Diag. bearing	.. ..	9·922 2983
log diagonal (*feet*)	.. .. ..	4·745 1547

To convert actual distances in feet into their corresponding lengths in inches on the plotting sheet, a constant log for the scale in use is determined as follows:

log 1/scale (= 12500)	4·096 9100
log ins. in 1 foot (= 12)	1·079 1812
$\therefore$ Constant log	3·017 7288

The plotting lengths of the diagonals and sides are then found:

	Diagonal.	Vertical.	Horizontal.
Log side (in feet)	4·745 1547	4·484 2998	4·667 4530
Constant log	3·017 7288	3·017 7288	3·017 7288
Log side (in ins.)	1·727 4259	1·466 5710	1·649 7242

These results give the plotting lengths of the sides:

Diagonals .. .. = 53·386 ins.
Vertical sides .. .. = 29·280 ins.
Horizontal sides .. = 44·640 ins.

The construction of the grid is now as follows:

(i) Set the above measurements on three beam compasses. Great care should be taken to see that these measurements are exact, a magnifying glass being used to examine the settings.

(ii) With a pricker draw a line somewhat longer than the diagonal and at an angle of about 57°, *i.e.*, the diagonal bearing, relative to the shorter sides of the paper.

(iii) Prick a hole on this line near one end to represent one of the grid corners, say, the S.E. corner.

(iv) With the beam compass set to the diagonal measurement sweep an arc to cut the line near the other end and prick through the intersection. This point will be the N.W. corner.

(v) With the beam compasses set to the vertical and horizontal side measurements, sweep arcs from these corners over the approximate positions of the other two corners. Prick through the intersections of these arcs which will be the N.E. and S.W. corners.

(vi) With the beam compass set to the diagonal measurement, check the length of *this*, the N.E.—S.W. diagonal. The agreement should be perfect; if it is not, start the whole plot again.

(vii) With a pricker join the N.E.—S.W. corners and draw in the four sides. Great care is necessary to see that the pricker lines pass exactly through the centres of the corner holes.

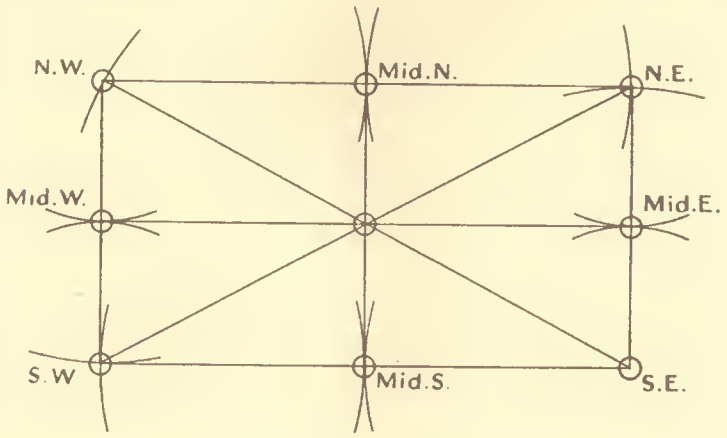


FIG. 107

(viii) Set a beam compass to *half* the length of the vertical sides and another to half the length of the horizontal sides, and sweep arcs from the four corners to bisect the vertical and horizontal sides respectively.

(ix) The intersections of these arcs with the sides give the Mid. N., Mid. S., Mid. E. and Mid. W. points. Prick these through and join them as shown in Fig. 107.

(x) The intersection of the four cross lines at the centre should be perfect. Examine the cut carefully with a magnifying glass to make sure that there is no suggestion of a "cocked hat." If any error is found it will be best to start again and replot the whole grid, since, if left, it will lead to endless trouble and inaccuracy when plotting the stations.

The operations described should not take more than 15 to 20 minutes and, within reason, the quicker they are done the better, as there is less likelihood of distortion taking place in the paper whilst the plot is in progress.

The longest straight edge usually available is 72 ins., and this is therefore the limiting length of a diagonal. The longest scale is usually about 60 ins. If it is necessary to have a diagonal of more than 60 (but less than 72) ins., the initial procedure must be modified as follows:

Set one of the beam compasses to *half* the length of the diagonal instead of

the whole length and, after drawing the first diagonal, prick a hole to mark the *centre* of the grid. The first two corners are then fixed by sweeping arcs from the centre across the diagonal with this half measurement. The subsequent procedure is the same, but the length of the second diagonal must be checked from the centre as it cannot be measured in one.

The corners, mid-points and centre usually provide a sufficient number of points from which to plot the main stations, but when very large sheets are plotted, it is sometimes convenient to subdivide the grid into further rectangles. It is clear that the co-ordinates of the mid-points and centre can be at once deduced from those of the corners, *e.g.*, co-ordinates (x'', y'') of Mid-South :

$$x'' = \frac{x + x'}{2} = \frac{+ 42500 - 4000}{2} = + 19250 \text{ ft.}$$

$$y'' = y' \qquad = - 18000 \text{ ft.}$$

For convenience, the corners, etc., should be circled in ink, as shown in Fig. 107.

(*b*) **Plotting of Main Stations.**

The grid constitutes a framework from which the main stations or points of the survey are plotted. The positions of the stations with reference to the corners and mid-points of the grid are known from their rectangular co-ordinates and the plotting of these positions is effected by the protraction of angles which are calculated from the differences of the co-ordinates. If, for instance, it is required to use a line drawn from the N.E. corner of the grid as one of the rays necessary for plotting the position of a station, the co-ordinate bearing of the station from that corner can be found by differences of co-ordinates, and a line in this direction can be drawn by protraction of the angle which the bearing makes with the vertical, horizontal or diagonal sides of the grid.

In plotting the main stations it will often be necessary to draw lines of considerable length, up to as much as 60 ins., on the paper. The ordinary types of protractor cannot be used for plotting long rays ; the largest size of instrument normally available has a radius of only about 10 ins., and two prick holes this distance apart do not serve as a sufficiently rigid alignment for the straight edge when a long line has to be drawn. Angles are therefore protracted by means of *chords* unless the required ray will reach the station *within* the radius of the protractor employed.

Chords.

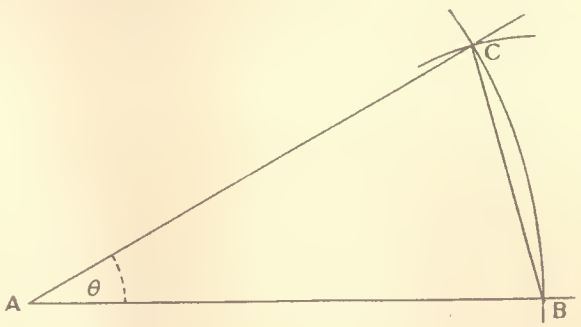


Fig. 108

If two radii of a circle of given radius r, containing between them a given angle θ, be drawn to cut the circumference of the circle, the length of the chord thus cut off is $2r \sin \frac{\theta}{2}$. From this formula are calculated the chords

from 0° to 60° for a radius of 10 which are given in Table XVIII. From them chords for any required radius can readily be calculated.

In Fig. 108, suppose it is required to draw a line AC, about 40 ins. in length (this distance being sufficient to reach beyond a station located in the direction AC) making an angle (θ) of 30° 10′ 15″ with the initial line AB.

(1) Set 40 ins. exactly on a beam compass and with centre A sweep an arc from B, carrying it through the estimated position of the line AC. Prick through the intersection of the arc and the line AB at B.

(2) Table of chords gives :

Chord for 30° 10′ 15″ at 10-in. radius = 5·20518 ins.

Hence, ,, ,, ,, ,, 40 ,, ,, = 4 × 5·20518 ins.

= 20·821 ins.

(3) Set the measurement 20·821 ins. on a beam compass and with centre B sweep an arc to cut the previous one. Prick through the intersection of the arcs at C.

(4) Join AC. CAB is the required angle.

Before plotting the main stations it is convenient to prepare a small-sized diagram of the grid, preferably on squared paper. On it should be marked the approximate positions of the main stations, and a scale of inches, corresponding to an equivalent scale on the actual plotting sheet, should also be added.

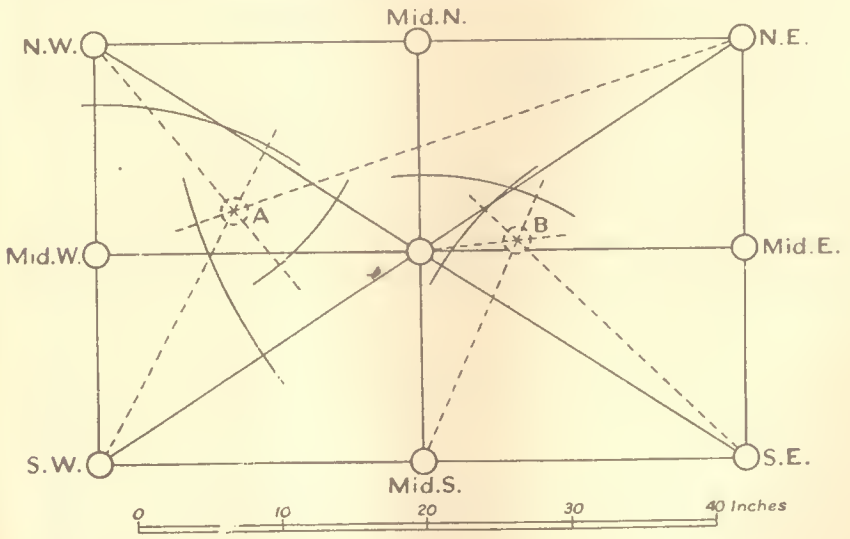


FIG. 109

Each station is plotted by three shots from suitably disposed corners or mid-points of the grid. The ideal cut for the shots is an angle of 60° and though this is not usually obtainable it is *always* possible to get a well-conditioned cut if the stations are located within the grid or not too far outside its edges. The small scale diagram shows at once which shots will give a good cut and the scale of inches enables suitable radii for the chords to be taken off. To simplify the calculation of the chords, it is convenient to use, if possible, half or even multiples of 10, *e.g.*, 15, 20, 25, etc.

In Fig. 109, A and B are the approximate positions of two main stations. A good cut at A can be obtained by plotting shots from the N.E., N.W. and S.W. points of the grid ; at B by shots from S.E., centre and Mid.S. Shots

from other points might do as well and there is usually a considerable choice of points from which shots will give almost equally good cuts at a station.

When using chords, it is easier and more accurate to protract *small* angles rather than large ones. With small angles the arcs, which are swept, cut almost at right angles, and provide very definite points of intersection, and so, slight inaccuracies in the measurement of the radii will have the minimum effect. Furthermore, distortion of the paper produces less change in small chords. For these reasons the shots to A and B are laid down as follows :

To A from N.E. : by protraction from the N.E.—S.W. diagonal.
,, ,, ,, N.W. : ,, ,, ,, ,, N.W.—S.E. ,,
,, ,, ,, S.W. : ,, ,, ,, ,, Vertical.
,, B ,, S.E. : ,, ,, ,, ,, S.E.—N.W. diagonal.
,, ,, ,, centre : ,, ,, ,, ,, Horizontal.
,, ,, ,, Mid. S. : ,, ,, ,, ,, Vertical.

The radii of the chords must be long enough for the arcs to be swept *outside* the stations which are being plotted. Suitable lengths may be taken off the diagram, *e.g.*, to A from N.E. 40 ins., from N.W. 20 ins., etc. In the case of the shot to B from the centre since B lies only about 7 ins. away, the angle can, if desired, be laid off with a protractor without using chords.

To calculate the co-ordinate bearings from which the plotting angles are deduced :

Take the co-ordinates of the grid sides and of the stations A and B to be as follows :

N. side, y_N = + 105000 feet.
S. ,, y_S = − 12000 ,,
E. ,, x_E = + 220000 feet
W. ,, x_W = + 42500 ,,
A ,, x_A = + 79860 ,, : y_A = + 59261 ft.
B ,, x_B = + 157516 ,, : y_B = + 48553 ft.

By calculation, the diagonal bearing of the grid=056° 36′ 32″.

Computation of bearings and plotting angles.

Grid Point.	N.E.	N.W.	S.W.
x	+ 220000	+ 42500	+ 42500
x_A	+ 79860	+ 79860	+ 79860
Δx	140140	37360	37360
y	+ 105000	+ 105000	− 12000
y_A	+ 59261	+ 59261	+ 59261
Δy	45739	45739	71261
log Δx	5·146 5621	4·572 4069	4·572 4069
log Δy	4·660 2867	4·660 2867	4·852 8519
L. tan bearing	10·486 2754	9·912 1202	9·719 5550
	° ′ ″	° ′ ″	° ′ ″
Bearing	71 55 28	39 14 32	27 40 00
Bearing of zero	56 36 32	56 36 32	00 00 00
Plotting angle	15 18 56	17 22 00	27 40 00

Grid Point.	S.E.	Centre.	Mid. S.
x	+ 220000	+ 131250	+ 131250
x_B	+ 157516	+ 157516	+ 157516
Δx	62484	26266	26266
y	− 12000	+ 46500	− 12000
y_B	+ 48553	+ 48553	+ 48553
Δy	60553	2053	60553
log Δx	4·795 7688	4·419 3939	4·419 3939
log Δy	4·782 1357	3·312 3889	4·782 1357
L. tan bearing	10·013 6331	11·107 0050	9·637 2582
	° ′ ″	° ′ ″	° ′ ″
Bearing	45 53 57	85 31 51	23 26 59
Bearing of zero	56 36 32	90 00 00	00 00 00
Plotting angle	10 42 35	4 28 09	23 26 59

It will be found convenient to keep a " Plotting " Book in which angles, chords and other details of the plot can be noted.

Example :

Station	From	Zero.	Left or Right.	Angle.	Chord (10″ radius).	Radius (ins.).	Final Chord.
				° ′ ″			
A	N.E.	Diag.	R	15 18 56	2·66510	40	10·660
	N.W.	Diag.	R	17 22 00	3·01947	20	6·039
	S.W.	Vert.	R	27 40 00	4·78196	25	11·955
B	S.E.	Diag.	R	10 42 35	1·86647	25	4·666
	Centre	Hor.	L	4 28 09	Not required	—	—
	Mid. S.	Vert.	R	23 26 59	4·06423	20	8·128
		etc.			etc.		

This provides the necessary data for plotting A and B, the procedure being as follows :

Set a measurement of exactly 40 ins. on a beam compass and, with centre the N.E. corner, sweep an arc across the diagonal as shown in Fig. 109. Prick through the point of intersection on the diagonal.

Set a measurement of 10·660 ins. on a beam compass and with centre at this point of intersection sweep an arc to cut the first one.

Prick through the second intersection and draw a line from this point to the N.E. corner. This will be one shot to A. Similarly with the other shots.

The three lines to each station should make perfect cuts at A and B respectively. If there is any suggestion of a " cocked hat," the measurements should be checked and, if necessary, a further shot should be calculated and plotted from another point of the grid.

All the main stations are plotted in this way ; and occasionally it may be necessary to calculate the co-ordinates of one or two of the secondary stations and plot them in a similar manner if well-conditioned cuts cannot be obtained by shots from the main stations. (See next page.)

Distortion.—It is impossible to manufacture a paper which will retain exactly the same size and shape under all conditions of temperature and humidity. If, however, the plotting sheet is carefully treated and prevented from undergoing drastic changes of temperature such as would occur if it received the direct rays of the sun, it is fair to assume that expansion and contraction take place nearly equally in all directions, so that only the scale and not the shape of the plot is altered. Slight alterations in scale are of no great moment, and the actual scale at any time can easily be found by remeasuring one of the grid diagonals and comparing its length with the original measurement.

It is owing to the existence of these changes in the paper that the method of plotting by angles, in preference to plotting by distances is adopted. In the example given, Fig. 109, it would be possible to calculate the distances in feet of A from the N.E., N.W. and S.W. corners of the grid and, converting these distances to the corresponding lengths in inches on the scale of the survey, to plot A by swinging arcs of these radii from the three corners. The intersection of the arcs would be the position of A. In practice, however, it is found that as a rule the intersection will not be so good as that obtained by the angle method.

On account of distortion of the paper the arcs of the radii once measured and scratched on the paper, cannot be considered as having been done once and for all. If some period has elapsed since scribing an arc, the radius should be remeasured to ascertain if it has altered and, if necessary, the arc must be redrawn.

(*c*) **Plotting of Secondary Marks.**

The main stations of the triangulation form a framework from which the secondary and sounding marks can be plotted by the protraction of the observed angles to them.

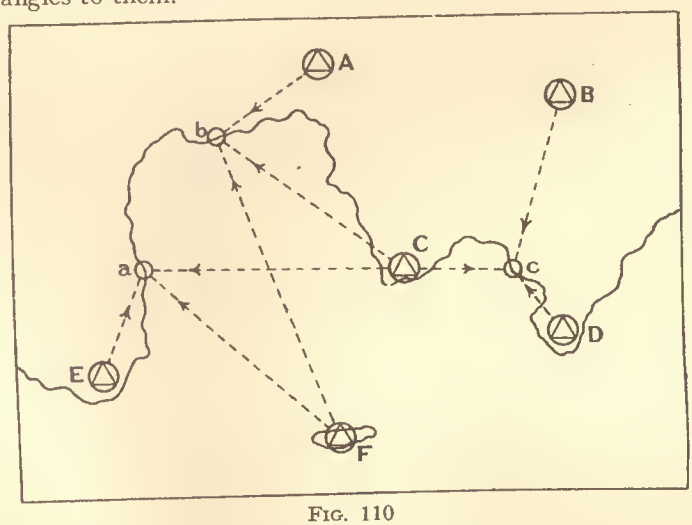


FIG. 110

In Fig. 110, A, B, C, etc., are main stations of a survey and a, b and c are secondary marks. Each secondary station must be plotted by not less than three shots making as good a cut as possible. If the three shots intersect in exactly the same position on the paper, the correctness of the observations and the plot will be proved. The figure shows that the shots indicated by pecked lines will make good cuts at a, b and c respectively and, assuming that these rays have been observed, those stations can be plotted from them.

In plotting any angle from a main to a secondary station, the protraction

should be made from a line *longer* than that necessary to reach the latter, *e.g.*, in Fig. 110 the shot to b from F should be protracted from the line joining FA, not from that joining FE or FC. In this way the plotting is carried *inwards* all the time and it is unnecessary to extend the zero line beyond the further station. If F and A are not intervisible, the necessary angle can be obtained from the calculated differences of co-ordinate bearings. If, for instance, the angle bFC was actually observed, the angle bFA can be found by subtracting from bFC the difference of the bearings FA and FC. If the main triangulation system forms a fairly dense control, it is probable that most of the shots to the secondary stations will be short enough to be laid off with a protractor. All angles and other details of the plotting of secondary stations should be entered in the Plotting Book in the form used for the main stations.

It is only exceptionally that all the secondary stations and sounding marks can be plotted entirely by shots from main stations; usually it is necessary to include some shots at least from other secondary points and sometimes resort must be made to less regular methods such as graphic resection or the use of angles obtained from ship and boat stations, *vide* Chapter IV.

Other Methods of Plotting the Main Stations.

(i) **By distances measured on a closely subdivided grid.**

A method frequently adopted when there are many points to be plotted, but ones which are only being fixed progressively as the survey proceeds, is to subdivide the main grid into small squares and to plot the main stations and other co-ordinated points, in their appropriate squares, by direct measurement east and west, and north and south from the enclosing borders of the square.

The method of constructing the main outer grid is the same as before, except that there is no necessity to draw in the full length of the diagonals; but the co-ordinates of the main grid corners should be chosen to facilitate subsequent subdivision into conveniently sized squares, *e.g.* in the example given on p. 170, more suitable co-ordinates for the sides of the grid would be, $y = +12000$ ft. and $y' = -20000$ ft.; $x = +44000$ ft. and $x' = -4000$ ft.; giving vertical grid sides of 32000 ft. and horizontal sides of 48000 ft. The vertical sides can then be subdivided into eight equal parts, each representing 4000 ft., by successive bisections, and the horizontal sides can be subdivided, almost as easily, into twelve parts, each, also, representing 4000 ft. By joining opposite subdivision ticks, the plotting sheet will be covered with a network of squares, each square having sides representing 4000 ft., *i.e.* 3·840 ins. on the selected scale of 1 : 12,500. Each grid line must be labelled, on the border, with its co-ordinate value, +12000, +8000, +4000, etc.

One of the squares (which is unlikely to have any point plotted in it) is then selected and a diagonal scale, as shown in Fig. 111a, constructed in it.

The plotting of any point can now be carried out by direct measurement in its appropriate square. *E.g.*, to plot a point whose co-ordinates are $x = +5063$ ft. and $y = -14796$ ft.; the appropriate square is that bounded by x co-ordinates +4000 and +8000, and by y co-ordinates —12000 and —16000. A plotting length corresponding to 1063 ft. is measured from the diagonal scale and set on the chord dividers, and short arcs, cutting the horizontal grid lines, are scribed, eastwards from the NW. and SW. corners of the square; a plotting length corresponding to 2937 ft. is now scribed westwards from the NE. and SE. corners of the same square, and should coincide with the first arcs; the intersections are pricked through and joined by a line (which will represent a small portion of the $x = +5063$ grid line). Next an arc, with plotting radius of 2796 ft. is measured on the diagonal scale and drawn, with centre, the intersection of the $y = -12000$ and the $x = +5063$ grid lines, to cut the latter; an arc, with radius corresponding to 1204 ft. is then drawn northwards from the south side of the square, and should coincide with the previously drawn arc.

The intersection of these two arcs on the $x = +5063$ grid line is pricked through and is the required position (Fig. 111b).

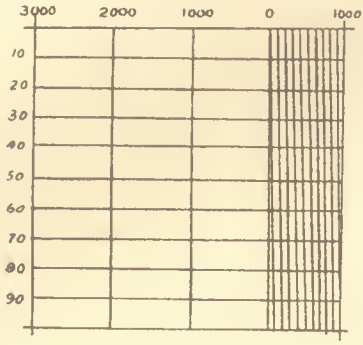


FIG. 111a

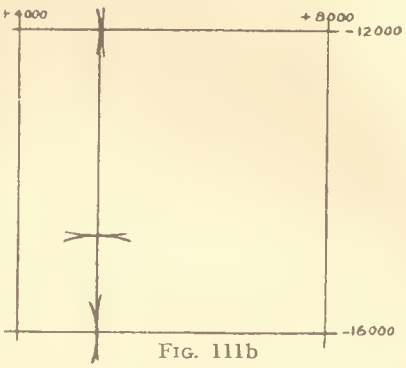


FIG. 111b

To save a myriad of small scratches on the plotting sheet, the process outlined above can be done on tracing paper on a square traced from the plotting sheet ; the position is then pricked through on to the plotting sheet from this small tracing.

The disadvantages of this method of plotting are the longer time spent in preparing the grid and the greater technical experience needed for its accurate construction ; and also the danger of plotting a point in the wrong square. The advantage of being able to plot points directly from their co-ordinates without any calculation of angles, chords, etc., is obvious.

(ii) **From a Long Side.**

Another method of plotting the main stations, which may sometimes be used for small harbour plans or in running and sketch surveys, is by plotting from a " Long Side." This method is not so satisfactory as that previously described, but it does not involve the calculation of co-ordinates and therefore saves time. It is also rather simpler to understand. Example :.

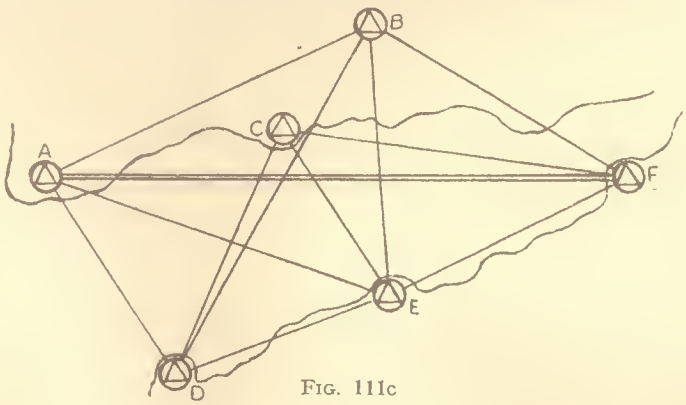


FIG. 111c

Figure 111c represents the triangulation of a small harbour. The longest side in this triangulation is AF and this is used as the initial side from which the plot is begun. By starting from the longest side the plot is carried *inwards* to shorter sides and errors of plotting tend to diminish rather than increase. The approximate bearing of AF should be known, and the procedure is then as follows :

(1) Draw a line with a pricker on approximately the correct bearing AF (relative to the edge of the paper).

(2) Calculate the plotting length of AF corresponding to its true length; *e.g.*, if AF = 25,000 ft. and the scale of the survey = 1/12,500, the plotting length of AF = 24·000 ins.

(3) Set this measurement on a beam compass and prick off on the line already drawn two points to represent A and F.

(4) The rays from A and F make a good cut at B: therefore plot B by protracting the angles BAF and BFA, and prick it through. At this stage there is no check on the accuracy of B's position.

(5) Plot E by protracting angles from A, B and F. If the intersection of the three shots at E is satisfactory, as it should be, a check is provided on the accuracy of the previous work.

(6) D can now be plotted from A, B and E, and C from D, E and F.

(7) Secondary points and sounding marks are plotted in the usual manner from the framework provided by the main stations.

(iii) **Plotting stations by Graphic Resection.**

It is sometimes necessary to plot stations by graphic resection, *i.e.*, by means of the angles taken *at* those stations. This, of course, cannot be done until a sufficient number of main stations have been fixed and plotted in the ordinary way, after which it may be found that one or two additional points are required, but are hardly of such importance as to justify time spent in revisiting a number of main stations in order to observe the shots to them. The co-ordinates of such resected points can, if necessary, be calculated in the manner described in Chap. IV, and they can then be plotted from those co-ordinates. If, however, their importance does not warrant this, a very good position can be obtained by protracting all the observed angles, if necessary with chords, on tracing paper. The tracing is then laid over the plotting sheet and adjusted as necessary till the rays cut the respective main stations when the point can be pricked through. This method gives a more accurate position than that obtained by using a station pointer on which only two angles can be set simultaneously.

Graduation

All charts and plans should be graduated, *i.e.*, the borders of the sheet should represent meridians and parallels drawn in conformity with the projection employed. Each corner of the sheet represents a definite geographical position and the borders are subdivided and numbered in such a manner that the latitude and longitude of any point may be easily ascertained. The graduation enables the mariner to fix himself by astronomical observations in relation to the detail on the chart and by reference to it a chart on one projection may be redrawn on another. Besides defining the location of the survey on the earth's surface, the graduation also shows the orientation of the work so that bearings between various points can be measured directly off the sheet.

The graduation should be done on the plotting sheet and subsequently transferred to the fair chart by means of a tracing. The process involves the protraction of angles, generally by chords, and it is undesirable to spoil the surface of a fair chart with scratches of the pricker and beam compass. The final subdivision, however, *e.g.*, to 6″ of arc, can be conveniently done on the fair chart itself.

Three cases must be considered:

(1) A regularly plotted survey (*i.e.*, by the use of co-ordinates) in which the geographical position of the Point of Origin is *not* known at the time the plot is made.

(2) A regularly plotted survey in which the geographical position of the Point of Origin *is* known at the time the plot is made.

(3) Surveys plotted from a "Long Side."

Case (1).

When no information regarding geographical co-ordinates is available, it is necessary to determine these by means of astronomical observations. This work is usually carried out during the progress of the survey and the results are not always available when the plot is made. In consequence, an assumed true bearing may have to be used and it is necessary to assign arbitrary co-ordinates to the grid corners and corresponding co-ordinates to the stations. On completion of the astronomical observations for geographical position and true bearing, the rectangular co-ordinates of the corners and stations must, if necessary, be recalculated to bring them into conformity with the true directions of the axes through the Point of Origin.

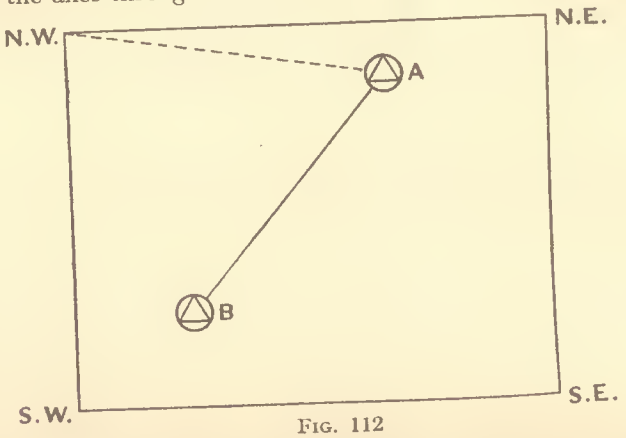


FIG. 112

In Fig. 112, A and B are two main stations. A has been selected as the Point of Origin for the rectangular co-ordinates. The true bearing AB is *assumed* to be 220° 00′ 00″ and the grid is constructed in the usual way, with suitable co-ordinates assigned to its sides so that they will enclose the necessary area. For example, let the co-ordinates of the N.W. corner be, $x = -65{,}000$ ft., $y = +11{,}000$ ft. Astronomical observations show later that the true bearing AB is 221° 10′ 45″, a difference of + 1° 10′ 45″.

The co-ordinates of the N.W. corner may be recomputed as follows:

Assigned co-ordinates of N.W. corner	$x =$	$-\ 65000$, $y = +\ 11000$
,, ,, A	$x =$	0 , $y =$ 0
	$\Delta x =$	65000, $\Delta y =$ 11000

log Δx	4·812	9134
log Δy	4·041	3927
∴ L . tan bearing	10·771	5207

∴ False bearing A—N.W. = 279° 36′ 19″
Add difference 1 10 45
True bearing A—N.W. = 280 47 04

log. dist. A—N.W.: 4·819 0450
True bearing: L . sin 9·992 2610
log. Δx ($= x$) 4·811 3060

log Δx	4·812 9134
L . sin	9·993 8684
log dist.	4·819 0450
	4·819 0450
L . cos	9·272 1077
log Δy ($=y$)	4·091 1527

∴ True co-ordinates of N.W. corner are:
$x = -\ 64{,}759{\cdot}9$ ft.
$y = +\ 12{,}335{\cdot}4$ ft.

Similarly, the true co-ordinates of the other corners and of the main stations may be recalculated. In the case of the latter the computation is shorter, since the log lengths of the sides will be available from the triangulation.

The geographical position of the Point of Origin is obtained from the astronomical observations; that of any other station may be calculated from the recomputed rectangular co-ordinates, using the formulae given in Chapter IV. Suitable geographical co-ordinates may then be selected for the four corners of the graduation, their positions being such that the sides of the graduated figure will enclose the limits of the survey, and as much additional space as may eventually be needed for the inclusion of a title, memoir and other details on the fair chart. It is convenient, though not essential, to place the corners at an exact minute, or in the case of large-scale surveys at an exact decimal of a minute, of latitude and longitude. This facilitates the subsequent subdivision of the sides at the required intervals, *e.g.*, at every 6″ of latitude and longitude. The geographical co-ordinates of the graduation corners having been selected, their rectangular co-ordinates are calculated and their positions are plotted in precisely the same manner as that adopted for the main stations. The result is exemplified in Fig. 113, in which the corners of the graduation, A, B, C and D, are circled and are joined with pecked lines, the co-ordinate grid being shown in firm lines. It will be noted that the corners of the graduation can be plotted in the usual manner, *e.g.* the S.E. corner, D, with shots from the centre, Mid.S. and S.E. points of the grid.

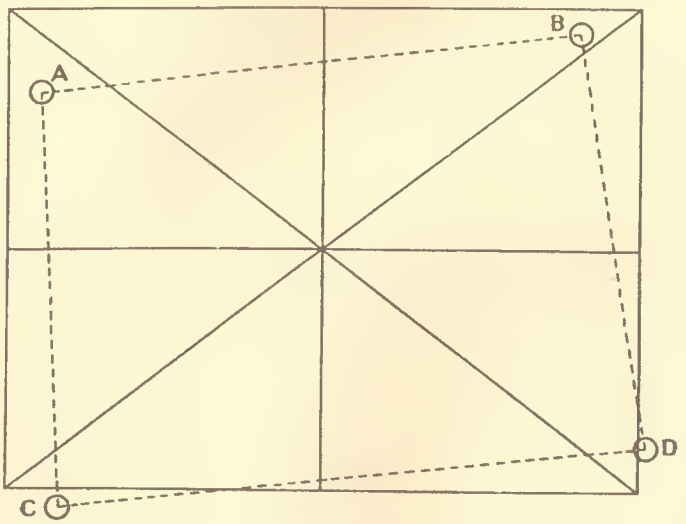


Fig. 113

The drawing of the bounding meridians and parallels joining the corners of the graduation must now be considered.

The bounding meridians are drawn by joining AC and BD (the western and eastern sides of the graduation) with straight lines. As previously explained, meridians are not truly represented by straight lines on the Cassini-Soldner projection, but the divergence is inappreciable on the plot. The parallels are curves, concave towards the pole, and to draw them it is necessary to have recourse to a table of abscissae and ordinates (*vide* Table XIV). This table is calculated for the gnomonic projection, but it may be used without sensible error for the Cassini projection within the limits of any normal hydrographic survey.

The method of drawing the parallels of latitude is illustrated in Fig. 114, in which for the sake of clearness the convergence of the meridians and the curvature of the parallel is greatly exaggerated.

PAQ and RBS are two bounding meridians of a sheet, A and B being the corners of the graduation and therefore in the same latitude. Join AB, bisect it

at E and draw EF at right angles, making its length equal to the ordinate for *half* the difference of longitude between A and B at the latitude of those points. The abscissae and ordinates in Table XIV are given in inches and decimals of an

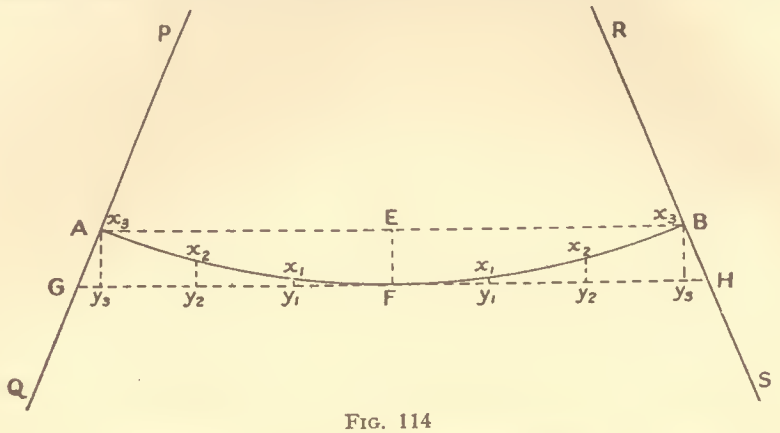


Fig. 114

inch for a scale *of* 1 *inch to* 1 *nautical mile* (*approximately* 1/73,000) ; for any other scale the figures must be increased or decreased in proportion. For instance, if A and B are in latitude 52° 20′, their difference of longitude is 1° and the scale is 1/50,000, then :

Ordinate for $\frac{1}{2}\ \Delta_\omega$ (= 30′) from table = 0·063 ins.

$\therefore$,, ,, scale of 1/50,000 $= \dfrac{0{\cdot}063 \times 73}{50}$ ins.

$= 0{\cdot}092$ ins.

Therefore, draw EF = 0·092 ins. F is the centre point on the parallel but more points are needed before the complete parallel can be drawn, the number depends on the curvature of the parallel (which decreases towards the equator), the scale, and the difference of longitude between the corners. In this example it would be sufficient to mark points at 10′ intervals of longitude. Therefore, proceed as follows:

Through F draw a line GFH parallel to AB and on it mark off the abscissae Fy_1, Fy_2 and Fy_3, corresponding to 10′, 20′ and 30′ of longitude at latitude 52° 20′ and on the given scale. Lay off at right angles the corresponding ordinates $y_1\ x_1$, $y_2\ x_2$ and $y_3\ x_3$. It is clear that the points x_3 should coincide with A and B. x_1 and x_2 are the other required points on the parallel.

It is impossible to draw a curve of very large radius which in theory is necessary to represent a parallel, and in practice it is drawn in by joining Ax_2, $x_2\ x_1$, etc., by straight lines. Provided there are enough points, x_1, x_2, etc., of subdivision, the series of chords will approximate sufficiently closely to the true curve.

The appearance of the completed graticule is shown in Fig. 115.

The lower bounding parallel CD is drawn in the same manner as AB ; within the limits of a single sheet on the scales normally used in hydrographic surveys, the ordinates of the two bounding parallels will be sensibly equal so that $AC = x_2\ x'_2 = x_1\ x'_1$, etc. The abscissae, however, will not be the same ; they are greater for that parallel which is farthest from the pole (in this figure, CD) and they must therefore be laid off independently for each. The scale of latitude along the meridians is practically constant for distances up to 2°, *i.e.*, at least as far as the difference of latitude in any one sheet, and the meridians may therefore be subdivided into as many *equal* divisions as are convenient and intermediate parallels may be drawn with the same curvature as the bounding parallels.

Intersections of parallels and meridians are transferred to the fair chart by means of a tracing and, to minimise the effects of distortion, such intersections should not be more than 6 ins. apart. The final subdivision of the border, (*e.g.*, to 6″ of lat. and long.) may be done on the fair chart itself.

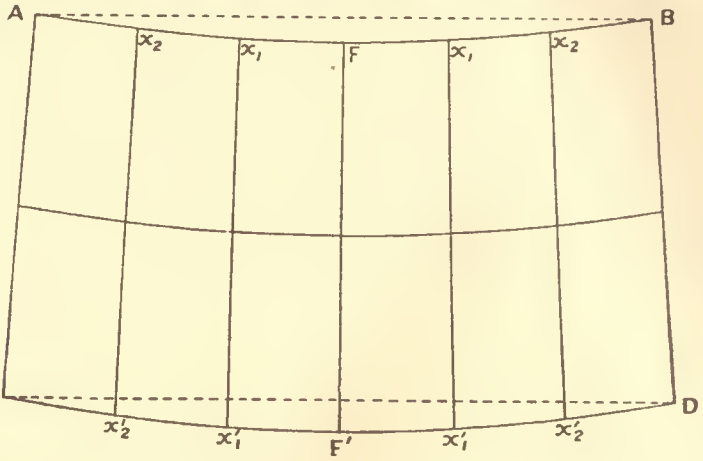


FIG. 115

Case (2).

When the geographical position of the Point of Origin is known at the commencement of a survey, the process of graduation may be simplified in the following manner.

Instead of selecting suitable *rectangular* co-ordinates for the sides of the grid, choose *geographical* co-ordinates for two diagonally *opposite* corners of the grid and calculate the rectangular co-ordinates of these points. Example :

Data :—Geographical co-ordinates of Point of Origin :

Latitude, 51° 16′ 10″·380 N.

Longitude 0° 39′ 55″·564 E.

Geographical co-ordinates selected for two opposite corners of the grid :

N.E. Corner : Latitude 51° 34′ 00″ N. Longitude 1° 44′ 00″ E.

S.W. Corner : Latitude 51° 19′ 00″ N. Longitude 1° 23′ 00″ E.

The rectangular co-ordinates of these corners and the convergence between the meridians through them and that through the Point of Origin are calculated by the usual formulae and found to be :

N.E. corner : $x = +\ 242904{\cdot}2$ ft., $y = +\ 110215{\cdot}1$ ft., convergence 50′ 05″.

S.W. corner : $x = +\ 164190{\cdot}3$ ft., $y = +\ 17999{\cdot}3$ ft., convergence 33′ 37″.

The x co-ordinates of the N.W. and S.E. corners are the same as those of the *S.W.* and *N.E.* corners respectively, and the y co-ordinates are the same as those of the *N.E.* and *S.W.*

The grid is constructed and the stations are plotted in the manner already described. To graduate the sheet, proceed as follows :

(*a*) From the N.E. corner draw a line representing the meridian through that point by laying off an angle of 50′ 05″ from the vertical side of the grid. A chord with a radius approximately equal to the length of the vertical side should be used. The direction in which to lay off the meridian, *i.e.*, to the right or left of the grid side, is obvious when it is remembered that all meridians, including that through the origin, converge to meet at the pole.

(*b*) Similarly from the S.W. corner draw a line representing the meridian through that point by laying off an angle of 33′ 37″ from the vertical side of the grid.

(*c*) From the N.E. and S.W. corners draw lines normal to the meridians; this is most easily accomplished by laying off angles equal to the respective convergences from the horizontal sides of the grid.

(*d*) From each of these normals lay off towards the pole an angle equal to *half* the difference of the convergence at the corners, *i.e.*, in this example $\frac{50' 05'' - 33' 37''}{2} = 8' 14''$. The points at which these lines intersect the meridians will be the other two corners of the graduation. The parallels may be drawn and the subdivision carried out in the manner previously described.

The construction is shown in Fig. 116, where the angles of convergence have been greatly exaggerated for the sake of clearness. The corners of the graduation are shown circled.

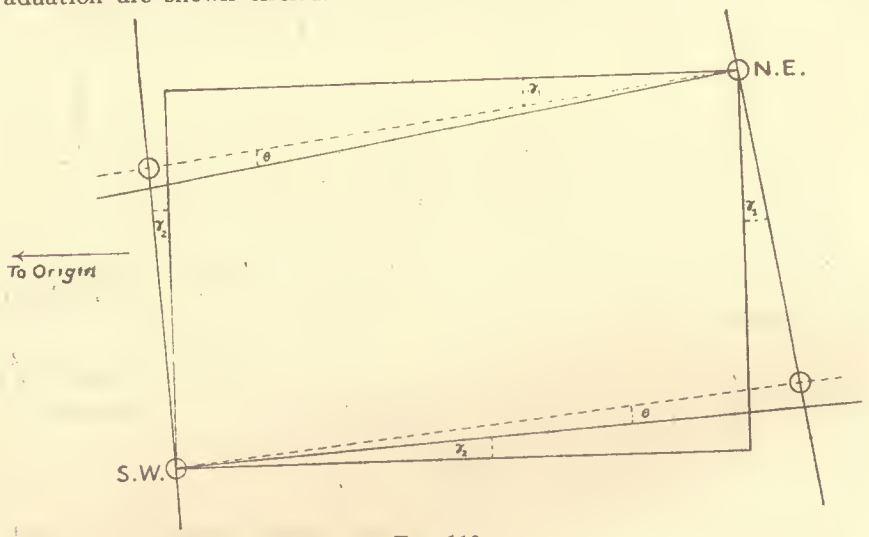


FIG. 116

Angle γ_1 = convergence between N.E. corner and the Point of Origin.
Angle γ_2 = convergence between S.W. corner and the Point of Origin.
Angle $\theta = \frac{\gamma_1 - \gamma_2}{2}$.

Case (3).

When the triangulation has been plotted from a "long side," different methods of graduating must be employed. The simplest is as follows:

In Fig. 117, A and B are two trigonometrical stations situated near two opposite corners of the survey. Their geographical positions and the bearing AB and reverse bearing BA are calculated from the triangulation. It is required to graduate the sheet.

(*a*) Join AB and lay off the meridians AM and BN through A and B; the angles will be the bearing AB and reverse bearing BA.

(*b*) Obtain the exact scale of the plot at the time by measuring AB and comparing the measurement with its actual length.

(*c*) Select two corners D and F for the graduation at even minutes or decimals of a minute of latitude and longitude in the vicinity of A and B respectively and so disposed that the graduation figure will enclose the area of the survey. In this example, the station A is excluded.

(*d*) On AM and BN mark off AC and BE, the distances, according to scale, corresponding to the differences of latitude between A and D and between B and F. The length of 1″ of latitude and its logarithm are given in Tables III, IIIa and IIIb in the column headed ρ sin 1″.

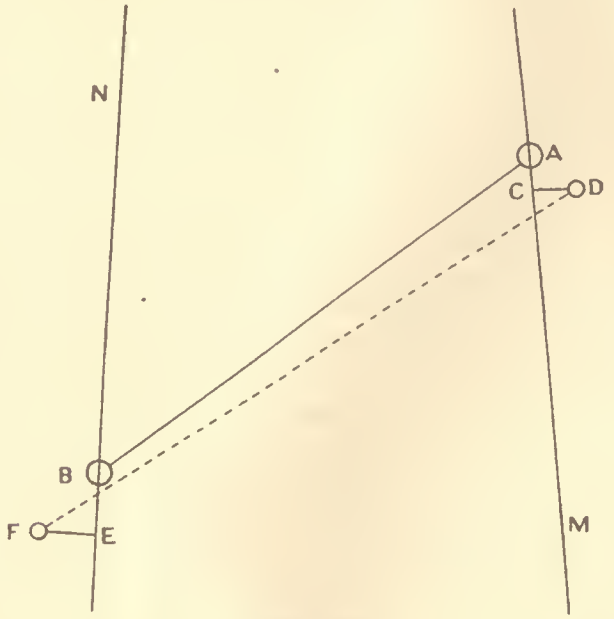


FIG. 117

(*e*) Draw CD and EF normals to the meridians at C and E, their lengths, according to scale, corresponding to the differences of longitude between A and D and between B and F. The length of 1″ of longitude can be found from Tables III, IIIa and IIIb by multiplying ν sin 1″ by cosine latitude. The latitudes of D and F must be used as the argument.

(*f*) D and F are two required corners of the graduation. In the construction it is assumed that the normals CD and EF coincide with the parallels through C and E; this will be sensibly the case if the distances CD and EF are short as they *must* be when this method of graduation is employed.

(*g*) Calculate the bearing DF and reverse bearing FD and draw the bounding meridians through D and F.

(*h*) Draw normals to these meridians through D and F and lay off angles towards the pole equal to *half* the convergence between D and F as in case (2). The intersections of these lines with the meridians are the other two corners of the graduation. The parallels can then be drawn and the subdivision effected in the manner already detailed.

BOARDS AND TRACINGS

Field Boards.

The plotting sheet should not be removed from the chart-room during the progress of the survey and it is not therefore available for plotting and recording the detail of the survey in the field. For this purpose use is made of wooden field boards which are supplied in various convenient sizes and are mounted with cartridge paper cut as necessary from a continuous roll. Zinc sheets, in lieu of wooden boards, have also been tested experimentally with satisfactory results and in time may be more extensively used.

To mount a board :

(1) Selecting the smoother side of the board, rub its surface thoroughly with sandpaper to remove all irregularities.

(2) Cut off a piece of cartridge paper the shape and size of the board, with about 1½ ins. overlap all round.

(3) Prepare a fairly thick paste by mixing ordinary flour and warm water and coat the surface of the board with it.

(4) Damp the under side of the paper well and lay it on the board : then smooth it down thoroughly with a hand-roller to remove creases and to squeeze out surplus paste.

(5) Cut a nick at each corner and paste the overlap to the edges and underside of the board ; this prevents the paper being torn away at the edges. Allow the mounted board to dry thoroughly before use.

The various stations and marks on the plotting sheet are transferred to the field boards by means of a paper tracing on which their positions are pricked through and circled. This *Pricking-through Tracing* should be the same size as the plotting sheet in order to cover the whole survey, and it must be carefully handled to prevent damage and distortion. In addition to the trigonometrical points and sounding marks, the corners and mid-points of the plotting grid should be transferred to the field boards for use as fitting-on points for the fair and collector tracings. A meridian and scale should be drawn on each board.

When the work for which the board was prepared is finished, the paper may be " floated off " intact by allowing the board to soak in a bath, or over the ship's side on a rope's end, for an hour or so, ofter which it can be prized away gradually without much difficulty. The " float-offs " should be retained until the fair chart has been drawn and checked.

It is usually convenient to prepare separate boards for the sounding and topography respectively ; the former get a great deal of wear and tear in the boats and soon become dirty—when they should, of course, be replaced by fresh boards.

Whatman's Drawing Boards.—These are made in various sizes and are made of stiff cardboard, one side of which has a specially prepared surface. They may be used in lieu of or to supplement the ordinary wooden field boards for many purposes and have the advantage of being much lighter. Their lack of rigidity, however, makes them inconvenient for use in boats or wherever wear and tear is likely to be considerable.

Fair Tracings.—The work on the field boards is transferred to the fair chart by means of paper tracings which should be of moderate size, say, not more than about a foot square. Tracing paper is made in rolls 30 ins. or 40 ins. wide and a sufficient number of pieces should be cut off to cover the area of the survey, each piece overlapping the adjacent ones an inch or so. Enough trigonometrical and grid points should be pricked through onto each tracing from the plotting sheet to ensure that it can always be fitted to the field boards in its correct relative position. To minimise the effect of distortion such points should not be more than 6″ apart and, if necessary, a few arbitrary " fitting-on " points should be placed on the plotting sheet and transferred to the boards and tracings for this purpose, but intersections of parallels and meridians or of grid lines will usually provide a sufficient number of these.

A series of small tracings for transferring the work from the boards is preferable to one large tracing covering the whole area since distortion is minimised and there is less likelihood of damage occurring in use. The labour of tracing is also expedited as the detail can be transferred from the various field boards simultaneously, and, to facilitate this process yet further, it is convenient to make a separate series of tracings for the soundings on the one hand and for the coastline and topography on the other. Separate tracings

should be made for the examination of shoals, and work in the vicinity which appears on a sounding tracing should be encircled in some distinguishing colour and transferred to the examination tracing.

As the work on each tracing or field board is completed, all details must be carefully checked to eradicate any errors of transcription (particular care being taken that work on the common overlap of tracings is in exact agreement).

Collector Tracing.—In addition to the field boards and fair tracings which show small isolated portions of the survey, a comprehensive record is essential as a guide to the progress of the work and as a duplicate in case of accidental destruction of the fair chart or tracings. This record, the " collector tracing," is kept on tracing cloth which is made in rolls 30 ins. or 43 ins. wide. All trigonometrical, grid and graticule intersections and sounding marks are pricked through on this tracing from the plotting sheet and it is then kept up to date, as the survey progresses, by tracing off the work from the field boards.

Tracing cloth distorts a great deal and must not therefore be used for transferring points from the plotting sheet to the field boards or detail from the boards to the fair chart, processes which must always be done with tracing *paper*.

When finally completed, the information on the collector tracing should be identical with that on the fair chart in every particular.

CHAPTER IX

GRIDS

The general term "grid" can be defined as any systematic network of lines superimposed on a chart or map and lettered and numbered in such a way that the position of any feature can be precisely defined.

A grid is designed to fulfill the following requirements :

(*a*) To provide a concise method of reference to positions.

(*b*) To remove any possibility of ambiguity in such reference.

(*c*) To enable positional references to be made by inspection.

(*d*) To facilitate the rapid calculation of bearings and distances between positions which may or may not appear on the same chart or map.

It is apparent that a grid that cuts corresponding points on a chart and on a map will provide a ready means of relating the information shown on the one with that of the other. The paucity of topographic detail on a chart and the absence of hydrographic information from a map often make it necessary to use the two in conjunction. By means of the same grid plotted on both, chart users and map users are able to co-ordinate positional references.

Systems of reference employed on charts and maps fall into two classes, those which are based on parallels of latitude and meridians of longitude and those in which the basis is a rectangular network of lines formed by measurement from some fixed point. The former is usually referred to as a "Graticule" system and the latter as a "Grid" system. The grid system has been widely adopted as the more convenient for most operational and intelligence purposes.

The Grid or Co-ordinate System.—This is simply a network of squares whose orientation is fixed by the direction of a meridian through a point called the "Point of Origin." This meridian is called the "Initial meridian."

The size of the squares forming the network varies with the scale of the chart or map and the values assigned to the lines forming the squares are some convenient round numbers of whatever units of measurement are employed.

The grid is related to a point of origin which is usually selected somewhere within the area covered by the particular grid system. Following the normal algebraic convention, grid lines and references East and North of the origin would have plus values, those to the West and South minus values, but, in order to avoid the use of signs in giving grid references, a "false" origin with zero co-ordinates is chosen to the West and South of the area and arbitrary co-ordinates (*e.g.*, 500 km. East and 610 km. North) are assigned to the true origin, so that all references within the area have plus values and it is therefore unnecessary to indicate their sign. This simplifies grid references and, of course, will not affect the use of the grid for the determination of bearings and distances since the calculations depend on differences of grid values and not the actual figures of these values. It should be noted, however, that grid bearings are not true bearings but are angular distances from the "initial meridian," *i.e.* that through the *true*, not the false, origin.

In the British Modified System the area covered by the grid is, on small scale maps, divided into squares of 100,000 units. The 100,000 squares are lettered from A to Z (neglecting I) in an easterly and descending order and the

grid lines are numbered. The first unit of the 100,000 numbers is printed in small figures.

Fig. 118 shows a part of such a grid. The point X is referred to by the letter B followed by a four figure group, *i.e.*, "B 1429"; the first two figures of the group specify the easting of the point and the second two its northing. In each case the first figure is obtained from the value of the grid line to west or south of the point and the second figure is obtained by visual interpolation in the square to east and north of these grid lines.

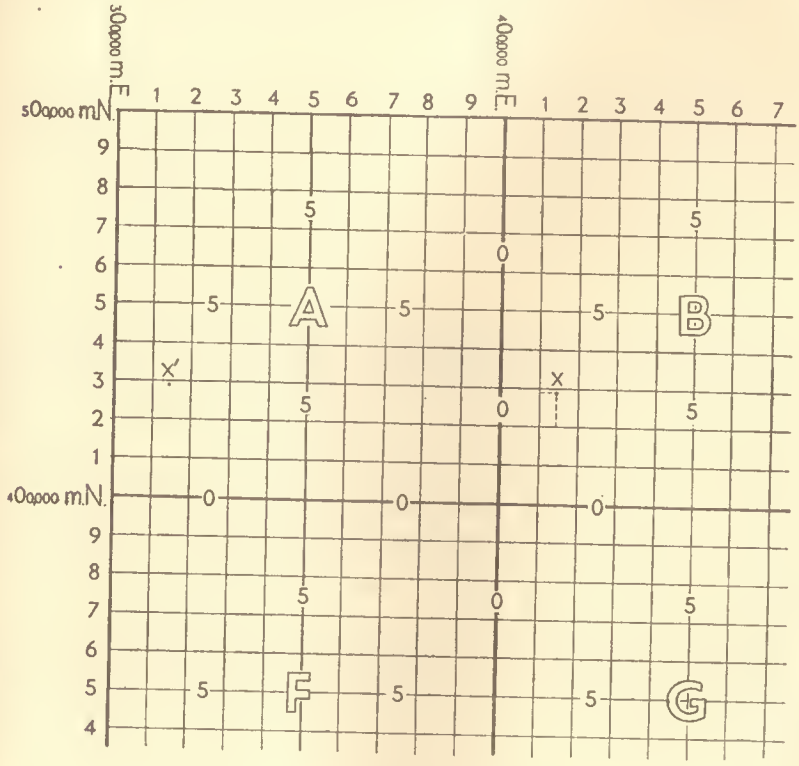


Fig 118

It will be seen from Fig. 118 that without the prefixed letter B, the reference would hold good also for the position X'.

On larger scale maps or charts the grid is printed in squares of 10,000 units, each square being subdivided into squares of 1,000 units and references are given by six-figure groups. The first three figures in such grid references refer to the easting of a point and the last three to its northing. In each group of three figures the first two are taken from the nearest grid lines to the west and south of the point and the third by visual interpolation within the square to the east and north of the selected grid lines.

Fig. 119 shows a part of such a grid, the point X being at 549574. The grid letter is omitted from the map as a rule, excepting in cases where the boundary between 100,000 squares appears on the sheet. Even in these cases it may sometimes be only shown on a key diagram. It is important to remember that the first figure in each half of a numerical reference, whether on large or small scale grids is always the 10,000 digit.

In the system which has been described it will be seen a singly lettered grid will cover an area of 500,000 units to East and North of the false origin

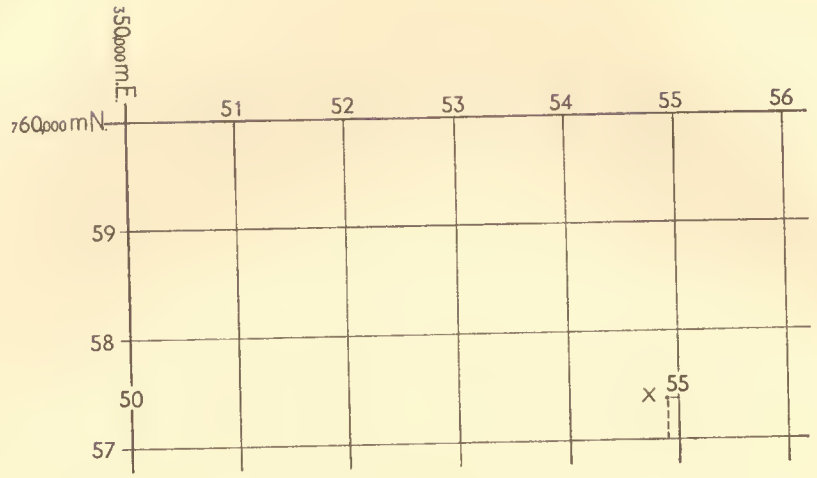


FIG. 119

and that, therefore, if the grid is extended beyond these limits the letters must repeat themselves at every 500,000 units. In many grid systems, therefore, the first subdivision is into lettered squares at 500,000 units and each of these lettered squares is further subdivided into 100,000 unit squares. A full grid reference on such a grid will then consist of two letters followed by a six-figure group.

To distinguish between the letter which refers to the 500,000 unit square and that which refers to the 100,000 unit square the former letter is printed on charts and maps smaller than the latter. Where no doubt can possibly arise of what 500,000 unit square is being referred to it is not necessary to quote the small (500,000 unit square) letter when giving a grid reference.

It is important to note that in some tables and diagrams giving grid data the 500,000 unit square letters are printed in large letters and the 100,000 square letters in small. This is the reverse of the procedure adopted on charts and maps.

In the majority of cases the unit of a grid is the metre, although the Malta, India, Ceylon and Australian grids, for example, are in yards.

World Polyconic System.—This system was introduced by the United States Army authorities and has been adopted in certain areas of the Pacific; its use may spread to other areas.

The world is divided into five bands of longitude and each band is subdivided into nine zones. Limits of the bands and zones are shown in tables of grid data.

No letters are used and a full reference is given by an eight figure group of which the first three in each half of the group are obtained from the values of the grid lines printed on the map and the last by interpolation.

The band numbers and zone letters are shown on the map in a "box" containing grid data.

GRIDDING OF CHARTS.

As a rule it will only be medium scale charts which will require gridding, though large scale charts of harbours have in the past been gridded.

One of the principal uses of gridded charts is for bombardment purposes and such a chart should, ideally, meet the following requirements:

(*a*) The scale should be as large as possible.

(*b*) All possible targets should be shown.

(*c*) As much sea room as possible must be included.

These requirements are at times mutually conflicting and it has been found that, in practice, a scale of about 1/50,000 provides the best compromise.

On charts (as distinct from maps) of 1/75,000 and larger, grid lines corresponding to a distance of 1 kilometre apart should be drawn to enable positions to be read by estimation to the nearest 100 metres. On charts of smaller scales than 1/75,000 grid lines 10 kilometres apart, enabling positions to be read to the nearest kilometre should suffice.

The general responsibility for deciding the type of grid, the projection and the spheroid to be used in any part of the world rests with the Geographical Section of the General Staff of the War Office. In any area where land operations are envisaged, the G.S.G.S. provide the necessary data for constructing the chosen grid and lay down the geographical limits of the zones which each grid system will cover. To ensure uniformity it is essential that the same grid systems with the same numerical values and colour should be used by other services, but black should *not* be used for grids on charts owing to the possibility of confusion with meridians and parallels.

The grid co-ordinates of any point derived from measurement on a gridded chart should agree with that derived from measurement on a gridded map. In the ideal case the grid on a chart assumes a regular pattern and this can be done if :

(i) The difference between the chart and the map is solely one of graduation, *i.e.*, a bodily shift of the graticule on the map or chart would bring both into complete harmony.

(ii) Only small, irregular differences in the topographic detail between the chart and the map exist.

Complete agreement could only be obtained by making a multitude of small distortions to the grid but, as this is impracticable, small inconsistencies between grid references on charts and maps must be regarded as inevitable.

If, however, the initial comparison between the chart and the map shows that in certain areas, more or less well defined, a series of comparisons gives one result, whilst elsewhere they give a considerably different result ; or if there is agreement only at one point, and, from that point, discrepancies increase, it may be necessary to distort the grid or even, in extreme cases, to make it discontinuous in places. If it is necessary to grid such charts for use in combined operations when other Services are making use of modern and presumably more accurate maps, it is as well to place a cautionary note on the chart to the effect that inconsistencies may be expected.

It has been customary, except in a few cases where the chart has been demonstrably superior to the map, to bring the grid on the chart into sympathy with that on the map. In general, therefore, the first step in gridding a chart is to decide what correction, if any, is necessary to bring the chart graticule into sympathy with that on the maps. Military maps do not always have a graticule drawn on them but geographical positions of the corners of each sheet are given or else can be found from data printed on the sheet. If the grid co-ordinates only of the sheet corners are given, their geographical co-ordinates should be calculated from the formulae applicable to the projection of the grid and a skeleton graduation should be drawn to enable geographical positions to be read.

The latitudes and longitudes of a number, say 10, of corresponding points of detail should be measured off the chart and map and mean differences

obtained to provide a correction which will bring the chart graticule into sympathy with that of the map. In cases where individual differences disagree largely with the general run, clearly examples of inaccurate survey or plotting, they should be rejected.

Suppose, for example, that the north-east corner of the graticule of a chart was shown as being in Latitude 52° 10′ N., Longitude 71° 05′ E., and that, on an average, the latitudes of points of detail on the maps were 10″ greater than those of corresponding points on the chart and longitudes 35″ less, then, to bring the chart graticule into sympathy, the north-east corner of the chart must be assumed to be in Latitude 52° 10′ 10″ N., Longitude 71° 04′ 25″ E. The same corrections would, of course, be applied to the other corners of the chart.

Modern hydrographic surveys are normally based on the national trigonometrical system (if one exists) of the area concerned and in such cases no corrections should be required apart from any correction necessary to longitudes based on a meridian other than that of Greenwich.

The calculations required before the grid can be drawn are not difficult, but the construction may be rather troublesome as, in most cases, the grid is not on the same projection as the chart. Maps, on the other hand, are usually on the same projection as the grid and the latter can be plotted as an equidistant system of straight lines at right angles to each other. On a chart drawn on a projection different from that of the grid, the grid lines are not necessarily parallel, equidistant or straight.

The majority of charts which require gridding are drawn on Mercator's or Gnomonic projections. The most common projections used for grid systems are Cassini's, Transverse Mercator (Gauss conformal) or conical orthomorphic projections such as Lambert's. Formulae for the conversion of geographical co-ordinates to rectangular grid co-ordinates and vice versa for these three projections are given in " Survey Computations "* and elsewhere. Whichever formula has to be used, constants for the spheroid appropriate to the particular grid system must be employed. The G.S.G.S. usually publish tables to facilitate conversion of co-ordinates in the various zones and these will save a good deal of labour, particularly if a calculating machine is available.

The following steps are necessary whatever type of grid is to be drawn on a chart :

(*a*) Obtain corrections to the graticule of the chart as already described.

(*b*) With the corrected geographical co-ordinates calculate the rectangular co-ordinates of the four corners of the chart graticule and the convergency between the meridians at each corner and that through the *true* origin of co-ordinates, using the formulae, constants and units appropriate to the grid. (On the Lambert projection the convergency of the grid is the same for all points on any particular meridian, so that the convergency at one point only on each marginal meridian of the chart need be calculated.)

It is usual to draw the grid over the whole of the chart but in cases where a very large expanse of sea is shown this may not always be necessary. In such cases intermediate positions can be selected to cover the area required.

(*c*) Find out on what projection the chart is drawn. This must usually be done by inspection. If the chart is a Mercator, the parallels will be represented by straight lines, the scale of latitude will increase towards the poles and the top and bottom of the sheet should be

* " Survey Computations, 1932 " (Ordnance Survey). H.M. Stationery Office.

equal in measurement. If the chart is a Gnomonic, the parallels will be curved, concave towards the poles, the meridians will converge towards the poles and the scale of latitude will be practically constant on a medium or large scale chart.

If the chart is drawn on Cassini's or a conical projection, the difference in the shape of the graticule from that of a single, medium scale Gnomonic chart is very small and cannot be detected with certainty by measurement on the paper as there is sure to be some distortion during printing. No appreciable errors will be introduced by assuming that such a chart is, in fact, drawn on the Gnomonic projection.

(*d*) A number of intersections of whole number grid lines are now chosen within or near the co-ordinate limits of the chart corners. These points must now be joined but the joining lines may be either straight lines or curves according to the shape that the grid lines will assume : and interpolation must be made within the computed points to obtain the intermediate values of the grid lines. The grid, when drawn, consists of a series of trapezoidal figures. On small scale charts (less than 1·75,000) grid lines at every 10 kilometre interval should be drawn ; on larger scales, lines at each kilometre interval.

The procedure is best illustrated by an example :

It is required to grid Chart No. 1585. " Bay of Acre."

The following data are available :

Scale of Chart.	1/50,000.
Projection of Chart.	Gnomonic. (Not indicated under the title but presumed to be so by inspection.)
Projection of Grid.	Transverse Mercator.
Spheroid.	Clarke 1880.
Unit.	Metre.
Point of Origin.	Latitude 31° 44′ 02″·749 N. Longitude 35° 12′ 43″·490 E.
False co-ordinates of Origin.	170,251·555 metres E. 126,867·909 metres N.

Comparison with the maps shows that no correction is necessary to bring the chart graticule into sympathy with that of the map.

The corners of the chart are :

North-east.	Lat.,	33° 01′ 27·3″ N.,	Long.,	35° 09′ 01·2″ E.
North-west.	,,	33° 01′ 27·3″ N.,	,,	34° 52′ 58·5″ E.
South-east.	,,	32° 44′ 00·0″ N.,	,,	35° 09′ 00·0″ E.
South-west.	,,	32° 44′ 00·0″ N.,	,,	34° 53′ 00·0″ E.

Computation gives the rectangular co-ordinates as :

North-east.	164,482·412 m E.	269,932·336 m N.
North-west.	139,522·948 m E.	269,980·314 m N.
South-east.	164,432·292 m E.	237,670·164 m N.
South-west.	139,435·700 m E.	237,717·798 m N.

A rough " mock-up " can now be made of the position of the grid lines. This is only to ascertain what cutting points will be required for computation and so need not be accurate nor to full scale. Fig. 120 shows such a " mock-up."

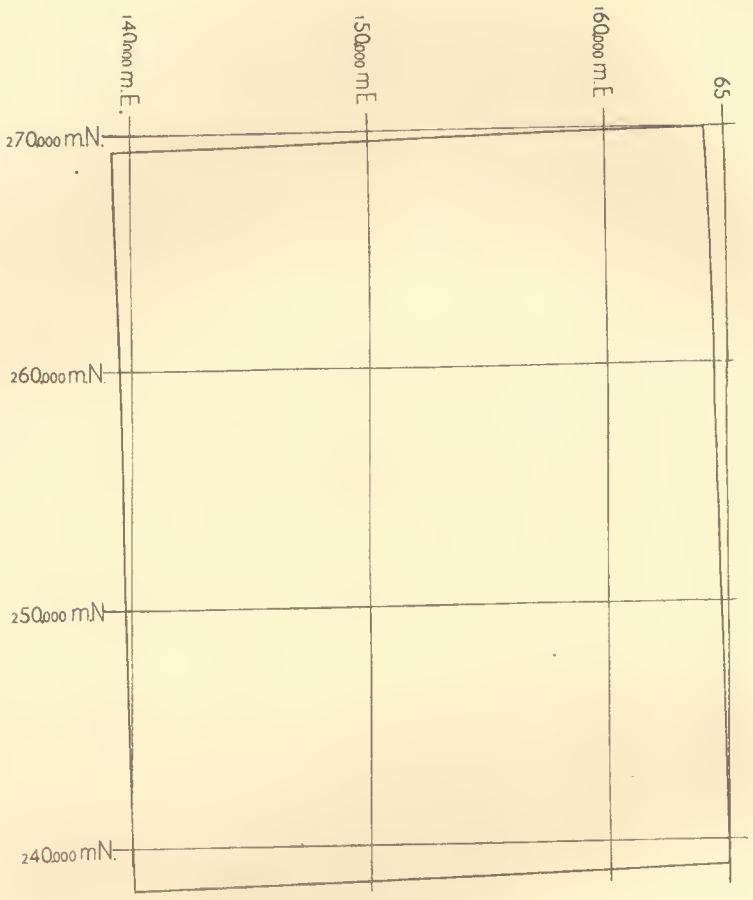


Fig. 120

The geographical co-ordinates of the 16 grid line cutting points shown in the figure are now computed. The table below shows the results of these computations.

Cutting Points—Latitude N.

Grid Line	140000		150000		160000		165000	
	° ′ ″	Diff.	° ′ ″	Diff.	° ′ ″	Diff.	° ′ ″	
270000	33 01 27·99	0·83	33 01 28·82	0·51	33 01 29·33	0·13	33 01 29·46	
	5 24·61		5 24·61		5 24·62		5 24·62	Diff.
260000	32 56 03·38	0·83	32 56 04·21	0·50	32 56 04·71	0·13	32 56 04·84	
	5 24·62		5 24·62		5 24·62		5 24·62	Diff.
250000	32 50 38·76	0·83	32 50 39·59	0·50	32 50 40·09	0·13	32 50 40·22	
	5 24·62		5 24·62		5 24·62		5 24·63	Diff.
240000	32 45 14·14	0·83	32 45 14·97	0·50	32 45 15·47	0·12	32 45 15·59	

CUTTING POINTS—LONGITUDE E.

Grid Line	140000		150000		160000		165000	
	° ′ ″	Diff.	° ′ ″	Diff.	° ′ ″	Diff.	° ′ ″	
270000	34 53 17·87	6 25·30	34 59 43·17	6 25·32	35 06 08·49	3 12·65	35 09 21·14	
	1·19		0·80		0·40		0·21	Diff.
260000	34 53 19·06	6 24·91	34 59 43·97	6 24·92	35 06 08·89	3 12·46	35 09 21·35	
	1·17		0·79		0·39		0·20	Diff.
250000	34 53 20·23	6 24·53	34 59 44·76	6 24·52	35 06 09·28	3 12·27	35 09 21·55	
	1·18		0·78		0·40		0·21	Diff.
240000	34 53 21·41	6 24·13	34 59 45·54	6 24·14	35 06 09·68	3 12·08	35 09 21·76	

Consideration of differences in the two tables shows that :

(*a*) The Latitudes and Longitudes of the cutting points of intermediate northing lines on the easting lines can be obtained by direct interpolation.

(*b*) The Longitudes of the cutting points of intermediate easting lines on the northing lines can be similarly obtained.

(*c*) The Latitudes of the cutting points of intermediate easting lines on the northing lines cannot be so simply obtained since the differences change along the northing lines ; *i.e.*, on Line 270000 differences are 140000–150000, 0·83 seconds and 150000–160000, 0·51 seconds.

The steps in latitude, between successive cutting points along the northing lines, decrease from west to east. Calling these steps L, (L — d), (L — 2d), etc., their value can be found from the simultaneous equations :

$$L + (L - d) + (L - 2d) \ldots\ldots\ldots\ldots\ldots + (L - 9d) = 0''{\cdot}83 \text{ of lat.}$$

$$(L - 10d) + (L - 11d) \ldots\ldots\ldots\ldots\ldots + (L - 19d) = 0''{\cdot}51 \text{ of lat.}$$

giving (in secs. of lat.), $L = 0''{\cdot}0974$ and $d = 0''{\cdot}0032$.

In the example above, the decrease in the differences is very small and direct interpolation would be within plottable limits.

It is customary to show the value of the grid convergency on two meridians situated near the east and west borders of the chart. In the case of large scale charts whose east-west extent is not great, this is sometimes shown for the central meridian of the chart only.

Computation in the above example gave the following figures for the convergency of the grid :

North-east corner	2′ 01·15″
North-west corner	10′ 45·18″
South-east corner	2′ 00·85″
South-west corner	10′ 39·95″

In this case it would be sufficient to show on the chart, in the " box " containing grid data :

" Grid North 0°6′ E. of True North at Meridian 35° East.

Experience will soon show how many cutting points are to be selected for computation. In the example 16 were used although it is probable that 9 would have sufficed.

The following table shows the form that will be taken by grid lines on a Lambert, Transverse Mercator or Cassini grid when plotted on a Mercator or Gnomonic Chart.

Projection of Chart.	Projection of Grid.	N–S lines.	E–W lines.	Scale along Meridians.	Scale along Parallels.
Mercator.	Lambert.	Straight.	Curved, convex towards pole.	Practically constant along meridian, *i.e.*, grid length of a kilometre is the same number of secs. of Latitude everywhere.	Increases very slightly, *i.e.*, length on paper of a kilometre increase is hardly appreciable.
Mercator.	Transverse Mercator.	Slightly curved, convex towards meridian of True origin which is straight. Can be neglected up to 150 miles E or W of origin.	Curved, convex towards pole.	As for Lambert grid.	Scale along parallel is practically constant.
Mercator.	Cassini.	Curved, as for Transverse Mercator Grid. Can be neglected up to 100 miles from True origin.	Curved, convex towards pole.	As for Lambert and Transverse Mercator grids.	As for Lambert Grid.
Gnomonic.	Lambert.	Straight.	Practically straight.	Practically constant.	As for Mercator with Lambert grid.
Gnomonic.	Transverse Mercator.	As for Mercator with Transverse Mercator grid.	Practically straight.	Practically constant.	Practically constant.
Gnomonic.	Cassini.	As for Transverse Mercator grid.	Practically straight.	Practically constant.	Practically constant, unless distance from origin is very great. Increases very slightly E–W or N–S of origin.

CHAPTER X

SOUNDING

Sounding is perhaps the most important part of the hydrographic surveyor's work. Safe navigation demands an accurate knowledge of the depth of the sea, particularly in harbours and their approaches where vessels may often be compelled to use anchorages, channels and fairways where the depths may allow little margin for error. As the bed of the sea is hidden from view, its form can only be determined by systematic soundings at regular intervals over the whole water area of a survey. It is assumed, unless there are indications to the contrary, that the slope of the bottom is uniform in the spaces between adjacent soundings, a postulate which has no strict justification; however, no better method than that of systematic and evenly-spaced sounding has yet been devised, and it can at least be said that the results have been on the whole satisfactory. Nevertheless, the possibility of the existence of irregularities, dangerous to navigation, between adjacent soundings cannot be lightly dismissed, and it is often necessary to supplement sounding by " sweeping " part or parts of the survey area at some pre-determined depth; this should at least ensure that no dangers remain undetected but unfortunately, unless bottom sweeping is resorted to, it can only give this negative information which does not completely fulfil the requirements of navigation.

The operation of sounding is usually the most arduous and sometimes the least interesting part of the hydrographic surveyor's work. Considerations of practicability, safety and economy demand that much of it should be done in small boats in which the surveyor is necessarily somewhat exposed, cramped and uncomfortable; if the weather is against him, his difficulties are, of course, greatly augmented. Modern inventions have done much to relieve the purely manual labour involved; sounding under oars or sail is now a thing of the past and the use of the hand lead is giving way to various types of echo sounding apparatus which have increased the output and the accuracy of the work.

Lines of Sounding.—In theory there is no limit to the number of soundings which might be considered necessary to make certain that the contours of the sea bed are accurately represented on a chart. It is conceivable that, even if soundings only 10 ft. apart were taken over the entire water area, important irregularities might be missed, but except in isolated cases, *e.g.*, more or less enclosed basins and harbours of very small extent, it would be quite impracticable to attempt any such detailed examination as this, nor as a rule, would much be gained by it. In practice the density of the soundings must be dependent on the scale and nature of the survey.

The normal method of sounding an area is to run lines of soundings with the ship or boats approximately at right angles to the trend of the coast. The fathom lines, or contours connecting isometric depths, are, as a rule, roughly parallel to the coastline and the drawing of these on the chart is somewhat facilitated by running the sounding lines perpendicular to them. It would be practically impossible to fix every individual sounding and the method adopted is to fix the position at regular intervals by sextant angles to marks on shore or floating beacons. Assuming that the ship or boat is properly handled, speed and course between successive fixes will be for all practical purposes constant, so that intermediate soundings can be plotted by interpolation with all required accuracy. When echo sounding apparatus fitted with an automatic recorder is used, the record of depths along the line is continuous, for the soundings are taken with such rapidity that there is no appreciable interval between them. If a hand lead, or any type of mechanical sounding gear is

used, there must be some interval between consecutive soundings dependent on the personal factor and/or the efficiency of the machine. The usual procedure in depths of 20 fathoms or less is to adjust the speed *according to the scale of the survey* so that at least as many soundings are taken as can be conveniently shown on paper. For instance, about ten soundings can be inked in legibly to an inch of paper, so that on a scale of 1/6,250 (about 12 ins. to the nautical mile) it will be necessary to take soundings at intervals not more than 50 ft. apart, whilst on a scale of 1/50,000 (about $1\frac{1}{2}$ ins. to the nautical miles) sounding at intervals of 150 *yards* would suffice. As the depth increases, the chances that a *dangerous* shoal exists (and might go undetected) between two consecutive soundings become correspondingly reduced, and the normal sounding intervals can be extended, at any rate on large scales. If, for instance, a survey on a scale of 1/6,250 included an area with depths of 50 fathoms or more, it would be a waste of time to sound that part of it at such close intervals as 50 ft.; sounding 300 ft. apart (*i.e.*, equal to the depth) should be quite sufficient to ensure that some indication of a possible danger was obtained. With echo sounding gear (fitted with an automatic recorder) the depth record is continuous and the speed of the ship or boat is only limited by the ability of the surveyor to fix and plot his position quickly enough as the work proceeds.

The distance between adjacent *lines* of sounding is normally made dependent on the scale of the survey. The usual practice in depths of 20 fathoms or less is to run the lines $\frac{1}{5}$ or $\frac{1}{4}$ of an inch apart *on the paper*: on a scale of 1/6,250, $\frac{1}{5}$ of an inch represents about 100 ft., on a scale of 1/50,000 about 800 ft. This may be expressed by saying that the degree of thoroughness with which an area is sounded is made contingent on the scale of the survey.

In deeper water, *e.g.*, over 20 fathoms, it is customary to open out the lines of sounding to, say, $\frac{2}{5}$ or $\frac{1}{2}$ in. apart, which will in most cases provide a sufficiently detailed examination, and this practice is sometimes followed in very *shallow* water where navigation will be confined to boats and no useful purpose will be served by any very exhaustive investigation. In depths of over 100 fathoms, the principle of running lines of sounding is usually abandoned, as being neither justified by navigational requirements nor feasible without special apparatus and an unwarranted expenditure of time. Sufficient soundings are taken in such depths at more or less regular intervals, *e.g.*, so many cables or miles apart, to provide a general picture of that part of the sea bed and to ensure that no large irregularities exist.

The lengths of the lines of sounding, *i.e.*, the distances to which they will be carried out from the shore in a coastal survey, are generally dictated by the instructions with which the surveyor is provided. Such instructions may define the limits of the water area to be surveyed in pictorial form, by latitude and longitude, or by measurement from the coast; alternatively, the surveyor may be directed to extend the sounding to some definite depth, *e.g.*, the 20-fathom contour. The lines of sounding will obviously be extended to these prescribed limits. In the absence of instructions, the limits of the soundings are usually determined by the purpose and scale of the survey and by the time available. On large-scale plans the sounding will naturally be confined to a comparatively small area as the plotting sheet cannot cover more than this. In other types of survey it may be possible to extend the sounding to the 100-fathom line, a useful limiting contour, as this is about the greatest depth to which soundings can be taken with the navigational appliances normally available in ships.

Off straight or only slightly sinuous, stretches of the coast the lines of sounding should in ordinary cases be at right angles to the general trend of the shore, and parallel to one another. A narrow ridge is best examined by lines at right angles to its longitudinal axis. Off points on the coast where there is a marked change of direction, in deep bays and round off-lying islets radial lines of sounding are generally the most suitable.

Example :—

Fig. **121** shows part of a coast running north and south as far as the point A and then turning north-eastwards : B is an off-lying islet. The lines of soundings which should be run are shown as pecked lines ; southward of A they run east and west, but are " starred " round B. They are again starred round A to turn roughly north-west and south-east beyond this point. The scheme of sounding provides for *a methodical and systematic examination of the area*, this being the one essential condition of any system which may be adopted.

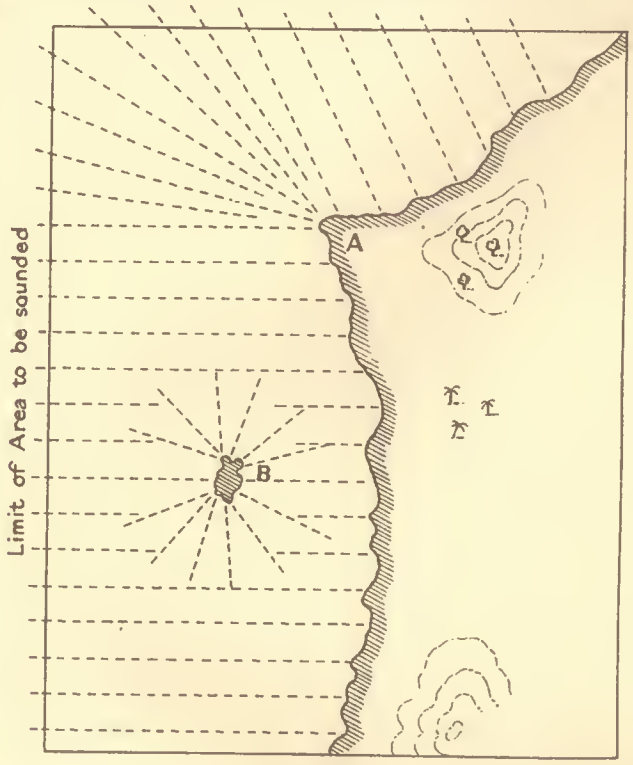


FIG. 121

Although running lines at right angles to the trend of the coast is the *usual* and ordinarily the most *convenient* method, it is unnecessary to adhere too rigidly to it. A channel, for instance, is often more easily sounded by running the lines parallel to its axis, and there is no reason why it should not be done in this manner.

It will sometimes be possible to ascertain the least water over a bank by running a line of soundings along instead of at right angles to its axis and casting the boat from one side to the other of the line of shoalest water as previously indicated by the lines of soundings which have already been run across the shoal.

Particular care is necessary where an abrupt rise or fall of the sea bed occurs and the speed of the boat must be controlled so that sufficient fixes may be obtained to enable the position of such changes to be accurately determined.

Interlining.—Lines of soundings, as in the example given, provide a record of the depths and enable contours of the sea bed to be drawn. Whether this density of sounding is sufficient to ensure that no dangers remain undetected in any part of the work must be decided from the scale and nature of the survey and from a rigorous examination of all the soundings when they have been inked in on the board. Where the bottom consists entirely of mud or sand, its slope will not, as a rule, be very great at any point and a shoal must therefore

occupy a considerable area in proportion to its height ; it is unlikely that a shoal involving any great change in depth could lie wholly in the space between two adjacent lines and so remain undetected. In such cases lines of sounding normally spaced will generally fulfil all requirements. In this connection it may be noted that off-shore shoals of mud and sand usually have their longer axes parallel to the coastline, and, as consecutive soundings in a line are always much nearer to each other than the adjacent lines themselves, there is this additional reason for running the lines at right angles to the trend of coast. Where the bottom is known or suspected to be irregular, it is advisable, especially on small scales, to increase the density of the sounding over the whole or part of the survey area by running " inter lines " between those normally spaced. By so doing, the chances of missing a shoal are, of course, much reduced.

Where there is a shoal, it is unlikely that any line of soundings will have passed directly over its shallowest part (by which the " least water " would have been obtained) but it *is* anticipated that the nearest line or lines will give some *indication* of its existence. Such indication will be a break in the continuity of the slope of the sea bed. If, for instance, a series of soundings (in fathoms and feet) was as follows :—

8_1, 8, 7_4, 7_4, 7_3, 6_1, 7_3, 7_2, 7, 7,

it is clear that the sounding 6_1 represents a sharp and sudden change of slope, and its vicinity must be further examined. A similar indication of the shoal may be found in the adjacent lines of sounding and interlines should be run to determine its extent more accurately and to obtain the least water. Though a shoal is generally indicated by an unusually shallow sounding, it is also necessary to regard unexpectedly deep soundings with suspicion since they may mark the scour which is always to be found in tidal waters by the side of a rock or any other obstruction rising steeply from the bottom.

Interlining is also advisable in important shipping channels and similar places where the depth of water is only just sufficient for safe navigation and small irregularities in the bottom might constitute dangers.

Cross Lines.—Lines of sounding run at right angles to those of normal direction are frequently useful when examining shoals, particularly so off points of land where narrow tongues of shoal ground are likely to extend at right angles to the shore. Such shoals might be missed by radial lines of sounding starred on the point, and cross lines should find them and define their limits in the most satisfactory manner.

Examinations.—Where irregularities of depth are found to exist, or where, from the rocky nature or formation of the coast, they may be expected to occur, the smallest indication of a danger must be thoroughly examined. Over banks of sand or mud a few interlines run between the normal lines of sounding will probably suffice to define the limits of a shoal and show the least water on it, but over rocky ground numerous lines should be run as close together as possible, *e.g.*, a few yards only apart. Except very close inshore, a close examination such as this is generally a matter of some difficulty, and the most satisfactory way to go about it is as follows :

Prepare a fresh field board for the work and mark on it the position of the shoal, as nearly as it is known. Anchor a small buoy with a flag at some convenient distance, say, 100 yards shorewards from the shoal and, if possible, in line with some recognisable part of the coast, assuming this to be visible. Fix the buoy by sextant angles to the survey marks. Run lines of sounding radiating from the buoy ; by bringing the buoy in line with a succession of marks on the coast, straight lines can be run at very close intervals. It may be necessary to erect one or two artificial marks for the purpose, but there are usually a sufficient number of recognisable natural objects. The speed of the boat (if using a hand lead) should be as slow as possible in order that the soundings on the lines may be closely spaced and a good look-out should be kept

over the side as the shoalest part may be visible in clear water and the lead can then be dropped on it. Fixing can usually be effected with sufficient accuracy by a bearing and range-finder distance of the buoy. It will usually be impossible to ink in more than a small portion of the soundings owing to the contiguity of the lines, but, provided a characteristic selection including the shoalest is recorded, this does not matter.

On small scales thorough examinations are hardly possible, and it is often necessary to make a large-scale plot in order to show the work clearly. Assuming that a buoy is anchored near the shoal, the fixes and soundings may be plotted on a scale of, say, 1 in. to 100 yds. by bearing and distance from the buoy, and the more important soundings can be afterwards re-plotted on the scale of the survey.

When the bottom is very irregular, even the closest sounding cannot always be relied on to reveal the least water on a shoal; in such cases the surveyor must resort to sweeping over the ground. (See p. 242 *et seq.*)

Leading and Clearing Lines.—No aids to coastal navigation are more useful than leading and clearing lines, and the surveyor should be constantly on the look-out to recognise any that may be of value. Before placing them on the chart and recommending them for use, such lines should be carefully sounded or swept over; this is especially important in the case of incomplete surveys. The most essential requirement for leading and clearing marks is that they should be easily recognised by a stranger and the seaman will often be much assisted if views of them and adjacent coastal features are prepared for inclusion in the appropriate publications. Such views should show the appearance of the marks from the point where they will first become useful. The two marks which indicate the leading or clearing line should be well separated so that the transit is very sensitive, *i.e.*, a slight divergence from the line will then be immediately apparent. Nevertheless, very distant or high back marks should be avoided, as they may often be obscured in haze or cloud.

Ship and Boat Sounding.—The *modus operandi* of ship and boat sounding must vary slightly in differing conditions and with different types of sounding apparatus, *e.g.*, the hand lead, steam or motor sounding machine, or echo sounding gear with or without an automatic recorder.

The area to be sounded is usually divided between the ship and the boats in such a manner that the work of each will be approximately equalised in time. The ship is naturally restricted to areas in which she can safely navigate and, in general, she is most usefully employed in the deeper and more exposed portions of the survey where boats may often be prevented from working by the weather and, with their low height of eye, may be unable to see the survey marks. On large-scale work, however, most, if not all, the sounding may have to be done by the boats, for it is probable that space will not permit the ship being manoeuvred on the lines of sounding and, with mechanical sounding gear, at any rate, plottable errors (not easily allowed for) would be introduced by the distance between the bridge and the lead.

For example, suppose it is required to survey an area such as that shown in Fig. 122, and that the sounding is to be done by a boat using the hand lead.

Three of the fixed points in the survey, Mug, Spire and Tall, are shown in the figure, and in the particular portion of the area under consideration angles to these marks will provide a good fix for the boat. The pecked lines *a*, *b*, *c* and *d* are drawn (in pencil) on the board, and it is proposed to start the work by running these. To sound out an area in a methodical manner, it is necessary as has already been explained, to run a series of *straight* evenly-spaced lines. In practice it is impossible to make good an absolutely straight course for any length of time, and the surveyor must follow the lines he had drawn on the field board *as nearly as he can* by fixing himself and plotting his position at

frequent intervals and, when he finds himself off the line, by making small alterations of course which will bring him back to it.

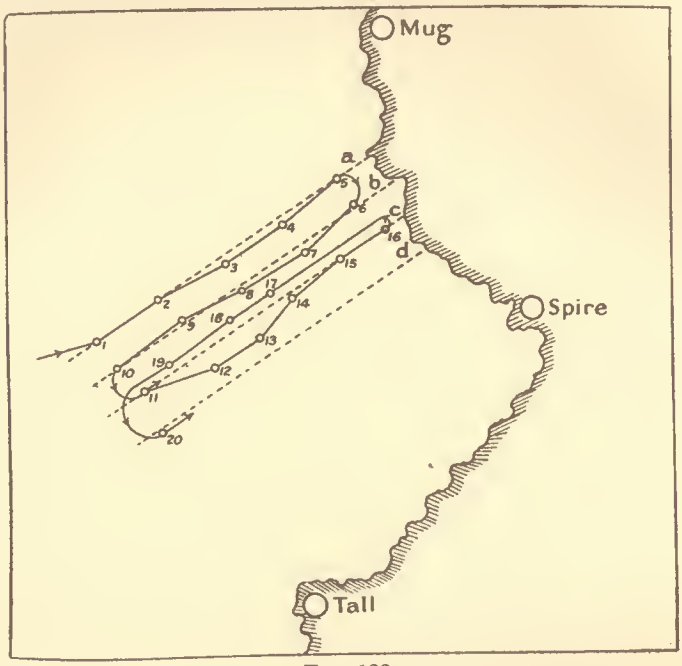


FIG. 122

Suppose, now, the boat is approaching from the westward to the point marked " 1 " on line " a." The position of the boat is ascertained at short intervals by means of simultaneous sextant angles to the shore marks, and the speed of the boat is adjusted to suit the scale and nature of the survey. In shallow water, soundings can be obtained fairly rapidly with the hand lead, and it may be possible to maintain a speed of 6 or 7 knots. In deeper water speed must be reduced as the rate of sounding is slower and the leadsman cannot obtain an up-and-down cast at high speeds. On large scales the speed must always be more moderate than on small scales, as more soundings are required over any given distance and the space-interval between fixes must be correspondingly less.

Having arrived at a position as nearly as possible on line " a " at its outer end, the boat is set on a course which it is anticipated will make good the direction of the pencilled line, an estimated allowance being made for wind and current. Sounding is then commenced, and simultaneously the first fix is observed and plotted. After a given number of soundings—usually five or so (but, for convenience when inking in the day's work, preferably an odd number)—have been obtained, fix 2 is observed and plotted. This procedure is continued along the line. Referring to the diagram, it will be seen that fixes 1 and 2 plot practically on the line, but that at fix 3 the boat has been set off a little to starboard, either by a change in the wind or current or by bad steering. So at that point course is altered a few degrees to port. Fix 4 shows that this was not quite sufficient to bring the boat back to the line, and a further small alteration is made, and at fix 5 the boat is back on the line. The boat then turns onto line " b " and a similar procedure is continued. Sometimes she may be set so far off the line that a gap is left much in excess of the normal spacing between the lines of sounding ; in such cases that part of the line must be re-run or the gap must be filled with another line of soundings, as shown at fixes 17 to 19, so that the principle of covering the ground methodically is preserved.

The sounding book must contain a record of the angles observed and objects used at each fix, the soundings, the nature of the bottom and, when they have been calculated, the tidal reductions. The usual method of recording this information is as follows :

1.	2.	3.	4.
1	VII. 21	Mug 43° 20′ Spire 55° 42′ Tall	$\frac{11/4}{S}$
		11/– $\underline{10/5}$ 10/4 $\underline{10/2}$ 10/–	
		M	
2	VII. 24	,, 52° 13′ ,, 63° 01′ ,, ...	$\underline{9/3}$
		etc. etc.	

Column 1 shows the number of the fix, column 2 the times, column 3 the objects and angles of the fix, underneath which are written the soundings *between* that fix and the next, and column 4 the sounding obtained *at* the fix. Soundings are usually written as 11/4, 10/5, etc., indicating 11 fathoms 4 ft., 10 fathoms 5 ft., and so on. To distinguish those taken in the port and starboard chains, the latter may be underlined ; this precaution is necessary in case different errors are found in the lengths of the lead lines. The nature of the bottom is recorded by the symbols normally used on charts under the sounding at which it is obtained.

Inking in the Soundings.—On completion of each day's work the soundings should be inked in on the fair tracings ; this may be done either by inking in the fixes on the board together with their numbers and the lines joining them, and then inking in the soundings on the relevant tracing superimposed on the board ; or by tracing the fixes in pencil on the tracing and then removing the tracing from the board for inking in. The latter method will be a little more laborious but may prove worth while when the area sounded is in an inconvenient position on the board. Soundings must first be corrected for the height of the tide to reduce them to a common datum. Tidal observations are always made during the periods of sounding and provide the necessary data for the reduction. To take the example illustrated in Fig. 122, suppose the tidal reduction at the times of fixes 1 and 2 is 15 feet. The series of soundings at present reads 11/4, 11/–, 10/5, 10/4, 10/2, 10/–, 9/3 : when reduced for the tidal correction, the series will read 9/1, 8/3, 8/2, 8/1, 7/5, 7/3, 7. The reduced soundings 9_1 and 7 at the two fixes are first inked in and then a convenient number of soundings between them. In this example three soundings may be legibly written between the fixes and the ones selected should be the centre sounding 8_1 and the means of the two soundings on either side of it. The two before it are 8_3 and 8_2 : as soundings are only required on the chart to the nearest foot, the shoaler, *i.e.*, 8_2, should be selected. The two after the middle are 7_5 and 7_3 ; therefore 7_4 should be inked in. On the tracing, then, the series will appear as $9_1\ 8_2\ 8_1\ 7_4\ 7$, the first and last appearing at fixes 1 and 2 respectively, and the intermediate ones on the line joining these two positions. In this case it will be noted that the slope of the bottom is practically even, but, if it is irregular, the surveyor must exercise his judgment and select such soundings as will best portray the character of the irregularities. If, for instance, the series between two fixes was $9_3\ 8_2\ 9_1\ 9\ 8_5$, and that three of these can be inked in, it is clear that the shoal sounding 8_2 must not be omitted, and the series will be recorded on the tracing as $8_2\ 9_1\ 8_5$.

Owing to small delays in fixing, and other causes, soundings in lines run towards the shore frequently appear to be slightly deeper than those in lines run outwards from it. If this is found to occur consistently, the fathom contour lines, when being inserted after the soundings are inked in, should be drawn as a smooth line to accord with nature, rather than zig-zagging in strict agreement with the inked-in soundings.

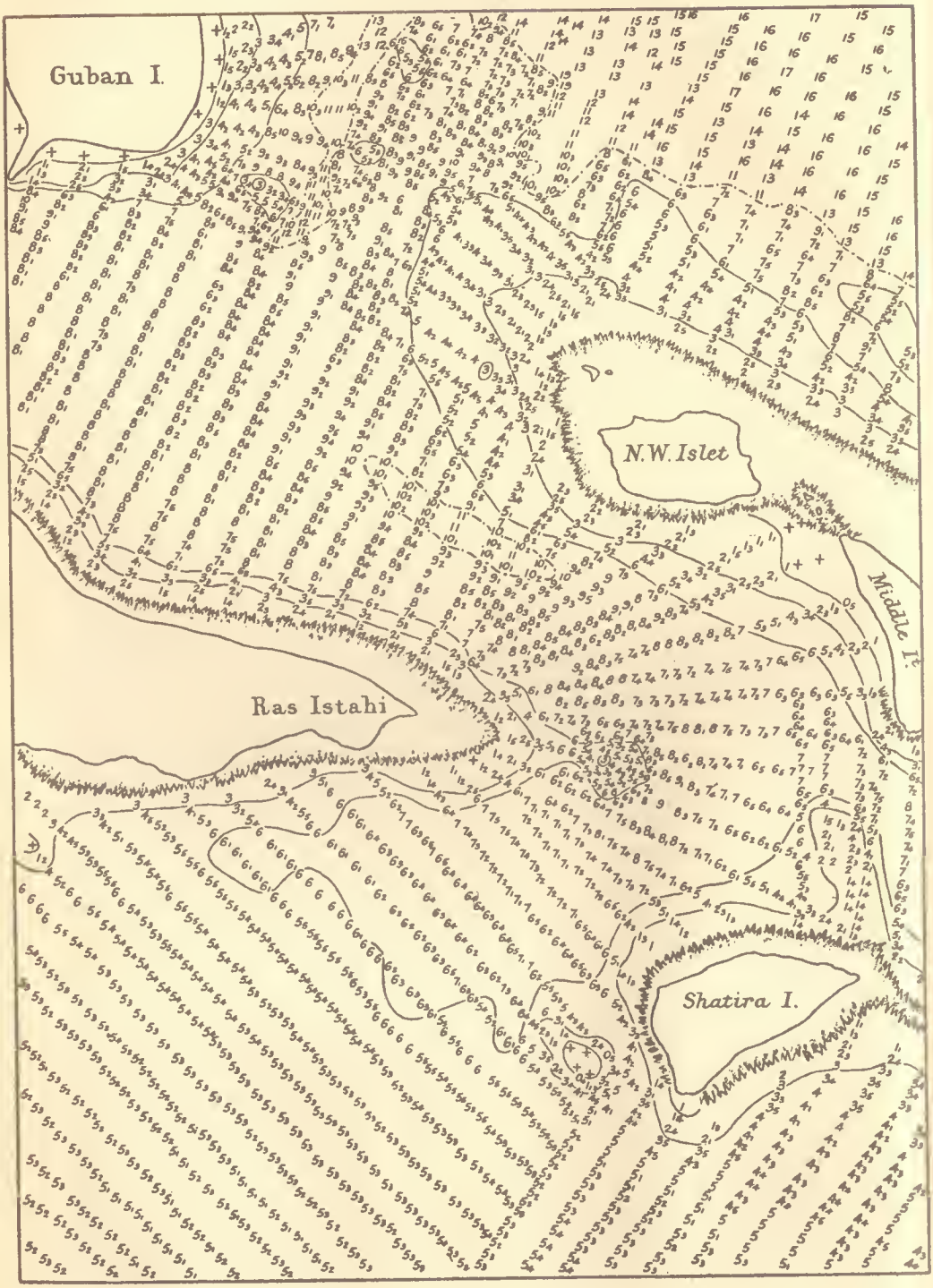


FIG. 123

Fig. 123 shows lines of sounding as they appear on a modern original survey ; it will be seen that in general they run at right angles to the fathom lines. The whole water area has been methodically sounded over and interlines have been run to examine more closely the irregularities and shoals eastward of Guban Island and Ras Istahi.

Management of Ship and Boats for Sounding.—Except in minor details there is no difference in the methods employed for ship and boat sounding. The higher speed of the ship permits a given area to be sounded in less time than is taken by a boat and the whole operation is in general rather easier ; the extra height of eye and the steadiness of the bridge as a platform for the observers are a great advantage. On the other hand, the ship is not so quickly and easily manoeuvred as a boat, so that errors of position take longer to correct, and more sea room is required. Nothing is gained by attempting to sound with the ship in too close proximity to shoals and dangers ; such work should be left to the boats.

The complement of a boat for sounding should include two observers for taking the simultaneous sextant angles which are required for a fix. The division of the work is usually as follows : One observer is in charge of the boat, takes one of the angles and plots the fixes ; the other observer takes the second angle and writes down all information in the sounding book. Whenever these duties do not wholly occupy them, both observers should keep a look-out on the echo reader or leadsmen to detect errors in the calling of soundings, and on the surrounding sea to note any signs of shoal water. This visual examination is a most important part of the surveyor's work, for it is a fact that as many dangers have been discovered by surface indications, such as tide-rips, eddies and weed, as by the soundings themselves. The even spacing of the lines of sounding ensures that a comprehensive visual examination of the whole area can be made.

On small scales sounding is usually a fairly simple operation, as small errors of position are hardly apparent and can in any case easily be corrected. On large scales, however, it is often a matter of great difficulty to make good a course with the degree of accuracy required to avoid leaving gaps in the work. The speed of the boat must be slow, and she is consequently more affected by wind and variations of current, and the time-interval between successive fixes is much less. Under these conditions good results can only be obtained after long practice and by the exercise of unremitting care on the part of the surveyor.

Transits.—The difficulty of making good a straight course whilst sounding is greatly lessened if use can be made of transits. In hydrographic parlance the word " transit " describes the relative position of two objects when they appear in line with one another, and it is obvious that, if the alignment is maintained as the ship or boat runs along a line of soundings, the course made good will be a straight line. When a course can be set only by compass, any divergence from the line which it is required to run will only be apparent when the fix has been plotted ; but, if a coxswain can pick out two objects on shore on the extension of the line, *e.g.*, a rock near high-water mark in line with a tree in the background, he has only to maintain the transit and the boat must make good a course directly towards or away from the objects. Any divergence from the line will be *immediately* apparent as the objects will no longer be in transit and the error can be rectified far more quickly than is possible, when its discovery has to await the plotting of a fix. The efficiency of the sounding work is greatly increased when transits are available and are properly used, and every effort should be made to instruct those in charge of boats in the principle involved. A practised coxswain should not only be able to steer along a transit, but under good conditions should be able to pick out suitable transits for himself on successive lines of soundings. A very distant back object is particularly useful, as when used with different front objects it may be

employed for a large number of lines which will be sensibly parallel or nearly enough so to fulfil the principle of covering the ground systematically.

Natural objects of various descriptions must generally be used as transits, but it will occasionally pay to erect temporary artificial marks for this purpose when working on large scales in waters where tidal streams are strong and variable. Running a transit *towards* two objects presents little difficulty and the use of the compass may be entirely dispensed with, but in running a stern transit, *i.e.*, away from the objects, the coxswain should steer by compass, glancing back frequently and correcting his course a few degrees whenever the boat is off the transit. If the coxswain watches the transit continuously, he cannot appreciate the direction of the boat's head and may correct errors by over-violent alterations of course. In ship sounding it is usually impossible for the quartermaster to see the transit marks ; they must therefore be watched by the officer in charge, and kept in line by altering the compass course as necessary.

Fixing

As the ship or boat runs along a line of soundings, she must be fixed at regular intervals in order that the soundings may be plotted in their correct positions. Fixing is carried out by observing the angles between three objects by sextant, use being made of the fact that the angle subtended by the chord of a circle is the same at all points on the circumference in each segment. The measurement of an angle between two fixed points defines the observer's position as being on the circumference of a certain circle passing through those points ; a second angle measured from the same spot between one of these two points and a third provides another " circle of position." So long as the two circles do not coincide, the intersection of their circumferences completely defines the observer's position.

Example :

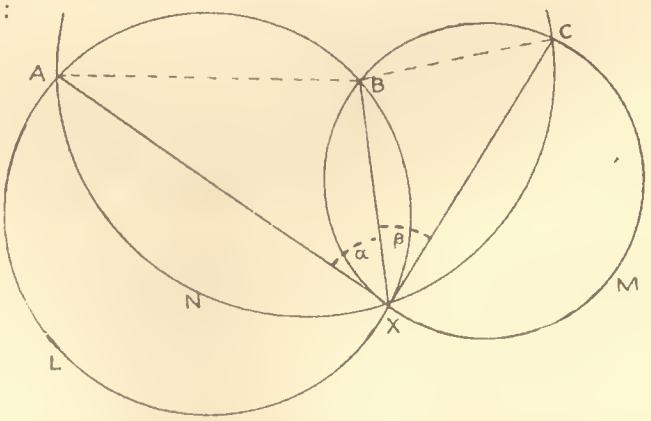


Fig. 124

If A, B, and C, in Fig. 124, are three fixed marks of a survey and the angles AXB and BXC (α and β) are measured simultaneously from a boat at some, as yet unknown, position X ; the angle α defines the observer's position as being somewhere on the circumference of the circle ABL, the angle β defines his position as being somewhere on the circumference of the circle BCM, and the combined angle $\alpha + \beta$ defines his position as lying on the circumference of the circle ACN. The intersection X of the three circles must be the required position. It would be possible with a knowledge of this data, *i.e.*, the positions of A, B, and C and the angles α and β, to draw the circles as they are shown in the figure and find the position X graphically. Such a procedure, however, would be laborious, and in practice the position is found by setting the angles α and β on a *Station Pointer*, which is then manoeuvred so that its legs coincide

with the points A, B, and C. The intersection of the legs will then be the required position X.

The above method of fixing is only valid when the objects, to which the angles are measured, are suitably disposed. A circle through the three points, A, B and C, can always be described, and the observer's position cannot be completely defined if it lies on, or close to, the circumference of this circle.

Fig. 125 illustrates a thoroughly bad fix, with A, B and C most unsuitably disposed for the purpose of fixing at X. The circles of position through A and B and through B and C, corresponding to the angles α and β, are nearly coincident (the third circle AXC is not shown but clearly almost coincides with the other two) ; the cut made by their circumferences at X is very ill-conditioned and any small error in measuring α or β will result in a large displacement of X. It is clear that X lies very close to the circumference of a circle drawn through A, B and C.

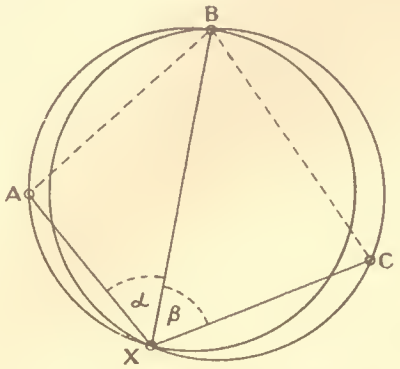


FIG. 125

An experienced surveyor visualises almost automatically the locus of a circle passing through any three points plotted on a field board and sees at once whether his position is too near its circumference to enable them to be used. The beginner may find some difficulty in selecting the most suitable objects of those available but will find the following rules useful :

(1) The sum of the two angles of the fix should not be less than about 50°.

(2) The centre object of the fix should be the closest.

(3) When the three objects are nearly in a straight line, the angle between any two should not be less than 30°.

(4) When the observer is inside the triangle formed by the three objects, the fix is always good.

The size of the individual angles admissible in a good fix depends on the relative positions of the three objects. If two of them are equidistant from the observer, the angle between them must not be very small, since any error of observation will make a great difference in the position of the plotted fix. If, however, one object is much farther off than the other, a very small angle will suffice provided rule (1) is observed.

If the short portions of the arcs of the three circles passing through the fix are drawn (this can be done quite simply with the station pointer), the pattern of the intersection can be seen, and the soundness, or otherwise, of the fix is at once apparent ; a simpler test is the purely mechanical one of noting the ease or difficulty with which the legs of the station pointer can be made to coincide with the three marks. With a bad fix it is very difficult to obtain perfect coincidence.

The angles required for fixing are the true horizontal angles at the eye level of the observer. Normally, only marks situated near high water level will be used for sounding, but occasionally it is necessary to include in the fix a mark situated at an appreciable elevation. The correction required to convert

observed angles to true horizontal angles is explained in Chap. XVII, page 395, and the correction itself is given in Table XIX. However, as it is impracticable to correct sextant angles during the preoccupation of sounding, should it be necessary to make use of an elevated object, angles to it should be observed to a point at water level vertically below the mark, instead of to the mark itself.

Fixed Angle Sounding.—This method consists of sounding along the arcs of circles subtended by two marks. For each arc one angle remains constant and, by varying this angle successively by equal amounts, a series of lines covering the area systematically may be run. These angles are the "fixed angles" for each line. A second angle between another pair of marks will define the position of the boat along the arcs. (Note it is *not* necessary for one central mark to be common to both angles of the fix.)

The selection of suitable marks is governed by two factors.

(*a*) The direction in which it is desired that the sounding lines should cross the area. (This choice may be limited by the position of the marks available.)

(*b*) The distance of the area from the marks. They should, if possible, be close to the area so that *any* wandering of the boat off the selected arc will produce a well-defined change in the fixed angle. Such a change will be greater, relative to the movement of the boat, when the angle is large.

Fig. 126 shows four sounding marks with arcs subtended by suitable pairs, crossing an area to be sounded.

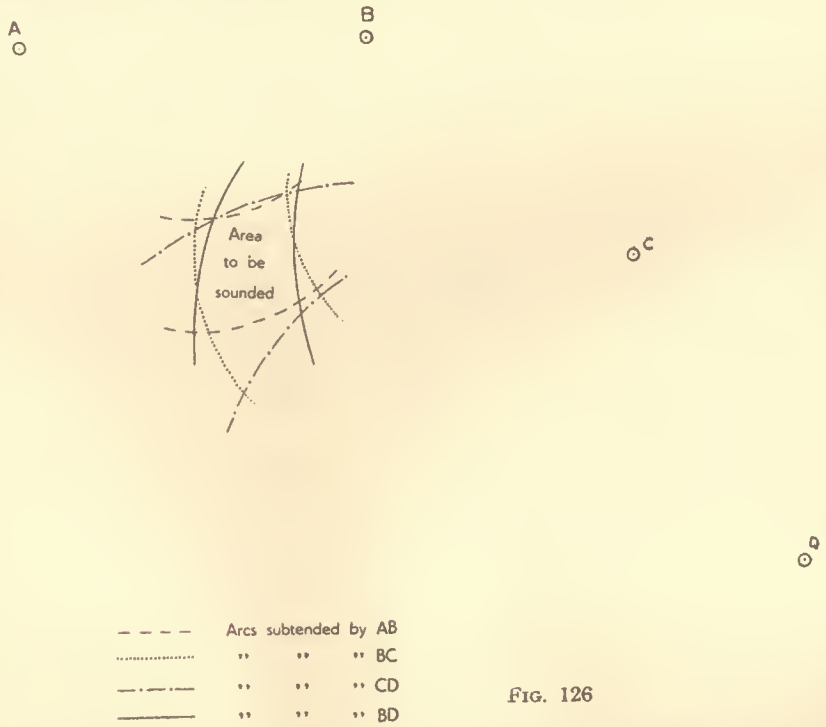


FIG. 126

Fig. 127 shows the method of constructing the arcs, along which it is desired to sound, on the board. A and B are the selected marks. The perpendicular bisector of the line joining them is first drawn, since this is the locus of the centres of *all* circles passing through A and B. Rough subtended angles of the limiting arcs (those nearest and farthest from AB) are measured by station pointer, and found to be, say, 96° and 68°; on the scale of the survey six lines of sounding will suffice for a first covering of the area, accordingly arcs at 5° intervals are drawn, with their centres lying on the bisector, at a distance of

s cot θ from the mid-point of AB, where θ is the subtended angle and $s = \frac{1}{2}$ AB.

When sounding in the boat, the officer in charge sets the angle of the arc selected as the first line on his sextant and, watching the angle continually, cons the boat onto, and then along, the line. There is no difficulty in keeping on the line since any digression from it is at once apparent; frequent alterations of course are necessary but, usually, only by small amounts at a time. When one line is completed the " fixed angle " for the next is set on the sextant and the process repeated.

The assistant observes the angle between the other pair of marks as necessary for fixing the boat along the line, but since the officer in charge knows continually whether the boat is on the line or not, plotting may be deferred, if desired, until the line has been completed. Limiting angles, previously taken from the board, will enable the assistant to report when the end of a line is reached.

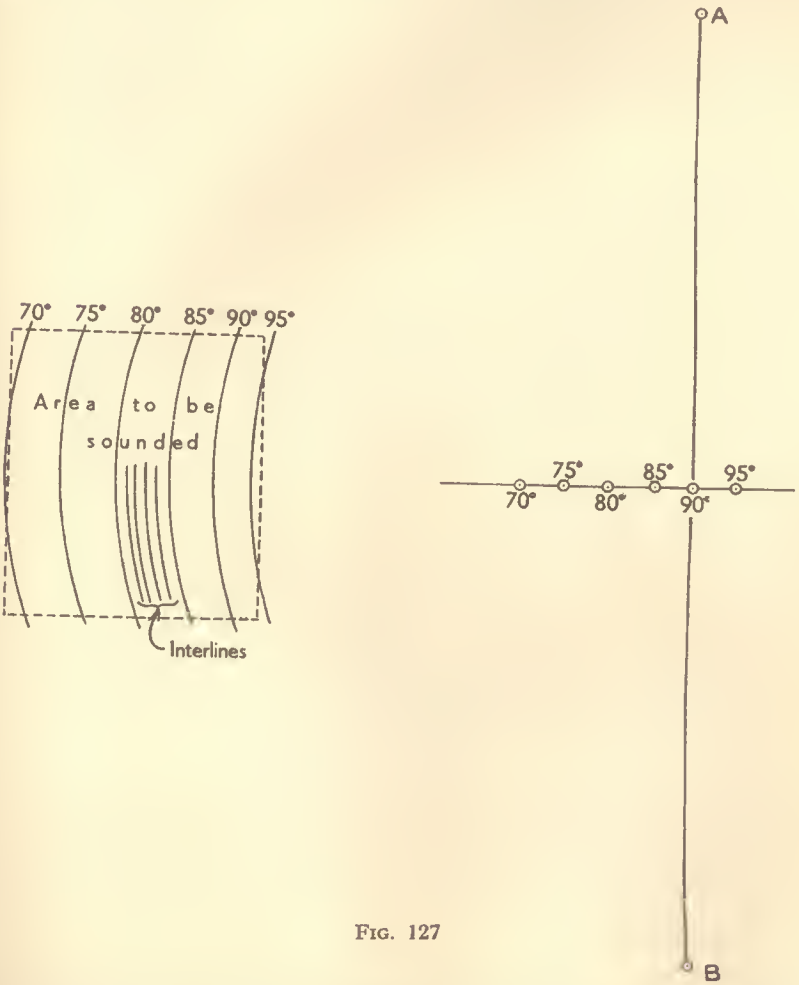


FIG. 127

Any interlines required may be readily interpolated; *e.g.* in Fig. 127 four interlines at 81°, 82°, 83° and 84° could be run between the arcs for 80° and 85°.

The method is generally suitable for the examination of small areas or for locating a pinnacle rock. In the example illustrated in Fig. 127, soundings could not be shown clearly at closer " fixed angle " intervals than 1°, whereas

it might well be possible, in good weather, to run accurate lines at 5′ intervals. If Fig. 127 is at a scale of 1 : 50,000 such lines would be about 20 ft. apart. If no shoaling was found using the arcs shown, lines could be run along the arcs subtended by another pair of marks, chosen to give arcs crossing the lines already run, at a wide angle. A pinnacle, if it exists, could hardly escape detection if searched for thus, a second time.

The principal disadvantages of the method are that the choice of direction of the lines is limited by the positions of the sounding marks, that the sounding board takes a little longer to prepare than for normal methods, and that, after a time, " fixed angle " sounding becomes rather tiring to the officer who is watching the fixed angle. Against the last may be set the advantage that the lines are accurately run and there is no worry that the boat may be straying from the line during the interval between fixes since any deviation is instantly apparent.

The lines converge as they approach either of the two marks used. This convergence may be minimised by selecting the marks so that the sounding area lies near the perpendicular bisector. It may, however, be of value where it is necessary to increase the sounding density close inshore and can be utilised, in such a case, by choosing one of the marks so that it lies close to the area to be sounded.

Instruments for Fixing

Sextant.—The general principles of the sextant are too well known to require description here. An ordinary observing sextant may be used when sounding, but it is not entirely satisfactory for use with terrestrial objects which are often distant and difficult to see. A more suitable type of instrument is that illustrated in Plate I.

This plate shows a modern type of sounding sextant of 6 ins. radius. The graduations are engraved on a brass arc. The index arm carries a worm shaft which engages in teeth on the arc and a micrometer head, showing minutes of arc, is provided for quick reading. The worm is released from the teeth, when rapid movement of the index arm is required, by squeezing the two clips which extend beyond the arm. Both mirrors are silvered all over and are mounted in circular holders which are completely damp proof so that re-silvering should hardly ever be necessary. The telescope is of about 5x magnification with a large object glass and field of view.

The errors and adjustments of the sextant are fully described in other publications, but in the case of this instrument the following are important points :—

(1) The index glass holder is rigidly attached to the arm so that no adjustment for perpendicularity can be made. It is the maker's business to see that this adjustment is correct.

(2) No cross-wires are fitted in the telescope and the usual method of testing for collimation error cannot therefore be used ; the following method is recommended :—

Set an angle of about 130° on the arc and, looking at the sea horizon, bring the reflected part of the horizon into coincidence with the direct image. The reflected image should be in perfect alignment with the direct image ; if it is tilted there is collimation error which is probably due to bending of the collar carrying the telescope. No adjustment is provided, but the collar can usually be tapped lightly with a hammer until it is back at its proper angle to the frame of the instrument.

Station Pointer.—To plot the fixes whilst sounding, the angles are set on a station pointer. A description and illustrations of this instrument will be found in most works on navigation, and it is unnecessary to describe it again here. The most useful sizes of station pointer for surveying purposes are those with arcs of 6 ins. and 8 ins. diameter, divided on brass and fitted with a vernier by which angles can be set to an accuracy of 1′ of arc. A larger station pointer

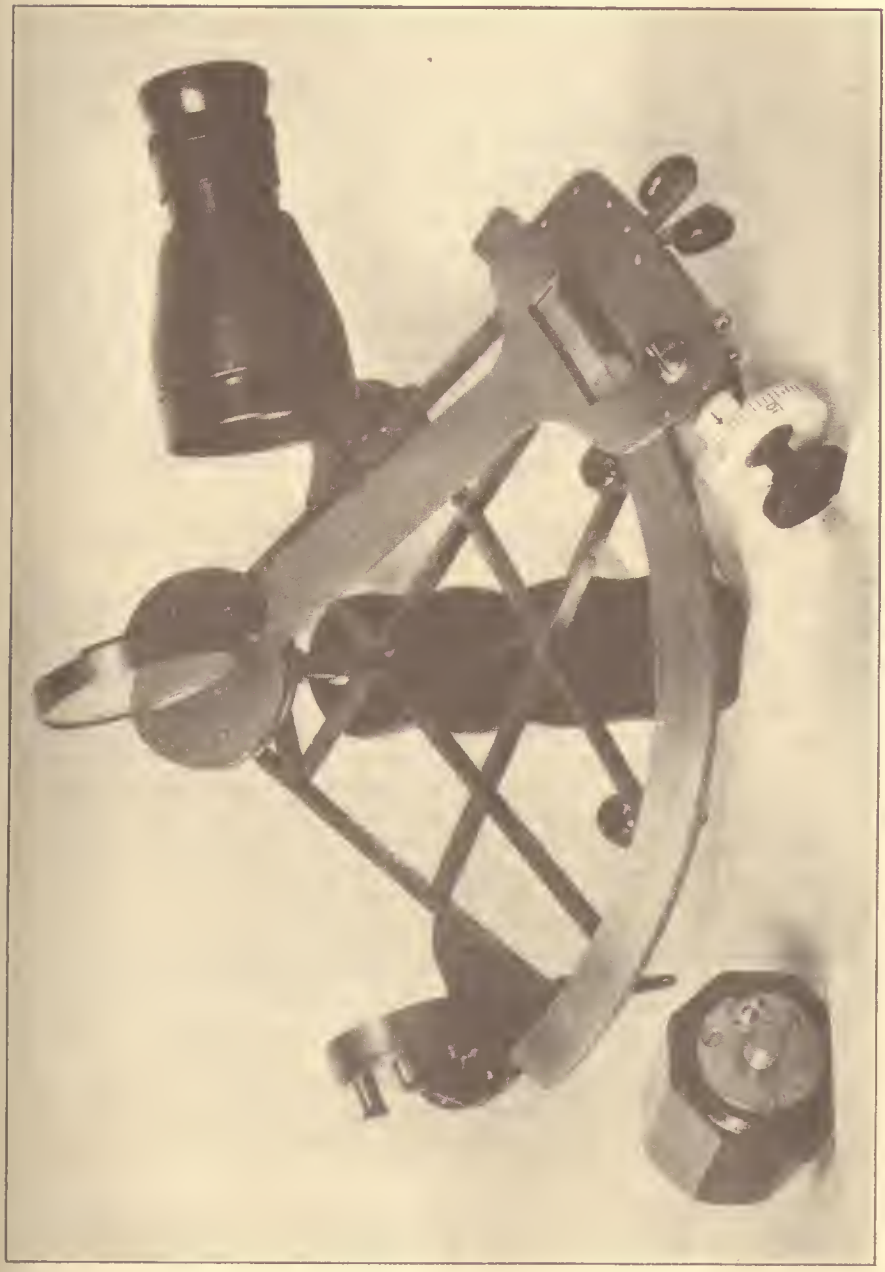


PLATE I

with an arc of 12 ins. diameter is also useful occasionally when stations are beyond the radius of the legs of the smaller instruments ; it is, however, too cumbersome and heavy for use in a boat. When stations are very close (as measured on the paper) to the observer, they are liable to be hidden by parts of the brass station pointer which cannot, in consequence, be correctly placed on the board, and in these circumstances use may be made of *Cust's station pointer*. This consists of a square or rectangular sheet of transparent xylonite engraved with a circle graduated in degrees and half degrees (in whole degrees only in the smaller types). At the centre of the circle is a hole large enough to take the point of a sharpened pencil. The observed angles are placed on it by drawing pencil lines on its under side which has a specially prepared surface ; it is then inverted when placed on the board so that parallax is eliminated. It should be noted that, in view of this inversion of the instrument, the angles must be reversed, left for right and right for left, when they are protracted.

Station pointers should be tested periodically on a board on which a series of angles have been carefully protracted by means of chords. A diagram for this purpose, published by the Hydrographic Department, Admiralty, is also available. The readings of the station pointer should not be in error by more than 2′ of arc on any setting, adjustments being effected by slacking off the screws holding the verniers and moving them if necessary. The legs of station pointers are easily bent by careless handling and their straightness should be carefully verified on the testing board at the same time as the accuracy of the readings is checked.

Sounding Gear

Lead Lines.

Sounding with the hand lead, although practically superseded by Echo Sounding, is still required for checking the latter and for obtaining the nature of the bottom. The soundings on Admiralty charts are given to the nearest foot up to depths of 11 fathoms. As sounding may be carried on during the whole period of the tidal movement from high to low water, lead lines must be marked with foot marks up to 11 fathoms *plus* the maximum rise of the tide ; *e.g.*, if spring rise is 21 ft., foot marks must extend to 14½ fathoms. Beyond this, marks at every ½ fathom will suffice. For use in boats, lead lines about 15 fathoms long are as much as a leadsman can conveniently manage ; in any case, he can hardly obtain deeper soundings unless the boat is stopped for each cast.

In past years a great deal of trouble was experienced with the stretching and consequent inaccuracy of rope lead lines which had to be frequently re-marked, but this difficulty can be almost completely overcome by inserting a wire heart in the rope. The materials used for making up lead lines are 1⅛-in. hemp rope and galvanised wire (about ⅛ in. diameter) similar to that used on a Kelvin sounding machine. When making up a lead line, the rope should be well stretched, *e.g.*, by towing it over the stern, before the wire is worked into its centre. Lead lines made in this manner do not stretch more than about 2 per cent. They should be checked daily on return to the ship and, if the error exceeds that amount, should be re-marked correctly.

The uniform system of marking lead lines is shown on Plate J.

Somerville Sounding Gear.

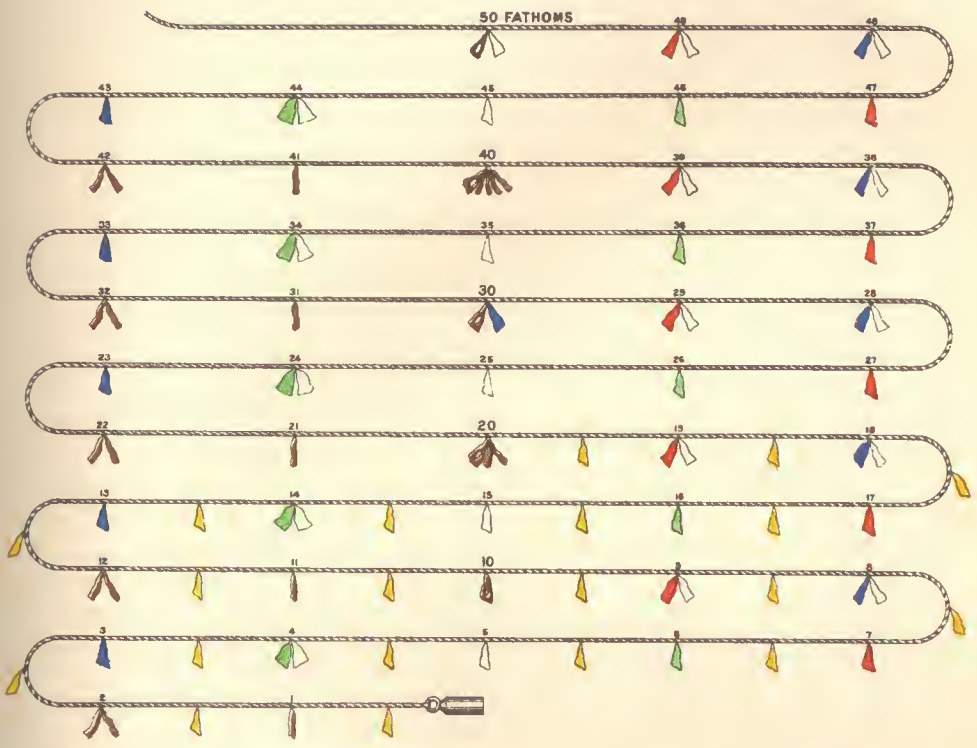
Ship sounding in these days is almost entirely done with Echo Sounding apparatus, but, in the absence of this, it may be necessary to rig some kind of mechanical sounding gear, the hand lead being too slow a method to be of much value. A suitable lay-out, originally suggested by Admiral B. T. Somerville, is shown in Fig. 128.

This gear consists of two main parts :

(1) A fore-and-after wire (1″ F.S.W.) passing from a winch aft through a

Plate J.
(To face page 212)

UNIFORM SYSTEM OF MARKING LEAD LINES



Lines (marked as above) to have in addition one knot inserted at every 1, 2, 4 and 5 feet of each fathom for a sufficient length to ensure that at least 65 feet (reduced) may be measured; eg. given springs rise 25 feet, 25+65=90 feet, therefore line to be marked in feet to 15 fathoms

UNIFORM SYSTEM OF CALLING SOUNDINGS

For that portion of the line which is marked, viz.	*Soundings are to be called*	
In fathoms and feet	*To the nearest foot*	*"Five three, five two, five one, five fathoms, four five, &c. &c."*
In fathoms and half fathoms	*To the nearest estimated half fathom*	*"Nineteen fathoms, eighteen three, eighteen fathoms, &c."*
In fathoms only (20 to 50 fms.)	*To the nearest estimated fathom*	*"Thirty fathoms, thirty one fathoms, &c."*

Note. *The only exception to the above is that 9 and 11 fathoms are to be called "deep nine" and "deep eleven", the word fathom being omitted, in order to distinguish them from 5 and 7 fathoms thus:— "Deep nine one, deep eleven, deep eleven three.*

ed by the Hydrographic Dept. Admiralty 10th May 1932. under the Superintendence of Vice Admiral H.P.Douglas, C.B., C.M.G., *Hydrographer.*
Corrections 1933 - [5.16]

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convenient leading block, *e.g.*, in the main rigging, to another block on the lower boom and thence to the lead which should weigh about 2 cwt.

(2) A depth line ($\frac{1}{2}$" F.S.W.), marked in fathoms and feet in the normal manner, attached to the lead and passing through a block on a small davit or spar near the sounding platform and then led over the quarter to a counterpoise weight of about 50 lb.

The winch should have some type of friction gear and some means of enabling it to be reversed quickly from the heave-in to the veer positions, and *vice versa*. The method of using the gear is as follows :

The lead is hove close up to the boom and, when the ship reaches the beginning of the line the winch lever is put to the veer position. The lead then falls

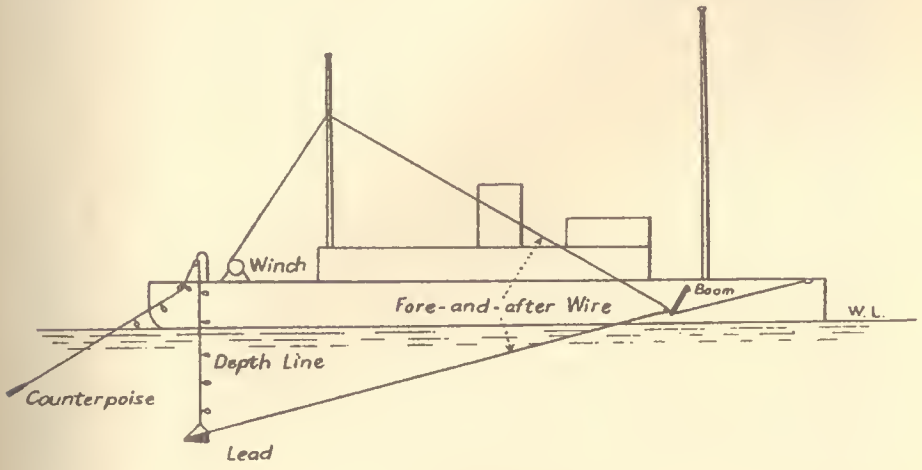


FIG. 128

to the bottom and draws aft. At the same time the slack of the depth line is taken up by the counterpoise. As the lead comes abreast the sounding platform an observer there reads off the sounding on the depth line which should then be in a vertical position. As soon as the sounding has been called, the winch lever is put to heave-*in* and the lead hauled up to the boom again, when the process is repeated.

The speed of the ship should not be more than about 7 knots ; otherwise the wash of water is liable to disturb the verticality of the depth line at the moment the sounding is registered. As the water deepens, speed must be reduced below this figure to give the lead time to reach the bottom. In shallow water, say, up to 10 fathoms, soundings can be obtained at intervals of 10 to 15 seconds as it is unnecessary to heave the lead right forward to the boom between each sounding. The maximum depth in which the Somerville gear can be conveniently used is about 50 fathoms.

In twin-screw ships there is some danger of the depth line or fore-and-after wire fouling the near-side propeller when sounding in a tide-way. Under these conditions it is best to run with the off-side propeller only in action and to have the gear rigged in duplicate, so that port or starboard side can be used as convenient. If the lines of sounding run athwart a strong tidal stream, the sounding gear should always be used on that side of the ship *to* which she is being set. If the reverse practice is followed, the ship will be set some distance away from the lead during the interval between the time it reaches the bottom and the time it comes abreast the sounding platform, the depth line will grow out at a wide angle and the soundings will consequently be erroneous. If,

however, the rule given above is obeyed, the ship is set *towards* the lead, but as it has been dropped from the end of the boom, a position well out from the ship's side, it is not likely to get appreciably inside the sounding platform, *i.e.*, under the ship, when it draws aft.

Sounding Machines.

The ordinary types of mechanical sounding gear are limited to use in depths of about 50 fathoms and under. In deeper water special machines are required and Plate K shows the Lucas Deep-Sea Sounding machine, which can be used to obtain the greatest depths of the ocean.

The nomenclature of the various parts of this machine are :

A Reel or Drum.
B Brake.
C Brake Lever.
D. Springs.
E Regulating Screw.
F Hand Wheel.
G Swivelling Frame.
H Measuring wheel.
J Indicator.
K Stop.
L Wire-guiding Rollers.
M Handle for traversing Rollers.
N Bolt.
O Screw Clamp.

The machine should be bolted down on the fxle where it can be seen from the bridge and in such a position that the wire will hang well clear of the ship's side. The drum holds rather more than 6,000 fathoms of 20½-gauge wire which is led outboard from its under side and over the measuring wheel. A complete turn should be taken round the latter to make certain that the wire does not render round it. The brake B is automatic and its action on the drum can be regulated by the handwheel F. When the weight of the lead is hanging from the measuring wheel, the frame G is pulled down through a small angle against the action of the springs D. This releases the brake and, assuming the clamp O is off, the wire will run out until the lead reaches the bottom. As soon as this happens, the weight of the lead no longer acts on the measuring wheel and the frame G lifts, thereby putting on the brake and stopping the drum. A handle can be shipped on the drum for heaving in the first few fathoms and the remainder is wound in by passing a swifter from a winch or motor to a wheel attached to the drum. In more modern types an engine is incorporated with the machine. Guiding rollers L are fitted to enable the wire to be wound on evenly ; when running out, they can be raised out of the way.

Two smaller patterns of the Lucas machine are also used, one holding about 1,000 fathoms of wire and worked entirely by hand, and a boat's machine holding about 400 fathoms. In the latter a hand wheel is not fitted, the speed being controlled by the screw clamp as the wire runs out.

Sounding Wire.

The wire used for deep-sea soundings is of galvanised steel, 20½-gauge. When new it has a breaking strain of about 240 lb., but, if kinked, it breaks very easily and great care must be taken to prevent this happening. Though galvanised, the wire soon rusts after a little use and then loses much of its strength. It is hardly practicable to grease it properly after every sounding if a continuous series is being made, but if the machine is going to be out of use for some time the wire should be run through a piece of well-greased canvas when being wound in after the final sounding, and the outer turns on the drum should be liberally greased. Even so, it will usually be found advisable after long periods of disuse to run off and destroy the first 50 fathoms or so. Splices may be found in the wire at intervals of 5,000 fathoms or so. As these approach the measuring wheel, the speed of the drum should be reduced to dead slow to minimise the danger of a break occurring. A few fathoms of hemp stray line should be interposed between the lead and the outer end of

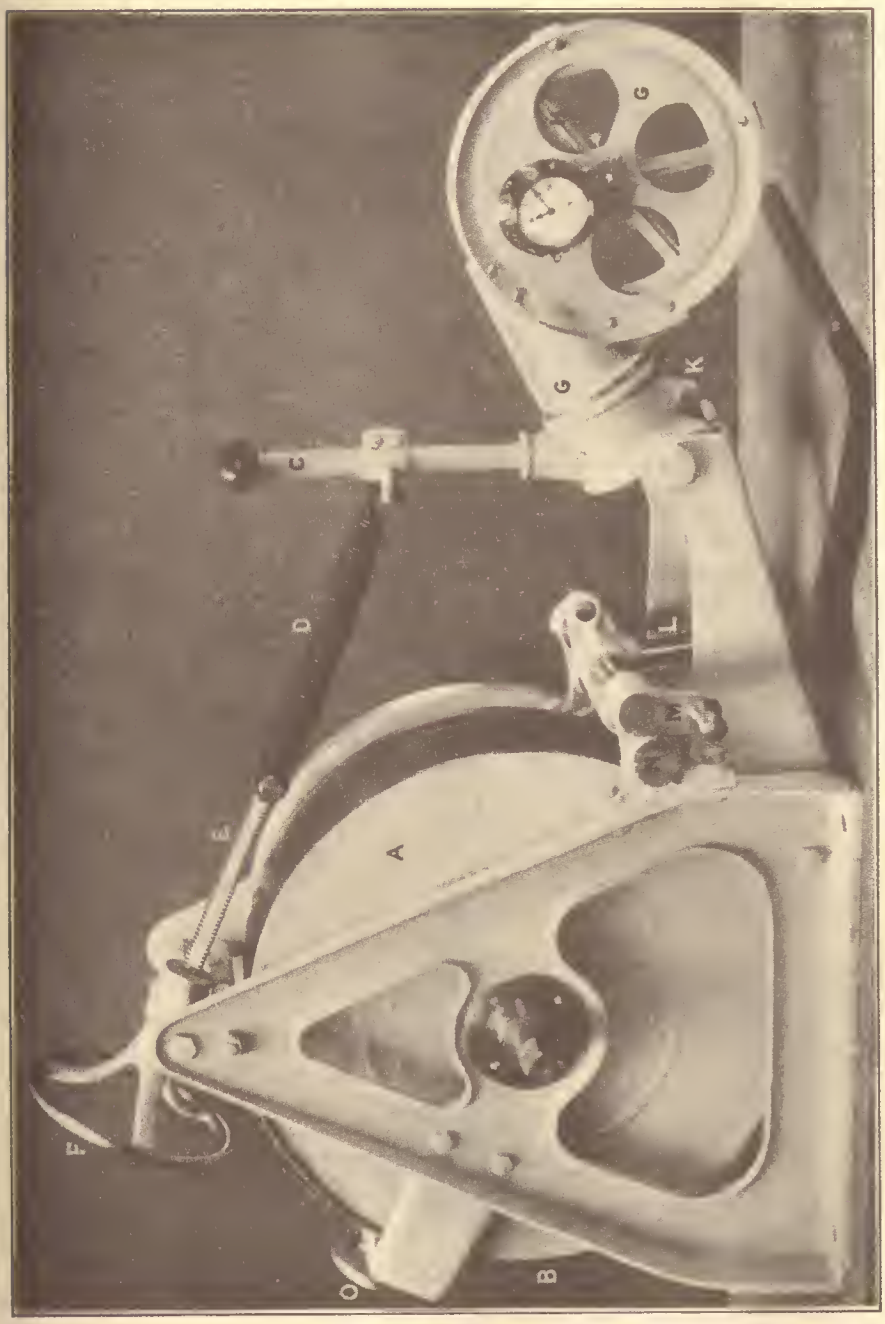


PLATE K

the wire, and the junction of the wire and hemp should be wrapped round with about 2–3 lb. of sheet lead ; this prevents kinking of the wire when the lead strikes the bottom.

Deep-Sea Sounding with Wire.

(*a*) **In Boats.**—Soundings with a boat's machine may be taken in depths up to about 400 fathoms. An ordinary boat's lead (14 lb.), bent straight on to the wire, is sufficiently heavy for use up to 100 fathoms or so ; in deeper water the work is expedited if a second lead is added. The machine is usually bolted down on the boat's quarter and the wire should be rove with a *complete* turn round the measuring sheave to obviate any possibility of rendering whilst running out.

Whilst sounding, the boat must be hove to and manoeuvred as necessary to keep the wire up and down. The latter should not be allowed to run out at more than about 100 fathoms a minute, or there will be a danger of overrunning when the lead strikes the bottom : the speed can be adjusted by operating the screw clamp. As soon as the sounding is obtained and the lead is off the bottom, the boat can proceed ahead towards the next sounding position whilst the wire is being wound in. In choppy weather care must be taken to prevent the wire jumping off the drum, and a stopper should be kept ready for attaching to the wire so that the weight of the lead can be taken whilst a foul is being cleared. Over an uneven bottom, and in rough weather, it is almost inevitable that the wire will part occasionally so a good supply of spare leads should be kept in the boat.

(*b*) **In the Ship.**—In depths up to 200 or 300 fathoms a boat's machine or a 1,000-fathom machine bolted down in any convenient position can be used, and the procedure is similar to that in a boat. Before going ahead, however, after sounding, care must be taken that the wire will not foul the ship's side or propellers.

In deeper water the large Lucas sounding machine should be used. In 1,000 fathoms or less a lead weighing about 28 lb. fitted with a flap valve in its base, for obtaining a bottom specimen, may be employed. In greater depths the

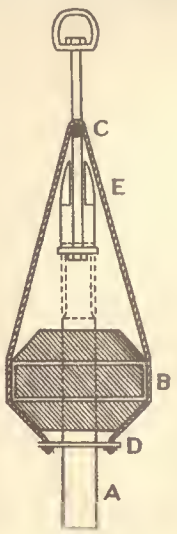


Fig. 129

friction of the water on the wire as it runs out makes a heavier lead necessary if a sounding is to be obtained expeditiously, and with a sinker as light as 28 lb. it would be difficult, if not impossible, to " feel " the bottom, partly

owing to the friction of the water on the wire and partly owing to the weight of the wire itself (about 15 lb. per 1,000 fathoms). However, it is undesirable to have to heave *up* a very heavy lead, and for this reason use is made of a " Baillie " sinker or Driver rod, the general design of which is shown in Fig. 129.

A is a galvanised iron tube ; its lower part, which can be unscrewed, contains a flap valve for retaining a bottom specimen. The upper end of the tube is fitted with brass shoulder pieces E between which a squared plunger rod passes. The lower end of this rod carries a cross-piece which can travel up and down in slots in the sides of the tube. Conical and cylindrical weights B are hung round the tube by means of a wire or cod-line sling rove through holes in a loose annular plate D and passed over a small lug C on the plunger rod. The outer end of the hemp stray line, attached to the end of the sounding wire, is bent to the shackle at the top of the plunger rod.

When the tube strikes the bottom, the weights pull the plunger rod down and, as the lug C passes between the brass shoulders E the sling is tripped and the weights, ring and sling are disengaged and left on the bottom.

In depths from 1,000 to about 2,500 fathoms, two conical weights, which with the rod aggregate about 60 lb., should suffice. From 2,500 to 4,000 fathoms, another 25 lb. should be added, and over 4,000 fathoms add 50 lb.

The procedure for taking a deep sounding is as follows :

Heave the ship to with the wind slightly on that bow to which the sounding machine is secured. This will ensure that, if she falls off the wind whilst the sounding is being taken, the wire will not grow under the ship. Turn the wire-guiding rollers up clear of the wire. Screw up the hand wheel and lower the lead over the side by hand, being careful to avoid jerking the wire. Set the dial for the amount of stray line below the water. Release the screw clamp and ease back the hand wheel slowly until the wire starts to run out. Let its speed increase until it runs at about 100 to 150 fathoms a minute, adjusting the hand wheel as necessary. When over 1,500 fathoms have run off, reduce the speed gradually to not more than 100 fathoms a minute, and in great depths, *e.g.*, over 3,000 fathoms, reduce it to about 50 fathoms a minute. As the wire runs out it must be carefully watched. The impact of the lead on the bottom will be indicated by a sudden pause or slacking in the speed of the drum ; as soon as this occurs screw up the clamp and read the dial. Ship a handle and wind in a few fathoms by hand to get the lead clear of the bottom as quickly as possible. After easing off the hand wheel and putting the guide rollers in position, the deck engine or motor can be connected and the lead hove up. As the wire comes in, work the guide rollers backwards and forwards to ensure even winding.

In depths up to 1,500 fathoms or so the machine will usually stop dead when the bottom is struck, but in deeper water the brake is not fully automatic and the moment of impact can only be detected by carefully watching the machine and wire. With a soft bottom the weights may not always be slipped, but, if a few fathoms are hove in by hand, their presence cannot escape notice and they can usually be detached by letting go again. If by some mischance the weights cannot be got rid of, the wire must be hove in *very* slowly and carefully. When using weights of 50 lb. or less, one brake spring can usually be unhooked, but this will depend somewhat on the condition of the brake band.

Whilst the lead is going down, every effort should be made to keep the wire vertical by manoeuvring the ship with the engines. Besides the necessity of counteracting leeway, it will often be found that there are surface currents which affect the ship but do not influence the lead after it has gone down a little way. In such cases the ship may have to steam all the time up to the lead if the wire is to remain vertical. However much care is taken, the wire is liable to grow out at an angle sometimes, especially in bad weather. When this happens, a fairly accurate value of the angle can be found by holding a protractor close to the wire with its longer edge parallel to the horizon, and the

sounding can be corrected for the " slope," but too much reliance should not be placed on the accuracy of such soundings as the direction of the wire *below* the sea surface is always, to some extent, uncertain.

Use of Somerville Sounding Winch for Deep-Sea Sounding.

Sounding with the Lucas machine is rather a slow process, especially if no direct form of drive is fitted, and more expeditious results have been obtained in depths up to 2,000 fathoms by making use of the steam sounding winch which is normally employed with the Somerville sounding gear. Five thousand fathoms of the ordinary 20·5 gauge wire are wound on one of the winch drums and led through blocks in the main rigging and on the lower boom. It is also led over a measuring sheave fitted in some convenient position near the winch. The " lead " consists of a 36-lb. pear-shaped lead sinker, with the lower portion of a Baillie rod embedded in it, or some other device, to enable a bottom sample to be obtained. A winding-on guide, similar to that on the Lucas machine, should be fitted to the base of the sounding winch, just before the drum. A wire jackstay is fitted on the lower boom so that the leading block can be hauled in and out and bottom samples removed from the lead. A few fathoms of hemp stray line should be interposed between the end of the wire and the lead.

To obtain a sounding with this gear, the drum should be clutched to the winch and as soon as the ship is stopped the lead is veered with the engine as fast as it will descend, care being taken to avoid over-running it. As soon as the bottom is struck the engine is stopped, the drum declutched and a few fathoms are hove in by hand until the lead is clear of the bottom. The drum is then clutched up again and the wire hove in at full speed by steam. The ship can proceed ahead towards the next sounding position as soon as the lead is clear of the bottom, and this is a great saving of time compared with the Lucas machine.

Echo Sounding

The general principles of Echo Sounding are now sufficiently well known to preclude the necessity of any very lengthy description here. In measuring the depth of water by Echo Sounding methods, an apparatus mounted in a ship or boat transmits an underwater sound impulse, which travels downwards through the water at a nearly uniform speed. On reaching the sea bed part of this sound is reflected and returns to the apparatus in the form of an echo, where its arrival is automatically recorded by graphic means.

The velocity of sound in sea-water is known with considerable accuracy, and so by measuring the time interval between the emission of the sound and the reception of its echo from the sea bed, the depth of water can be determined. It is obvious that, if v is the velocity and t the time interval, the depth of water is $\frac{1}{2} vt$. Various corrections, which must be taken into account, will be discussed later.

In shallow water the time interval is extremely short ; for example, the time taken for a sound impulse to travel to a depth of 10 fathoms, be reflected, and return to the ship is about 1/40th second. It is not possible to measure accurately an interval of time of this order by direct methods, such as by the use of a stop watch, and special apparatus—the Echo Sounding equipment—is provided to enable the measurement to be made. By means of a suitably graduated scale the time interval is for convenience converted into units of length (fathoms, feet or metres) so that the depth can be read off directly from the instrument.

Magneto-striction Sounding Apparatus

Great improvements in echo sounding have followed the introduction of a high frequency system of sound transmission used in conjunction with an

automatic depth recorder. The apparatus of this type adopted by the Admiralty for use in surveying motor boats is known as the " Universal " Recording Echo Sounder, type M.S. XII, and its principal features are :

(1) **The Transmitter.**—This consists of a pile of nickel stampings wound toroidally so that an oscillating discharge from a condenser passing through the windings creates an oscillatory magnetic field in the stampings. This causes the mean diameter of the stampings to vibrate radially, an effect called magneto-friction. The mechanism is fitted in a tank filled with fresh water and secured on the inside of the hull. The vibrations are transmitted to the water, reflected downwards by a conical reflector so that the sound energy is concentrated into a conical beam, and the impulse passes through the hull into the sea and is reflected from the sea bed back to the receiver.

(2) **The Receiver.**—The construction of this is similar to that of the transmitter and it is fitted in another tank secured to the hull close to the first. The process which occurs in the transmitter is reversible so than when the incident sound energy sets the receiver into vibration, the magnetisation of the nickel stampings is altered periodically and this sets up an oscillatory electric current in the windings. This current is amplified, rectified and passed on to the recorder.

(3) **The Recorder.**—This houses the " brain " of the apparatus and is fitted in an alloy case secured in any convenient position inboard. Plate L shows a typical recorder.

Fig. 130 shows diagrammatically the essential features of the apparatus.

The motor A is supplied by a battery and is fitted with a governor B so that it runs at a practically constant speed under normal variations of voltage. An adjustment by means of which the speed can be varied within small limits is fitted on the governor. The motor is coupled to a two-speed gear box Q and through this drives a shaft on which is mounted a drum D built up of insulated discs ; an arm holding the stylus F is mounted on this drum. The front ring on the drum is continuous and is for bringing the echo impulse to the stylus by means of the brush *a* ; the next two rings operate the make-and-break by which an electrical impulse is sent to the transmitter once in each revolution of the drum. In practice this make-and-break is operated by means of cams on these two rings, but is shown for convenience in the diagram as if it was operated by the brushes *c* and *d*. A device is fitted to operate an initial suppression switch which is sometimes required for suppressing the interference caused by the transmission impulse.

The paper H, on which the record is made, is made in rolls which are kept in sealed tins until required for use. Before being sealed up, the paper is chemically treated with a solution of potassium iodide and starch and is consequently moist. When a small impulse of electrical current passes from the stylus through the paper, it produces a brownish-purple stain at the point of contact of the stylus. The roll of paper is contained in a " tank " G (so called because in earlier models the paper was supplied dry and moistened in the machine), and led over the face of the tank and through rollers R at its base. One of these rollers is driven by shafting (not shown in Fig. 130) from the motor and the paper is thus drawn down at a regular speed over the face of the tank whilst the machine is running.

The battery also supplies a rotary converter C which transforms the battery voltage (12 volts) to 400 volts D.C. This voltage is used to operate the transmitter J. Current for operating the valve filaments of the amplifier I is obtained from the same battery. The high tension supply to the amplifier is from a tapping off the rotary converter.

The action of the gear may be briefly explained as follows :

When the motor is switched on, the drum D, carrying the stylus F, revolves

anti-clockwise at a constant speed, and once in each revolution the contacts *c* and *d* are "made." At this moment an oscillatory current is sent *via* the transmitting condenser L to the transmitter J, and a sound impulse is sent out

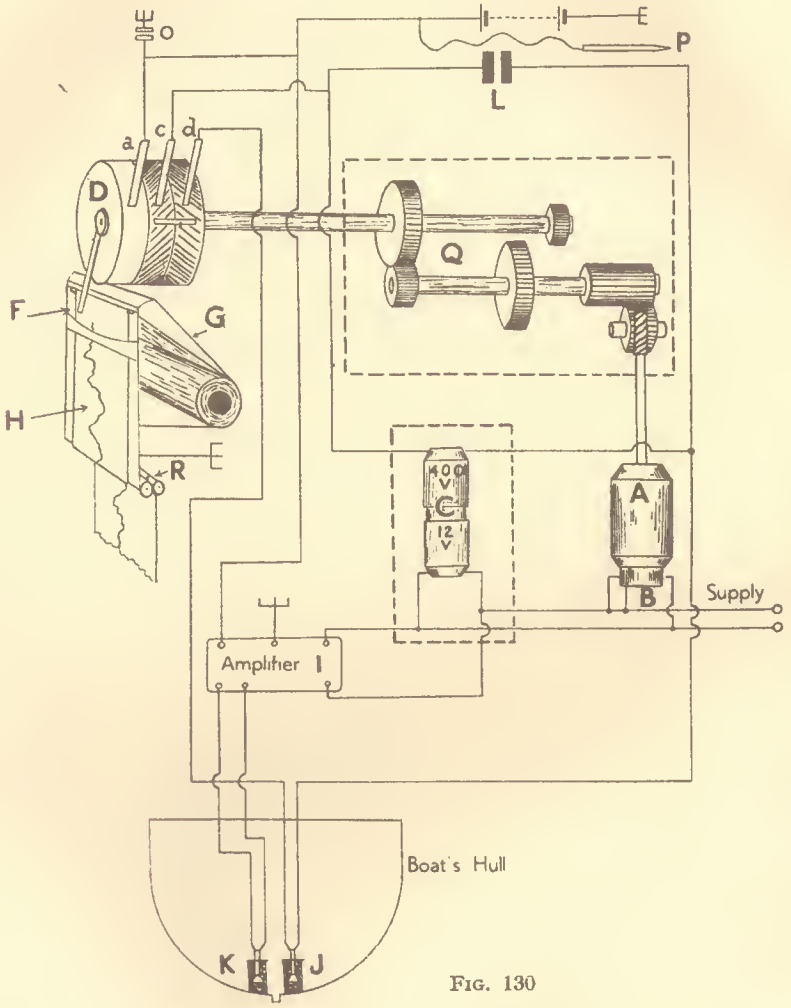
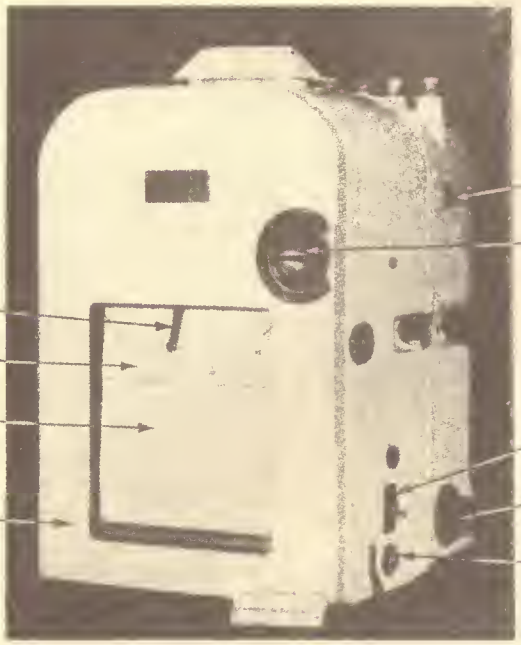


FIG. 130

as previously explained. After reflection from the sea bed, the sound impulse reaches the receiver K and a corresponding impulse of electrical current is passed *via* the amplifier I to the brush *a* and thence to the stylus which will by that time have rotated through a certain angle corresponding to the echo interval. From the stylus the current passes through the paper to the tank face and thereby completes the circuit to earth, and the position which the stylus occupied at this moment is shown by a brown stain on the paper. This process is repeated at each revolution of the drum D. A press-button O is fitted for marking the fixes; when it is pressed the stylus draws a continuous line across the record, and an electric pencil P is used for numbering the fixes on the record.

In the boat's echo gear a two-speed arrangement, worked by the switch shown on the face of the instrument in Plate L, is fitted, whereby at the *high* speed setting the stylus traverses the breadth of the paper in an echo interval

RECORDER - FRONT VIEW



RECORDER - VIEW WITH DOOR OPEN.

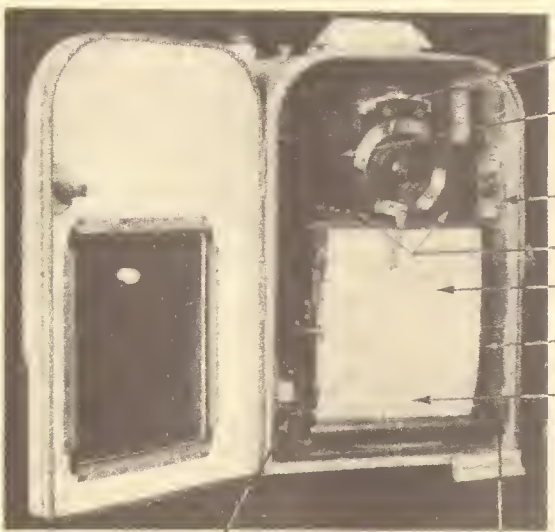


PLATE L

corresponding to a depth of just over 60 ft. At the *low* speed setting the speed of the drum is one-sixth that of the high speed setting and the stylus therefore traverses the breadth of the paper in an echo interval corresponding to a depth of just over 60 *fathoms*.

The appearance of a typical record is shown in Plate M.

The " fix " marks appear as continuous arcs and are numbered with the electric pencil. The depth marks are recorded automatically and are spaced at intervals corresponding to 10 ft., horizontally on the record, and 30 secs., vertically (10 fathoms and 3 minutes, with the low speed setting).

Double or treble echoes are often obtained, but there is never any difficulty in distinguishing the first or true echo. With the rotating stylus type of echo gear the slopes of the bottom, besides being exaggerated, are distorted ; the sides of a vertical wall under the sea would appear as a curve following the arc traced by the stylus. This distortion is not of material significance from the surveyor's point of view.

When the gear is fitted in a ship or when different requirements in performance are wanted, certain additions and modifications are necessary. The motor in the recorder box is, of course, wound to suit the ship's mains and the rotary converter is removed, transmission voltage being obtained from the ship's mains discharged through a choke and condenser circuit by a contactor. Instructions for setting this are contained in the handbook issued by the makers. Where a different range of depths is required to be recorded the set must be modified in the gear box so that the stylus speed is correctly related to the range of depths and phasing steps required.

The breadth of the paper on which the soundings are recorded is about 6 ins., and it is clear that, as so far described, soundings could only be obtained down to quite moderate depths ; the stylus speed might be slowed down so that 6 ins. of paper were equivalent to, say, 500 fathoms, but the scale would then be so compressed that soundings could not be read with sufficient accuracy. The machine fitted in ships, therefore, has a device whereby the transmitting contacts can be rotated, relative to the operating cam, about the axis of the spindle which carries the cam and stylus arm. This enables transmission to take place at some point *before* the stylus reaches the zero line. In this case the record no longer shows the true depth but the depth *plus* the amount by which transmission has occurred before the stylus crosses the depth zero.

This is the " phase correction " or " amount of phase " and in the ship type whose scale reads from 0–150 ft. phasing can be accomplished in steps of 100 ft., the amount of phase being shown by a number which appears in the front of the recorder box.

In the boat type, one phase (of +40 ft. or fathoms) only is possible. This is accomplished by a switch which brings into operation an alternative pair of transmitting contacts.

Operation of Boat's Echo Sounding Gear.

Apart from the possibility of false echoes and incorrect tidal reduction (both of which are discussed later in this chapter) the accuracy of echo soundings depends, in general, on the correct functioning of the Echo sounding machine, and in particular :

(i) that the stylus (or motor) speed, in revolutions per second, at the time of sounding, is in correct ratio to the velocity of sound in the water of that particular area.

(ii) that the Transmission line remains constant and indicates correctly the draught of the transmitter at all times.

The velocity of sound in water is given in Admiralty publication H.D. 282, " Tables of the Velocity of Sound in Pure Water and Sea Water for use in

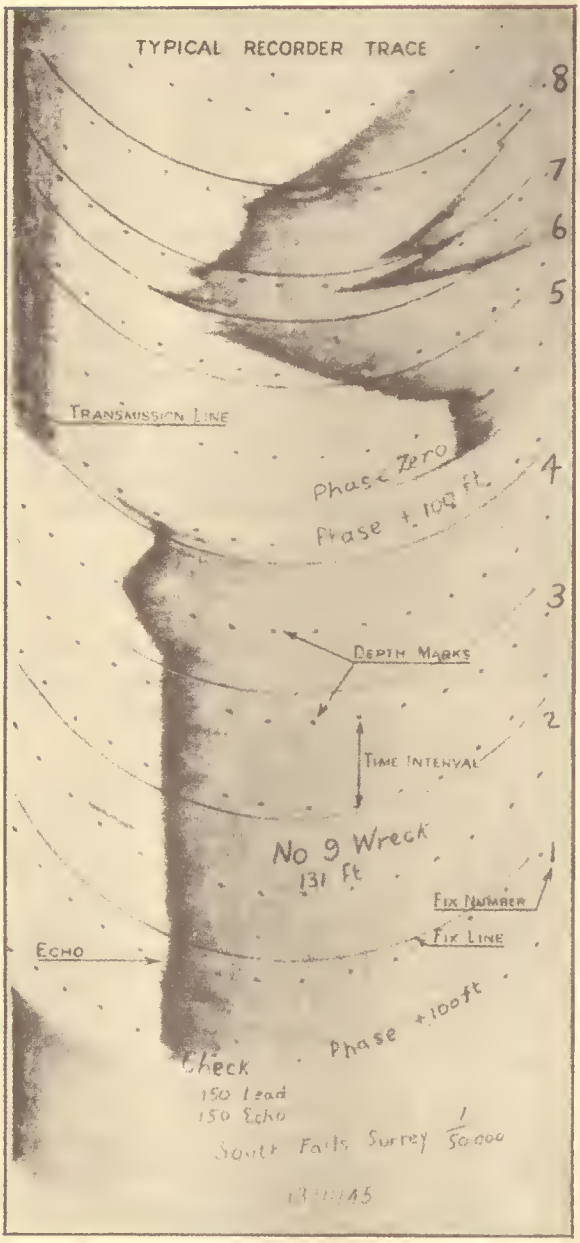

TYPICAL RECORDER TRACE
8
7
6
5
TRANSMISSION LINE
Phase Zero
Phase +10ft
4
3
DEPTH MARKS
2
TIME INTERVAL
No 9 Wreck
131 Ft
1
FIX NUMBER
FIX LINE
ECHO
Phase +100ft
Check
150 Lead
150 Echo
South Falls Surrey 1/50000

PLATE M

Echo-sounding and Sound-ranging " (2nd edition, 1939) ;* the other provisions of (i) and (ii), above, must be determined at frequent intervals by calibration and, if error is found, must be either corrected on the spot, or allowed for before the soundings are inked in.

Surveying ships are supplied with *tachometers* for checking the speed of the stylus (or recorder motor). The method of using the tachometer varies with different types of recorder, but in the Type 762 the speed is taken from the end of the stylus carriage shaft, on the front face of the recorder.

The stylus speed is in direct ratio to the motor speed, according to the setting in use, high speed (feet scale) or low speed (fathom scale). The standard speed of the stylus, which is stamped on the stylus carriage, corresponds to a sounding velocity of 800 fathoms per second (4,800 ft./sec.), but the necessary stylus speed corresponding to any other sounding velocity can be determined by simple proportion, *e.g.* for a sounding velocity of 840 fathoms per second, and a stamped stylus speed of 533·5 revs. per min., the stylus speed must be adjusted to $\frac{840}{800} \times 533{\cdot}5$, or 560·2 revs. per min.

Should there be no tachometer available, the speed of the stylus can be found by timing the Interval markings as they appear on the record. The machine must be in the high speed (feet scale) setting, and a " run " of not less than three intervals should be timed with a stop watch. For a sounding velocity of 800 fathoms per second the markings should appear at intervals of exactly 30 seconds ; for other sounding velocities the correct time intervals can be found by simple proportion, as above.

Calibration.—At the beginning of every new survey, and at frequent intervals thereafter, careful calibration of the echo gear is essential. The procedure with the Type 762 machine should be as follows :

(*a*) Run the machine in the high speed (feet scale) setting for at least 30 minutes, so that it may warm up and attain a steady speed.

(*b*) Check the speed of the motor as detailed above, and correct it, if necessary, by adjusting the governor.

(*c*) Proceed into shallow water, say 2 or 3 fathoms, where the bottom is reasonably flat and, if possible, hard. With the boat stopped, and the machine running in the high speed setting, take careful soundings by " dobbing " with a lead line, held as close to the transmitter as possible. Compare the called soundings with those shown on the echo record. In such shoal water it can be assumed, if the speed of the stylus has been corrected that any discrepancy between the lead line and echo depths is due to an incorrect setting of the point of transmission.

An adjustment is provided by means of which the relative positions of the transmitter contacts and the stylus arm can be varied within small limits. The setting of this adjustment must be altered until the echo depth agrees with the lead line. This operation will correct for both lag in transmission and for the draught of the transmitter.

(*d*) Proceed into a depth of 50–60 ft. (*i.e.*, near the limit of the feet scale at zero phase) and, observing the same precautions as in (*c*), check the echo sounding against the lead line. Any discrepancy between them must now be ascribed to an imperfect estimate of the sound velocity used in (*b*) and the speed of the motor must be adjusted accordingly. If the echo is *deeper* than the lead line the speed must be reduced by screwing *up* on the governor and vice versa.

(*e*) If it will be necessary to use the low speed (fathom scale) setting in any part of the survey (*i.e.*, if there are depths of over 10 fathoms) proceed to the deepest part (but not over 60 fathoms) and check the echo

* *Note.*—If Echo Sounding Velocity Charts (published by the Admiralty) are available they should be used in preference to H.D. 282.

against a careful wire sounding. It is unlikely that any measurable discrepancy will be found, but if one does exist, it must be due to a change in transmission lag caused by shifting over to the low speed setting. The error should be allowed for by correcting the soundings when inking in.

Wear of the carbon annular ring situated in the motor governing gear will cause the motor to increase its speed, and will introduce a percentage error in the soundings. Prolonged use may cause an alteration in the setting of the transmitting mechanism which will affect the instant of transmission and introduce a zero setting error. In addition, the sound velocity is not always known with accuracy and, in any case, is subject to local or seasonal change. Consequently the echo soundings must be checked at frequent intervals during each day's work by comparison with up-and-down casts of the lead, as described above.

Depths of 7–10 fathoms should be selected for these daily checks, if practicable, but they should be carried out in different parts of the area of the day's work to obviate any changes in water conditions, such as would be found in estuaries, or where large temperature differences exist (such as at the junction of the Labrador Current and the Gulf Stream).

Great care must be taken when checking to obtain the first strike of the lead, especially when the bottom is soft.

Provided the motor is running at the correct speed for the deduced sounding velocity and the transmission is correctly aligned, any errors found in these periodical checks can be accepted as a percentage, and allowed for accordingly. However, unless the assumed sounding velocity differs greatly from its true value, there should be little or no appreciable error in soundings under 50–100 fathoms.

(The ship's echo soundings must be checked in a similar manner, but, of course, the Lucas machine will be used for these checks more frequently than the lead line.)

(As far as deep sea soundings are concerned, it is not customary to adjust the speed of the motor for different sounding velocities, but to run it for a standard velocity value of 800 fathoms per second. Variations from this figure, as given in H.D. 282, should then be applied as a correction to each recorded depth. Such corrected deep sea soundings should be checked periodically against wire soundings, but, except under ideal conditions, such checks cannot be regarded as really accurate ; they will, however, be a guard against gross errors, and may also furnish data for assessing the reliability of previous wire soundings.)

All checks should be recorded by electric pencil on the echo record as well as in the sounding book.

False Echoes, caused by dense shoals of fish, or by layers of water differentiated by sudden changes of temperature or salinity (or both), are occasionally recorded, and care must be taken not to accept them as genuine soundings.

Echoes from shoals of fish should be recognised without much difficulty, as their character will probably differ considerably from the genuine bottom echoes, the beginning and end of the shoal will hang in " mid air " on the record, and usually the true bottom echo will be recorded as well as the false echo.

The reflecting power of a layer may, owing to its nearness to the surface, give as strong an echo as the bottom, but in many cases it may be distinguished by being several times as long as the outgoing signal. If, however, the layer is thin it will be indicated by a sharp echo.

It is possible that the " falseness " of the echo given from a layer can be appreciated more readily on the slow speed setting, and in cases of doubt, a temporary change over should be made. A check with the lead line, however, is the only satisfactory method of proving or disproving such false echoes.

Running the Gear.—For use in boats, power for operating the Type 762 gear is supplied by a 12-volt battery. A detailed description of the various parts of the apparatus and instructions for maintainance and location of faults are contained in handbooks supplied by the makers, Messrs. Henry Hughes & Son, Ltd. It will be sufficient to say here that a high standard of mechanical and electrical efficiency is essential and that the machine must be carefully protected from the weather.

The method of covering the ground by systematic lines of sounding with fixes at regular intervals is the same as that employed with the hand lead or any other sounding apparatus. The only material difference is that with the hand lead the speed of the boat is generally limited to that at which the leadsman can obtain accurate and sufficiently numerous soundings, whereas with echo gear the speed is only limited by the ability of the observers to fix and plot the position with sufficient rapidity. When using the hand lead, it is usual to fix after every so-many soundings ; with the echo gear, fixes should be obtained at regular intervals of *time*.

The echo machine should be started up fifteen minutes or so before it is actually required for use so that it may warm up and attain a steady running speed which should be checked before sounding commences to ensure that it is correct within, say, 1 per cent. The first of the daily checks described above should be made, and the machine adjusted, if necessary, before sounding is started.

The machine should run unattended, but it is more convenient to station a man at it to carry out the following duties :

(*a*) Report if the record tends to fade out or disappears at any time.

(*b*) Report if sensitivity control requires adjustment.

(*c*) Operate the phasing and gear changing mechanism as necessary to maintain the sounding range of the machine in conformity with the depth of water.

(*d*) Call out times for the fixes and, as they are taken, mark and number them on the record.

(*e*) Call out and enter in a sounding book the depths as read off the record at regular intervals, *e.g.* every 30 seconds, taking care to include also the shoalest depths at every point where a rise in the bottom occurs.

(*f*) Report unexpected or rapid shoaling.

The soundings can, of course, be read off the record later on when the boat returns to the ship, but this will be a very considerable labour and means that " inking in " takes much longer than necessary. By carrying out the arrangement suggested, all that is necessary, later on, is to glance quickly at the record after the soundings between successive fixes have been inked in and note that no shoal depths have been accidentally omitted.

As the boat proceeds along the line of soundings, the nature of the bottom should be ascertained with the hand lead or small Lucas machine at regular intervals, speed being reduced as necessary for this purpose. Approximately every half-hour the boat should be stopped and the accuracy of the echo record checked against a lead line or wire sounding. After running for an hour or so, the record may become slightly defective owing to deposit from the back of the paper accumulating on the face of the tank where the arc is traced by the stylus. The machine must be temporarily stopped so that this deposit can be cleaned off.

The following information must be recorded in the sounding book, whether the soundings are read off at the time or later :

(*a*) Marks used for fixing and observed Angles.

(*b*) Numbers and Times of the Fixes.

(*c*) Nature of Bottom.

(*d*) Notation of check Soundings. (These must also be noted on the record.)
(*e*) Tidal Reductions (when they have been ascertained).

Portable Echo Sounding Gear.—This gear has been developed to meet the needs of the Hydrographic Surveyor who may have to sound in a boat which has not been fitted with tanks to house the transmitter and receiver units.

The echo sounding unit consists of a standard 12-in. reflector which is mounted in a wooden casing of streamlined form. Inside the reflector are two 1-in. magneto striction packs mounted on the same spindle, the lower one functioning as a transmitter and the upper as a receiver. The unit is known as a " transceiver." The reflector is air-filled, its lower end being closed by a brass plate but the wooden casing which surrounds it is open to the sea.

The streamlined housing is carried at one end of a 9-ft. steel tube about 3 ins. in diameter through which the leads to the Transceiver are passed. The tube is flattened to an oval shape at its lower three feet and carries suitable lugs and a pad piece for mounting. Plate N shows the outboard unit mounted on the stem of a motor skiff.

The recorder and amplifier are of normal type arranged for 12-volt battery operation and are mounted in a light steel box so that they may be transported and installed together. The weight of the recorder and amplifier and the framework on which they are mounted is approximately 170 lb. and that of the Transceiver with its steel tube about 60 lb.

A lighter type of portable set is under development.

Reduction of Soundings

Soundings may be carried on at all states of the tide but the measurements give the depth only at the particular time they are taken. The surface level of the water is in most places undergoing constant periodic changes and, as it is not feasible to produce charts showing the actual depths at any and every state of the tide, soundings must be " reduced " to some arbitrary plane so that they indicate the depths at some particular phase of the tidal movement, *e.g.* at mean low water springs. The selection of this arbitrary plane, *i.e.* chart datum, is discussed under the heading " Tides and Tidal Streams " in Chapter XV. It is sufficient to remark at this juncture that the datum should be a plane so low that the tide will but seldom fall below it. Adoption of this principle ensures that the chart will show approximately the *least* depths to be found in any locality, *i.e.* the mariner will very rarely have to *subtract* from the charted figures to find the true depth at any time.

Predicted heights of the tide are always subject to some uncertainty and should not be used in any regular survey.* To reduce soundings to a common datum an exact knowledge of the tidal movement (in a vertical plane) is essential and readings of the water level must therefore be observed on a tide gauge and recorded at frequent intervals during the periods sounding is in progress. (See also Chapter XV.) By plotting the times against the heights on squared paper, a series of points is provided through which a fair curve, representing the vertical tidal movement, can be drawn, and the reduction at any particular time can be read off.

In reducing soundings the general principle observed in Admiralty surveys is that soundings are never to be *increased* by more than one-quarter of that unit employed. It therefore follows that the tidal reduction changes to the next higher foot when the decimal of the height of the tide above datum is 0·3 ; *e.g.*, when the tide is 3·3 ft. above datum the reduction is 4 ft. By the application of the same principle to depths which are shown on the chart to the nearest fathom only (*i.e.*, those over 11 fathoms), a reduced sounding of 19_4 would be shown as 19 fathoms, whilst 19_5 would be shown as 20 fathoms.

Reduced soundings of less than 1 ft. are considered as being equivalent to

* But see pages 308 and 309.



PLATE N

" awash." When the reduction is greater than the original sounding, the resulting figures must represent a height above chart datum and would, for example, be inserted on the chart as $\underline{12}$ or " dries 12 ft." It may be noted here that " drying " soundings are now always recorded in *feet* not in fathoms and feet.

Nature of the Ocean Bottom

A chart should indicate by the recognised abbreviations the nature of the sea bed in every part of the water area. This information is of value to :

(1) The mariner ; to assist him in choosing the most suitable places to anchor, *i.e.*, in the best holding ground and clear of rocks. In some instances he will also be helped in determining his position.

(2) The harbour engineer ; in connection with dredging operations, the construction of breakwaters, etc.

(3) The fisherman ; to enable him to select areas free from rocks where he can safely shoot his trawl and where particular varieties of fish may be expected.

(4) The scientist ; for the advancement of oceanographical knowledge generally.

Special equipment in the form of self-closing grabs for obtaining samples of the bottom is sometimes used, but the surveyor is normally dependent on a lead " armed " with tallow or fitted with a non-return flap-valve in its base. Both of these should be used, since some materials, *e.g.*, fine sand, will adhere readily to the tallow but may be completely washed out of a flap-valve which is unlikely to be perfectly non-return in its action, whilst others, *e.g.*, soft mud, scarcely adhere to tallow at all but leave a good sample in a valve lead. Failure to appreciate this fact has often led to the bottom being erroneously and indefinitely described as " hard " or even as " rock." The best indications of rock are abrasions on the base and sides of the lead. Bottom specimens should be obtained at regular intervals, *e.g.*, every five minutes, whilst sounding ; in boats the hand lead or boat's machine is used, in the ship the Kelvin machine is usually the most convenient for depths up to 200 fathoms or so, whilst in deeper water a driver tube or valve lead on the deep sea sounding machine is employed.

The samples found on the lead come from the surface layer which may be of very different character to that just underneath, but some clue to the composition of the latter may often be obtained by examining the material brought up on the flukes of the ship's, or beacon, anchors. Surveying ships are usually provided with a microscope and a sample box of bottom specimens. Comparison with the labelled samples should ensure proper identification of the various types likely to be found. The following paragraphs, which give a short summary of the general nature of the ocean bottom, are based on the very comprehensive study of marine sedimentation as described in " The Oceans " (Sverdrup, Johnson and Fleming ; Prentice-Hall Inc., New York, 1942), to which further reference should be made.

Marine deposits are derived from a variety of sources, of which the following contribute the greater part :

(1) Detrital material, *i.e.* material from the breakdown of rocks, carried into the sea, mainly by rivers.

(2) Products of volcanic activity, either from above or below the sea surface.

(3) Skeletal remains of organisms, both plant and animal, such as normally comprise a large part of the deep-sea deposits.

(4) Inorganic precipitates from sea water, which are not abundant but which may form conspicuous components in certain localities.

(5) Products of chemical transportation, *i.e.* substances formed by the interaction of solid particles and sea water.

(6) Extraterrestrial materials, such as the small black or brown spherules which may be found in sediments which accumulate at extremely slow rates, such as red clay.

These deposits may be subdivided into two main groups called *pelagic* and *terrigenous*. Pelagic deposits are those found in deep water far from shore and may be either organic or inorganic in origin. They are usually light-coloured, fine-grained and, generally, contain some skeletal remains of plankton organisms. The inorganic deposits are called *red clay*, and the organic sediments *oozes*. Terrigenous deposits are those which are found near shore, and generally contain at least some material of terrigenous origin. They cover a wide range in depth, and vary greatly in colour, texture and composition. They are known as sands, silts or muds ; inshore sediments may contain remains of plankton animals but, in addition, skeletons of bottom-living organisms may also be abundant in relatively shallow water.

Pelagic Deposits

The pelagic deposits are restricted to the large ocean basins and are classified according to their composition ; those with less than 30 per cent. of organic material being known as *red clay*, and those with more than 30 per cent. of material of organic origin, as *oozes*. This latter class is further subdivided into *calcareous* and *siliceous oozes*. Calcareous oozes are those which contain more than 30 per cent. of calcium carbonate, mainly in the form of the skeletons of plankton animals, whereas siliceous oozes are those pelagic deposits which contain a large percentage of siliceous skeletal material produced by planktonic plants and animals.

The main types of pelagic deposits are thus as follows :

Red Clay.—This deposit is one of the most predominant and covers 38 per cent. of the sea bottom. It has a world-wide distribution, but is confined to those portions of the ocean floor at moderate or great depths. In the deeper waters of the Pacific Ocean red clay is the most extensive sediment, comprising 49 per cent. of the total deposits. The minimum depth at which it is found is 2,200 fathoms, the maximum, 4,530 fathoms ; in the Atlantic Ocean it occurs at an average depth of 2,950 fathoms. The colour of the sediment varies from brick red in the North Atlantic to chocolate brown in the Indian and South Pacific Oceans.

Calcareous Oozes.

(*a*) **Globigerina ooze.** This deposit consists mainly of the skeletal remains of pelagic foraminifera. It has an extensive distribution and is predominant in the Atlantic Ocean, where it covers about 65 per cent. of the ocean bed, but it is also found in considerable quantity in the Pacific and Indian Oceans. The sediment varies considerably in colour ; near land it may be dirty white, grey or blue, but in the open ocean, rose, yellow or brown tints may be observed. The distribution of globigerina ooze normally is confined to the lesser depths and, in the Atlantic Ocean, the range of the deposits lies between 420 and 3,300 fathoms, with an average depth of occurrence of 1,975 fathoms.

(*b*) **Pteropod ooze.**—This sediment contains the conspicuous shells of pelagic molluscs. It only occurs in significant amounts in the Atlantic Ocean. It ranges in colour from white to light brown, with a reddish, pink or yellow tinge. The depth at which it is found varies from 390 to 1,930 fathoms, with an average depth of 1,130 fathoms.

Siliceous Oozes.

(*a*) **Diatom ooze.**—The characteristic feature of this deposit is the large amount of diatom frustules, *i.e.*, the siliceous remains of microscopic plankton plants. The distribution of this ooze is restricted to a virtually continuous belt

around Antarctica, a band across the North Pacific Ocean, and a small belt parallel to the west coast of South America. The depth-range of occurrence is between 600 and 3,150 fathoms, with an average depth of 2,300 fathoms. The colour of diatom ooze when wet is yellowish, straw or cream-coloured, but when dry it is dirty white. Near land the deposit is darker.

(*b*) **Radiolarian ooze.**—This ooze contains a large proportion of skeletons of radiolaria, a group of minute protozoa. Its distribution is almost entirely limited to a wide band in the equatorial regions of the Pacific Ocean. It occurs at depths between 2,350 and 4,500 fathoms, with an average depth of 2,900 fathoms. In colour it may be straw, chocolate brown, or red.

Terrigenous Deposits

The classification of terrigenous deposits is not so satisfactory as that of pelagic deposits in that so much may depend on local conditions. Relative to the pelagic sediments, the terrigenous deposits cover only a small proportion of the sea floor, and they show a wide range in properties within a relatively short distance from the coast. They are classed usually as gravels, sands, silts and muds, and their texture, generally, is coarser than that of the pelagic deposits. Terrigenous deposits have a wide range in colour, from white to black, with the addition of blue, yellow or red, or mixtures of these. Generally, the sediments of the shelves and slopes are dark in colour, ranging from green or brown to blue. Colour varies greatly with local conditions, but off the mouths of large rivers, the deposits are usually red, yellow or brown, as the result of the debris carried seaward by the rivers, and are often referred to as *red muds*. Along most coasts the terrigenous deposits generally are green or olive-green, possibly due to the intermediate stages in the oxidation or reduction of the iron present. In general the light-coloured sediments are relatively coarse-grained and are composed of quartz or limestone. A grey colour is due probably to the presence of black minerals, while black deposits are typical of the stagnant conditions which may prevail in fjords, basins and other isolated places. Near large glaciers, the near shore deposits will consist almost entirely of detritus carried out to sea by the ice, while in volcanic regions much of the inshore sediments will be ash and pumice.

CHAPTER XI

SWEEPING

The determination of the contours of the sea bed by means of sounding only is always open to the criticism that dangers may pass undetected if they happen to lie in the inevitable spaces which must exist between adjacent soundings. The introduction of echo sounding methods has met this criticism in *one* respect, *i.e.*, soundings are so closely spaced that no danger could possibly remain undetected on any particular *line* run by the ship or boats ; nevertheless there still exist the comparatively large spaces between adjacent *lines* of sounding where for want of better information it is assumed that the slope of the bottom is more or less even. If, however, the bottom is irregular in form or if obstructions such as wrecks exist, soundings alone cannot be relied upon to indicate the shoalest depths and it is necessary to resort to sweeping. In this operation a wire is towed horizontally or carried at a set depth below the surface of the water over the area where the shoals or obstructions are believed to lie. If the wire does not foul anything during the process, it provides positive evidence that no shoals or obstructions in that area rise from the bottom to a greater height than the set depth of the sweep. This information is of great value as it shows the depth which may be *safely* carried in all parts of the swept area. A sweep is also employed for obtaining the exact depth of shoals and obstructions which are of such small size that there could be no certainty of locating their shallowest parts with any type of sounding apparatus. In this connection it must always be remembered that the most satisfactory method of finding the least water over an obstruction of small extent, and known position, is by sending down a diver.

The various types of sweeping gear commonly used in surveying and the methods of operating them are described below.

Mark IV Oropesa Sweep

This is a single-ship sweep for use at speeds up to about 12 knots and was originally designed for mine clearing. For surveying purposes it has been modified in certain particulars and has proved extremely valuable in locating wrecks and shoal patches. It can be used as a single sweep, *i.e.*, on one quarter only, with a maximum useful spread of about 600 ft., or as a double sweep with twice that spread. The principal items of the equipment are :

(1) Three **Kite Otter Multiplanes :** rectangular steel box-frames fitted with vanes. Two are used as *otters* for pulling the sweep out on either quarter and the third as a kite for keeping the ship end of the sweep down ; the construction of all three is identical.

(2) Two **Sweep Wires :** each of 450 fathoms of $1\frac{1}{4}''$ British mine-mooring wire, one right hand laid for use on the starboard quarter and one left hand laid for use on the port quarter.

(3) **Kite Wire :** 100 fathoms of $1\frac{3}{4}''$ E.S.F.S. Wire.

(4) Two **Floats :** for supporting the otters.

(5) Two **Snatch Blocks and $1\frac{1}{4}''$ Wire Pendants :** for connecting the sweep wires to the kite.

(6) Two **four-legged chain Slings :** fitted on a beam with bottle-screw adjustment, for securing the sweep wires to the otters.

(7) **Three-legged Chain Slings :** one leg fitted with bottle-screw adjustment, for securing the kite wire to the kite.

(8) **Float Wires** ($1\frac{1}{4}$″) **:** in various lengths up to about 30 fathoms.

To work the gear, two winches (or a double winch) to take the sweep wires and fitted with friction brakes are required on deck, and a third winch or capstan to take the kite wire. Hand cranes for hoisting the floats and otters in and out should also be fitted.

The manner in which the sweep operates is shown in Figs. 131 and 132. Fig. 131 shows the sweep in plan and in elevation from astern ; Fig. 132 shows it in side elevation. As the ship steams along, the kite pulls down the snatch blocks through which the sweep is rove and so keeps the ship end of the sweep down to a set depth, whilst the otters pull out on each quarter and spread the sweep. The otters have a tendency to dive but are restrained by the floats to which they are connected by float wires. The natural tendency of a wire being towed through the water would be to take the form of a catenary, but, by using right-hand laid wire on the starboard leg and left-hand laid wire on the port leg, the wash of the water on the lay provides a lifting action which increases with speed. The speed of the ship can be adjusted in relation to the spread of the sweep so that a practically flat sweep is obtained. For example, if 140 fathoms of sweep wire are veered (giving a spread of about 435 ft. on each leg), the speed through the water to obtain a flat sweep must be 7·7 knots ; with 240 fathoms veered (giving a spread of about 570 ft. on each leg) the speed must be 8·3 knots.

The depth of the sweep wire at the otter ends depends primarily on the amount of float wire used and to a lesser extent on the speed at which the sweep is towed. To alter the set depth of the otter end, it is necessary to heave in the sweep and hoist in the otter and float so that the length of float wire can be changed. The depth of the sweep at the ship end is regulated by the depth of the kite which can be altered at any time by veering or heaving in the kite wire which should be marked at every fathom of its length.

Each float carries a staff and flag so that the outer ends of the sweep are indicated and can be seen from the bridge of the ship. Thus by bearings and rangefinder distances of the two floats, the spread of the sweep can be checked at any time.

Depth recorders are available for testing the accuracy of the depth-keeping of the sweep wire, to which they can be shackled between puddings at any convenient point. They are intended for use only when testing the gear and should not be attached at other times.

Preparation of Oropesa Sweep Gear for running.—The various items of equipment may be considered *seriatim* as follows :

(*a*) **Kite Wire.**—This should be marked in a similar manner to a lead line at every fathom from 5 to 50. An eye and thimble for shackling to the kite slings should be worked at its outboard end. The kite wire is usually operated from an after capstan in surveying ships.

(*b*) **Kite.**—If the gear has not been run previously, it will be necessary to " balance " the kite, an operation which is carried out as follows :

Lay the kite flat on deck, leading edge forward and with towing eye bolts on top. No difficulty will be experienced in recognising the leading edge if it is remembered that the lug on the centre strut (*see* Fig. 133) is on the *after* edge. Shackle on the *three*-legged slings : the shortest leg shackles on at the lug on the centre strut, the leg with the bottle screw (which should be adjusted to about its central position) to the ring of the *port* leading eye bolt, the third leg to the starboard leading eye bolt. All legs must be entirely free from turns.

Lead the kite wire through the after fairlead and shackle it to the towing ring of the kite slings, put the ship to slow speed and veer until the 5-fathom mark is awash. Increase to 6 knots and if the kite wire tows centrally, *i.e.*, fore-and-aft, increase gradually to, say, 10 knots.

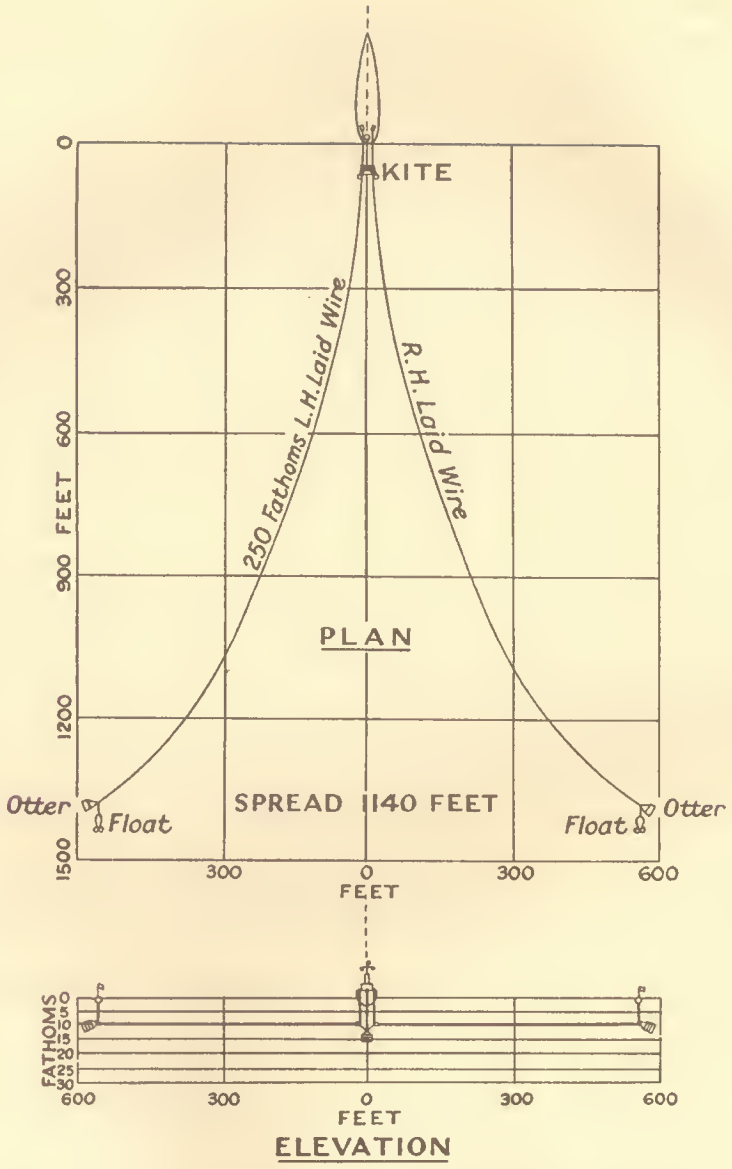


FIG. 131

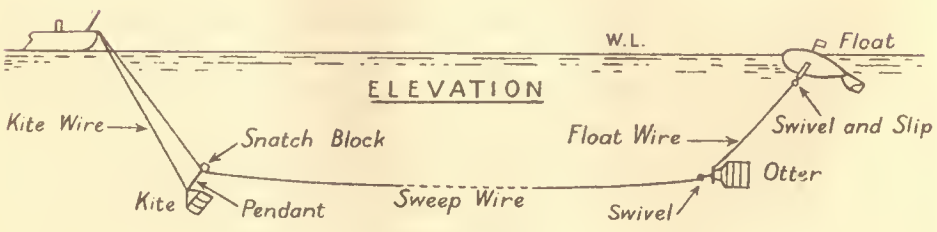


FIG. 132

If, however, the kite sheers to port or starboard, slow down, heave it up and adjust the bottle screw. If the sheer is to port, lengthen the port leg and *vice versa*, turning the bottle screw as much as three or four turns if the kite sheers at low speed. Lower the kite again and repeat the process as necessary with smaller adjustments until the kite runs true at full speed.

(*c*) **Kite Pendants and Blocks.**—Two kite pendants are required for the double sweep. These should be made of 1¼″ British mine-mooring wire and fitted with an eye and thimble at each end. Each pendant should be about 5 ft. in length, but *must* be long enough to reach across from the midship fairlead, which takes the kite wire, to the quarter fairlead, which takes the sweep wire. One end of the pendant is shackled to the ring of the kite slings and the other to a snatch block through which the sweep wire is rove.

(*d*) **Floats.**—A wooden stave with a small flag should be fitted in the flagstaff socket on the top of the float. A swivel-piece is shackled to the towing saddle and a small Senhouse slip should be fitted below the swivel so that the float is quickly detachable from the float wire. The floats are hoisted out by hand cranes or davits : tripping hooks should be fitted on the whips for dropping them.

(*e*) **Float Wires.**—A sufficient number of float wires (1¼″ British mine-mooring wire) should be made to allow for various lengths from 5 to 30 fathoms being made up. Lengths of 20, 10, 5, 4, 2 and 1 fathoms should be cut off and fitted with an eye and thimble at each end : these will be required in duplicate when the double sweep is used.

(*f*) **Sweep Wires.**—To wind a wire onto the sweep winch, the coil should be hung on a turntable so that the whole coil can be turned as it is wound on ; if an attempt is made to wind on directly from the coil as it lies on deck, the wire will become hopelessly kinked. Wire with *right-hand* lay must be wound on the starboard winch, and *left-hand* lay on the port winch. The outboard ends of the sweep wire can be secured to the otter slings by means of bulldog grips. The wire is difficult to splice and, as frequent breakages may occur when sweeping for obstructions, it is a waste of time to eye-splice it.

(*g*) **Otters.**—If these have not been previously used, it will be necessary to attach the slings and adjust them so that the otters run properly. To attach the slings, place the otter on its edge, on the side of the ship it is going to be used, leaning it up against the bulwarks or rail with its leading edge forward and with the sling attachments inboard. The *four*-legged slings are used for the otter and these are shackled on with the bottle screw *below* the towing beam and the *longer* legs to the *forward* edge of the otter. The link for attaching the float wire is at the top of the beam. See Fig. 133.

When shackling on the slings, it is most important to see that all legs are free from turns. Assuming the otter has not been run before, place the bottle screw in about the central position.

To " balance " the otter, get the sweep out and run it as later described. Each otter should be balanced independently. For correct running, the float should tow in a moderately calm sea almost wholly submerged but with the whole staff showing. The water should be seen lifting over the nose of the float but not breaking until the tabernacle of the flagstaff is reached. If the float rides lighter than this, or if the otter breaks surface, heave up the sweep and shorten up the bottle screw ; if the float tows with the flagstaff partly or wholly submerged, lengthen the bottle screw and continue adjustments until the float

runs correctly. After these adjustments have been carried out, a depth recorder may be used to test their accuracy.

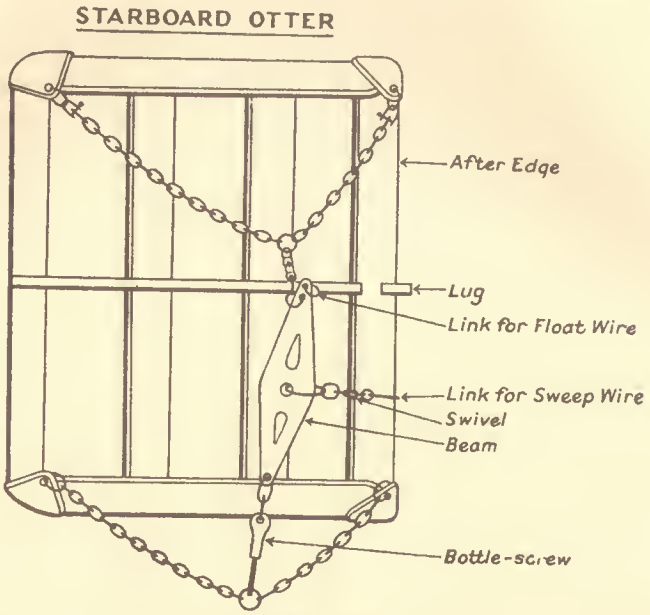


FIG. 133

Operation and Management of the Oropesa Sweep.—For surveying purposes the sweep must normally be run as a flat sweep at some pre-arranged depth below the datum of soundings and it is therefore necessary to predict the tidal movement for the period that the sweep will be in use. A predicted tidal curve should be drawn and the heights above datum at, say, each hour taken off, so that the required running depth of the sweep *below the surface* at any time will be known. This information determines the amount of kite wire to be veered (see Tables C and D in this chapter) and the length of the float wires (see Table B) which must be altered as the tide rises or falls. As alteration of the float wires involves heaving up the sweep and takes some time, it is hardly feasible to alter the setting very frequently; as a rule settings within, say, 2 ft. of the required mean can be accepted. For example, if it was intended to start sweeping at 0700 at a depth of 8 fathoms below datum and if the predicted height of the tide was 16 ft. at 0700, falling to 12 ft. at 0800, the sweep might be set at 10 fathoms 2 ft. during the first hour, giving depths below datum varying from 7_4 to 8_2.

To get the sweep out, the sequence of operations is as follows:

(1) Reeve the sweep wires through their respective quarter fairleads and attach to the slings of the otters; the latter can then be lifted over the rails and the sweep wires hove in until the otters hang close up to the fairleads with their leading edges forward and outboard, and without any turns in the slings. Set the winch dials to zero.

(2) Hoist the floats outboard with the cranes or davits and lower them level with the bulwarks. Then shackle on the required lengths of float wire joining the floats to the otters, keeping the bights clear of the water.

(3) Reeve the kite wire through the after fairlead, shackle it to the kite slings and lift the kite over the stern, leaving it hanging close up by the kite wire. Shackle on the pendants and snatch blocks to the ring of the kite slings.

(4) Steaming at about 6 knots, lower and slip the floats which will then tow astern by the float wires. See that the latter lead clear from the otter slings over the top of the otters.

(5) With the winch engine clutched up, veer about 30 fathoms on *one* otter at full speed. Stop the engine, and the sweep wire and float should draw out on the quarter. If they do not do so, it is probable that the float wire is foul of the otter which may have taken an unusually violent sheer when it entered the water or the otter may have struck the bottom whilst veering; alternatively there may have been an undetected twist in the slings. In this case, reduce to slow speed, heave the otter in close up to the fairlead where it can be reached, and clear the slings and float wire. Then increase to 6 knots and repeat the operation until the otter runs properly. Similarly with the other otter and sweep. It is not advisable to veer *both* at the same time at this stage as, if one otter runs foul, it will probably jump over the other sweep wire and lead to a bad foul. Disconnect the sweep engines and tow on the brakes.

(6) When 30 fathoms are out on each sweep wire, it may be termed the "short stay" position and everything is now ready to veer and set the sweep as the ship approaches the scene of operations. The short stay position is useful as the ship can be manoeuvred under almost full helm and the drag of the otters does not greatly increase her normal turning circle.

From the short stay position it takes five to eight minutes to veer 250 fathoms of sweep wire and to lower the kite, so that with the tidal stream behind the ship it may be necessary to arrange a "run up" of as much as two miles to the sweeping area to ensure that the sweep is set in time. It is obviously necessary to arrange for the run up to be made where there is sufficient depth of water to prevent the sweep fouling the bottom.

(7) Increase to sweeping speed. As previously explained, this must conform to the spread of sweep it is proposed to use, if a flat sweep is to be obtained: *vide* Table A in this chapter. It must be remembered that, when it is set, the sweep exercises a considerable drag on the ship and it may be necessary, with the double sweep out, to do revolutions for 10 knots to attain a speed of 8½ knots through the water. For surveying purposes the most useful speeds are 7 to 9 knots, corresponding to spreads of about 140 and 210 yards respectively, on each leg.

(8) Veer both sweep wires slowly "on the brake" until the required amount of sweep wire (as shown on the winch indicators) is out. This operation must not be hurried or the otters will probably hit the bottom and run foul. As the wires run out, the floats should be carefully watched to see that they always pull out a little on the quarter. If a float is seen to be coming almost dead astern, put the brake hard on and wait a few moments until it has pulled out again on the quarter. When the necessary amount of sweep wire is out, put the brakes on just hard enough to prevent the drums rendering *unless* any additional strain comes on the sweep wires.

(9) As the sweep wires run out, snatch the kite pendant blocks onto them and veer the kite with the after capstan, stopping it when the required length of kite wire is out, as noted by the fathom mark awash.

Manoeuvring with Sweep Set.—When the ship is under way with the double sweep fully veered and requires to make any large alteration of course, the sweep on the side to which the ship turns, should be hove in whilst the turning movement is in progress, and the kite should be lifted about one-third of the amount

of kite wire out. The ship should not use more than 20° of rudder and should straighten up temporarily whenever the inner float comes before two points abaft the beam. Having turned to the required course, the inner sweep can be veered again and the kite lowered to its correct setting. The drag of the sweep strongly opposes turning movement and the normal turning circle diameter is nearly doubled with the sweep out.

It is sometimes found more advantageous to use a single sweep (which is rigged in exactly the same way as the double sweep, except that only one snatch block and pendant are required on the kite), as the manoeuvring capacities of the ship are less seriously impaired. Using a single sweep, it may be possible to arrange any large alterations of course to be made away from the sweep and the ship can then turn under almost full rudder. The single sweep is especially useful when sweeping near the bottom in a small area, in the vicinity of shipping, or where obstructions are numerous.

Running the Oropesa Sweep.—Lines for the ship to follow should be drawn on a field board at distances apart about 40 yards *less* than the total spread of the sweep. This will provide a reasonable overlap to allow for failure to follow the lines exactly. Fixing and plotting are carried out by means of sextant and station pointer in a similar manner to that employed for sounding and, as the ship steams along, the flags on the floats which mark the ends of the sweep should be shot up from time to time by compass bearings, and rangefinder distances or depressions from the horizon. This will enable the actual position and spread of the sweep relative to the ship to be plotted at any time on the board and prevent gaps being left in the work. If beacons are used for fixing they should be moored ; otherwise the fix will not be sufficiently precise to ensure that no gaps are left, unless very large overlaps are allowed.

Any convenient scale may be used for sweeping work but it is not advisable to use one *smaller* than about 1/30,000 (2½ ins. to the nautical mile), or there may be difficulty in relocating an obstruction which has been found by the sweep.

No attempt should be made to sweep too close to the bottom. It must be remembered that the kite and otters extend some little distance below the sweep wire and, to allow for this and for minor variations in depth keeping, a margin of not less than 1½ fathoms clear of the bottom is required. As the ship steams along, the floats should be constantly watched. If one or both tend to come in towards the wake of the ship and the winch drums render in *little* jerks, it is probable that the sweep wire is foul of the bottom. When this happens, *increase* speed, if possible, and the sweep will probably clear itself as will be evident by the float pulling out again on the quarter. There is never any doubt when an actual obstruction is fouled, for the drum renders rapidly and continuously and the float is seen to draw quickly into the wake of the ship.

It is essential to have good communication from the quarterdeck to the bridge. As soon as an obstruction is fouled, the bridge should be informed and the engines put to full speed astern, and stopped as soon as the way is off the ship. About 40 to 60 fathoms of sweep wire will probably render off the winch whilst the way is being taken off the ship ; it is therefore advisable to have at least 75 fathoms remaining on the winches after the sweep has been veered and set. The coils of wire (450 fathoms) are amply long enough but, if obstructions are numerous, there will be a constant expenditure of small lengths of sweep wire. For example, if the sweep is to be veered to 250 fathoms, there must be at least 325 fathoms on the drum before veering, to ensure that there is enough spare wire to prevent it running out to a clinch.

When an obstruction is fouled, it is generally correct to assume that by the time the way is off the ship the otter has been pulled into the obstruction and that the float therefore marks its position. A fix should be at once obtained

and the bearing and distance of the float observed and, as a check, a series of these should be made whilst the sweep is being hove in and the ship is being pulled back towards the obstruction. When sweeping against the tide, the otter will sometimes fall clear of the obstruction as soon as the way is off the ship and the sweep wire has slacked up. The otter will then drift or be dragged away hanging from the float, so that it is important to lose no time in getting a fix and shot into the float before this can happen.

Clearing the Sweep from a Wreck.—When the sweep meets an obstruction, such as a wreck, the drum renders quickly and the float is seen to draw rapidly in towards the wake of the ship. As soon as the way is off the ship the winch engines should be clutched up and the sweep hove in. Normally only one leg of the sweep fouls ; the other falls to the bottom and can be hove in fairly quickly. The kite should be lifted at once and the blocks unsnatched from the sweep wires. As the ship is being hove astern towards the wreck, some help may be given with the engines but great care must be taken not to over-run the wire or to get the other sweep wire foul of the propellers. When the fouled sweep wire is nearly up-and-down under the stern, the otter may be pulled clear of the wreck, but more usually it will be too firmly wedged and, if there is any sea, the sweep wire will part or else may be cut with a hammer and cold chisel. Assuming that this has been done, the otter will be left hanging from the float by the float wire, and the ship should be turned head to tide or wind and brought up, head on, to the float which can then be hooked on to the fore derrick whip. The float can usually be lifted high enough out of the water to enable the slip on the float wire to be reached, but care is necessary to prevent too much strain being put on the derrick. A picking-up pendant should be shackled on to the float wire and, after knocking off the slip, brought to the forward capstan. On heaving in, the otter is often pulled in the opposite direction to that in which it was previously drawn by the sweep wire and, if so, will usually come clear of the wreck so that it can be weighed by the float wire. The only expenditure of gear then is the few fathoms of sweep wire which reached from the stern vertically down to the wreck.

It sometimes happens that, when the ship is being hove back to a wreck in which the otter is foul, the strain on the sweep wire will pull the otter through part of the wreckage and the float is dragged under water, remaining submerged after the sweep wire parts or is cut. In such cases the gear can often be recovered by running over the position with the other leg of the sweep set to a shoaler depth so that it clears the wreck but catches the float wire. The latter *may* part when the sweep wire fouls it, but the loss is then confined to the otter and a few fathoms of wire, as the float immediately rises to the surface.

Normally it is found that the otter receives little damage by contact with a wreck unless a " direct hit " is scored ; the vanes sometimes get bent but can easily be hammered back to shape again. A direct hit, however, will probably render the otter or kite unfit for further service.

Notes on Oropesa Sweeping.—In an area where obstructions are numerous it often pays to use a short sweep, *e.g.*, 150 fathoms of sweep wire veered, giving a spread of about 1½ cables with the double sweep. This saves time at each foul as heaving in and veering the sweep is quicker ; also, the ship is handier to manoeuvre than with the longer sweep out.

If it is necessary to sweep on a line between two known obstructions, approach the first one against the tide with the sweep fully veered, but the kite up. As soon as the first obstruction is passed, lower the kite at full speed. Just before the second obstruction is reached, stop the ship and heave in the sweep. By this means a minimum area is left unswept. It is, of course, essential that the ship should pass exactly over the first obstruction in the approach.

In rigging the Oropesa gear it is essential to mouse all shackles. If this is done and if care is taken to see that the kite and otter slings have no turns in them, little trouble should be experienced in running the gear.

It must be fully realised that the Oropesa gear is only suitable for a *Searching Sweep*; it should not be used for finding the least depth over an obstruction, nor for bottom sweeping.

The following tables give the necessary data for operating the sweep at speeds of 7 to 9 knots. The tabulated spread of sweep for various amounts of sweep wire veered are approximate only; they vary somewhat with the depth at which the sweep is run and should always be checked by direct observations of the floats from the bridge.

Tables for Mark IV Oropesa Sweep.

A. **Flat Sweep.**

Sweep wire veered less kite wire.	Spread of *single* Sweep.	Speed through the water.
Fathoms.	Yards.	Knots.
140	145	7·7
190	170	7·9
240	190	8·3
290	205	9·3

B. **Amount of Float Wire in fathoms.**

Speed through water in knots.	Swept depth in fathoms.											
	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.
7	4	5¼	6½	8	10	12	14¼	16¾	19	21	23	25¼
8	4½	5½	7	8½	10½	13	15½	18	20¼	22½	24¾	27½
9	4½	5¾	7½	9¼	11¼	14	16¼	19	21½	24	26½	29

C. **Amount of Kite Wire awash in fathoms for Single Sweep.**

Speed through water in knots.	Swept depth in fathoms.											
	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.
7	9½	11¼	13½	15	17	19	21	23½	25¾	28	30½	32¾
8	9¾	11½	13½	15¼	17¼	19¼	21½	23¾	26¼	28½	31	33½
9	10¼	11¾	13¾	15¾	17¾	19¾	22	24¼	26¾	29	31¼	33¾

D. **Amount of Kite Wire awash in fathoms for Double Sweep.**

Speed through water in knots.	Swept depth in fathoms.											
	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.
7	14¾	17¾	21	24	27¼	30¼	33½	36¾	40	43½	47	50¼
8	15½	18½	22	25¼	28½	31½	35	38	41¾	45¼	49	53
9	16	19	22½	26	29¼	32½	36	39¾	43½	47	50¾	55

Motor Boats' Sweep

The use of the Mark IV Oropesa sweep is obviously restricted to areas in which the ship can safely navigate. For shallow water a smaller type, working on similar principles, has been evolved for use in 28-ft. surveying motor boats. Owing to the low power of these boats and the absence of space for engine-operated winches, the gear is much lighter and is modified in the following respects :

(1) The sweep is used as a *single* sweep only, *i.e., either* to port or starboard, with a maximum of 80 fathoms of sweep wire (from kite to otter) veered, giving a spread of about 80 yards.

(2) The speed of the boat with the sweep set *must* be over 2 knots ; speeds of 3 to 4 knots, according to the conditions, should be obtainable with the power available.

(3) The sweep wire ($\frac{3}{4}$-in British mine-mooring wire) is operated from a hand-worked reel secured in the stern sheets. The kite-wire ($1\frac{1}{4}$-in.) is veered by surging round any convenient cleats and is hauled in by hand.

(4) Three floats and float wires are necessary ; one, as with the Mark IV sweep, on the otter, one shackled on between puddings half-way along the sweep wire as the latter is veered, and one shackled on to the kite slings.

(5) The length of kite wire veered should be two and a half times that of the kite float wire ; the length of the latter and of the other two float wires for various sweeping depths are given in Table E.

(6) If a recorder is used, it should be shackled on by a tail to the sweep wire above a pudding placed about 3 ft. from the otter slings.

(7) If there is no central fairlead at the stern, the sweep wire is led through the port or starboard one and the kite wire on the opposite side. A chafing plate, fitted over the coaming in the way of the kite wire, which is most conveniently worked from forward, is a necessary fitting.

Manoeuvring the Boat with the Sweep out.—Small alterations of course can be made with the rudder in the usual way, but once the sweep grows on the quarter, the speed of altering course is very slow indeed. By partly closing the kitchen rudders, however, as large an alteration of course as desired can be made in either direction, but whilst this is in progress the sweep wire will drop much below sweeping depth and is liable to foul the bottom. With the wind on the beam difficulty may be experienced in steering, and it is therefore best to sweep up and down wind when possible.

Boat's Oropesa Sweep.

Table E. **Length of Float Wires for Depths of Sweep.**

Depth of Sweep in fathoms.	Length in fathoms of		
	Otter Float Wire.	Kite Float Wire.	Mid Float Wire.
2	$1\frac{1}{4}$	$2\frac{1}{2}$	$3\frac{1}{2}$
3	$2\frac{1}{4}$	$3\frac{3}{4}$	$4\frac{3}{4}$
4	$3\frac{1}{4}$	5	6
5	$4\frac{1}{2}$	6	$7\frac{1}{4}$
6	$5\frac{1}{2}$	$7\frac{1}{4}$	$8\frac{1}{2}$
7	$6\frac{1}{2}$	$8\frac{1}{4}$	$9\frac{3}{4}$
8	$7\frac{3}{4}$	$9\frac{1}{2}$	11
9	$8\frac{3}{4}$	$10\frac{1}{2}$	$12\frac{1}{4}$
10	$9\frac{3}{4}$	$11\frac{3}{4}$	$13\frac{1}{2}$

Drift Sweeping

The least water over pinnacle rocks, shoals of small extent, wrecks, etc., can best be ascertained by means of a drift sweep operated from the ship or boats. When there is sufficient depth of water over the shoal or obstruction and enough room to manoeuvre, the sweep should be worked from the ship in preference to the boats, as a much wider spread can be used and there is a correspondingly greater certainty of passing over the required position.

As the name implies, the drift sweeping is effected by allowing the ship (or boat) to drift with the tidal stream or wind over the required position, carrying a sweep at a set depth below the surface of the water.

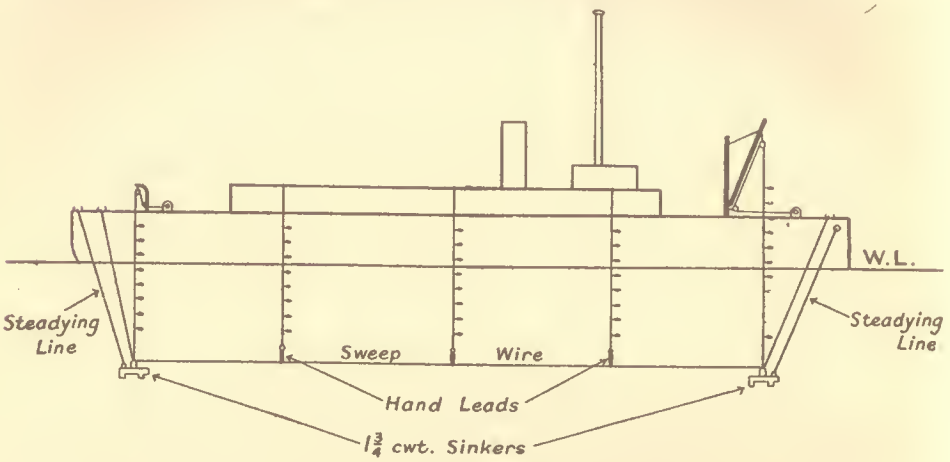


Fig. 134

Fig. 134 shows the general arrangement of the sweep in a ship about 230 ft. long. The sweep wire is supported at intervals of about 40 to 50 ft. by depth lines which are marked in feet in the same manner as a lead line. The foremost and aftermost depth lines should be of 1½-in. wire; they are shackled to 1¾-cwt. sinkers, and are worked from the capstan forward and a sounding or sweep winch aft. The intermediate depth lines are ordinary hand lead lines, fitted with wire hearts, and are worked by hand from the boat or upper deck. The leads should be fitted with screw shackles in their bases, to take the sweep wire. The 1¾-cwt. sinkers should each have an eye bolt screwed in near its perimeter, so that steadying lines may be attached; these are led inboard as far forward and aft as possible and prevent the sinkers turning as they are lowered; they also act, to some extent, as guys should it be necessary for the ship to move ahead or astern. The sweep wire must be light so that it will not hang in any appreciable catenary between the depth lines; ⅜-in. or ½-in. F.S.W. or Kelvin sounding wire are all suitable; it is rove through the centre rings of the sinkers and through the screw shackles on the leads, the forward end is belayed on the forecastle, and the after end, which should be fitted with a soft eye, is put on a small slip on the quarter-deck, so that it can be let go and the sweep wire allowed to unreeve itself whenever the sweep fouls. (A spun yarn sweep has been used with success, and has the (possible) advantage that it can be belayed at both ends and allowed to part when a foul occurs.)

To lower the sweep, about seven men and two winch drivers are needed. The sweep wire is rove at the upper deck level with the sinkers and leads hanging ready over the side. As it is lowered, one man tends each depth line, one forward eases out the sweep wire and foremost steadying line, and one aft eases out the after steadying line. When the sweep has been lowered to the required depth, as noted by the marks, the depth lines can be belayed and

the sweep and steadying lines pulled hand-taut and secured. As the ship drifts, three watchers should be stationed, forward, amidships and aft, to report when and where the sweep fouls. The watcher aft should be responsible for knocking off the slip when a foul occurs. To raise the sweep, the full crew will be required again.

Management of a Drift Sweep in the Ship.—The normal purpose of drift sweeping is to find by experiment the depth below datum at which the sweep will just clear a shoal or obstruction, hence finding the least water over it. It is first necessary to predict the vertical tidal movement for the period of sweeping so that each setting of the sweep can be made with reference to sounding datum and adjusted from time to time, as the work proceeds, to compensate for changes in the water level. The tidal predictions may not be quite accurate, but the exact depth below datum, at which the sweep just cleared the object, can be ascertained later on when the tidal readings for the period have been received.

When the least water over a shoal or obstruction is to be found by drift sweeping, it is desirable first to ascertain as accurately as possible the position where the shoalest depth may be expected. In a few cases the position will be fairly obvious from surface indications such as eddies and swirls, otherwise it must be found by close sounding or with the Oropesa Sweep. If there is uncertainty of position amounting to more than about 50 ft., there will be a doubt as to whether the sweep actually passes over the shoalest part. It is true that this doubt might be removed by drift sweeping over a comparatively large area as is often done to check the results of dredging operations, but this is always a very slow process if it is to be sufficiently thorough to ensure that no gaps are left unexamined. It is therefore better to determine the position by other means, leaving the drift sweep to find the least water only.

Suppose now that an obstruction has been located by the Oropesa sweep set to a depth below datum of 8 fathoms and that the least water over this is to be found by drift sweeping. Having plotted the position of the obstruction on the sweeping board, a dan buoy should be anchored 300 to 400 yards directly up stream from the obstruction and carefully fixed and plotted. This buoy acts as a mark to enable the ship to be stopped in such a position that she will drift down over the obstruction. It may be useful first to carry out a test drift to make certain of the course the ship will follow, and the buoy can then be laid accordingly. A test run is usually worth making when the tidal stream is weak or negligible and the wind is the more powerful agent in moving the ship. When the tidal stream is very strong, it will be necessary to lay the buoy further from the obstruction in order to allow time for lowering the sweep. From the sweeping board, take off the relative positions of the obstruction and buoy by bearing and distance and plot them on a large scale, *e.g.*, 400 ft. to an inch, as shown at A and B in Fig. 135.

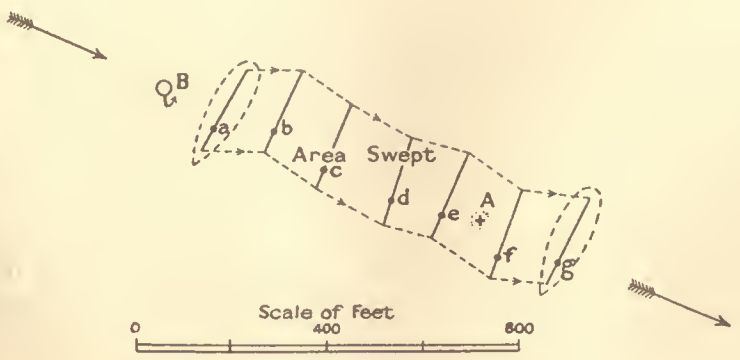


FIG. 135

The arrows show the direction in which it is anticipated the ship will drift, *i.e.*, in this case about ESE. Assuming the sweep to be rigged on the starboard side, bring the ship up to the position *a* in the figure, stopped and headed as nearly as possible SSW., *i.e.*, at right angles to the line of drift so that the sweep presents the greatest possible spread in the approach to the obstruction. The sweep may then be lowered to the required depth below datum ; in this example it would be best to set it at 8 fathoms below datum, so as to be *certain* of fouling the obstruction on the first drift. This will engender confidence that the ship has really drifted over the right place.

As soon as the sweep is set, a fix by bearing and distance of the buoy should be obtained, the direction of the ship's head noted, and the position of the sweep plotted, this being repeated at *b*, *c*, *d*, etc., as the ship drifts down. (For convenience, the sweep is shown at fixes *a* and *g* as if it hung directly under the ship.) If, however, the large-scale plot will embrace enough land stations to fix by, these should be used for fixing, but the buoy will still be found convenient as a mark showing where the ship should be hove to for starting the sweep. If it is found that the ship is not drifting directly towards the obstruction, it may be possible to rectify matters with a slight touch ahead or astern on the engines ; in the figure, for example, a touch ahead might have been given after fixes *b* and *e*. As the ship drifts down, a careful watch should be maintained on the depth lines, the moment any are seen to grow out from the ship's side, the nearest observer should inform the bridge, *e.g.*, " sweep foul between Nos. 4 and 5 depth lines," and the slip is then knocked off so that the sweep wire can unreeve itself. The information that the sweep fouled between Nos. 4 and 5 gives a valuable indication of the exact location of the obstruction with reference to the buoy, and these relative positions can be replotted more accurately for use in the subsequent drifts.

As soon as the position of the obstruction has been passed, the sweep is raised and the ship goes back to the buoy to repeat the operation. For the next drift it is best to raise the sweep a fairly large amount, say 8 ft., *i.e.*, to 6_4 below datum ; if it now passes over without fouling, lower 4 ft. and continue halving the corrections until the least water is ascertained correct to the nearest foot ; if it still fouls at 6_4, raise a further 8 ft., and so on. Sometimes, however, a good indication of the required correction will be obtained by observing the action of the sweep when it fouls ; if it grows out strongly, a large correction is probably necessary ; if, however, it grows out a little and then slips over the top of the obstruction, it is obvious that it need only be raised a foot or two to pass clear over it.

Suitable weather conditions are almost essential for drift sweeping. Most ships, when stopped, lie nearly beam on to the wind and, except in very light breezes, it is not possible to keep them headed in any other direction for long, especially with the sweep down when the engines can then only be used with great discretion. The best conditions are with a light breeze blowing in the same direction as the tidal stream is setting ; with strong winds the sweep will grow out a long way when drifting with the wind, or will grow right under the ship when drifting against it. With the wind right athwart the tidal stream little can be done, and it is best to defer sweeping until more suitable conditions prevail. Operations should be started soon after the stream begins to make, thus ensuring, as far as possible, that the work will be completed on one tide ; this will save moving the buoy and anchoring it again on the other side of the obstruction.

The sweeping book should contain a record of the fixes (including direction of ship's head) and times and notations of the place and depths at which the sweep fouls. If a recording type of echo sounding gear is fitted, it should be run whilst the drifts are in progress ; the record will show the outline of that part of the shoal or obstruction which passed under the transmitter and receiver and may even indicate the least water over it in some cases.

Visual indications, such as surface swirls and eddies, are rarely exactly over the objects which cause them ; with a strong tide, they may be 100 yds. or more downstream, so that the drift must be started a long way upstream. It is hardly ever satisfactory to work on visual indications alone ; there will always be some uncertainty as to whether the ship really drifts over the required position, and it is much better to fix frequently as the drift is in progress, either by bearing and distance of a mark buoy or, if close to the land (so that a large scale can be used) by station pointer fixes to shore objects.

Drift Sweeping in a Boat.—When manoeuvring space is restricted or when the depth of water over a shoal or obstruction is insufficient for the ship, a drift sweep can be operated from a boat in much the same way as from the ship. Surveying motor boats are only 28 ft. long, which is hardly enough to give the sweep a satisfactory spread. Two bamboos, or light spars, may be lashed in the boat so that they project 12 to 15 ft. over the bow and stern. Each bamboo should be fitted with a wire jackstay, and with an outhaul rove through a thimble or block secured at its head. The blocks which take the depth lines and sweep wire are supported on the jackstay and can be hauled into the boat or out to the head of the bamboo as desired. Leadlines supporting ½ cwt. sinkers are suitable for the depth lines ; the sweep wire (Kelvin sounding wire, cod line or spun yarn) being rove through the rings of the sinkers.

Other types of drift sweep may be improvised without much difficulty for use in boats to suit varying circumstances. When a short sweep will do, an iron bar or a weighted pole suspended below the boat may be more convenient than a wire sweep. The procedure whilst drifting is the same as that in the ship and, if boat's echo sounding gear is fitted, this should be run whilst the sweep is in progress.

Drift sweeping is often carried out in important harbours or over dredged areas where it is essential to ascertain that a certain depth can be safely carried. In these cases the procedure is the same as that already detailed, except that fixing will be by angles to shore marks and the sweep will always be lowered to some prearranged depth below datum. A series of drifts is then undertaken until the whole area has been covered.

A modification of boat drift sweeping, of great use for ensuring that the required minimum depth alongside a quay has been obtained by dredging or demolition, is carried out by rigging a boat for drift sweeping, as described above, and towing her, broadside on, parallel to the quay, by two dinghies. Or, alternatively, the sweep may be rigged between the dinghies, but in this case, a surface distance line must be secured between the boats to ensure that the sweep is kept taut below the water. (In this respect, it should be noted that if the boats close together the sweep will drop and will search at a greater depth than that shown by the depth lines ; consequently, if the objective is merely to ensure that a minimum depth exists, any error from this cause will be on the safe side though, of course, the area swept will be narrower.)

Wire Drag Sweeps

It is a drawback of the Oropesa sweep that it cannot be used for sweeping nearer than about 1½ fathoms to the bottom and many experiments have been made with other types of sweep which can be towed at slow speeds. The most ambitious of these is a wire drag sweep, the lay-out of which is shown in Fig. 136.

This sweep has been used with a spread of 12,000 ft. and can be towed at about 1½ knots through the water. In view of this slow speed it should always be worked *with* the tidal stream, should there be any. A considerable amount of special gear is essential for a sweep of this size and the principal items of the equipment are as follows :

(1) Two 50-ft. power launches to act as " guide " and " end " vessels ; one of these must be fitted with a power winch and a mast and derrick capable of lifting the heavier parts of the sweep.

(2) A smaller launch to act as a tender for setting the depth of the sweep, as the tide rises and falls, and for clearing it when it fouls a shoal or obstruction.

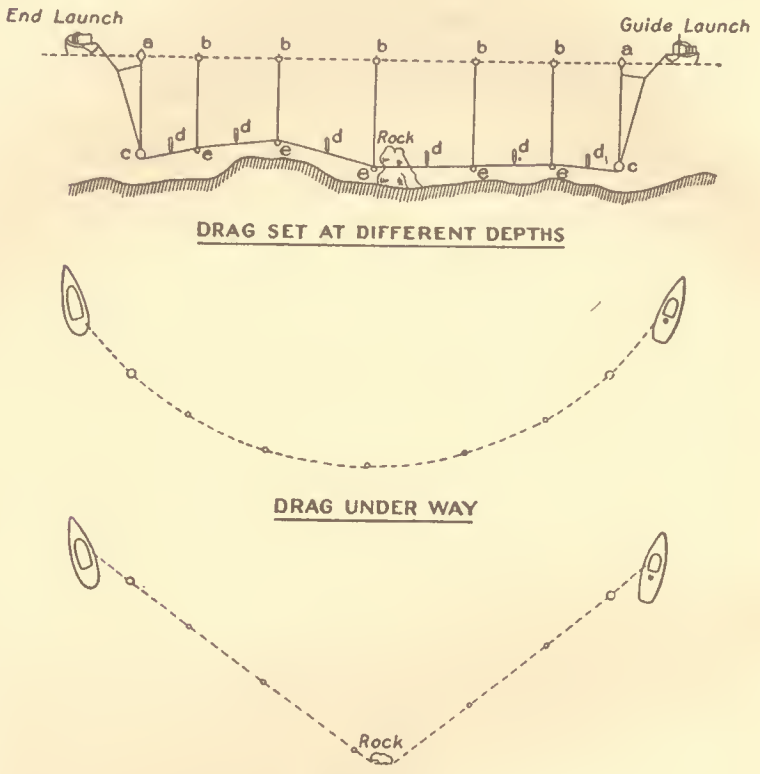


FIG. 136

(3) Two large buoys (*a*, *a*) to support the ends of the sweep, fitted with hand winches by means of which the length of the depth lines may be adjusted.

(4) Smaller buoys (*b*, *b*, etc.) similarly fitted, for supporting the sweep wire at intervals of about 100 ft.

(5) Two $1\frac{3}{4}$-cwt. sinkers (*c*, *c*) for the ends of the sweep and intermediate weights (*e*, *e*) of about 35 lbs.

(6) Metal floats (*d*, *d*) to support the wire between the buoys.

(7) Sweep wire (7-stranded $\frac{3}{8}$-in.), depth lines ($\frac{3}{16}$-in. wire) and wire and rope for towing spans. The sweep wire must be cut into 100-ft. lengths each end of which is fitted with a swivel, long link and shackle.

Management of Wire Drag Sweep.—An accurate knowledge of the general depths prevailing in the area to be swept must be known before sweeping starts and the tidal movement must be predicted. All the gear is stowed in the guide launch and the sweep is streamed under way, the end launch towing it out and buoys, floats and sinkers being attached as it is paid out. When the sweep is set, the launches proceed, each heading 10° to 20° outward from the line of advance. The guide launch follows the proposed track as closely as possible, fixing and plotting her position at frequent intervals, and the end

launch concentrates on preserving the spread of the sweep and keeping on a constant line of bearing from the guide. When the sweep fouls anything, an extra strain on the tow will be apparent and the buoys in the vicinity of the foul will capsize. It will then probably be necessary to anchor the guide and end launches temporarily by the stern, whilst the attendant launch lifts the sweep wire clear by means of the nearest depth lines. The obstruction or shoal should be buoyed as soon as the foul occurs and the least water over it can be obtained afterwards by drift sweeping or close sounding in the ordinary way.

CHAPTER XII

COASTLINING

Accurate delineation of the coastline is an essential feature of marine surveys and, although this part of the hydrographic surveyor's work might be regarded merely as one of the many topographical details which go to make up the completed chart, its importance is considered to justify a separate chapter being devoted to the subject.

The mariner is frequently under the necessity of fixing himself by bearings and angles to promontories and similar features of the coastline, and it is therefore imperative that every small sinuosity should be faithfully represented and that both high and low-water lines with the nature of the intervening foreshore should be clearly distinguished by the appropriate symbols; this will also assist the mariner in recognising the coast and in selecting the best landing places, etc. In land surveys, which include portions of the coast, the coastline is sometimes treated in rather a perfunctory manner. For this reason, and also because the coastline in many parts of the world frequently changes its position and shape, either by accretion or erosion, the hydrographic surveyor can scarcely ever justify the practice of reproducing it from land maps in the way he might employ with other topographical details, such as hill contours.

Marking the Coast.—The location of the stations and marks of a marine survey is chiefly considered from the point of view of their convenience for fixing the soundings, but some regard must also be had for the topographer and coastliner in *their* work. Soundings *must* be fixed by sextant and station pointer and, although other ways of fixing are available on land, the work of the coastliner will be much facilitated if he can be provided with a sufficient number of fixed marks to check his position from time to time by that method. When the shore is fringed with trees and there are no off-lying islets, it may be impossible to provide marks suitable for station pointer fixes, and in such cases the best that can be done is to place them in such positions that they see along the coast as far as possible in each direction, adjacent ones being intervisible when this can be arranged.

The marks provide the control by which the coastliner works, and the density of this control should depend chiefly on the scale of the survey. For example, on a scale of 1/50,000, marks spaced at intervals of a mile or so (*i.e.*, 1½ ins. on the paper) along the coast should generally suffice, whilst on a scale of 1/12,500 marks at intervals of ⅓ to ½ a mile (*i.e.*, 2 to 3 ins. on the paper) may be required. These figures should be regarded only as a rough guide; if station pointer fixes can be obtained here and there between adjacent marks, they could be spaced at wider intervals, whereas under other conditions, and particularly if the configuration of the coastline is very involved, marks may have to be spaced more closely.

Field Boards and Instruments for Coastlining.—Separate field boards should be prepared for plotting the coastline and topography; those used for sounding become soiled in use and will probably be required for that purpose at the same time as coastlining is in progress. Whatman boards, being light to carry, may often be used with advantage; their surface is particularly suitable for plotting fine and intricate work. Coastline boards should have the main and secondary stations and sounding marks pricked through on to them, and it is useful to plot also any rays previously obtained to topographical features, *e.g.*, during the triangulation. The boards should have one or more magnetic meridians drawn on them and scales of miles, cables and yards.

The instruments required for coastlining are as follows :

(*a*) A level (fitted with a horizontal circle) and levelling staff,
or, a theodolite and levelling staff,
or, a sextant and 10-ft. pole or portable range-finder.

(*b*) Linen tape measure.

(*c*) Pocket compass.

(*d*) Station Pointers (including Cust's).

(*e*) Dividers, spring bows, protractors, pencils, watch, field book, etc.

Whenever possible, a dinghy or whaler should accompany the coastlining party. It is useful for carrying any gear not actually in use at the time and may be required for crossing rivers, negotiating awkward parts of the coast, or for visiting off-lying rocks, banks, etc., from which it is often convenient to observe.

METHODS OF COASTLINING.—Delineation of the *high* water line is the principal business of the coastliner, but he must also record the character of the shore, both immediately above and below high water, and should include any information regarding coastal features, which he can conveniently obtain in the course of his work. For example, tangent shots should be observed to off-lying islands and rocks, especially those which dry, these should be fixed by angles from the coastal marks and their heights computed from vertical angles to them ; also the heights of cliffs, buildings, etc., should be measured with a tape or deduced from vertical angles taken, if necessary, from a boat anchored offshore. The coastliner should also be responsible for obtaining full descriptions of all navigational aids and any information of this type required for inclusion in the Sailing Directions. The extent to which the coastliner should concern himself with topographical detail in general must depend on the time available and on individual judgment, but he should usually be regarded as responsible for fixing houses, roads, railways, etc., when they lie close to the shore and he can often usefully include angles to hill summits and other features which lie farther inland.

Whenever possible, the whole coastline should be walked over ; if this is not done, important features, such as small rivers, may escape observation unless good air photographs of the coast are available.

Methods of coastlining :

(*a*) **Station Pointer Fixes.**—In some circumstances it is possible to get good station pointer fixes at almost any point on the coast. The observer occupies a series of stations on the high-water line and at each obtains sufficient angles to the fixed marks of the survey (with a sextant, theodolite or level) to fix himself and to determine the trend of the coast on towards the next station and back towards the previously occupied one. If the stations are sufficiently close, the coastline can be drawn in, when they have been plotted, with all required precision. This method is chiefly applicable to work on small scales and where the coastline is not too intricate. When the configuration is irregular, it is usually more convenient to combine it with the methods described below in (*b*).

(*b*) **Traverse.**—A much used method of fixing the coastline is by means of traverses between adjacent surveying marks. Starting the traverse from some known position, *e.g.*, a sounding mark on or close to the coast, a sufficient number of points are fixed along the high-water line to enable it to be drawn in accurately. Each leg of the traverse is limited to a length of about 300 yards, *i.e.*, as much as can be measured accurately by tachymeter or sextant and 10-ft. pole (*vide* Chapter VI), and observations are made at each end and each turning

point of the traverse to as many intermediate points as may be necessary for plotting the high-water line. The distance apart of the intermediate points must depend on the scale of the survey and on the configuration of the coast. If the latter is involved, it may be necessary to fix a series of points not more than $\frac{1}{4}$-in. apart on the paper so that it can be accurately drawn ; on a scale of 1/6,250 this would correspond to a distance of about 40 yds., whilst on a scale of 1/50,000 it would be about 300 yds. On a straightforward coastline, however, the points could be much more widely spaced.

Various methods of running traverses have been described in Chapter VI, and these are applicable to coastlining, with the following important modification. In coastlining it is not usual to calculate the co-ordinates of the turning points and adjust the traverse mathematically between the terminal points. The positions of the turning points are plotted direct on the field board by protracting the bearing and length of each leg in succession, starting from the initial station, and the accuracy of the work is established by the close of the plot on the final station. This practice can be accepted on the understanding that the coastliner will *never* be required to run traverses of any great length without independent checks at intervals on his position, *e.g.*, by station pointer fix. Unless such checks are available, a sufficient number of fixed coastal marks should be provided to keep the total length of any coastline traverse between the marks within a limit of 6 in. or so, as measured on the paper, so that with reasonable care no appreciable error should accumulate in the plotting.

The angular measurements to determine the directions of the traverse legs and of the intermediate stations from the turning points may be made with a theodolite, a level (fitted with a horizontal circle) or a sextant. In most of the work it is only necessary to protract short rays on the paper so that angles need only be measured with an accuracy of, say, 10′ of arc. This, however, does not apply to angles observed for station pointer fixes used for checking the work, as these may often have to be taken to distant objects. On small scales (and even on large scales for short rays) a pocket compass will often give the required direction with the necessary precision ; for example, on a scale of 1/50,000 an error in bearing of 1° in a ray a mile long would transpose the terminal point only ·025 in. when plotted on the paper.

On scales of 1/12,500 (5·8 in. to a nautical mile) and larger, it is best to measure the distances between the turning points of the traverse by tachymeter (see Chapter VI) as this is a rather more accurate method than the sextant and 10-ft. pole, or a portable range-finder. On smaller scales, either of the latter methods will do well enough. Direct measurements with a tape are often necessary in surveys of harbour basins and large-scale work of that description.

Example of coastlining by the traverse method :

In a survey on a scale of 1 : 12,500, it is required to fix the coastline between two sounding marks " Nut " and " Oak " ; as shown in Fig. 137.

As the scale is fairly large the observer is provided with a levelling staff and a level fitted with a horizontal circle, for obtaining angles and direction, and distances by tachymetry. A small range-finder is also taken. His field board has marked on it " Nut " and " Oak " and also " D," one of the main triangulation stations situated in the vicinity.

Landing at " Nut," the level is set up over the mark and the observer first looks along the coast and obtains angles to any prominent objects on or near it, using " Oak " or " D " as his zero, or another fixed point if neither of these is visible. In this instance, he sees an islet *p*, and obtains tangents and an

elevation to it ; a rock awash *q*, the end of a pier *d*, and a summit *s* are also shot up. A man with the levelling staff is then directed to proceed along the high-water line and stop at every point where the coastline makes an appreciable

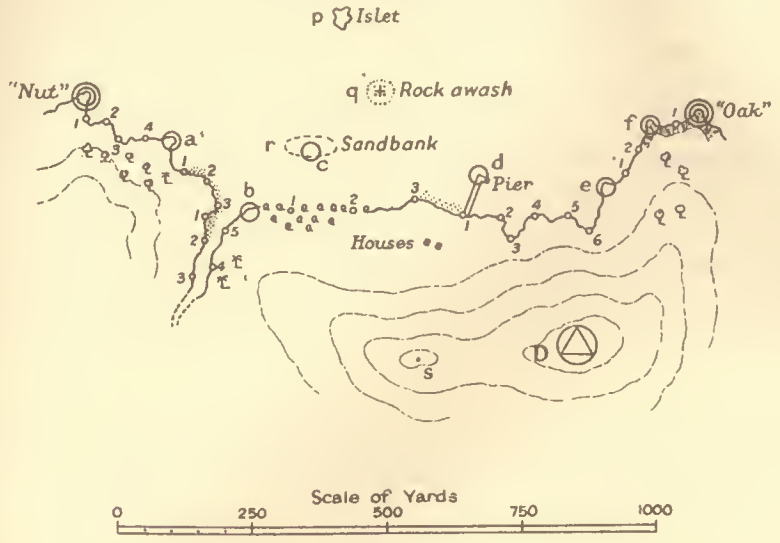


FIG. 137

change of direction. At each of these points, marked 1, 2, 3 and 4 in the figure, an angle and distance are observed to the staff. On arrival at *a*, the staff man can go no farther without disappearing from sight. So an angle and distance are obtained to this point, which becomes the first turning point of the traverse, and the staff man remains there to mark the position until the observer brings his instruments along. At *a*, a similar procedure is carried out, further shots being obtained to *p* and *q*, and the next intermediate stations numbered 1, 2 and 3. A river is found to lie beyond No. 3 and as most of it is out of sight from *a*, a second turning point is established at *b* where angles and distances to the intermediate points 1, 2, 3, 4 and 5 on the river banks are obtained. Beyond *b*, the coastline is found to consist of mangroves and cannot be walked over. The observer, however, notes an off-lying sandbank *r*, sends the staff man to it and returns to *a* where he obtains an angle and distance to the man at *c*. As *b* cannot be used for continuing the traverse eastward, *c* will be made the second turning point of the traverse and *b* will be regarded as an off-shoot from the main traverse for fixing the river entrance. At *c* angles and distances by range-finder are obtained to the points 1 and 2 on the mangrove shore line and a check angle is obtained into *b*. The staff man then lands at the point No. 3 close west to the pier and then goes to *d* on the pier head, this becoming the third turning point of the traverse. The observer follows him to *d* where a similar procedure is enacted ; and also at *e* and *f* for fixing the rest of this part of the coast ; angles are obtained to *p*, *q* and *s*, and other prominent objects, from each turning point. The observer " zeros " his angles on one of the fixed marks of the survey if possible (when practicable, a fairly distant one, for ease in plotting later (see below)), but if none can be seen, he must leave a mark at the last turning point and zero on that. As he progresses he notes in his field book the nature of the foreshore and the country immediately inland and measures, as far as the state of the tide permits him, the extent of drying ledges and banks by sending the staff man out to their limits ; also no opportunity is lost of obtaining check angles to any points which have been or will be occupied so that the accuracy of the work can be tested.

For small scale work (about 1 : 30,000 or smaller) a method of traversing the

coastline, which has been found both quick and reliable, is by sextant and a 300-ft. wire. Two sailors, with a length of Kelvin sounding (or similar) wire, 300 ft. long and marked every 50 ft. with bunting, measure the lengths of the traverse legs, working ahead of the officer and keeping their own line by walking past each other alternately. (The " front " man remains still until the " rear " man has walked ahead to the full extent of the wire and taken his line by a transit on the old " front " man and the terminal mark.) The end of each 300 ft. measured is marked by a small pile of stones or sticks. The legs are made as long as possible consistent with keeping within 100 yards of the coast ; when it is necessary to change direction the " wire " men wait until the officer has caught up. The direction of each leg is measured by sextant by the officer who follows behind ; large angles at the turning points are measured in two parts making use of some convenient distant mark, or a pole stuck in the ground, as a middle object. Offsets to the H.W. line and other detail are paced out, at right angles to the leg, at every 300-ft. mark, or as often as necessary. Records can conveniently be kept on a rough sketch in the field book, as shown in Fig. 137a.

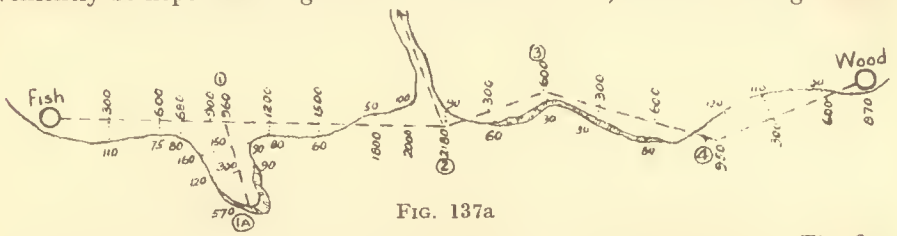


Fig. 137a

The traverses can be adjusted graphically as shown in Fig. 137b. The firm line shows the traverse as first plotted, terminating at W′ instead of " Wood." W′ is joined to " Wood " by a line, and short lines parallel to this are drawn through each turning point. The lengths of these short lines are made proportional to their distance from the terminal point. The pecked line joining their lower ends is the adjusted traverse.

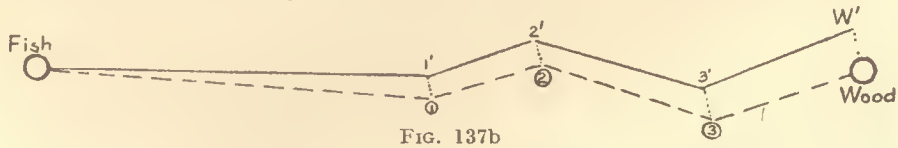


Fig. 137b

(*c*) **Coasting.**—It is necessary to resort to a variety of expedients when the coastline is of such character that it cannot be walked over. In these circumstances, it is often possible to " coast " along the shore in a boat and to obtain a series of station pointer fixes close inshore ; from these the shape of the coast can be plotted with all required accuracy. Alternatively, though it may not be possible to fix close inshore, a series of stations can be made in a boat at distances of 200–300 yds. from the land and at each of these sextant angles and range-finder distances will be observed to all recognisable features.

(*d*) **Shooting up the Coastline from Boats.**—When the shore cannot be approached owing to breakers, the surveyor must be content to shoot in as many points as he can identify on the high-water line from boat stations, at least three rays being obtained to every point. The results are not so satisfactory as those obtained by walking over the coastline, but on small scales, where minor sinuosities are not appreciable in the plot, a reasonably accurate representation can be attained. Often the necessary angles can be obtained whilst sounding, by stopping the boat temporarily at the inshore end of each line of soundings.

Plotting the Coastline.—Unless weather conditions are bad, it is advisable to plot the results of most of the observations in the field, finishing off each small section, *e.g.*, between adjacent sounding marks as soon as the observations are

complete. By so doing, the observer is less likely to make mistakes and can fill in the detail whilst it is still fresh in his mind. There is the further advantage that, if errors of measurement have been made, they will be discovered at the time and the observer can retrace his steps and reobserve the angles and distances without much loss of time. Also, if any essential observations have been omitted, they will be discovered at once and can be made. On intricate parts of the coast, it is often helpful to draw a rough diagram on a large scale in the field book, and this can be consulted when the points have been plotted, and used for filling in the more involved features.

In the example (Fig. 137) the plot is started at " Nut " and the first stage in the work is to plot the turning points of the traverse in order, *i.e.*, *a* is plotted by protraction of an angle and distance from " Nut," *b* and *c* from *a* with a check angle into *b* from *c*, *d* from *c*, and so on. At *f*, an angle and distance to " Oak " will have been measured and these should agree precisely with the angle and distance taken off the board. If they do not, it is evident that some mistake in observation or booking has occurred and the previous work must be checked. It is clear that the plot must be made with great care if agreement is to be obtained, since any errors are cumulative. The board must be provided with a scale on which distances (on the paper) can be taken off with an accuracy of 1/100th of an inch and set on the springbows or dividers. As the distances are short—only an inch or so on the paper—the angles can be laid off with an 8-inch or 12-inch protractor with sufficient accuracy, but care must be taken to align the protractor exactly on the zero object. In this connection it should be noted that accurate protraction is easier from a long zero line and the coastliner should therefore endeavour to measure his angles from a reasonably distant object.

When the turning points of the traverse have been laid down and a satisfactory closure obtained on the fixed terminal station, the intermediate points and the rays to other objects can be plotted from them. If some of the latter lie outside the limits of a protractor, a station pointer may have to be used. The high-water line should then be drawn in with a hard pencil and symbols to indicate the character of the foreshore added, the work being inked in on return to the ship.

Coastline Records.—It is unnecessary to lay down any stereotyped method of recording the observations taken in coastlining, but the rule must be that *all* the observations used for plotting the work should be entered in the field book, so that a comprehensive record is available and could, if necessary, be used for a re-plot at a later date. Assuming, as in the example, that an observer is working along the coast from " Nut " to " Oak," using a level and staff, the observations might be recorded as follows :

At *Nut*— : 10 yards within H.W. line : time 1030.

To	Zero.	L. or R.	Angle	Distance	Remarks.
			° ′	Yds.	
Nut 1	Oak ⊙	R	85 30	40	H.W. line : rock as far
Nut 2	,,	R	49 00	65	as *a* : rocky ledges
Nut 3	,,	R	48 30	102	extending 25 yds.
Nut 4	,,	R	33 10	142	between Nut 1 and
L.T. Islet } (*p*) ..	,,	L	20 50		Nut 3 : drying sand
R.T. Islet } (*p*) ..	,,	L	17 10		below this. Small
Rock awash ..	,,	L	3 05		trees and bushes
Pier head (*d*) ..	" D "	L	15 50		above H.W. line.
Hill summit (*s*)	" D "	R	11 20		
T.P. (*a*)	Oak ⊙	R	27 05	183	
At *T.P. a* : on H.W. line : time 1100.					
a 1	" D "	R	39 10	62	Sandy shore.
etc.	etc.	etc.	etc.	etc.	etc.

Heights of Coastal Features.—The simplest method of measuring heights is usually by means of vertical sextant angles from a boat anchored and fixed a little distance off shore. The height can then readily be calculated by the formula :

$$\text{Height in feet} = \frac{\text{Angle in seconds} \times \text{distance in nautical miles.}}{34}$$

The vertical angles should be measured above the high-water line (the height datum) if this can be identified from the boat ; if not, they can be measured from the water level at the moment and subsequently corrected when the tidal observations are received. Other methods of determining heights will be found in Chapter VII ; of these the aneroid is probably the most useful for coastlining purposes. (When inking in cliffs it is usually necessary to exaggerate them to some extent so that they will show clearly ; the *base* of the cliff must be correctly charted, but the top can be drawn far enough back to allow the distinctive penwork to be inserted.)

Use of Maps and Air Photographs.—The method of using land maps and air photographs for the purposes of topography is discussed in Chapters XIII and XIV. Though the use of land maps as a basis for reproducing the coastline cannot often be justified, this does not apply to the less important features such as narrow unnavigable waterways, the limits of swamps, etc. : these add to the completeness of the chart but, being of no great importance to the mariner, meticulous accuracy is not essential.

Air photographs, provided they are of recent date, are often of considerable assistance in filling in coastline detail, particularly when an overlapping vertical series is taken along the line of the coast. From the hydrographic surveyor's point of view, however, there are generally difficulties in making fully effective use of such photographs for coastlining purposes, for

(*a*) Each photograph only shows the water's edge at the moment of its exposure ; on shelving beaches this may be some distance from the high-water line, and it may not be possible to identify the position of the latter.

(*b*) The photographs give little indication of the character of the foreshore and should therefore be supplemented by an inspection on the ground.

(*c*) It is probable that existing air photography of the area being surveyed will not be suitable since the " principal points " of such photographs will often fall in the sea and unless ground control is dense and readily identifiable on each photograph there is no satisfactory method of fitting consecutive photographs together.

For these reasons it is best to regard *random* air photography as a valuable *addition* to the normal methods of coastlining and to employ it chiefly for filling in the smaller and less important detail. It may, for instance, be particularly useful for plotting such work as the following :

Outline of small islands.

Shape and extent of drying reefs.

Irregular sections of the coastline between fixed sounding marks.

Narrow waterways.

Mangrove-fringed coast which is difficult to fix from the ground in the ordinary way.

CHAPTER XIII
TOPOGRAPHY

Requirements of a Hydrographic Survey

From a cartographic standpoint, the term " topography " implies the fixing and delineation of the land features. A chart is a guide over the sea, and in consequence, soundings, which indicate the form of the sea bed, are the most important part of it ; nevertheless, in coastal waters the mariner is chiefly guided by the *land* and a chart must therefore represent the visible land features in such fashion that they will be readily recognisable and will enable him to fix the position of his ship in relation to them. The amount of topography needed in a hydrographic survey depends on circumstances, but as a general rule the aim should be to show all the land features which are visible from the sea, and as much additional detail between the most distant features and the coast as time allows, in order to give completeness to the picture. It is obvious that on large-scale charts distant hill summits, etc., will be outside the borders of the sheet and cannot be shown, whilst on small scales it may be necessary to leave between such summits and the coast a considerable amount of " dead ground " which may be described in some general manner, *e.g.*, " undulating country, densely wooded."

In hydrographic surveys the governing factor in topographical work is usually the time available. Anything very elaborate, involving the detention of the surveying unit for a long period after the sounding and coastline are completed, cannot often be justified. Assuming, however, that sufficient staff is available to undertake topography simultaneously with sounding, there will generally be time, before the latter is completed, to produce a satisfactory map of the principal land features consisting of natural objects, such as hill summits, rivers, lakes, marshes, woods, etc., and artificial objects such as buildings, towns, flagstaffs, roads, railways, etc. In addition to these, contours or form lines are required to indicate the height and shape of the ground with a reasonable degree of accuracy, though not necessarily with the precision found in a good land survey. From the hydrographic surveyor's point of view it is fortunate that where the topography is complicated by the existence of towns, railways, etc., the results of intensive civilisation, and where the work would consequently be laborious, there usually exist good large-scale maps from which the detail can be transferred to the chart with all required accuracy.

The representation of the land features is effected by the use of recognised symbols, and in the employment of these a certain latitude must be allowed to the draughtsman in order that essential detail may be clearly shown. This is particularly the case on small scales where isolated houses, roads, bridges, etc., cannot be drawn true to scale without making the symbols very minute, and it is permissible to exaggerate their size to some extent in order that they may be easily seen on the map. Similarly, in the case of towns, it is impossible to show all the individual streets clearly on any scale smaller than about 1/12,500, and it is usual to block in areas between the main streets, or even whole towns, so that only the more important and conspicuous buildings are clearly defined, the limits to the other houses being merely approximate.

Use of Existing Maps.—The hydrographic surveyor should always endeavour to obtain copies of any land maps covering the coast of the survey. Such maps will usually show much more topographical detail than could normally be fixed during the progress of the hydrographic work and their use will be a great saving of labour. Some discretion, however, must always be exercised in their selection and employment, particularly if it is necessary to square *up*, *i.e.*,

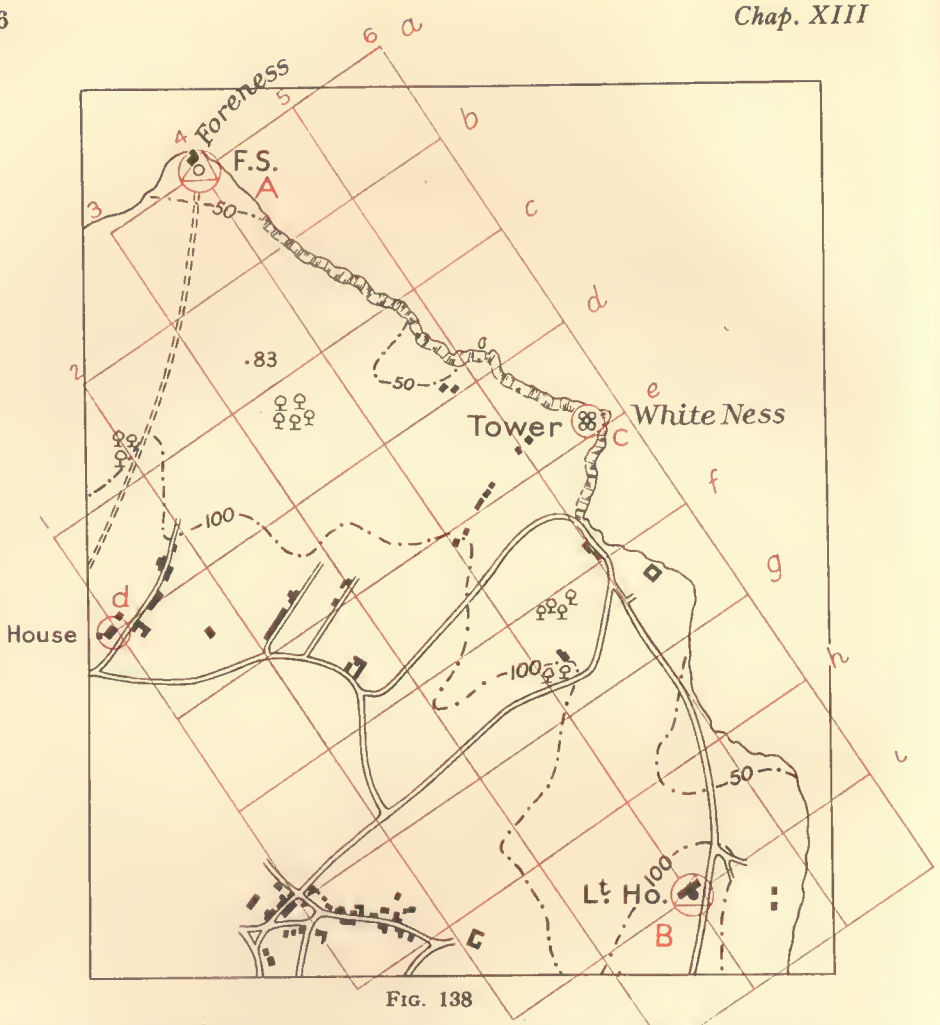


FIG. 138

increase the scale of a map. Surveys executed under government auspices can generally be relied on to be sufficiently accurate for hydrographic purposes, but the same cannot always be said for other types. In any case it is desirable to fix during the progress of the hydrographic work a few points appearing on the land map which it is proposed to use for the topography. The angles and distances between these points can be checked on the map against those on the plotting sheet, and a very good idea of the accuracy of the land survey can then be formed. It is also necessary to inspect on the ground, as far as this can be done, the features appearing on the map in order to ensure that, where changes have occurred, out-of-date material is not reproduced on the chart.

The usual method of transferring topography from a map to a chart is by the process of squaring up or down. Two points shown on the map and situated near opposite ends of the survey area are fixed in the hydrographic survey and are joined on the map and on the field board by a straight line which is common to both surveys. This line is used as a " base " for the squaring up or down process and the relative scales of the two surveys are found by a comparison of its length on the map and field board. It is essential to make certain that the selected points on the map have been correctly identified when they are fixed in the hydrographic survey. The base lines are divided into any convenient number of equal parts (the same number on each) in such a way that on the *smaller* scale the divisions are not more than $\frac{1}{4}$ in. apart. From the divisions,

systems of corresponding squares are drawn on either side of the base lines extending sufficiently far to cover the area. In addition to the base line terminals it is advisable to fix on the ground one or two other points appearing on the map and the position of these in their respective squares should be checked to ensure that there is absolute agreement. Example :

Fig. 138 is a portion of a map on a scale of 1/21,800, which it is required to square down to 1/50,000, the scale of the hydrographic survey shown in Fig. 139. A and B, a flagstaff and a lighthouse, trigonometrical points of the hydrographic

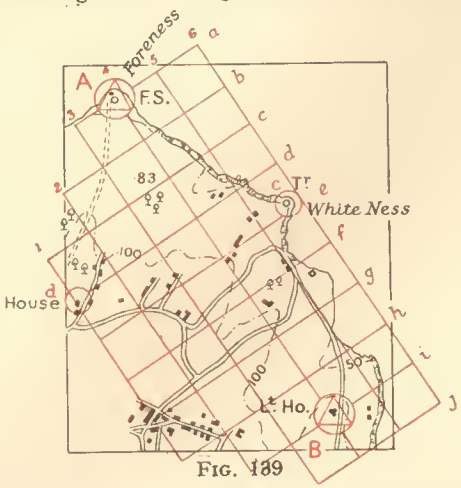


FIG. 139

survey, lie near the limit of the area and can be identified on the map and the line AB is therefore used as the " base." This is divided up as shown in the figures and a system of corresponding squares is drawn to cover the area ; it is convenient to number and letter the lines or squares 1, 2, 3, and *a*, *b*, *c*, etc., to prevent mistakes in transferring the detail. Great care must be taken in the construction of the squares ; any error in drawing them will result in inaccuracy when the detail is transferred. In this case the positions of two sounding marks *c* and *d*, plotted on the field board, are also identified on the map and it will be noted that, when the squares are drawn in Fig. 139 the marks are found to occupy the same positions as they would if transferred from the map, thus furnishing a satisfactory demonstration of its accuracy and a proof that the points A and B have been correctly identified. The topography, consisting in this example of roads, houses, contours, trees, etc., is transferred to the field board by means of a pair of proportional dividers adjusted to the ratio of the two scales, the detail appearing in each square on the original being transferred by measurements on the dividers from the sides and corners of the square in which it lies, to its position in the corresponding square on the copy. In making the copy it is necessary to omit many of the less important topographical details which cannot well be shown on the reduced scale and some features, *e.g.*, the width of roads and cliffs, must be exaggerated.

The process of squaring *up* is done in a similar manner to squaring down, but any error in the original is *increased*, so that the results are not always satisfactory and careful consideration must be given to the question of whether they will fulfil the requirements of accuracy.

Enlarging and reducing may be done more easily and quickly with an eidograph or pantograph if available. A description of these instruments is to be found in most books on surveying instruments and in makers' catalogues. Their operation presents no difficulty and the only adjustment required is that of setting the scale-bars for the required ratio of the scales of the original and copy. The ordinary photographic processes of enlarging and reducing are, of course, the most convenient when the necessary apparatus is available.

Topographical Work in the Field

Methods Used in Hydrographic Surveys.—The methods adopted for filling in the topographical detail of a hydrographic survey must be varied to suit the character of the land features, the scale of the survey and the time available. The more important hill summits and other conspicuous objects should be fixed and should have their heights determined by theodolite angles from the main and secondary stations and sounding marks. When these points have been plotted, they provide a framework of control in the horizontal and vertical planes and are of great assistance in interpolating other detail which is usually of less importance on the chart. A separate field board should be prepared for the topographical work and should show the positions and heights of all the fixed stations in the survey and of any other points to which observations have been obtained ; the coastline should also be shown, as a knowledge of its position and shape is usually a valuable guide to the form of the hill features in its vicinity.

The normal procedure in topographical work is for an observer to examine as much as possible of the required area by occupying a number of points on the ground where he can fix his position in relation to the marks of the survey and can obtain a good view of the surrounding country. At each fixed point the observer determines the height and observes rays, elevations and depressions to any other prominent and easily recognisable features. With the aid of a theodolite, clinometer or telescopic alidade, the heights of any intersected points can be found and will provide data for sketching in the contours. It is usually best to work along the highest part of each ridge of hills and on the spurs which extend from them, occupying any marked eminence and from there observing rays to other summits and to any recognisable features at the lowest points of the neighbouring valleys. With a few additional observations to marks at intermediate levels, *e.g.*, conspicuous trees, houses, outcrops of rock, etc., there should then be enough data to draw in the contours with sufficient accuracy for the purposes of a chart.

The observer may fix his position by sextant or theodolite angles or with a plane table, whilst on small scales a compass will often do well enough. He can determine his height at each occupied point by vertical angles, with a theodolite or telescopic alidade, to other points of known position and height, or with an aneroid. No hard-and-fast rule can be laid down as to the best type of instrument to use in topography. In wooded country it will be impossible to occupy many points on land that command a good view, and it may be necessary to plot most of the land features from an analysis of a series of views taken from ship or boat stations off shore. For the mapping on large scales of towns and villages and other involved topographical features, it is usually necessary to resort to traverse methods and a level fitted with a horizontal circle or a theodolite may be the most convenient instruments to employ. The land surveyor, however, does the greater part of his topographical work with a plane table and this is undoubtedly the best instrument to use in most cases.

Plane Table.—This instrument consists of a board mounted on tripod legs in such a way that it can be adjusted to the horizontal and turned in azimuth, and an alidade for observing rays to the various objects it is required to fix. The positions of the fixed marks of the survey, magnetic meridians and scales of miles and cables and feet should be transferred from the plotting sheet to the plane table which is mounted with paper in the same manner as a field board.

Plate O shows a plane table and a telescopic alidade. The various parts are :

A, telescope levels.
B, levels for adjusting the board to horizontality.
C, parallel bar.
D, compass.

The board is levelled by levelling screws on the tripod head. A clamp and slow-motion screw are provided for orienting the board. The telescope pedestal

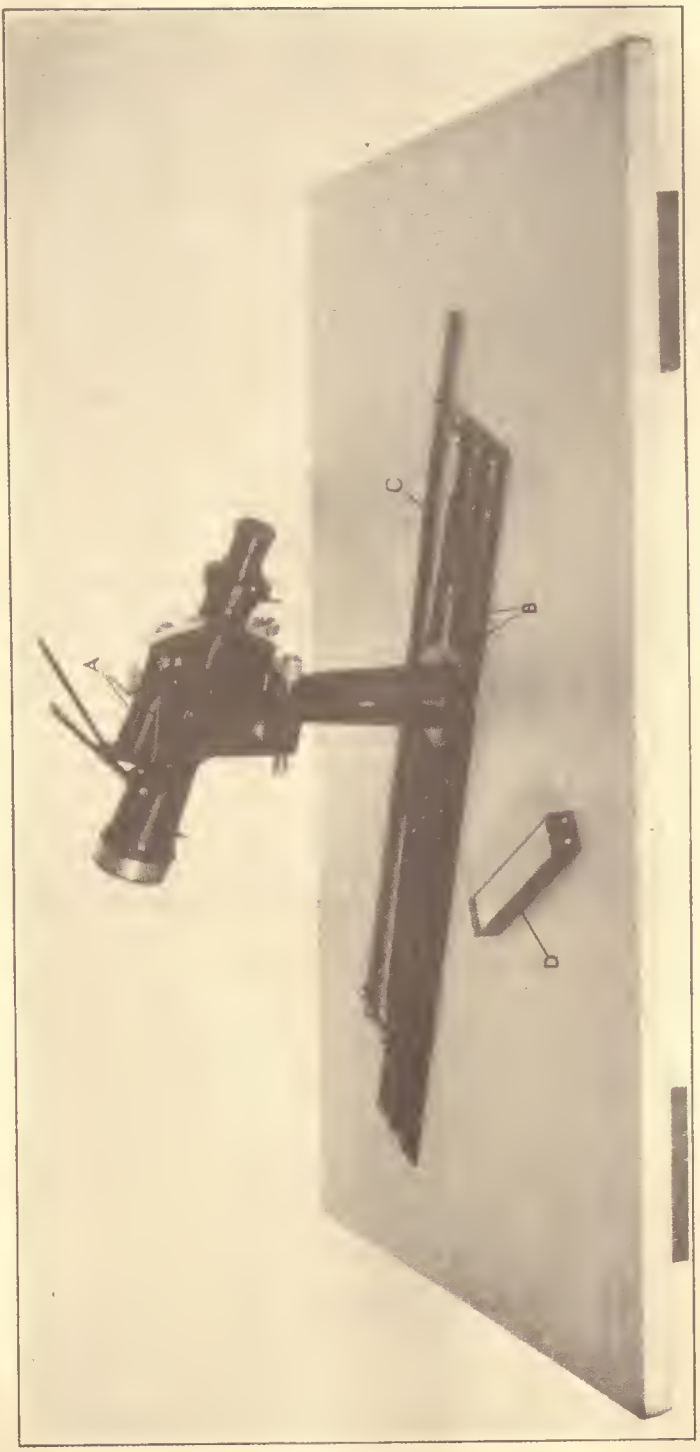


PLATE O

carries a vertical arc, reading to minutes, and sensitive levels. The telescope is fitted with stadia wires for tachymetric measurements.

Many simpler types of plane table are made. In these an open-sight alidade is used and the board is generally secured to the tripod head by a ball-and-socket arrangement which permits adjustment in tilt and azimuth; a loose spirit level placed on the board enables it to be adjusted into the horizontal plane. Such a plane table, fitted with an open-sight alidade, is much lighter than that illustrated in plate N and serves for many purposes just as well. The drawback to it is that it may be difficult to intersect a distant object accurately with the open sight and, as heights by vertical angles and distances cannot be measured unless a clinometer is provided, some auxiliary equipment, *e.g.*, an aneroid and range-finder, must be carried when such measurements are required.

Setting Up and Using the Plane Table.—If the instrument is to be set up at some previously fixed point, the procedure is very simple. Having levelled the board, the alidade ruling edge or parallel bar is aligned on the prick holes marking the observer's position and some other distant mark in the survey. The board is then turned in azimuth until the distant station is intersected with the alidade sights and the table is then correctly oriented and can be clamped in that position. To prevent any possible mistake in identification, the alidade should be directed on another fixed point and its ruling edge should be parallel to the line joining the prick holes representing the observer's position and this second object.

When the instrument is set up at some as yet unknown position, it is necessary to fix by the method of graphic resection. As with the sextant and station pointer, three marks must be selected which will give a good fix and the observer's position is determined from the directions and intersection of the rays to them. Suppose, for example, three stations A, B and C (Fig. 140) are visible and are suitably disposed to afford a good fix.

Having set up and levelled the table, place the compass trough along a magnetic meridian (which should have been previously drawn), turn the board until the needle is central and then clamp the board. Intersect station A with the alidade sight, and aligning the ruling edge or parallel bar on the prick hole A on the board, draw in the ray A*a*. Similarly with the stations B and C, draw in the rays B*b* and C*c*. The result will usually be a small triangle of error, as shown in the figure. The position of the observer must be a point

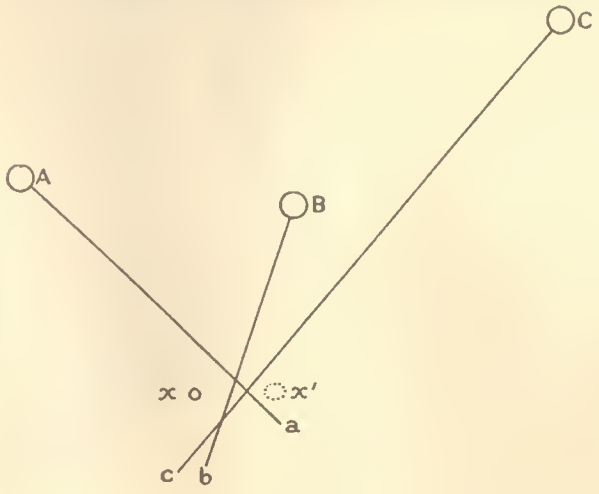


FIG. 140

where the three rays would meet if they were all turned through *equal angles* in the *same* direction, clockwise or anti-clockwise, the distant station being

regarded as the centre of the clock in each case. It will also be noted that the distance of this position from each ray is proportional to the observer's distance from each station. The true position can be interpolated fairly accurately by eye at the point x (if the triangle of error is not too large), and is the *only* one which fulfils the above requirements. At first sight it might be thought that the observer's position could be in the space marked x', since all the rays might be swung anti-clockwise in that direction, but a moment's inspection will show that no position in this space can be at a distance from the rays proportional to that of the observer from the stations. It is convenient to remember that, if the observer is situated *outside* the triangle formed on the earth by the three distant stations, his position is *outside* the triangle of error, and *vice versa.*

Having obtained a fairly accurate position for x by this means, align the alidade edge on prick holes marking this position and the most distant station, in this case C, and turn the table in azimuth until C is intersected with the alidade sights, clamp up and draw in the ray Cx. Intersect A and B successively with the sights, at the same time aligning the alidade edge on the prick holes marking those stations, and draw in these two rays. If the previous interpolation has been good, the three rays will now meet in the point x; if not, there will be another triangle of error, but this will probably be of such small proportions that a final interpolation can be made by eye with all required accuracy and a small further alteration in azimuth will orient the board correctly. As a final check, align the alidade edge through the accepted position and the prick-hole marking a fourth station and see that the latter is then intersected with the sights. When only two distant stations are visible, it is necessary to rely on the compass for correct orientation and fixing. This method is not so satisfactory, but can be used when the rays are short, *i.e.*, are represented by measurements of only a few inches on the paper. When the observer's position has been determined and marked on the board, rays to any other points may be drawn in with a chisel-edged pencil by intersecting the distant objects with the alidade sights and at the same time aligning the alidade edge or parallel bar through the observer's position.

The chief advantage of using a plane table in preference to a theodolite for topographical work is in the saving of clerical work. Directions or rays are drawn as soon as they are observed and there is no need to observe and write down a large number of horizontal angles and plot them with a different instrument.

Suppose it is require to sketch in the topographical detail shown in Fig. 141 with the aid of a plane table and an aneroid.

Landing near F, the aneroid reading is checked at sea level and the observer proceeds to the point a. Here he obtains his own height with the aneroid, sets up the table and resects from A, C and B, with a check ray to F. Rays are observed to a small waterfall a' on the stream south-westward of him and to any other conspicuous and easily recognisable marks, and the contours in the immediate vicinity are sketched in. Proceeding on to b, on the shoulder of the hill, he sets up and fixes himself and carries out the same procedure. If the instrument has a telescopic alidade the height of b can be determined by a depression to the water-line of the islet B (*see* Chapter VII). Similarly at the other points c, d, e, etc., the track followed being shown with a pecked line. " e," it will be noticed, is the previously fixed point C, so at that point no resection of position will be necessary and if, as is probable, its height has already been determined, it will be possible to check the aneroid. Rays to intermediate points such as the houses c', a conspicuous rock d', the tree clumps b' and e' are of great assistance to the observer in drawing in the contours correctly. On reaching sea level the aneroid is again checked, and on return to the ship a curve should be drawn of the mercurial barometer readings during the day, so that the atmospheric pressure changes can be applied as corrections to the observed aneroid heights. (See also Chap. VII, pp. 153 and 154.)

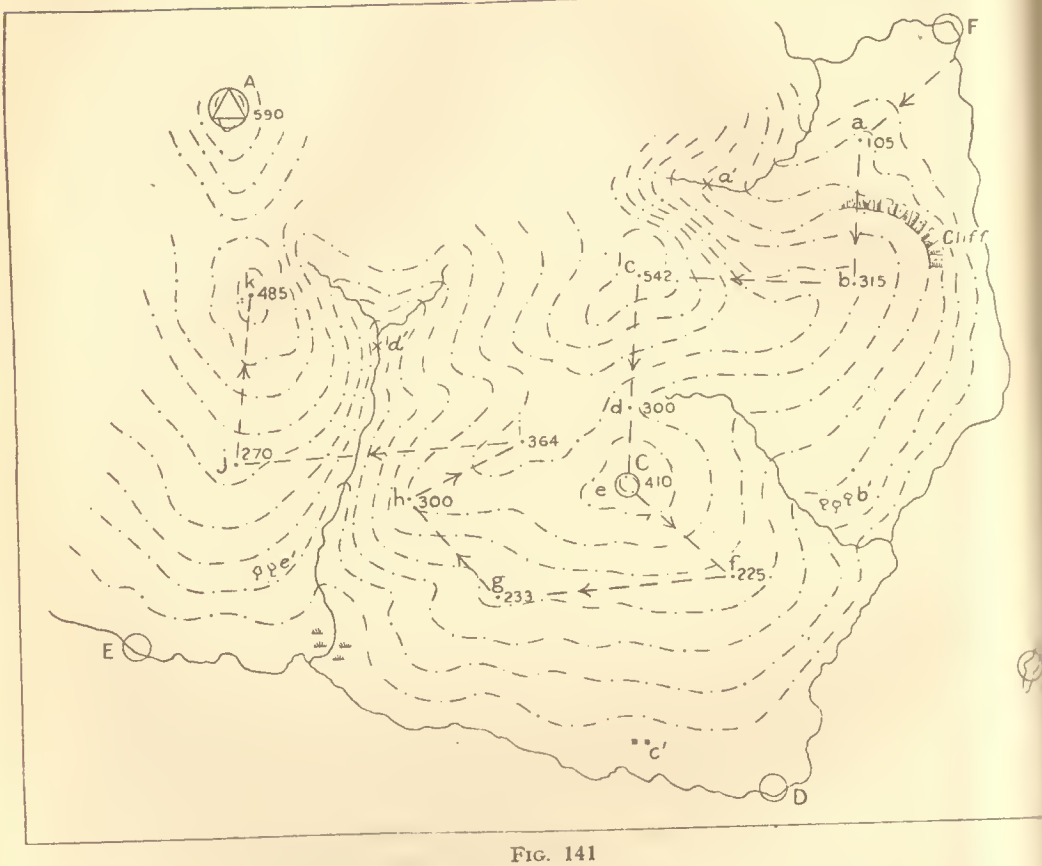


FIG. 141

Contouring and Form Lines.—Contours are generally recognised as the most suitable means of depicting the shape of the land on charts, though other methods, *e.g.*, hachures, were often used in the past. Graded colour washes to supplement the contours such as are used on land maps are hardly practicable on charts, but shading on the south-east slopes of the hills is a useful adjunct. The vertical interval between contours must be varied to suit the character of the land and the scale of the survey. In low-lying country a hill 50 ft. high is an important feature and it may be desirable to have contours spaced at only 10-ft. vertical intervals. In mountainous country, however, contours at 50, 100 or even 250 ft. intervals may be all that are necessary to show the shape of the ground, and on small scales it is obvious that very close spacing is impracticable.

The ordinary method of sketching in the contours from the ground is that described in the previous paragraphs, and this is generally sufficient for the purposes of a chart. It is clear, however, that no very great accuracy is obtainable unless the number of plane table stations is multiplied almost indefinitely. When little time is available for the topographical part of the survey, it is best not to attempt contouring in the strict sense of the word. The shape of the hill features can in such cases be shown with form lines which may be regarded as generalised contours. The only essential requirements for form lines is that they should indicate different degrees of slope reasonably well by being crowded up closely on steep slopes and opened out on gentle ones and that they should show the limits of each feature with tolerable accuracy.

Instrumental Contours.—This is a much more precise method than the above

but is necessarily slow and laborious. The procedure is as follows. Starting from some known height—usually sea level in the case of the hydrographic surveyor—the observer runs a line of levels (*see* Chapter VII) until he arrives at some selected contour, *e.g.*, the 100 ft. level. Setting up the level at this point a fix is obtained and the height of the telescope above the ground is measured : say, this is 4 ft. An assistant with a levelling staff is then directed to walk with it as nearly as possible along that contour, stopping and holding up the staff wherever a marked change in the direction of the contour occurs. At each stopping place, the levelling staff must be moved up or down hill until the telescope intersects the 4-ft. mark on the staff and to that point a horizontal angle and tachymeter distance is observed. When the assistant has covered a distance of 300 yards or so, he stops and the level is brought forward and set up at this new station. The process is then repeated and thus a traverse is run along this 100-ft. contour. Similarly, with other contours, *e.g.*, at 200 and 300 ft., and, when these have been plotted, intermediate contours are drawn in by eye from the ground. This method of contouring may sometimes be used by the hydrographic surveyor when working on large scales and in open country, but usually time will not allow the use of such elaborate methods.

Mapping of Towns and Villages.—On small scales any attempt to make an accurate plan of a town or village can hardly be justified in a hydrographic survey. It will be sufficient to show the more conspicuous buildings, which can usually be fixed by angles from points in the surrounding country or from boat stations, and to indicate the limits of the built-up area approximately. On large scales, however, it is necessary to show at least the main streets and blocks of houses correctly on the chart and the best method of effecting this is usually by means of a traverse through the streets. The most suitable instruments are a plane table with open-sight alidade, a small range-finder, a pocket compass and a tape measure.

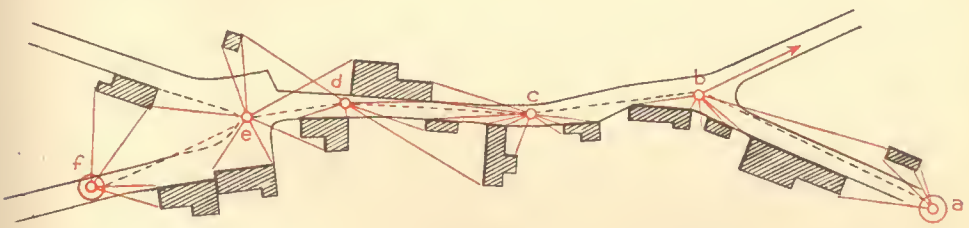


FIG. 142

Suppose it is required to make a plan of the village shown in Fig. 142. Setting up the plane table at *a*, the observer fixes his position by resection from suitably disposed marks and measures the lengths and bearings of the rays (shown in red) to corners of the nearest houses. Similarly at *b*, *c*, *d*, etc., a traverse being run through the village to *f* where the observer finds he can again see to fix his position by resection from the survey marks and satisfies himself that accurate closure of the traverse has been made. When possible, two or more rays from different stations should be obtained to each building whilst its exact shape and orientation can be determined with the compass and tape measure. Owing to the large amount of detail, this form of topographical work is very slow, but fortunately for the hydrographic surveyor sufficiently accurate town plans are generally obtainable from local sources and he can confine his labours to minor corrections and additions to the material shown on the existing maps.

The methods of running traverses are described in Chapter VI. They may be applied to topographical features such as the above and are also useful for fixing roads, railways, boundaries, etc.

Topography from Sketches.—In wooded country it is hardly feasible to do much topographical work by traverse methods or plane table. A very useful alternative in such cases is afforded by making sketches of all the visible land features from fixed positions on land or from ship and boat stations off the coast. The outline view of all the hills and any other easily recognisable objects is drawn on squared paper approximately to scale, and vertical and horizontal angles are observed to all the more prominent features. On the sketches, the scale used for the vertical angles should be somewhat greater than that for the horizontal angles (except perhaps in very hilly country), a vertical scale 1½ or 2 times the horizontal scale is convenient. Too much time need not be spent in attempting to make a very perfect sketch; certain features may often be a little exaggerated with advantage either in height or breadth if thereby it will be easier to deduce their character from the picture. It *is*, however, important that every object should be correctly placed in the sketch, to the right or left, above or below other neighbouring objects, as they are actually seen from the observer's view point. The horizontal and vertical angles to the hill summits and other conspicuous detail are measured and recorded on the sketch and provide data from which positions and heights may be determined.

A good representation of the topography of the coastal region can be produced from a series of sketches made from boat stations offshore.

The procedure is as follows:

A field board is first prepared showing the positions and heights of the main and secondary stations and sounding marks, the coastline and any other topographical detail already obtained. This board, drawing instruments, tables of logarithms (for computing heights), station pointers, and a sextant for the observations should be taken in the boat.

Anchor the boat about 1½ miles off shore, or rather more if the land is high, and fix in the ordinary way by sextant angles. Sketch in the topography as shown in View A, Fig. 143, observing horizontal and vertical angles to all the more prominent features. Proceeding a short distance along the coast, make a second sketch embracing roughly the same part of the land, as illustrated in View B, Fig. 143A. As the boat proceeds from A to B, the observer should watch the land features very carefully so that the hill summits, etc., which have been lettered in View A may be correspondingly lettered in View B. The appearance of the land often changes very rapidly as the view-point is altered, especially if some interval has elapsed between the occupation of the two boat stations and the sun has moved round. This is illustrated in Figs. 143 and 143A, where it will be noted that the light is coming from the right in View A and from the left in View B.

After completing View B, proceed to position C and carry out the same procedure until a series of views has been obtained. The time at which the various positions are occupied must be noted so that calculated heights may subsequently be corrected for height of tide. The number of views required will depend upon circumstances, but five will usually supply sufficient information to cover a fairly large area.

To the more important summits it is desirable to obtain three or more horizontal angles to insure against errors of observation and plotting, but for the minor detail two shots will generally suffice, as a useful check against mistaken identity of any point is provided by the agreement in its height as determined from the two independent vertical angles. All horizontal angles should be plotted before each station is left, and as soon as enough observations have been secured to fix a summit its height should be calculated by the methods given in Chap. VII, and provisionally recorded on the field board. The series of spot heights provide a basis for drawing in the contours.

FIG. 143

FIG. 143a

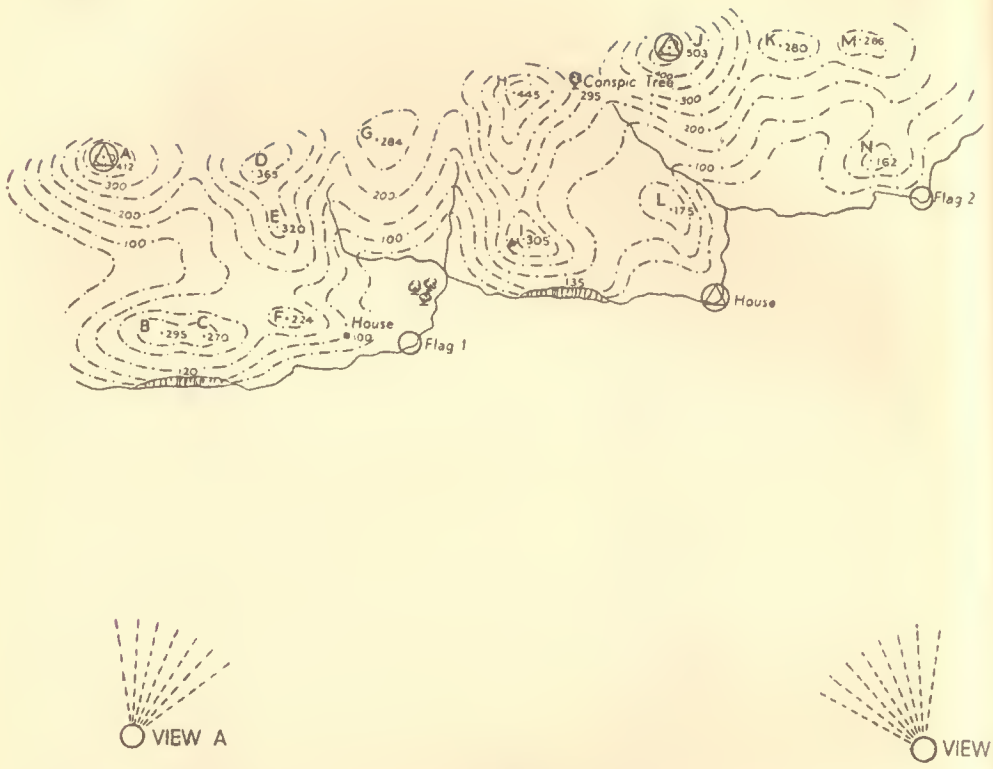


FIG. 144

The results of analysing the sketches from A and B (Figs. 143 and 143a) are shown in Fig. 144.

After completing a series of sketches from stations off shore, it will often be found advantageous to steam slowly down the coast on the opposite course, stopping as necessary from time to time to sketch in the general formation of the hills. On return to the ship the records of the tidal movement during the day should be consulted so that any necessary corrections can be applied to the heights determined from the vertical angles. When the country is wooded it may be necessary to take the vertical angles to the tops of the trees; in this case a note should be made on the chart that the heights do not refer to the ground level.

When sketches are made from land stations a theodolite should be used for observing the horizontal angles and the elevations and depressions; they have this advantage over sketches from boat stations that the observations are more accurate; but a better view-point is usually obtained from the sea and many more stations can be occupied within a given time. Such theodolite sketches should, however, always be made when opportunity occurs as they take very little time and are of great assistance in filling in the topographical detail.

Panoramic photographs offer an alternative to sketching, but for the type of work considered here they are hardly likely to be of great use. The advantages of a sketch over a photograph are that there are no restrictions as to scale and that it is possible to exaggerate any particular piece of detail which may be of importance but is hardly recognisable on the photograph. Unless the light is favourable, hills in the foreground will hardly be distinguishable against those in the background on a photograph, but in sketching they may be carefully examined with a pair of binoculars and drawn in correctly.

CHAPTER XIV

SURVEYING FROM AIR PHOTOGRAPHS.

This chapter deals only with methods of plotting topography from air photographs using such simple equipment as is likely to be found in H.M. Surveying Ships. The methods outlined are graphical since it is unlikely that the larger precision instruments will be available afloat.

For the sake of compression some definitions of terms, theory and many of the proofs of formulae have been omitted; reference to "A Simple Method of Surveying from Air Photographs," by Lt. J. S. A. Salt, R.E.,[1] and "Air Photography applied to Surveying," by Major C. A. Hart, R.E.,[2] should be made for these.

In addition to the time saved in obtaining topographic detail, the use of air photographs will often be found to overcome difficulties met with in ground surveying. Among these are:

(i) Access to difficult ground.
(ii) Restricted view in flat enclosed country.
(iii) Maintenance of detached parties.
(iv) Restricted activities of ground parties in bad weather.

An additional difficulty may arise in time of war, namely, the denial of ground by the enemy.

Procedure Preparatory to Photography.—The authorities who will undertake the photography must be supplied with as much information as possible of what is actually required.

A tracing or marked map should be sent, with a "brief," showing actual flight lines and areas to be covered by each strip of photographs. The importance of preserving straight courses, constant height and of keeping tilts of the aircraft to the absolute minimum should be stressed. If the lenses and cameras available are known, the focal length of the camera lens and the height at which the photography is to be carried out should be specified in order to obtain photographs at the required scale. It will save much time later if photography can be carried out at or near the scale required for plotting.

If the camera used for photography is not fitted so that it also photographs a clock and an altimeter, the authorities who carry out the flight should be requested to furnish full information of times of photography and heights flown.

The overlap of successive photographs of a strip should be not less than 60% and of adjacent strips 25%.

In organisations where a large number of reconnaissance photographs are received for processing it is customary for machinery to be used in the processing operations. This often results in slight distortion of the finished picture and for this reason it is desirable to request that the photographs should be hand processed. If topographic-base film and waterproof printing paper are available they are naturally to be preferred.

It will generally be necessary to order four sets of photographs of the area flown. One of these will be required for the graphical triangulation, one for spot heighting and contouring if this is done and one for forwarding with the results of the survey. A fourth set will prove useful for taking into the field for the identification of ground control points and for ground interpretation and from which to replace photographs which may be damaged inadvertently.

Ground Control.—This is a matter requiring careful attention, particularly

1. H.M. Stationery Office. 2. Longmans, Green and Co.

in uncivilized areas where maps are either unreliable or non-existent. Results obtained by the Air Survey Committee of the War Office indicate that a plan to a scale of 1/25,000 can be made as accurately as any ground survey, with ground control provided by a triangulation whose sides are 10–12 miles in length. For navigational requirements of the aircraft, however, closer ground control may be desirable.

It will be necessary, in the absence of easily identifiable ground objects, to mark on the ground the required control points so that they will be visible from the air, and appear on the photographs. Points used as main stations in the triangulation are most suitable, the photography being deferred until these have been selected and marked.

It is desirable that some control points be situated near the outer limits of the area to be surveyed since the necessity for extending the graphical triangulation by uncontrolled extensions will not then arise. Any method of marking in the field may be adopted. Whitewash on dark areas and ashes on light are quick and simple.

Plotting—General.—Although not absolutely essential it will be a very great advantage to make a " light table " as shown in Fig. 145. This consists

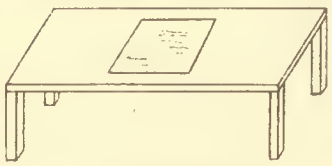


FIG. 145

simply of a low table with a sheet of glass let in flush with the top. The glass should be about 12 in. square and the height should permit a table lamp of the " Anglepoise " type to be laid on its side underneath. Prick-holes in photographs laid over the glass show very clearly as minute specks of light and greater accuracy can be achieved.

All lines drawn on photographs in colour should be as fine as it is possible to draw them. The medium used should be water colour paint not waterproof ink since if a mistake is made a damp cloth will effect a clean erasure of the former without in any way damaging the photograph.

Prick-holes must be very small and intersecting lines through them must be drawn with the greatest care. The most meticulous accuracy must be preserved throughout all operations if cumulative errors are to be avoided.

Preparation of the Photographs.—These should first be laid out to form a rough mosaic and it should be checked that the requisite area has been covered, the overlaps are adequate and that the photograph numbers are consecutive, *i.e.*, that the Veeder counter in the camera has not jumped two steps or that a photograph is not missing ; inspection of the overlap will show if either has occurred.

If more than one strip is being dealt with, " tie points " should be selected at this stage. These are points of ground detail lying in the common overlap between strips, the points at the ends of the strips being known as " end ties." They should be chosen bearing in mind the following requirements :

(*a*) They must form well-conditioned triangles when intersected from the principal points of three adjacent photographs of one strip. If possible they should also give good intersections from the principal points of photographs in the adjacent strip.

(*b*) They should be the points which will later be used for " minor control plotting ". If this cannot be arranged additional points will have to be selected later.

(*c*) At least one tie point must be intersected at every three overlaps, but as stated above it is better for points used in the minor control plot to be selected so that they will serve as tie points as well. (See under "Minor Control Points.")

No special attempt should be made to select points which will later be of topographic importance when detailed plotting begins. End ties and tie points should be circled with soft pencil on the face of the photographs and marked E.T. and T.P. respectively. The mosaic may then be broken up.

These points must now be accurately pricked on the photographs. They must be of such a size that the lightest of prick-holes will completely cover the feature selected. "The middle of that black bush" or "the apex of the shadow of that tree" will not be accurate enough. Ground control points should also be identified, pricked, enclosed within a pencilled triangle and their names written on the backs of the photographs.

The photographs should then be sorted into their respective strips in order.

It should be noted here that mass repetitive operations, such as drawing the base lines (see below) on *all* photographs, etc., leads to boredom and consequent inaccuracy, and therefore should be avoided. An assistant should carry out the minor control plot of a strip in its entirety. If the work is being done single-handed, one strip, as a rule, should be completed before the next is started.

Minor Control Plotting.

The principal point of each photograph (*i.e.*, the foot of the perpendicular from the internal perspective centre to the plane of the photograph) must first be pricked.

If the actual position of this is known from the camera calibration figures a template can be made on Kodatrace showing its position in relation to the collimating marks on the photographs and applied in succession to each photograph. In a great many cases this will not be known and the intersection of the lines joining the collimating marks is used. This intersection point may be pricked either by preparing a template to fit the collimating marks or by scribing short lines between them on the face of each photograph and pricking their intersection. The principal point having been pricked is marked in red with a small diagonal cross.

The *principal point base lines* (or base lines) have now to be drawn. These are lines drawn on each photograph joining its principal point to the photographic positions of the principal points of the preceding and following photographs. A quick and accurate method is as follows :

Fig. 146 shows two photographs A1 and A2 whose principal points are at P.P.1 and P.P.2.

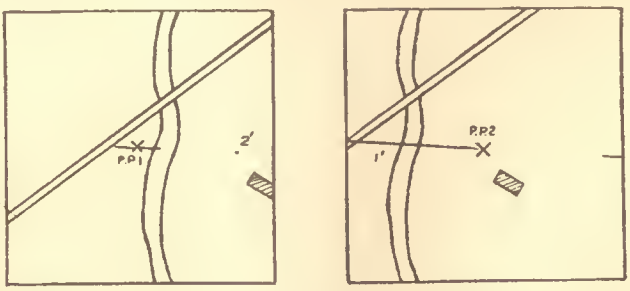


Fig. 146

These points will not generally fall on any well-defined point of detail but an attempt is made to estimate, and mark with a pencil, the position of P.P.2 on photograph A1 at 2′. Line up P.P.1 and 2′ against a straight edge and score with a pricker a short line through P.P.1. Any error in estimation of the

position of 2′ will not appreciably affect this short line. Instead of trying to estimate the position of P.P.1 on A2, select some well defined point of detail on the short scored line through P.P.1.

On photograph A2 join this point of detail to P.P.2 and the base line on photograph A2 is correctly established. The base line on A1 can now be drawn by drawing a red line from P.P.1 which everywhere cuts the same points of detail as that drawn on A2.

The base lines should be produced on the opposite side of the principal point and drawn as short " tails " at the edges of the photographs.

Fig. 147 shows a photograph with the base lines and tails marked on it. All photographs are marked similarly. The end photographs of each strip, of course, only have a single base line drawn on them.

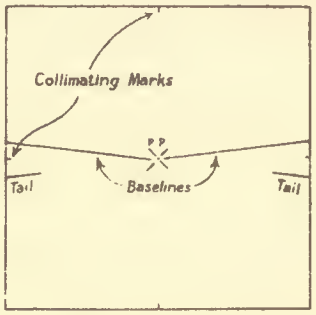


FIG. 147

As a check on the correct drawing of the base lines, each pair of photographs should be turned through 90° to the direction of their common base lines and viewed through a stereoscope. If the common base lines when in stereoscopic fusion appear to lie on the ground the marking has been correctly done. If they do not, another attempt must be made.

The base lines and their tails are now inked in red with the finest lines possible.

A difficulty sometimes arises that the principal point of a photograph falls on a point of no detail such as the middle of a lake. In such a case an accurate base determination may be made as follows (see Fig. 148):

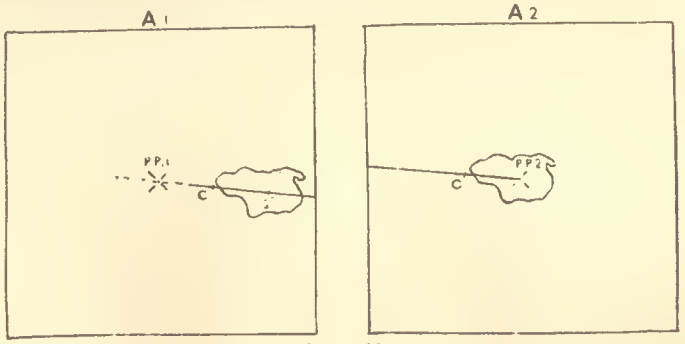


FIG. 148

Lay photograph A1 over A2, matching the detail as nearly as possible around P.P.2 and as before estimate the position 2′. Make a short score through P.P.1 and by observing detail along this line draw the base line on photograph A2. Now select a point of detail C on the base line of photograph A2, identify this point at c on photograph A1, join P.P.1 to c and produce it.

The base line on photograph A1 is not a good determination, but it can be strengthened as follows (see Fig. 149):

Select two points on photograph A2 which satisfy the following conditions :

(*a*) They should be at nearly the same height.
(*b*) They should lie at approximately equal distances from the base line.
(*c*) The line joining them should be roughly at 90° to the base line.
(*d*) They should be as close to the base line (and to P.P.2) as possible.

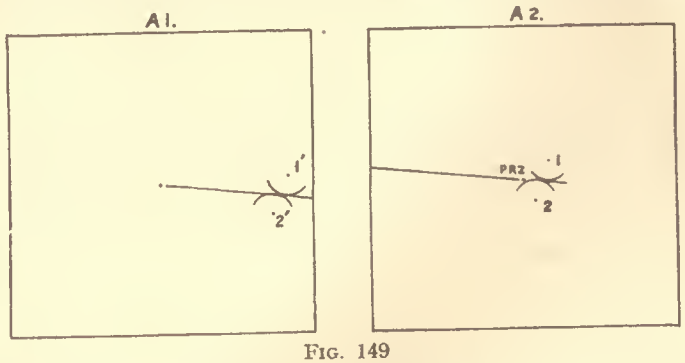


FIG. 149

On photograph A2, measure with springbow dividers the arc from 1 to the base line. Identify this point on photograph A1 (at 1') and scribe the arc with centre 1'. Repeat this process with points 2 and 2' and the two arcs and the base line on photograph A1 should be tangential. In practice the arcs will not always be tangential to each other but the mean of the two tangents will give the position of the base line. If the discrepancy is large the most probable cause is a difference of height between points 1 and 2 and another pair of points should be chosen.

Minor Control Points.—The minor control points must now be chosen to supplement those used as tie points. These minor control points should be points of detail lying in the common overlap of successive photographs on each side of the base line.

The photographs of a strip are placed in a pile, consecutively on the right of the observer and Nos. 1, 2 and 3 of the strip are taken and laid out with No. 1 overlapping the left side of No. 2 and No. 3 overlapping the right side of No. 2. Three strips of common ground are now lying alongside each other. Points A and B are chosen on photograph 2 opposite Q, the principal point, so that they are about equidistant from Q and that QA and QB are about equal to Qp or Pq. They are identified on photographs 1 and 3 at a_1 b_1 and a_2 b_2 respectively and these six points are carefully pricked.

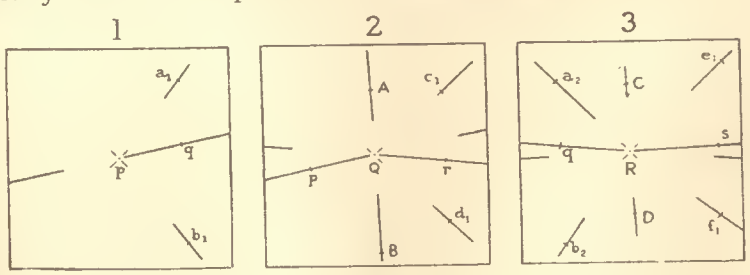


FIG. 150

Photograph No. 1 is now placed on one side to the left of the observer and photograph 4 is taken from the pile. Points C and D are chosen on photograph No. 3 and identified at c_1 d_1 on photograph No. 2 and c_2 d_2 on photograph No. 4. These six points are pricked and the foregoing operations are repeated

throughout the strip. It is to be noted that many of these minor control points will also be the tie points originally selected.

The rest of the work is done over the light table.

Each photograph is taken in turn and fine lines radiating from its principal point are drawn in red through the minor control and tie points. These lines should extend about half an inch on either side of the prick holes, their lengths being dependent on the variations in height of the country. Experience will soon show what length to draw them ; it is better to draw them too long than too short. The photographs will now appear as in Fig. 150.

It will be noticed that the principal point base lines and their tails are shown. The small ticks indicating the positions of the minor control points and their letters shown in the figure are not marked on the photographs. At this stage rays should also be drawn through any tie points which it has not been possible to use as minor control points, also any trig points which appear on the photographs.

Graphical Triangulation.—The photographs of a strip are laid out with a rough fit and a piece of Kodatrace is cut to include the area of the strip and leave a margin to spare.

The first photograph is then laid on the light table under the Kodatrace and orientated so that, working from it, the remainder of the photographs will fall within the limits of the Kodatrace.

The Kodatrace and photograph are then weighted down and the base line Pq drawn on the Kodatrace in blue. It should be produced for a distance rather more than its own length.

The position of a_1 on the photograph is pricked and circled on the Kodatrace and the radiating line through b_1 is drawn in blue, as is all subsequent work. The correct superposition of plotted work in blue over the lines in red on the photographs can be readily checked since the two colours will combine to form a clear heliotrope.

The first photograph is removed and the second placed underneath the Kodatrace. It is positioned so that its base line pQ lies under the plotted base line Pq, the extension of the latter giving a more precise orientation by coinciding with the tail on the photograph. The Kodatrace is now slid over photograph No. 2 along Pq until the radiating line through A passes through the pricked position of a_1 on the Kodatrace. The fit of the base lines is checked and the Kodatrace weighted down.

The forward base line Qr and its extension is now drawn and also the lines through B, c_1 and d_1. Fig. 151 shows the state of the minor control plot at this stage.

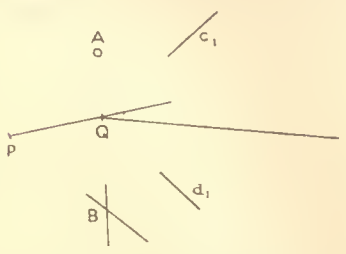


Fig. 151

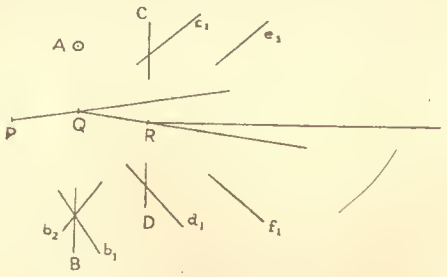


Fig. 152

The second photograph is now removed and the third placed under the Kodatrace. It is positioned as before so that the plotted line Qr lies over the line on the photograph qR and the Kodatrace is slid along qR until the ray through a_2 is under the pricked position of a_1 and A. The ray through b_2 should now be seen to make a perfect intersection with the plotted rays b_1 and B on the Kodatrace. If it does not, an adjustment must be made which is explained later under " Triangle of error."

The forward base line Rs and its extension is now drawn, also the rays to b_2, C, D, e_1 and f_1. Fig. 152 shows the minor control plot at this stage.

The third photograph is now placed under the Kodatrace and its control points intersected in this way and so on throughout the strip. Fig. 153 shows the appearance of the minor control plot after the first five photographs have been dealt with. It will be noticed that no letters are shown but that the principal points are cross ticked and marked with the numbers of the photographs to which they relate.

Any tie points not used as control points or Trig points encountered should be rayed in at the same time as the minor control points and should be lettered on the Kodatrace. It will be noted from the figure that since it is only minor control points that appear on the lower side of the principal point base lines the plot is obviously one of a strip having a strip adjacent on its upper side only. A single strip clearly requires only minor control points.

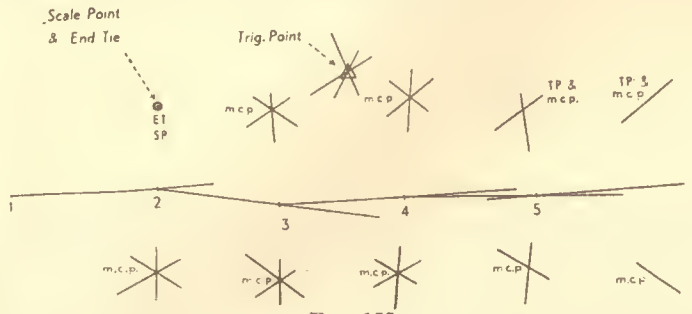


FIG. 153

Scale Point.—In constructing the plot only one length has been assumed, that of Pa_1 (see Fig. 150), and the whole of the plot will therefore be at this scale. Subject to small errors due to height distortion, the scale will be :

$$\frac{f}{H-h}$$

where f is the focal length of the camera
H is the height of the aircraft,
and h is ground height at the point A.

This point A is called the Scale Point and in order to minimise height distortion it should be chosen at the mean height of the country. If no suitable point can be found on the first photograph a point can be chosen farther on in the strip and the strip can then be plotted outwards from this point. It should be noted that *any* point can be chosen, the problem being analogous to a base measurement.

Triangle of Error.—So far it has been assumed that tilt and height distortions are radial from the principal point, but if photographs having fairly large tilts are taken of hilly country this assumption no longer holds good and discrepancies may be expected in the minor control plot.

For example when plotting the fourth photograph of the strip it may be found that, with the base lines in coincidence, it is impossible to make the rays Sc_2 and Sd_2 pass through the positions of C and D (already intersected from photographs 2 and 3) at the same time. In this case an adjustment must be made but before this is done it must be checked that the error has not arisen through inaccurate plotting of base lines or minor control points on previous photographs.

The adjustment depends on two assumptions :

(*a*) That for a given tilt the error in the radial direction to a point will be proportional to the height distortion along that ray, *i.e.*, to the distance, very nearly, on the plot, between the plotted position and the photographic position.

(*b*) That the tilt is equally likely to be in any direction.

The adjustment cannot be guaranteed to give an improvement for a single overlap, but when tilts are large and varying throughout the strip a considerable improvement is made in the strip as a whole. The procedure is illustrated in Fig. 154.

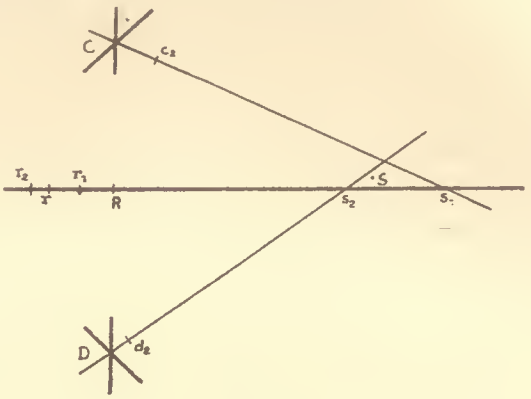


FIG. 154

The thick lines in the figure show the forward base line Rs and the intersected positions of the points C and D as they appear on the Kodatrace. rS on the next photograph is fitted under the Kodatrace under Rs and the radial direction Sc_2 made to pass through the plotted position of C. At this setting the ray Sd_2 will not pass through D. Draw the ray Sc_2 in pencil on the Kodatrace and mark its intersection with the forward base line Rs at s_1; the photographic positions of c_2 and r, the principal point of the preceding photograph are marked as shown at c_2 and r_1.

Now slide the Kodatrace so that the ray Sd_2 passes through the plotted position of D. As before draw the intersection of this ray with Rs at s_2 and mark the photographic position of d_2 and r at d_2 and r_2. There is now a triangle of error for the point S, whose most probable position is required. Divide r_1 r_2 internally at r so that r_1 r/r_2 r equals Cc_2/Dd_2 then select a position inside the triangle so that it is distant from the rays Rs, Cs_1, Ds_2 in the proportion Rr, Cc_2, Dd_2 respectively.

Fit the Kodatrace so that the principal point S on the photograph falls under this position and the rays Sr, Sc_2 and Sd_2 miss the positions of r, C and D by amounts proportional to Rr, Cc_2 and Dd_2 respectively. Draw the forward base line and radial rays as before. There will be a slight discontinuity at S.

The figure greatly exaggerates the size of the triangle of error that will be met with in practice. Even when comparatively large the position of S can usually be estimated by eye and in many cases the triangle is so small that its centre can be accepted immediately.

Short Overlap.—It sometimes happens that owing to camera or other difficulties the overlap between successive photographs falls below 50%.

Base lines may still be drawn, either by observing in the stereoscope or by pinning the photographs through their principal points and, with a straight edge against the pins, rotating them until the straight edge cuts identical points of detail on each.

Minor control points, however, cannot be chosen in the ordinary way and hence, although azimuth can be carried forward, though to a lower degree of accuracy, a method must be devised to carry forward scale. Fig. 155 shows a photograph (No. 3 of the strip) on which, because of a short overlap, no points can be found which are common to both No. 2 and No. 4 photographs.

Points are chosen, such as C_1, D_1 in the left overlap and C_2, D_2 in the right overlap which satisfy the following requirements :

(*a*) C_1 and C_2 (or D_1 and D_2) should be as close together as possible.

(*b*) They should be points which lie as nearly as possible at the same ground height. (This can be estimated by observing in a stereoscope.)

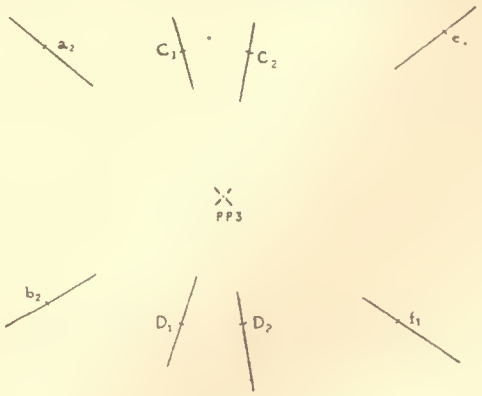


FIG. 155

Fig. 156 shows the minor control plot. Photograph No. 3 is resected in the ordinary way and the plotted positions of C_1 and D_1 intersected at C_1' and D_1'. Rays to C_2 and D_2 are drawn and the photographic positions of C_1, D_1, C_2 and D_2 marked on their respective rays.

Consider the case of C_1 C_2. The line joining their photographic positions, C_1 C_2 is first drawn. A line parallel to this is then drawn through C_1' cutting the ray to C_2 at C_2', this is accepted as the intersected position of C_2. The position of D_2, at D_2', is obtained in the same way and the forward rays to e_1 and f_1 drawn. The plot can then be continued as usual. If a short overlap occurs on one side of the base line only, the other side is treated as for a normal overlap.

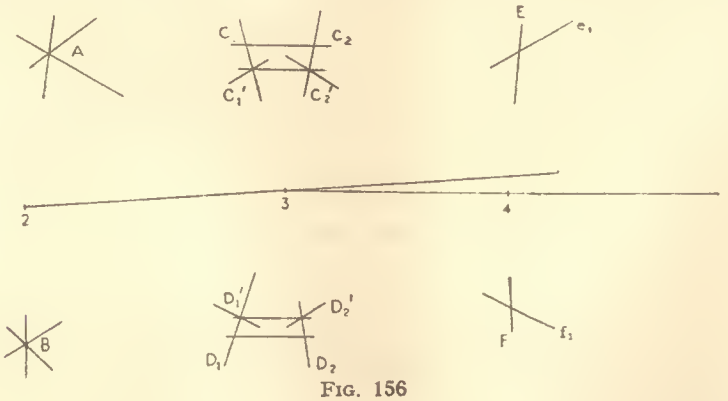


FIG. 156

The method is based on the assumption that height distortion of points at the same height is proportional to the distance of these points from the principal point. Provided that the points are close together, tilt distortion will not be appreciable. A loss of accuracy results, however, and if short overlaps continue is liable to be cumulative ; they should therefore be avoided.

Triangulation Points near the Base Line.—Whilst control and tie points are always selected so that a good intersection may be obtained from at least two principal points, it may happen that Trig points will fall on or near the base line

and their intersections will therefore be poor. Fig. 157 illustrates the procedure which should be adopted in such a case.

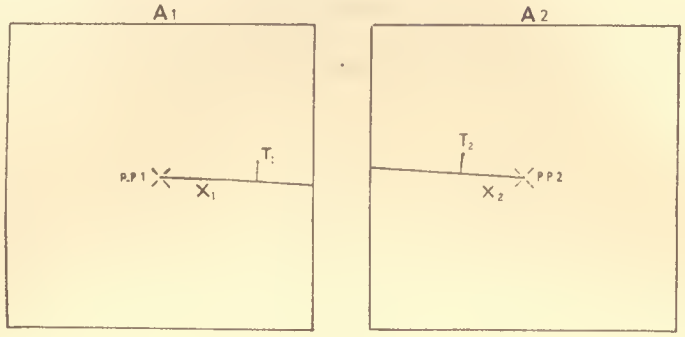


FIG. 157

Drop perpendiculars from the two photographic positions of the Trig points on to the base lines. Measure along each base line from the foot of the perpendiculars to the principal points of the photographs and call these distances X_1 and X_2. Measure the length of the air base B on the minor control plot (shown in Fig. 158).

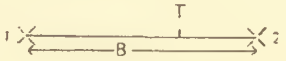


FIG. 158

Then the length from the foot of the perpendicular from T to the plotted position of P.P.1 will be given by $\frac{B.X_1}{X_1 + X_2}$ and to P.P.2 by $\frac{B.X_2}{X_1 + X_2}$. Erect the perpendicular from this point and an ordinary intersection from P.P.1 or P.P.2 will give its distance from the base line. This method should be used when rays from the principal points intersect at angles greater than 150°.

Intersected Points.—Any point of photographic detail in the strip may be plotted by intersection from two or three principal points. To do this photographs are taken in turn and placed under the minor control plot ; the latter is then adjusted so that the principal point of the photograph and its base lines coincide with those on the plot. A ray is then drawn on the plot from the principal point through the point of detail on the photograph which it is desired to intersect. (Except for detail lying in the small common overlap of three successive photographs, points will be intersected by two rays only.)

Adjustment of Minor Control Plots.—Single Strip Adjustment.—The minor control plot now consists of a framework on which any photograph may be set in its correct relation to others, and on which the correct relative positions of all intersected points have been established. This framework, subject to the accumulation of small errors which is inherent in the method, is true to shape but is on an unknown scale. To bring the plot to a predetermined scale the procedure is as follows :

Fig. 159 shows the plotting sheet on which the Trig marks A, C and S have been plotted. These also appear within the area covered by the strip and have been intersected on the minor control plot at A′, C′ and S′. The minor control plot is laid over the plotting sheet so that A and A′ are coincident and the line A′C′ on the plot is superimposed on AC on the plotting sheet.

A separate sheet of tracing paper is laid over the minor control plot and the plotting sheet and A′C′ are pricked on it and joined. The distance CC′ is measure on this tracing and set off along C′X at right angles to AC′. AX is joined and lines are drawn parallel to C′X along AC′.

To reduce the position of No. 6 principal point to the scale of the plotting sheet, for example, the tracing paper is pivoted about A until the line AC′ cuts principal point No. 6. The distance, at right angles to AC′ from this

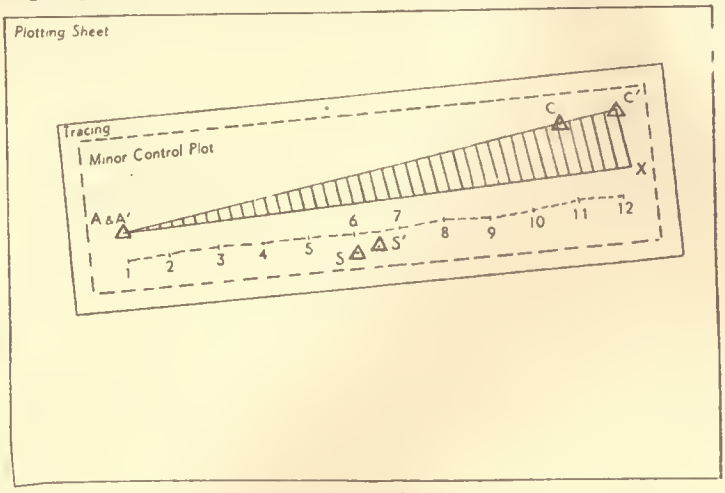


FIG. 159

principal point to the line AX is measured and set off from No. 6 along AC′ in the direction of A. The point so found is the scaled position of No. 6 principal point. All points on the plot, except minor control points which are no longer required, are treated similarly and circled on the plotting sheet in orange.

There will probably still be a small discrepancy between the Trig point S and the scaled position of S′ due to small errors which have accumulated throughout the strip. This error is distributed uniformly throughout the strip as follows:

Measure the difference in the East-West and North-South directions between S and the scaled position of S′; the sign convention is that a required shift of S′ to North or East is + and to South or West is —. A table is constructed as in the example below and, supposing that S′ must be moved 0·06 ins. East and 0·03 ins. South in order to agree with S, the table is filled in under principal point 6 (the nearest to S) putting + 0·06 in the X column and — 0·03 in the Y column.

	A′	S′										C′
P.P.s	1	2	3	4	5	6	7	8	9	10	11	12
X	0	+ ·01	+ ·03	+ ·04	+ ·05	+ ·06	+ ·05	+ ·04	+ ·03	+ ·02	+ ·01	0
Y	0	$\bar{\cdot}01$	$\bar{\cdot}01$	$\bar{\cdot}02$	$\bar{\cdot}02$	$\bar{\cdot}03$	$\bar{\cdot}03$	$\bar{\cdot}02$	$\bar{\cdot}02$	$\bar{\cdot}01$	$\bar{\cdot}01$	0

No. 1 and No. 12 principal points will receive no correction (being nearest to A and C respectively) and Nos. 2–5 and 7–11 receive proportionately distributed corrections as shown in the table. These corrections are applied to the orange positions on the plotting sheet and are circled in red. They are the finally adjusted positions of the principal points.

Two-Strip Adjustment.—As previously stated the strips must have an end tie point at each end and tie points throughout their common overlap.

If two Trig marks lie in one strip that strip is first scaled as previously explained and the second strip is scaled to the first by treating the end ties as though they were Trig points.

If the Trig points lie one on each strip the two strips are first brought to a common scale by using the end ties and are then scaled to the Trig points.

In either case, having brought both strips to the scale of the plotting sheet discrepancies may be expected in the positions of the tie points, common to both strips. The adjustment of these is explained below in the section dealing with the internal adjustment of a block.

Blocks of Strips.—Where more than one strip is necessary to cover the area photographed the procedure, after the minor control plots have been made of each strip, is as follows :

(*a*) Choose a strip containing two well-separated Trig points, situated if possible at its outer ends.

(*b*) Bring this strip to the scale of the plotted Trig points, as already described, and prick off a scaled strip.

(*c*) Bring other strips successively to this scale, using the end tie points in their common overlaps, as if they were Trig points, and prick off scaled strips.

(*d*) Fit the first scaled strip to the plotting sheet and prick on to the latter the tie points. Attach arrows to these to indicate from which strip they have been intersected. Similarly fit the next scaled strip by the end tie points, prick through the tie points and attach arrows. Continue with the remainder of the scaled strips.

(*e*) The scaled minor control plots may now be allocated to assistants for detail plotting to be carried out on them. (See p. 280.) The area to be plotted on each strip is demarcated by joining the tie points.

Internal Adjustment.

(*f*) Examine the discrepancies on the plotting sheet between the positions of tie points in each common overlap of strips. Discrepancies should show a regular and progressive error between the end tie points ; if a sudden irregularity occurs it will be due to a mistake at some earlier stage and must be investigated before going any further.

(*g*) Choose a strip where the aggregate discrepancies between the tie points are least. (Strip C in Fig. 160.) This strip is allowed to stand unaltered and tie points intersected from it are circled in orange.

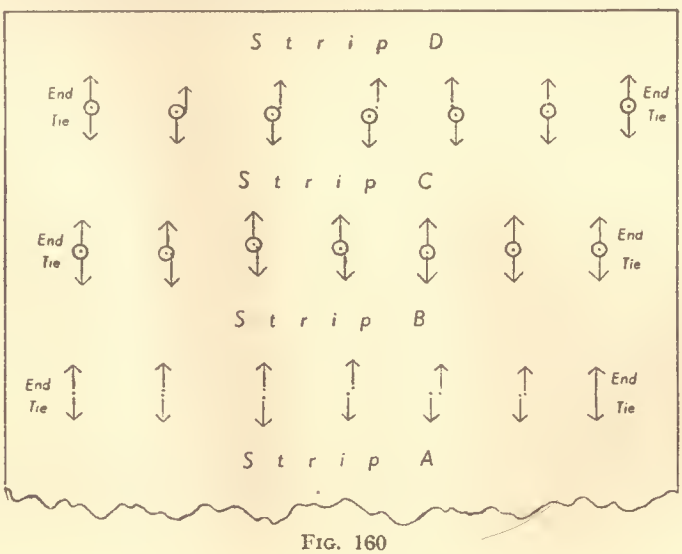


FIG. 160

(*h*) Strips D and B must now be shifted by an amount equal to the full amount of the discrepancy. To maintain the width of the strips the movement must be the same on both sides of them, it is necessary therefore in this example, to move tie points on the south side of strip B and any intersected points which may lie on the north side of strip D. The movement is effected by measuring the amount of the discrepancies of all tie points with those of strip C in terms of Eastings and Northings. Any point on the north side of strip D or the south side of strip B is replotted by a corresponding amount and circled in orange. Points not exactly opposite tie points of strip C receive a shift which is an interpolation of the discrepancies of the two (strip C) tie points most nearly opposite. The plotting sheet will now appear as in Fig. 161.

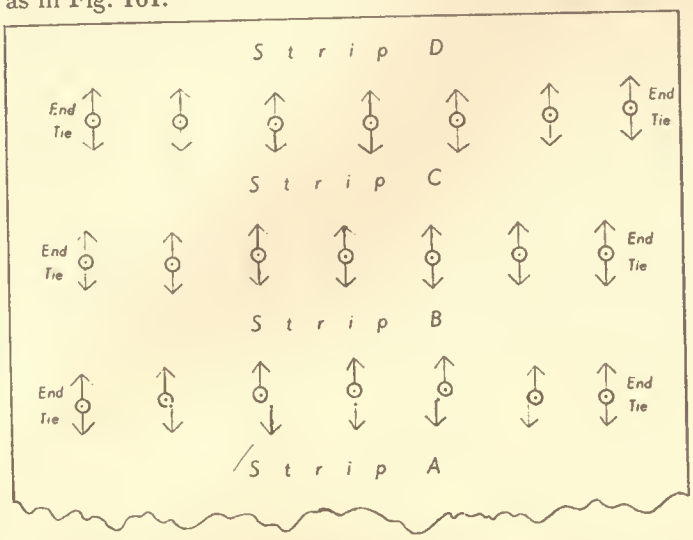


FIG. 161

(*i*) Discrepancies between the tie points replotted on the south side of B and those on the north side of A are adjusted similarly, the full amount of the shift of tie points on the north side of A being given to all points on the south side. These adjusted tie points are circled in orange and this completes the internal adjustment.

(*j*) Any Trig point on the minor control plot will be surrounded by four tie points or minor control points and these can be joined to form a "unit." By joining similar points on the plotting sheet a similar "unit" will be formed. Since the minor control plots have been scaled to the plotting sheet the two "units" can be compared and will often be exactly the same in which case the Trig point can be pricked through and circled in orange. More generally the two "units" will show slight discrepancies, though these should be very small (of the order of one millimetre or less) and the Trig point is plotted as follows (see Fig. 162) :

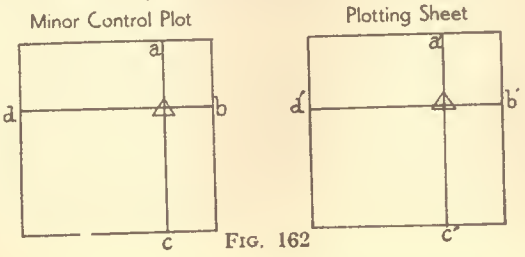


FIG. 162

On the minor control plot draw two lines ac and db through the Trig point. Interpolate the cutting point of these on the sides of a " unit " on the plotting sheet giving a', b', c' and d' and join a'c' and b'd'. All Trig points are plotted in this manner including the two used for scaling the first minor control plot and whose positions may have been altered very slightly by the internal adjustment.

External Adjustment.—The Trig points plotted in orange must now be adjusted to fit the same points which were already plotted (from their rectangular co-ordinates) in red on the plotting sheet. This adjustment will of course involve a relative shift of tie points and is done as follows :

A tracing is made of all the orange positions of Trig and tie points as obtained by the internal adjustment and the discrepancies at the Trig points between their internally adjusted (orange) positions and their true (red) positions are measured in terms of Eastings and Northings and written against their positions on the tracing. The sign convention used is, as before, that a required shift of the orange positions to North or East is plus and to South or West is minus.

The Trig points are now joined so that a network of well-conditioned triangles is formed on the tracing. By linear interpolation along the sides of the triangles, points are found at which the discrepancies are 0·01, 0·02, 0·03, etc., inches in Eastings and Northings respectively.

Points of the same value are joined by lines, in red for Eastings and green for Northings. When this has been done it should be checked that the lines in any one triangle are parallel and equidistant. If this is not so an error has been made in interpolation. Fig. **163** shows an example of such a diagram, in which the Northing corrections only have been inserted.

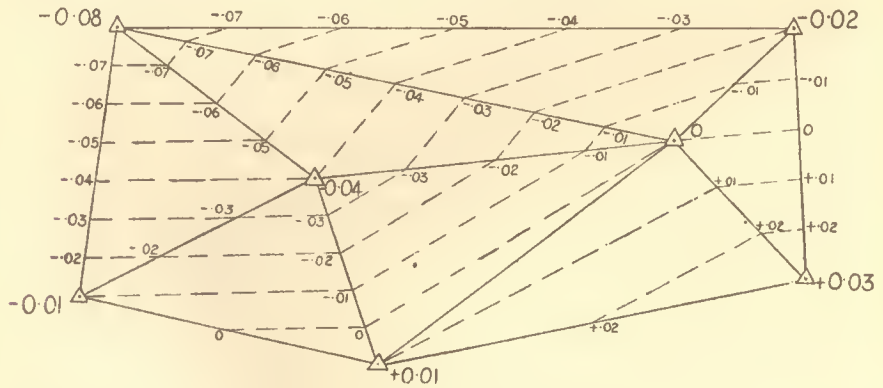


FIG. 163

Shifts of all tie points are taken from the diagram by inspection, set off on the plotting sheet from their orange positions and circled in red, *e.g.*, a tie point lying midway between the —0·03 and the —0·02 pecked lines, on Fig. **163**, would be moved 0·025 in. to the south.

It must be emphasised that the greatest care must be taken that measurements are correctly made and have their correct signs applied. Frequent independent checking is the only safeguard against errors.

DETAIL PLOTTING.—Plotting of detail is carried out on the scaled minor control plots, points of topographic detail being intersected from the principal points of photographs on which they appear. The photographs used for the minor control plotting will be the most suitable since these already carry on them the principal points and the base lines.

Generally speaking, intersections of points of photographic detail should be made every three-quarters of an inch. This density should be increased over

very hilly country and may be decreased in flat featureless country. If the scaled plot is at the same scale as the photographs, detail between intersected points may be traced directly from the photographs. If not, intersected points are taken in threes and the detail between them adjusted by a process analogous to " squaring-down." (See p. 256.)

Interpretation of Detail.—This can only be learned by experience and in cases of doubt as to the exact nature of objects seen on the photographs the area must be visited. Certain features such as main roads and broad rivers are usually obvious enough but secondary roads if tree-lined, for example, may be easily confused with small streams and vice versa. Shadows will often provide a clue to the nature of ground objects. A stereoscope should always be used to assist interpretation.

Compilation.—The finally adjusted control and tie points on the plotting sheet are pricked through on to a tracing, and the plotted detail of each scaled minor control plot is traced onto this. Points on the scaled minor control plots will not fit their finally adjusted positions on the tracing exactly, but by transferring the detail " unit " by " unit " (similar to those formed for the plotting of Trig points (p. 279)) the effect of such misfits will be negligible.

Twin Oblique Photographs.—These, also known as Split Verticals, are sometimes employed for mapping instead of true verticals. From the same height and with a lens of the same focal length a greater width of country can be photographed for each strip flown.

Two cameras are employed each having an equal and opposite lateral tilt. The photographs, which are taken simultaneously by each camera, overlap along the centre line of the strip. This arrangement is shown diagrammatically in Fig. 164.

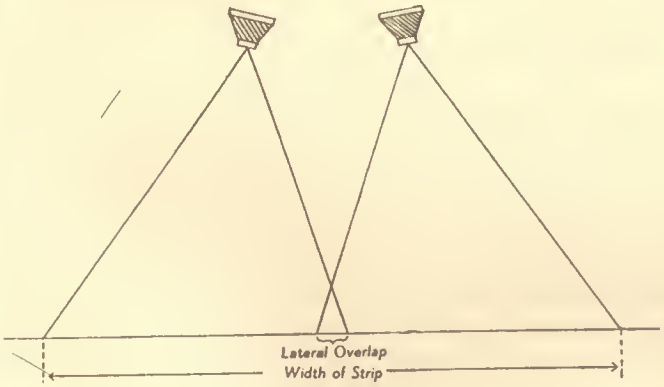


Fig. 164

The photographs obtained are " low " obliques (*i.e.*, the image of the horizon is not included) and since the amount of tilt is greater than can be dealt with by normal radial triangulation a different method must be used.

Fig. 165 illustrates the conditions obtaining in an oblique photograph. In this figure

O is the perspective centre.
OvVv′ is the vertical line through O.
APV is an oblique photograph tilted θ° from the vertical.
P is the principal point of the photograph.
OP is the principal distance f.
aPv is a horizontal plane intersecting the plane of the oblique photograph along an axis of tilt PT through P. (It may also be regarded as a vertical photograph whose principal point is at v.)
Ov′a′ is a vertical perspective plane.

It is clear from the figure that Pv = f sin θ and PV = f tan θ and it should be noted that the horizontal plane, the plane of the oblique photograph and the vertical perspective plane all intersect in a common point T.

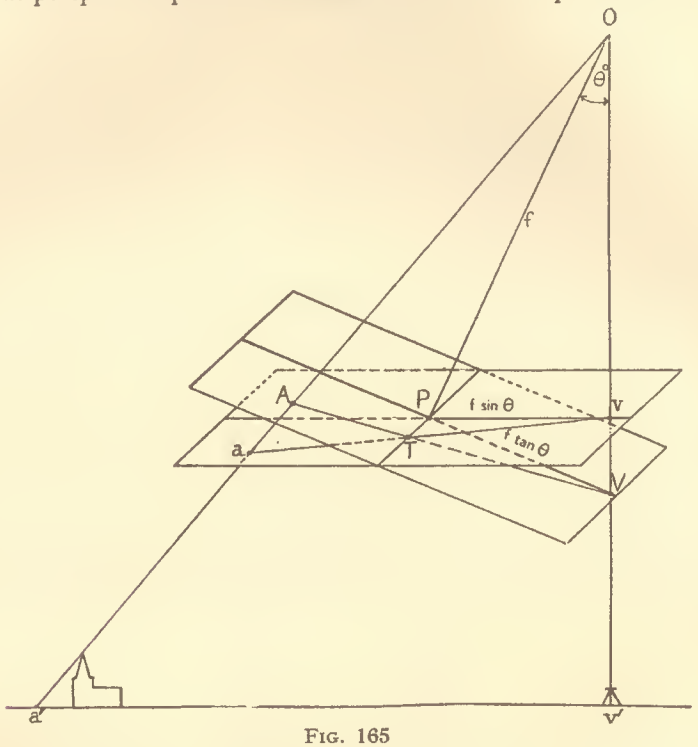


FIG. 165

If the vertical perspective plane is rotated about Ov′ to measure horizontal angles on the ground it is evident that angles will be unaffected by the height of ground objects and truly equivalent to horizontal angles measured on the ground by a theodolite at v′. Also that a perspective plane rotating about an axis that is not vertical (say about OP) cannot always contain the top and bottom of a vertical object at the same time. Such a plane would measure different angles according to the height of the object.

To revert to the case of angles measured by a vertical perspective plane ; if these are traced on the plane of the tilted photograph their tangents will equal the tangents of the angles on the horizontal plane multiplied by the cosine of the angle of tilt θ, for in the triangle PvT

$$PT = f \sin \theta \tan PvT$$

and in the triangle PVT

$$\tan PVT = \frac{PT}{f \tan \theta}$$
$$= \frac{f \sin \theta \tan PvT}{f \tan \theta}$$
$$= \tan PvT \cos \theta$$

(Since the cosine of a small angle is nearly equal to 1, there is very little difference between angles measured in the horizontal and tilted planes when the angle of tilt is small and this is the fundamental assumption in the " radial line " method of plotting vertical photographs which have small, and random, tilts.)

It is clear from the foregoing that if the position of the plumb point V appears on oblique photographs, and can be established, then, by drawing plumb point base lines, the radial line method, modified slightly, will hold good.

The Modified Marriott Method.—This is a graphical construction to establish true angles on a tilted photograph whose angle of tilt is known.

Fig. 166 shows a pair of twin oblique photographs whose overlap is indicated by pecked lines.

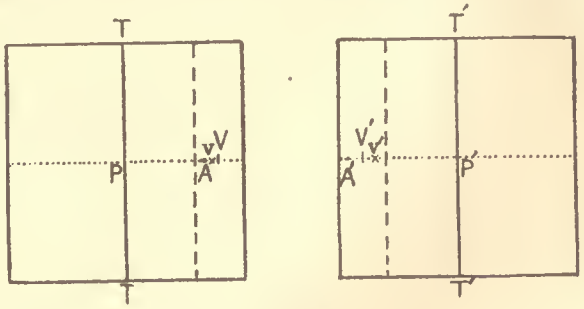


FIG. 166

P and P′ are their principal points, V is the plumb point, common to both photographs and TP and T′P′ are lines joining the fore and aft collimating marks of the photographs. These lines when the photographs are tilted squarely are also the axes of tilt through P and P′. The plumb point, V, and the f sin θ points, v and v′, are established as follows :

(*a*) Join in red on each photograph the lines joining the fore and aft collimating marks. Scribe with a pricker the lines joining the lateral collimating marks and prick the principal points P and P′.

(*b*) Identify in the common overlap of the pair of photographs a point of detail, A and A′, lying on the line joining the lateral collimating marks.

(*c*) Measure PA and P′A′. The plumb point lies at the mean of these two distances from P and P′.

e.g., if measurements give PA = 4·05 inches
and P′A′ = 4·61 inches
then PV = 4·33 inches = P′V′

(*d*) When the whole strip has been treated in this way the plumb point base lines are drawn on the starboard photographs in the same manner as principal point base lines are drawn on verticals. Plumb point base lines are then drawn on the port photographs by drawing them over points of the same detail as on the starboard photographs.

(*e*) All distances PV and P′V′ are measured on all photographs and their mean is used to compute a mean value θ° for the angle of tilt of the photographs. The formula used is $\text{Tan } \theta = \frac{PV}{f}$

(*f*) The positions v and v′ are marked by measuring the distance f sin θ from P and P′ along PV and P′V′.

Minor control points are selected, six on each photograph. Three of these should lie on the outer side of the collimating axes TPT and T′P′T′ and three on the inner sides, the conditions governing the selection of the outer points being the same as those governing the selection of points on vertical photographs.

The *Minor Control Plotting* in this method is similar to plotting from vertical photographs but rays are radial from the plumb point instead of the principal point and they are corrected as follows :

Fig. 167 shows No. 1 port photograph of a strip, with the plumb point base line drawn and a minor control point " a " selected. This point is to be intersected.

(*a*) On the photograph, mark the point y where the line TPT is intersected by Va.

(*b*) Place the Kodatrace plot over the photograph and draw the plumb point base line, mark the plumb point V and the photographic position of No. 2 plumb point.

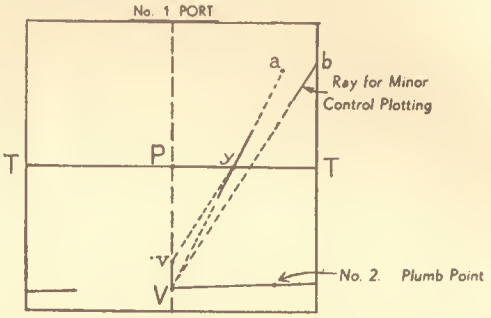


FIG. 167

(*c*) On the Kodatrace, draw the outer part of the plotting ray, Vb, from V, parallel to the line joining v, the f sin 0 point, to y.

(*Note.*—The pecked line vy and the pecked parts of the lines Va and Vb are shown in Fig. 167 to illustrate the construction; they would not be drawn in practice.)

(*d*) Repeat this construction for all other points on the photograph, lying on the outer side of TPT, which are to be intersected. It is not usual to correct the rays lying on the inner side of TPT in this manner though it may be done if desired.

(*e*) When this construction has been completed on the port photograph, control points on the starboard photograph are similarly intersected. The plotting is continued taking No. 2 port and then No. 2 starboard photographs. If the lengths of the plumb point base lines are not exactly the same on port and starboard photographs the mean length should be accepted.

Plotting is continued similarly throughout the strip. The correction to the rays is a very small one and, as stated before, it is not usual to apply it to points lying on the inner side of the fore and aft collimating axes.

Scaling the strip, the internal and external adjustments, and the detail plotting are carried out in the manner already described for vertical photographs.

The Four-Point Method of Plotting.—This method can be applied to any photograph, vertical or oblique, where ground relief is small in comparison to the height of the aircraft.

Whereas the radial line method is precise *irrespective of relief* when tilt is absent, the four-point method is precise *irrespective of tilt* when relief is absent. With tilts of up to 30° investigation has shown that the method can be used at a scale of 1 : 12,500 if ground relief is not more than 2% of the flying height. For photography at 30,000 feet it can be relied upon for planimetry with relief up to 600 feet, for any tilts likely to be encountered. Exactly where the four-point method breaks down is still being investigated.

Fig. 168 represents a badly tilted photograph, and the area covered by the photograph on the plotting sheet. Capital letters on the plotting sheet correspond to lower-case letters on the photograph. It is required to locate on the plotting sheet a point H, whose photographic position is h. A, B, C and D are the previously plotted positions of a, b, c and d.

From b draw rays on the photograph to a, c, d and h, and on the plotting sheet from B to A, D, and C. Lay a strip of scrap paper on the photograph across the rays from b and tick the intersections of these on one edge of the

paper strip. Transfer the paper strip to the plotting sheet and manoeuvre it so that the rays from B to A, C and D cut the edge of the strip at the tick marks. Draw the fourth ray from B, through the tick mark h to H.

Repeat the process with rays from a to d, c, b and h and from A to D, C and B, drawing the second ray to H. Check the plotting by a third intersection from d and D. A pleasing feature of this method is that when relief is small any triangles of error are due to faulty plotting and to no other cause.

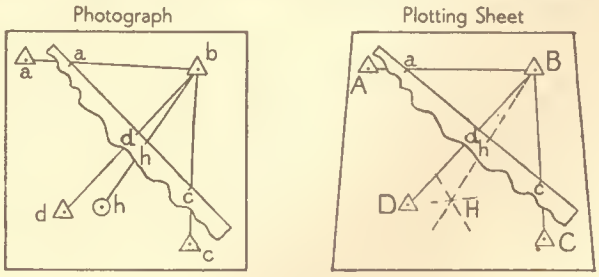


FIG. 168

Since a large number of lines on the face of the photographs will confuse the detail, the work should be carried out on the backs of the prints. The ground points plotted on the plotting sheet should be traced off on tracing paper which is then *turned upside down*, to agree with the inversion of the print, for detail to be plotted. The only point to watch is that when pricking the points through to the backs of the photographs, the pricker should be held vertically.

When a near vertical photograph contains four control points additional points may be established by this method and if these lie in the overlap between photographs, control may be extended.

The method also establishes a network of triangles on photograph and plotting sheet whereby detail may be transferred in a manner analogous to " squaring-down."

The method is particularly suitable where only random photography is available. Knowledge of focal length and height of aircraft is unnecessary ; and there need be only sufficient overlap to permit four common points to be identified on adjacent photographs, which may be taken with different cameras at different times, from different heights, and having different tilts.

DETERMINATION OF HEIGHTS

The *Absolute Stereoscopic Parallax* of a ground object lying in the overlap of two photographs is the algebraic difference, in the direction of the air base, of the distances of its two images from the principal points of their respective photographs, and it can be shown that it is given by

$$p = \frac{fB}{H-h}$$

where p = Absolute Parallax. f = Focal length of lens.
B = Length of air base. H = Height of aircraft.
h = Ground height.

Since f, B and H are constant for a pair of photographs, p depends only on the change in h. If dp is a small change in p and dh the corresponding change in h, then $dp = \frac{fB.dh}{(H-h)^2}$. and this is the *parallax equation.*

When two photographs of an object are placed under the stereoscope and fused they will appear as one single ground object, and a pair of dots superimposed on the photographs will appear as a single dot. If the distance between the dots is variable, the fused dot may appear to lie above, below, or at the level of the ground object. Such a fused dot is known as a " floating mark ".

The Parallax Bar.—This is essentially a beam carrying two glass plates, usually engraved with either a dot or a Y. One plate can be slid along the beam and clamped for gross adjustment, the other can be moved for fine adjustment by a screw carrying a micrometer scale engraved in millimeters. To measure the height, above ground level, of a single ground object such as a factory chimney the parallax bar is adjusted over the photographs so that the black dots on the glass plates are roughly superimposed over the images of the object whose height is to be measured.

Examination under a stereoscope will fuse the dot in some position in space above, below or at ground level. The micrometer is adjusted to bring the floating mark on to the ground at the base of the object and a reading made. It is important that the point on the ground which is selected should be as close as possible to the base of the object whose height is required.

The micrometer is then adjusted to float the mark on top, or very close alongside the top, of the object and the micrometer is again read. The difference of parallax readings should be obtained thus at least three times, and a mean difference arrived at, by which the height of the object can be computed. The following example illustrates the computation.

$B = 1{,}016$ feet $= 309{\cdot}7$ m.
$f = 5$ inches $= 127$ mm.
$H = 15{,}000$ feet $= 4{,}572$ m.
$h = 215$ feet $= 65{\cdot}5$ m.

Bar micrometer at ground level read 3·05 mm.
Bar micrometer at top of object read 2·97 mm.
therefore $dp = 0{\cdot}08$ mm.

$$dh = \frac{(H-h)^2 \times dp}{fB}$$

$$= \frac{4{,}506{\cdot}5 \times 4{,}506{\cdot}5 \times 0{\cdot}000{,}08}{0{\cdot}127 \times 309{\cdot}7} \text{ m.}$$

$$= 41{\cdot}3 \text{ m.} = 135{\cdot}5 \text{ feet.}$$

It is necessary that when observing with a parallax bar the following conditions should be satisfied:

(*a*) The photographs must be set in exact base alignment.

(*b*) They must be equally illuminated with the direction of illumination the same, if possible, as that obtaining at the time of photography. A study of the shadows will enable this to be observed.

(*c*) They must be held rigidly whilst observing.

Accurate observation is greatly dependent on practice. At the instant of taking a reading, attention must be concentrated on the object whose height is being obtained and not on the floating mark.

Contouring within Spot Height Control.—Heights of chimneys, etc., can be obtained from the parallax equation as previously explained since it is assumed that relative tilt errors within a very small area will be negligible. For obtaining spot heights over wider areas and, from these, interpolating contours the following method is used:

Scales of absolute parallax in relation to height are prepared from the equation $p = fB/(H - h)$. Since B, the air base, will vary slightly throughout the strip the photographs must be sorted and tabulated in groups of similar base lengths. In general, an interval of 200 feet, between the base lengths used for computing the scales, gives satisfactory results.

For example, suppose the area photographed varies in height from 0–200 metres; that $f = 127$ mm.; height flown is 3,581 m. and that the air base

length of the group of photographs being dealt with is 1,524 m. Then by substitution in the above equation the following table is obtained :

Ground height (h) in metres	Absolute parallax (p) in milimetres
0	54·05
50	54·81
100	55·60
150	56·41
200	57·25

A scale is now constructed from the above data, as shown in Fig. 169.



FIG. 169

A pair of photographs containing a point of known height is now set under a stereoscope and fixed rigidly, and a reading of the bar is taken on the point. Suppose the known height is 174 m. and gives a bar reading of 3·31 mm. This

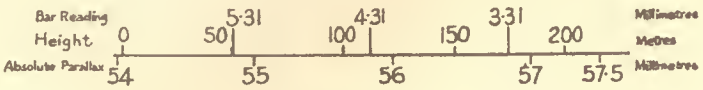


FIG. 170

is inserted on the scale as shown in Fig. 170, and used as a datum for plotting the readings 4·31 and 5·31, which are stepped off from it at intervals corresponding to one millimetre on the scale of absolute parallax. Whole number

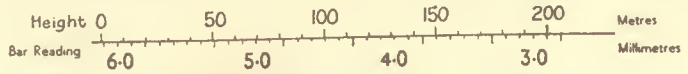


FIG. 171

bar readings are then interpolated ; the absolute parallax readings are deleted and the final scale appears as in Fig. 171.

Preparation of the Photographs.—Photographs are taken in turn and, the base lines having been drawn, known heights are marked on them with a small red dot, and a figure denoting the height ; these figures should be placed well clear of the dots. The known heights should be marked on only one photograph of a pair, consequently such markings will appear only on alternate photographs. Points whose heights are required are then marked with a blue dot. Points of known height which appear in the common overlap *between strips* should be marked on both strips (other heights in this overlap should also be inserted on both strips when they are determined later from the parallax measurements). At least four known heights are required in each overlap between consecutive photographs, and these should, if possible, lie in the four corners of the overlap.

The greater part of *Tilt Error* is linear, in a direction either parallel or perpendicular to the base line. For this reason heights should be determined by comparison with others lying in lines parallel and at right angles to the base lines, as follows :

Fig. 172 shows a common overlap with four points of known height, A, B, C and D close to the corners. K is a point whose height is required. It is necessary first to obtain heights at E from A and B, and at F from C and D. The height of K is then found from E and F.

The height of K could also be obtained from G, lying between B and C, and H, between A and D.

To obtain the height of E, the bar readings of A, E and B are taken. It is almost certain that the heights of A and B, as found from the bar readings and scale (Fig. 171), will not agree precisely with their known heights, so a correction must be made to the measured height of E, proportionate to the errors found at A and B. If a and b are the corrections to be applied to the measured heights of A and B to make them agree with their known heights, then the correction to be applied to the measured height of E will be $a - \frac{AE}{AB}(a - b)$. Example:

A, 120 m. Bar reads 4·31 mm. Height from scale = 124 m. Corrn. = —4 m.
B, 155 m. Bar reads 3·98 mm. Height from scale = 147 m. Corrn. = +8 m.

On photograph, AB measures 2·4 in. and AE measures 1·6 in., so correction to be applied to the measured height of E $= -4 - \frac{1{\cdot}6}{2{\cdot}4}(-4 - 8) = +4$ m.

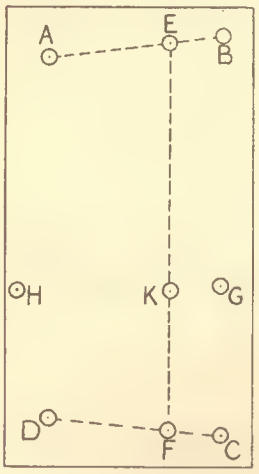


Fig. 172

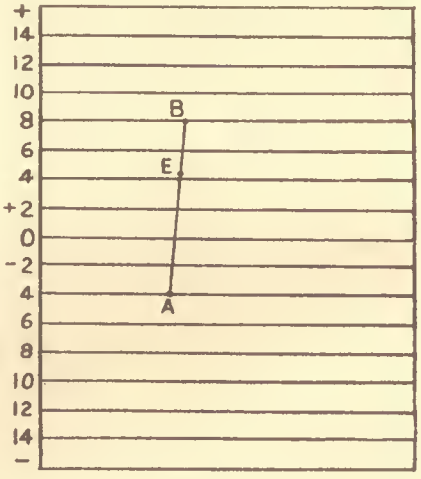


Fig. 173

It is simpler to find the correction for E graphically from an error graph as shown in Fig. 173. The positions of A, E and B are transferred from photograph to graph by means of a strip of paper. A and B are placed on their appropriate " correction " lines, and the correction to be applied to the measured height of E is then found by inspection.

When spot height control is limited, heights may be measured of objects selected along a straight line, joining two known heights, which is neither at right angles nor parallel to the base line. The assumption is made that all error is due to tilt error and that the corrections to be applied to intermediate points are proportionate to their distances from the controlling points, of known height, at the ends of the straight line. This method, however, is not as accurate as the previous one, if tilts are present.

Contouring.—This is done by interpolation between spot heights from observation in the stereoscope. Drainage lines are marked on the photograph by fine pecked lines, then the ruling contours, chosen (at first) with large vertical intervals between them, are sketched in. The ends of these where they pass from one overlap to the next are equalized, and additional contours are interpolated between them. Contours are plotted from the photographs and then traced from the plot on to the fair tracings.

TILT CORRECTION TO HEIGHT MEASUREMENTS.—Throughout the following section Height Error is taken to mean the error in a height which has been measured on the parallax bar and calculated by the ordinary parallax equation. It must not be confused with "Height Distortion" (which is defined as the error in the relative positions of the images on a photograph of points lying out of the reference plane).

Height *error* will be taken to follow the usual convention, *i.e.*, measured height *minus* true height. Height *correction* is taken to mean the amount which must be added to a measured height to obtain the true height. It is therefore of opposite sign to height error.

Tilt errors are of two kinds, linear and non-linear. A linear error has the effect of raising or lowering the ground and of tilting it about some axis without deforming it. Non-linear errors are those which do not progress uniformly across the overlap and as a result of these the ground appears warped.

In general both errors will be present in an overlap, but it is possible to separate them when dealing with vertical or near vertical photographs.

The following paragraphs explain the causes of height errors and their effects. (*a*), (*b*) and (*c*) produce linear errors; (*d*) and (*e*) produce non-linear errors.

(*a*) **Inclination of the air base,** see Fig. 174.

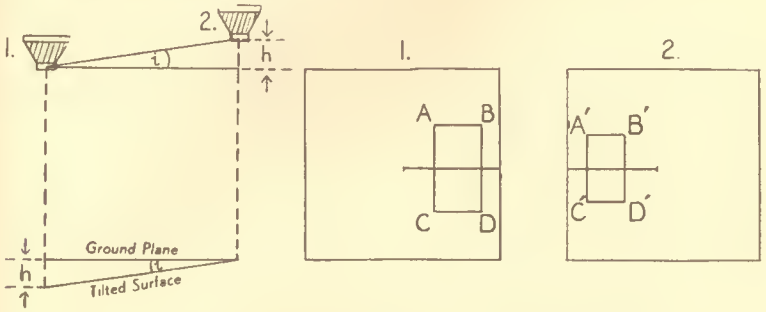


FIG. 174

If the air base in inclined, the plane of each photograph remaining horizontal, the ground will be tilted by an equal amount in the same direction. It should be noted that uniform shrinkage of one photograph relative to the other is not distinguishable from a small inclination of the air base (non-uniform deformations of film or paper are small and may be neglected in all rapid methods of plotting).

(*b*) **Equal fore and aft tilt of both photographs,** see Fig. 175.

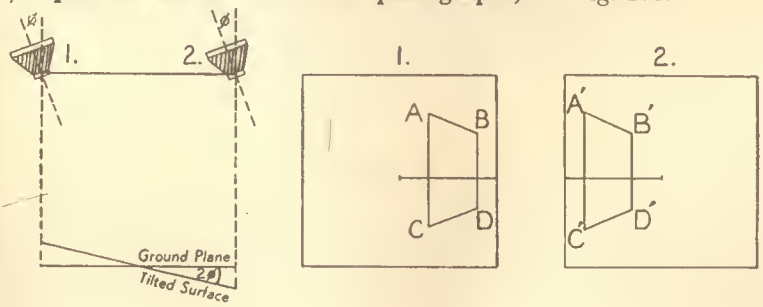


FIG. 175

If both photographs are tilted by the same amount about horizontal axes perpendicular to the air base, the ground is tilted about an axis halfway along the base and normal to it through an angle equal to twice the angle of tilt on each photograph.

(*c*) **Equal lateral tilt of both photographs,** see Fig. 176.

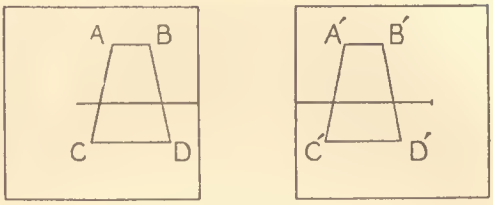


Fig. 176

The effect of this on the photographs is shown above. As in the case of equal fore and aft tilt, the ground is tilted through an angle equal to twice the angle of tilt on each photograph.

It should be noted that unless the photographs are correctly aligned under the stereoscope the effect of any departure from true alignment of their base lines will have the effect of tilting the ground about the base line and will be indistinguishable from equal lateral tilt.

(*d*) **Differential lateral tilt.**

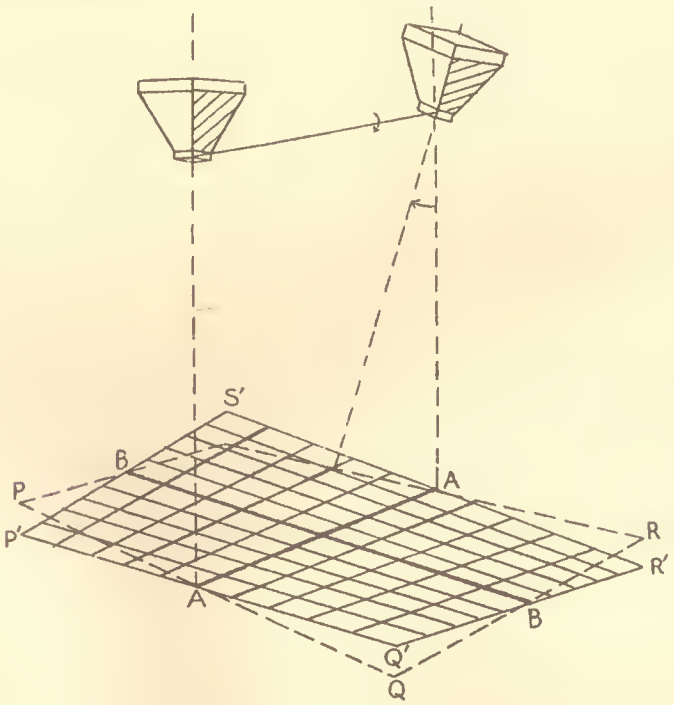


Fig. 177

A perspective view of the deformation of a horizontal plane (the overlap between a pair of photographs) P Q R S due to one photograph being tilted, relative to the other is shown in Fig. 177. The deformed surface is represented by P′ Q′ R′ S′.

This deformed surface is known, geometrically, as a Hyperbolic-Parabaloid. It has two properties which must be considered in eliminating tilt ; (the error due to tilt of this kind will henceforth be known as *Hyperbolic Error*) firstly the surface is what is known as " doubly-ruled," that is to say it may

be generated in two ways by moving a straight line. Application of a straight edge to the firm lines in the figure above will show that it has been drawn in this way. The importance of this property is that along any line parallel to or at right angles to the baseline, height error changes uniformly. Secondly, the contours of the height errors on this surface are Rectangular Hyperbolas. These are illustrated in Fig. 178 which has been drawn on the assumption that height errors at the points P Q R S of Fig. 177 are −10, +10, − 10, and +10 respectively.

To illustrate the effect of combining Hyperbolic Error with Linear Error, rectangles have been drawn over the hyperbolas.

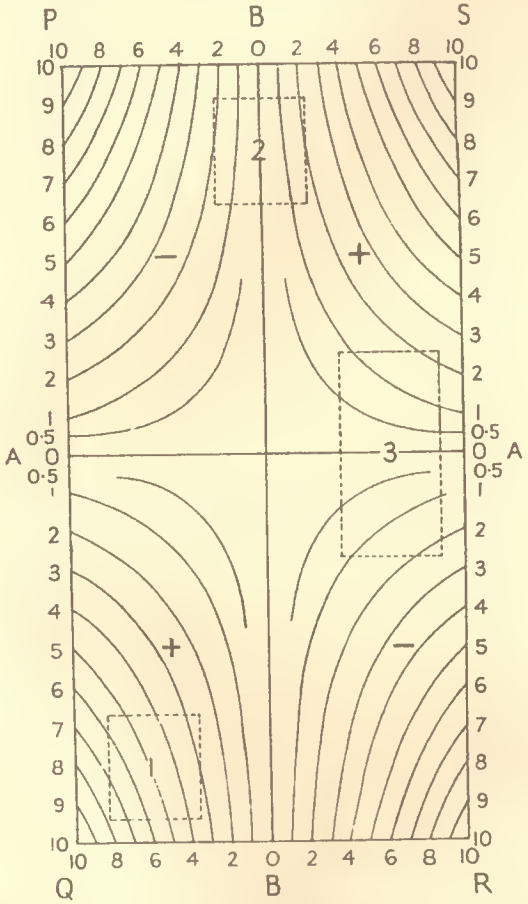


FIG. 178

If Hyperbolic Error is combined with both Lateral and Fore and Aft error the curves will cross the overlap in the way they cross rectangle 1. Rectangle 2 shows Hyperbolic and Linear Fore and Aft errors combined and rectangle 3 shows the combination of Hyperbolic with Linear Lateral error. The asymptotes must always represent the same height errors but they will not in general be zero.

Fig. 179 is an example to show how the hyperbolic contours may be drawn. The initial information is that the height errors at the corners P′, Q′, R′ and S′ are − 25, +25, 0 and + 20 respectively.

Using the property that error progresses linearly across and along the overlap, the sides of the rectangle are subdivided evenly and the points where

each 5 unit error contour cut the boundary are obtained. The asymptotes are obtained by joining points on the boundary which have the same value by lines parallel to or at right angles to the base lines, in this case $7\frac{1}{7}$ units. Unless they lie a long way outside the boundary they should always be drawn to help in obtaining the correct form of the hyperbolas.

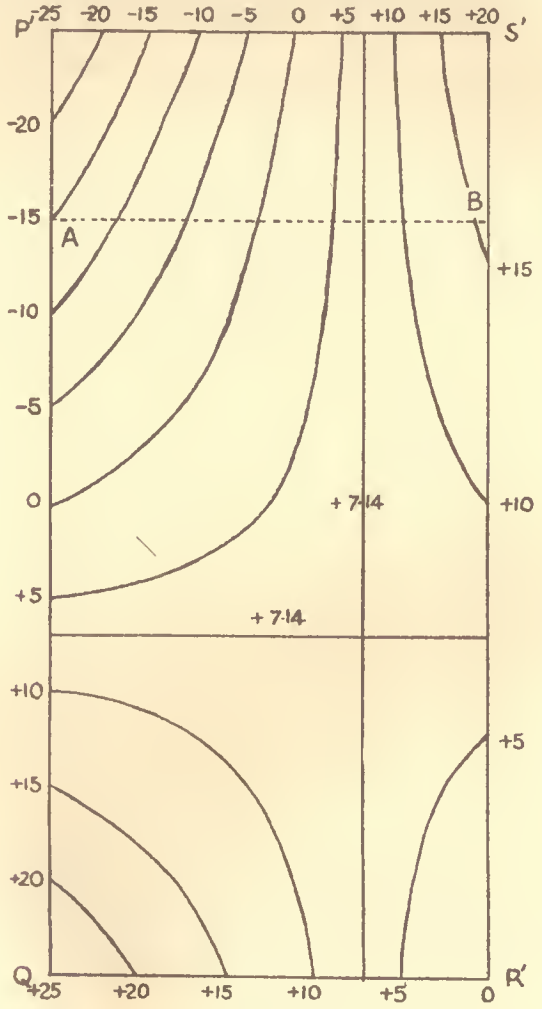


FIG. 179

With small height errors the subdivision illustrated will be sufficient to enable the curves to be drawn by eye. With larger errors more points will be necessary and these are obtained by interpolating linearly along lines parallel or normal to the base line.

For example, the dotted line AB cuts the boundaries at -15 and $+16$ units respectively. Interpolation between A and the asymptote will give cutting points for the $+5$, 0, -5 and -10 curves ; between B and the asymptote will similarly give cutting points for the $+10$ and $+15$ curves.

(*e*) **Equal and opposite fore and aft tilts.**

Fig. 180 shows the effect of equal and opposite fore and aft tilts, ϕ', of each photograph. The ground is raised or lowered bodily and deformed in such a

way that vertical sections parallel to the base are parabolas symmetrical about the mid point of the base line. The maximum deformation with which we are concerned is the amplitude $\frac{1}{2}$ B ϕ'. It will be seen later that ϕ' may be found from simple measurements on the photographs.

To draw the error contours, which are straight lines normal to the base line, a parabola of amplitude $\frac{1}{2}$ B ϕ' is made to pass through the two ends of the base. Lines parallel to the base line are drawn at the contour interval to cut the parabola. The contours are then lines passing through the intersections

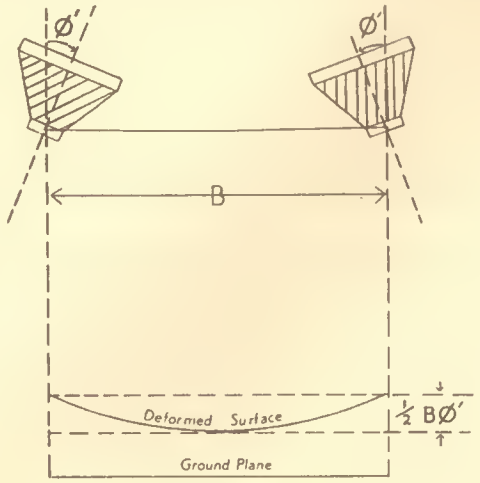


FIG. 180

of the parallels with the parabola and normal to the base line (Fig. 184, where these contours are shown in blue, makes this clear). This error will henceforth be known as *Parabolic Error* and will be taken to mean solely the deformation of the ground. It will not include the general raising or lowering,. which is fundamentally a linear error.

Before investigating this error, it should be noted that, so far, only equal-and-like and equal-and-unlike tilts have been considered, but it can be shown that any combination of different fore and aft tilts may be separated into equal fore and aft tilt (*b*) and equal and opposite fore and aft tilt (*e*), thus :

Let ϕ' be the hypothetical angle of equal and like tilt.

Let ϕ'' be the hypothetical angle of equal and unlike tilt.

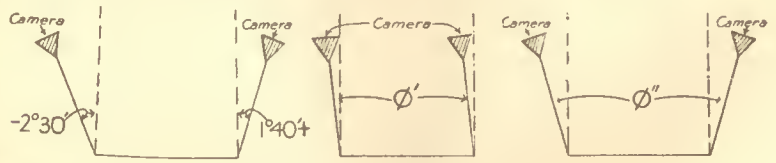


FIG. 181

then, in the case illustrated in Fig. 181, $\phi' + \phi'' = -2° 30'$

and $\phi' - \phi'' = +1° 40'$

therefore $\phi' = -0° 50' \div 2 = -0° 25'$

and $\phi'' = -4° 10' \div 2 = -2° 05'$

Determination of Parabolic Error. The amplitude of the parabola, $\frac{1}{2}$ B ϕ', may be found simply from measurements of "want of correspondence" (*i.e.*, the lack of alignment in one part of the photographs, when other parts, *e.g.*, the base lines, are in correct alignment) made on the photographs, without reference to any points of known height.

In the formulae which follow, signs are based on the assumption that the micrometer reading of a parallax bar *increases* with *increased* separation of the floating marks. If the readings decrease, the signs of the formulae (1), (2), (3) and (5) must be reversed.

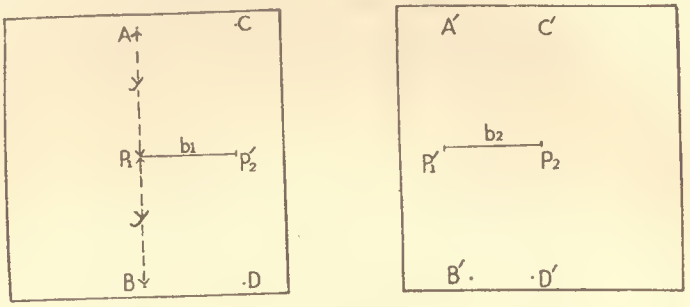


FIG. 182

Fig. 182 shows two overlapping photographs whose principal points are at P_1 and P_2. The photographic image of P_1 is at P_1' on the right-hand photograph and that of P_2 is at P_2' on the left-hand photograph.

A-A′, B-B′, C-C′ and D-D′ are four points common to the overlap lying at a distance y from the base line and opposite P_1 and P_2. It can be shown that to a first order,

$$\tfrac{1}{2} B \phi' = \frac{Bf}{8by} (A'B' - AB - C'D' + CD) \quad \ldots\ldots (1)$$

where B is the length of the air base

f is the focal length of the camera lens

b is the mean base length of the photographs $= \tfrac{1}{2}(b_1 + b_2)$

y is the distance of the points from the base line (it should not be less than b).

Fig. 183 shows the photographs turned through 90° in a clockwise direction and placed so that the pairs of points A-A′, B-B′, C-C′ and D-D′ can be fused in a stereoscope. $P_1 P_2'$ and $P_2 P_1'$ must be as near parallel as ordinary drawing methods will allow (though very small departures from parallelism will not materially affect the accuracy of the measurements).

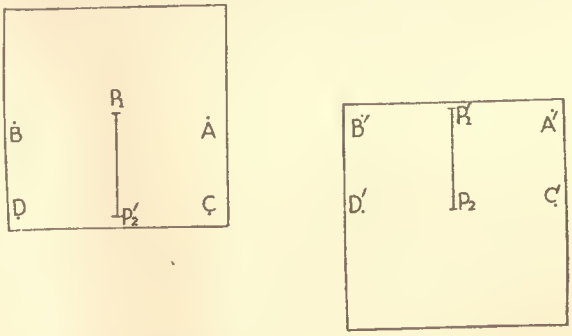


FIG. 183

Suppose that, when the floating mark is brought to the ground at the four points, the micrometer reads P_A, P_B, P_C and P_D then,

$$A'B' - AB = P_A - P_B$$
$$C'D' - CD = P_C - P_D$$

and the maximum parabolic deformation becomes,

$$\frac{Bf}{8by}\left[(P_A - P_B) - (P_C - P_D)\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

the *correction* to be applied to eliminate the error will be,

$$\frac{Bf}{8by}\left[(P_B - P_A) - (P_D - P_C)\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

When the correction is not too great (less than 15 metres) the assumption can be made that,

$$b = B\ \frac{f}{H\text{-}h} \text{ (which is very nearly true)}$$

hence (3) becomes,

$$\frac{H\text{-}h}{8\,y}\left[(P_B - P_A) - (P_D - P_C)\right] \text{ metres}$$

when H-h is given in metres and the P's in millimetres.

Within certain limits it is possible to choose any value for y so that if the value

$$y = \frac{H\text{-}h}{80} \text{ is chosen, where H-h is still in metres, formula (3) will}$$

become,

$$10(P_B - P_A) - (P_D - P_C) \text{ metres} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

thus eliminating all calculation except subtraction and multiplication by 10.

Certain combinations of H-h and the size of the photographs may make (H-h)/80 an unsuitable length for y. In such cases formula (3) must be used, or a suitable factor chosen so that a simple multiplier may be used in formula (4) instead of 10.

The same value for y may be retained throughout the strip provided H-h is reasonably constant. Equation (3), if it has to be used, may be computed on a slide rule.

Example of height correction when parabolic error is present :

A pair of photographs are orientated under the stereoscope and are then turned through 90° clockwise. (This setting will be helped by laying them over squared paper in the first instance so that their base lines lie in the direction of one set of lines on the paper.) Parallax readings are taken at A, B, C, D, in the order B, A, D, C, or " top left," " top right," " bottom left," " bottom right."

Point.	Bar reading (mm.)	Point.	Bar reading (mm.)
B	2·37	D	2·80
A	2·30	C	1·82
$P_B - P_A$	+0·07	$P_D - P_C$	+0·98

Whence $(P_B - P_A) - (P_D - P_C) = 0{\cdot}07 - 0{\cdot}98 = -0{\cdot}91$ mm.

If for this overlap $f = 127$ mm.

$B = 1540$ m.

y (measured with a scale) = 42 mm.

b (measured with a scale) = 42 mm.

then $\frac{Bf}{8\,by} = \frac{1540 \times 127}{8 \times 42 \times 42} = 13{\cdot}9$ and the maximum parabolic correction becomes $13{\cdot}9 \times -0{\cdot}91 = -12{\cdot}6$ metres.

The photographs may now be set in correspondence along their base lines and parallax readings made in the usual way on all points of known height and other points whose heights are required. The photographs should not on any account be disturbed until *all* the required height measurements have been made.

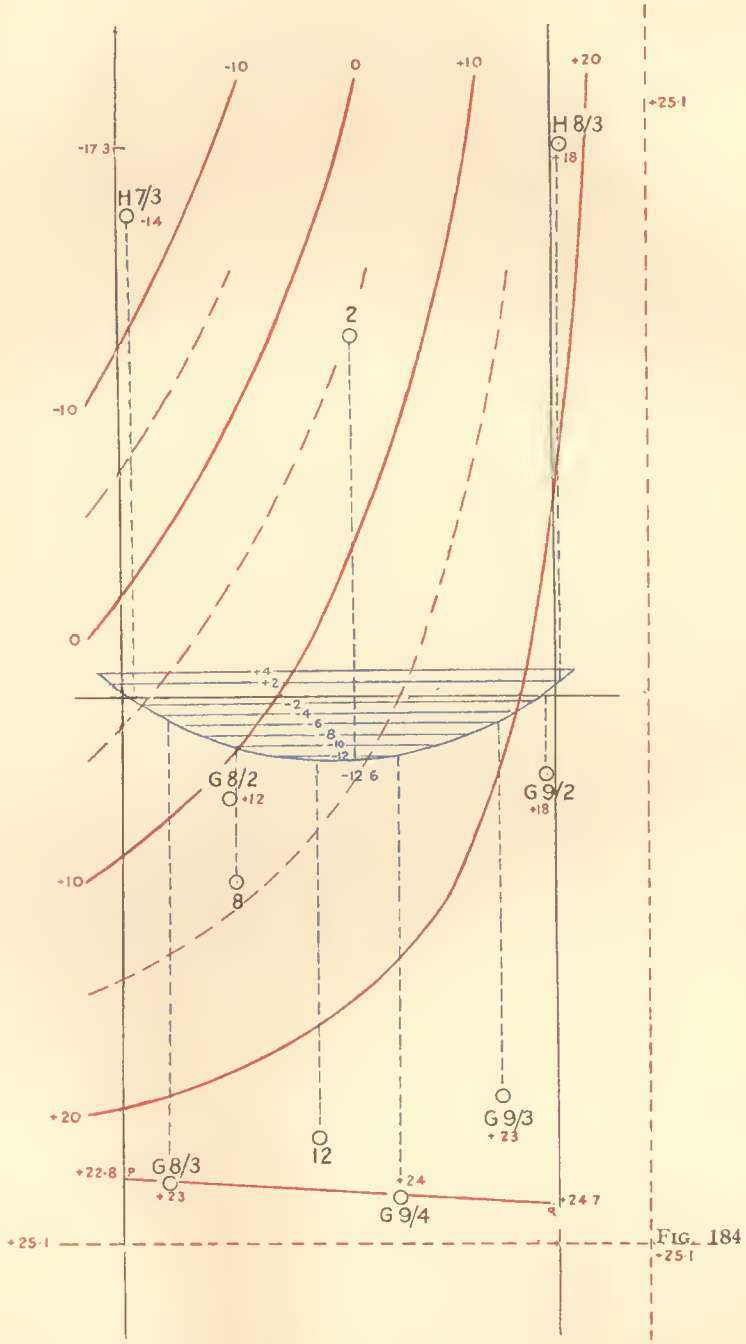


FIG. 184

A piece of tracing paper (see Fig. 184), large enough to cover the whole of the right-hand photograph is pinned down along its upper edge only, so that it may be lifted from the photograph. On this tracing, the principal point base line $P_1'\ P_2$ is marked together with two lines at right angles to and passing through the ends of the base line. The control points are then pricked through, circled and numbered (black in the figure).

A parabola is next drawn through the two principal points on the tracing

(blue in the figure). It is convenient to draw it convex upwards for positive maximum corrections and concave upwards for negative. It may be drawn by making a zinc template to the measurements given in Fig. 185 or by laying the tracing over the figure and carefully drawing the parabola. Since a standard parabola is being used its amplitude, which represents the maximum parabolic correction, depends on the length of the base line $P_1' P_2$; in this case it is —12·6 metres, which should be written against the curve. A series of lines are now drawn parallel to the base line to show corrections every 2 metres, shown in blue in Fig. 184.

The table below shows the parallax readings which have been made at the seven control points on the photograph.

1	2	3	4	5	6	7	8
Name of Point.	Micro. Reading.	Diff. from 10.	Calcd. Height.	True Height.	Total Corrn.	Parabolic Corrn.	Residual Corrn.
H 7/3	9·98	+0·02	120	104·8	−15	−1	−14
H 8/3	10·40	−0·40	88	107·7	+20	+2	+18
G 8/2	10·57	−0·57	75	76·6	+2	−10	+12
G 9/2	10·32	−0·32	94	111·9	+18	±0	+18
G 8/3	10·45	−0·45	84	102·0	+18	−5	+23
G 9/4	10·29	−0·29	97	108·6	+12	−12	+24
G 9/3	10·58	−0·58	74	90·5	+17	−6	+23

Column 1 shows the reference number of the control point and Column 2 the bar reading at that point. The micrometer readings have all been made purposely near to 10 mm. By this means datum error can be eliminated since, instead of referring six of the points to a seventh, all of them are referred to an arbitrary datum whose parallax reading is 10. This tends to reduce mistakes. Column 3 shows the parallax difference from 10.

Column 4 gives the height of each point as computed from the parallax measurements (or read from a scale similar to that shown in Fig. 171) and Column 5 gives its known height. Column 6 gives the total *corrections* which must be applied to the measured to obtain the true height, and contains the corrections necessary to eliminate some or all of the linear and non-linear errors ; the parabolic corrections can be separated, leaving only the hyperbolic and linear corrections.

By dropping perpendiculars from each control point to the base line to cut the parabola, the parabolic correction can be inserted in Column 7. If a point, such as H 8/3, lies outside the base line, the sign of the correction is reversed. As far as possible all points should lie inside the base line.

Column 8 gives the residual corrections after those in Column 7 have been subtracted from the total corrections in Column 6. These corrections are written in red against the control points on the tracing and the four best conditioned points selected. Ideally these should lie at the four corners of a rectangle whose shorter side is the base line. The best points in the example are H 7/3, H 8/3, G 8/3 and G 9/4.

Since parabolic corrections have been eliminated, the residuals must be made up of purely linear or hyperbolic corrections (which are linear at right angles or parallel to the base line). If G 8/3 and G 9/4 are joined by a straight line pq and extrapolated linearly, it can be deduced that the error at p is 22·8 and since p to H 7/3 is roughly normal to the base line, linear interpolation can be made to find the cutting points of the hyperbolas along this line. By a combination of interpolation and extrapolation, all other cutting points of hyperbolas can be found as well as the asymptotes, which are pecked in red on the figure. The hyperbolas can then be drawn as previously explained. The

three other control points which have not been used will provide a check on the accuracy of the method. It will be noted that point G 9/2 is in error by about 3 metres.

Where large discrepancies occur, they must be investigated. It is not difficult to make gross errors in parallax readings and these must be carefully watched. Other causes of discrepancies are incorrect determination of heights in the field and misidentification of points on the photographs.

Parallax readings of all points of unknown height are now entered in a table similar to that below.

1	2	3	4	5	6	7	8
Point.	Micro. Reading.	Diff. from 10.	Calctd. Height.	Parabolic Corrn.	Hyperbolic Corrn.	Total Corrn.	True Height.
2	10·10	−0·10	111	−13	+5	−8	103
8	10·27	−0·27	98	−10	+14	+4	102
12	10·55	−0·55	76	−12	+23	+11	87

Columns 1–4 are filled as in the previous table. Column 5 shows the parabolic correction and Column 6 the hyperbolic correction, both of which are obtained from the tracing. These corrections are combined in Column 7 and applied to Column 4 to obtain the True height in Column 8.

(It has been found, in practice, that time is saved by making all "want of correspondence" measurements, for the determination of the parabolic correction, throughout the strip before any height measurements are made. The greatest care must be taken to avoid gross errors of measurement or calculation.)

Fig. 185 shows a suitable template for drawing parabolas and constructing the hyperbolas. The parabolas may either be traced off this figure or plotted, from the dimensions given, on squared paper and then gummed to a piece of zinc which is cut and filed to the outline. The wider series of parallel lines will be found useful in subdividing the sides of the overlap when interpolating the cutting points of the hyperbolas, the narrower parallels for subdividing the amplitude of the parabola.

Summarising the previous ten pages, *Height Errors* consist of :

Linear

(*a*) Inclination of the air base.
Uniform deformation of photographic materials.

(*b*) Equal fore and aft tilt of both photographs.

(*c*) Equal lateral tilt of both photographs.
Incorrect orientation of the stereoscope.

Non-Linear

(*d*) Differential lateral tilt (Hyperbolic error).

(*e*) Differential fore and aft tilt (Parabolic error).

General.—Adhesive tape is better than drawing pins for holding the photographs down. Powerful lights, if brought too close to the photographs, will cause distortion of the paper.

Use of Air Photographs for Finding Under-Water Dangers

Under good conditions, *i.e.*, if the air photography can be done in clear weather and if the water is smooth and fairly free from sediment, under-water features, such as wrecks, reefs and sandbanks, are sometimes clearly visible in the photographs and can be studied in relation to the soundings to make sure that no dangers have been missed, but though a photograph may give positive evidence of the existence of a shoal it must not be assumed that deep water exists merely because no contrary indications are found on it.

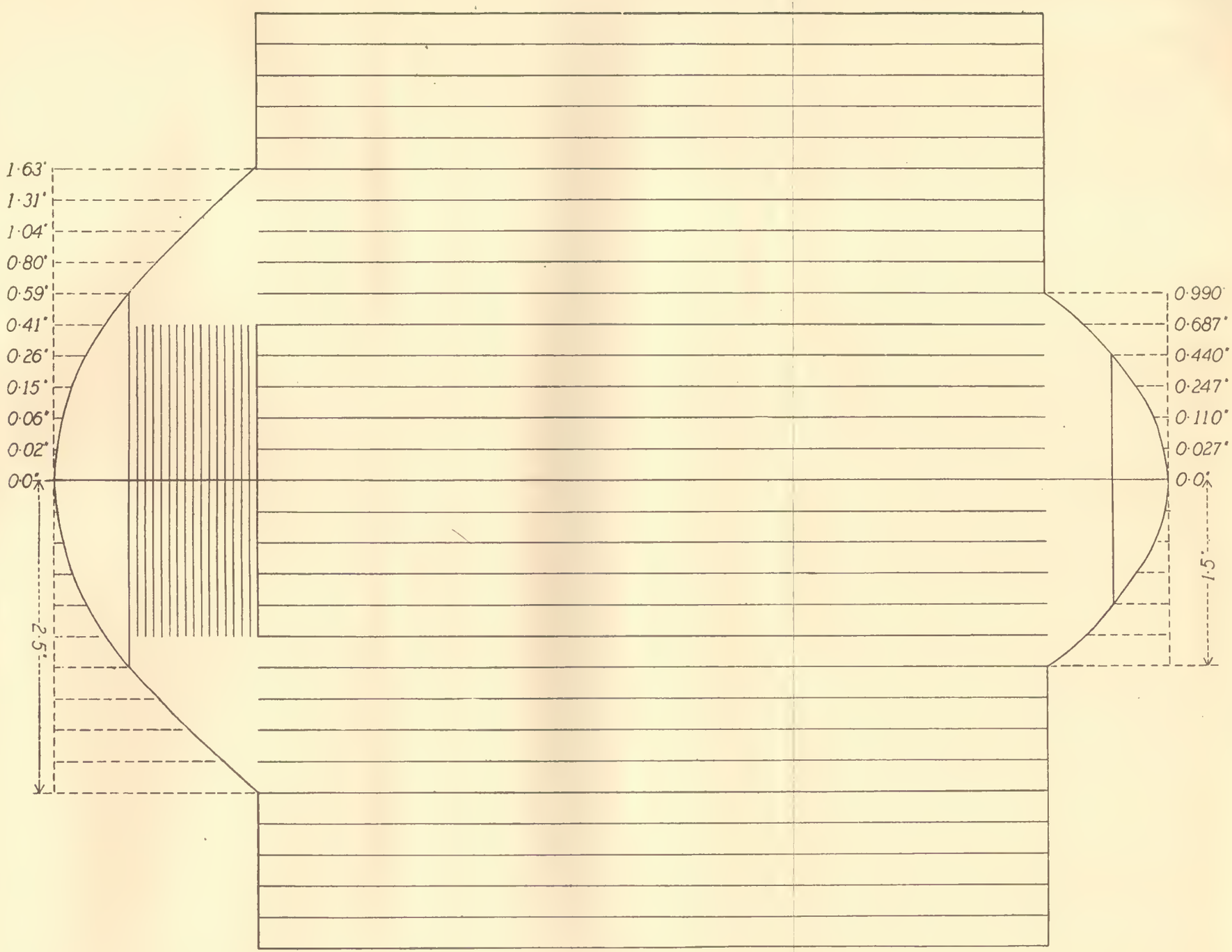

1·63"
1·31"
1·04"
0·80"
0·59"
0·41"
0·26"
0·15"
0·06"
0·02"
0·0"
2·5"
0·990"
0·687"
0·440"
0·247"
0·110"
0·027"
0·0"
1·5"

FIG. 185

CHAPTER XV

TIDES AND TIDAL STREAMS

The theoretical principles governing the phenomena of tidal movement in the ocean are fully discussed in the Admiralty Manual of Tides, and methods of prediction as well as instructions for observing and analysing the observations are contained in Admiralty Tide Tables, Part III.* The reader desiring more information on the subject than can usefully be included in this book is referred to those publications. The hydrographic surveyor should possess a general knowledge of the various types of tides and their causes, but he is more concerned with the practical business of making the necessary tidal observations for the reduction of soundings and analysis.

Admiralty Tide Tables, Part I, contain predicted times and heights of high and low water at a number of standard ports. Predictions at intermediate times based on this data are accurate enough for the purposes of navigation but must not be used for reducing soundings in a regular survey, since the method is only approximate and the effects of unusual meteorological conditions at any time cannot be allowed for in the tables and are not fully determinate except by actual observation on the spot.

The period of observation upon which the predictions at Standard Ports are based is given in Table V of Admiralty Tide Tables. At the great majority of these ports this period is at least a year, but at some it is only 29 days. The value of the data for predicting at the secondary ports in Part II of the Tide Tables varies widely and much of it is based on inadequate observations. At the great majority of the secondary ports and at those Standard Ports where the observations do not cover a complete year, any additional observations for periods suitable for analysis are of great value.

Chart Datum.—Reference has been made in Chapter X (p. 227) to the necessity of reducing soundings to a common datum which should be " a plane so low that the tide will but seldom fall below it." Heights on land maps are usually referred to mean sea level at some place on the coast, and this is an easily ascertained and fixed datum. The problem of the hydrographic surveyor, however, is more difficult both in choosing a datum and in selecting places suitable for the necessary tidal observations, for the range, times of high and low water and even the character of the tide may differ considerably at places not very far apart. For example, at Ramsgate spring rise is about 2 feet greater than at Margate, 6 miles away, whilst high water is about 25 minutes earlier and low water 27 minutes later. If, therefore, tidal observations at the former were used without correction for reducing soundings taken off the latter place, errors of more than 2 feet in the reduction may be introduced at half-tide.

Although the soundings in any particular survey must be reduced to a datum which is common in the sense that a low water which falls to datum in one part of the survey, falls also to datum over the whole survey, it will be seen that, owing to differences of range, chart datum is not necessarily at any fixed depth below mean sea level.

The problem of establishing chart datums for the survey falls naturally under three headings :

(1) Re-establishment of chart datum at a place where an accepted datum already exists.

(2) Transfer of chart datum from a place with an accepted datum to another place in the neighbourhood.

(3) Establishment of a new datum.

* *Note.*—Throughout this chapter frequent reference is made to Admiralty Tide Tables, Part III. This book is under revision and will shortly be reissued under the title " Admiralty Handbook of Tidal Prediction and Analysis for Seamen."

Re-establishment of Chart Datum.—In many parts of the world chart datum has already been established, usually by the original surveyors, and in such places this level should be used for a new survey unless instructions to the contrary are issued. The largest scale Admiralty charts will in such cases give the necessary information for recovering the old datum in one or more of the following forms :

(*a*) By a reference to one or more fixed bench-marks on the land, *e.g.*, " Soundings are reduced to a level 18·2 feet below a bench-mark cut on a wall. . . .", etc.

(*b*) By a reference to the datum used in the land survey of that area, *e.g.*, " Soundings are reduced to a level 10·1 feet below Ordnance datum."

Note.—(*a*) and (*b*) are now usually combined.

(*c*) By a reference to a particular state of the tidal movement, *e.g.* " Soundings are reduced to the level of mean low water springs." This form of reference is given on many old charts, but is now no longer used.

If the information is given in form (*a*), it is only necessary to locate the bench-mark referred to and to run a line of levels (see Chapter VII) from it to a tide pole erected at any convenient place in the vicinity. This will establish the difference in level between the zero of the tide pole and the bench-mark, and hence the tide pole reading of the required datum. For example, if chart datum is stated to be 18·2 feet below the level of the bench-mark, and the zero of the pole is found to be 21·6 feet below the bench-mark, then the datum for soundings will be a reading of 3·4 feet on the pole, and this figure will be subtractive from all readings on the pole to find the tidal reductions.

If the information is given in form (*b*), it will be necessary to find one of the bench-marks established by the land surveyors and level from this to the tide pole. The positions and heights above land survey datum of these bench-marks are usually shown on the larger scale maps, such as 25-in. and 6-in. ordnance survey plans of Great Britain. For example, if chart datum is to be 3·3 ft. below ordnance datum, and the zero of the tide pole is found to be 43·1 feet below a bench-mark shown on the map with the figures 39·0, then the required level of chart datum is 43·1—(39·0+3·3)=0·8 feet on the pole.

If the information is given only in form (*c*) or if no bench-marks or other points of known height can be located, it will be necessary to make a series of tidal observations in order to establish a new datum by the methods described later. Exact agreement with the old datum is usually out of the question, for datums of this description, such as " mean low water springs," " mean sea level," etc., in the older surveys were rarely established from a long enough series of observations to give any great accuracy.

Transferring an Established Datum.—There are now but few regions without established chart datums, and it is of great importance that neighbouring datums should be so co-ordinated that a tide which falls to datum at one, falls also approximately to datum at all. When, therefore, it is required to establish a new datum and there are established datums in the vicinity, or when a newly established datum is to be transferred to another part of the survey where there may or may not be appreciable differences in the tidal movements, transfer should be made by means of tidal observations obtained simultaneously at the place where datum is known and at that where it is to be established. The method employed to transfer the datum will depend upon the character of the tide.

(*a*) When the tide is mainly semidiurnal, that is, when, at the established datum, the harmonic constants show that H of M_2+S_2 is greater than 2H of (K_1+O_1), and when, in the absence of seasonal changes in mean level, the lowest tides may be expected to occur at equinoctial springs.

(*b*) When the diurnal component of the tide is relatively large, that is, when, at the established datum, the harmonic constants show that 2H of (K_1+O_1) is greater than H of M_2+S_2, and when, in the absence of seasonal changes in mean level, the lowest tides may be expected to occur on the days of maximum tidal range near the solstices.

Sometimes at the established gauge H of M_2+S_2 and 2H of K_1+O_1 may be nearly equal ; or the character may change so that the one which is the smaller at the established gauge may have become the larger at the new gauge. If the change in relative magnitudes is small, the formula applicable to the character of the tide at the established gauge should be used to transfer the datums.

In some parts of the world the characters of the tide in adjacent seas differ very considerably ; in consequence, the character of the tide in a strait joining those seas will be changing rapidly. In such cases datum cannot be transferred but must be established separately.

In most rivers the difference between the low water levels at springs and neaps decreases as the river is ascended and eventually the neap level may become the lower. Datum should not be transferred from the lower to the upper reaches in such cases. In the upper reaches of the river the changes in low water level due to meteorological effects are likely to be greater than those due to astronomical conditions and datum should be separately established from local information as to low river level.

In the case of (*a*), in order to make allowance for the meteorological effects upon the observations obtained for transferring datum, it is essential that at the established gauge, the heights of M.H.W.S. and M.L.W.S. should be known, either from Admiralty Tide Tables or from previous computations. From these heights the height of half-tide level at mean springs is computed ; this half-tide level will not necessarily be the same as mean tide level, given in Table V of Admiralty Tide Tables for Standard Ports, or computed from observations at other ports. The difference between half-tide level at mean springs and mean tide level is due to shallow water effects, and when these effects are large the difference may be considerable.

In the case of (*b*), in order to make allowance for the meteorological effects upon the observations obtained for transferring datum it is essential that at the established gauge, the height of mean sea level, Z_0, should be known either from Admiralty Tide Tables or from the harmonic analysis of a period of at least 15 days' observations.

(*a*) *To transfer datum when the character of the tide is mainly semidiurnal.*—At a spring tide, preferably a spring tide with not less than mean spring range, obtain comparative observations, at the two gauges, of the heights of four successive low waters and of the heights of the three intermediate high waters.

(1) From the low water heights a, c, e and g, calculate, for each gauge, the average low water height from $\frac{a+3c+3e+g}{8}$.

(2) From the high water heights b, d and f, calculate, for each gauge, the average high water height from $\frac{b+2d+f}{4}$.

(3) At the new gauge, from the average high and low water heights above zero of the gauge, calculate :

r′, the range
m′, the observed half-tide level

(4) At the established gauge, from the average high and low water heights above chart datum, calculate :

R′, the range
M′, the observed half-tide level

(5) At the established gauge, from the heights of M.H.W.S. and M.L.W.S. above chart datum, calculate :

M, the spring half-tide level

Then the height of the datum, d, above the zero of the new gauge will be :

$$m' + (M - M') - (M \times \frac{r'}{R'})$$

Example. Comparative observations for transferring datum :

	Established gauge (Heights above datum)				New gauge (Heights above the zero)			
	H.W.	L.W.	H.W.	L.W.	H.W.	L.W.	H.W.	L.W.
a	—	2·5 × 1		2·5	—	6·3 × 1		6·3
b	21·0	— × 1	21·0		20·8	— × 1	20·8	
c	—	3·5 × 3		10·5	—	7·5 × 3		22·5
d	20·2	— × 2	40·4		20·3	— × 2	40·6	
e	—	2·3 × 3		6·9	—	6·2 × 3		18·6
f	21·3	— × 1	21·3		21·0	— × 1	21·0	
g	—	3·2 × 1		3·2	—	7·1 × 1		7·1
Sum			82·7	23·1			82·4	54·5
H.W. = Sum/4, L.W. = Sum/8			20·7	2·9			20·6	6·8

Range = H.W. − L.W. . . . . R′ 17·8 r′ 13·8

Half-tide level = (H.W. + L.W.)/2 . . M′ 11·8 m′ 13·7

From A.T.T., or previous computation, M.H.W.S. = 20·0, M.L.W.S. = 1·0

Spring half-tide level, $M = \frac{20{\cdot}0 + 1{\cdot}0}{2} = 10{\cdot}5$

Datum on new gauge $= m' + (M - M') - (M \times \frac{r'}{R'}) = 13{\cdot}7 + (10{\cdot}5 - 11{\cdot}8) - (10{\cdot}5 \times \frac{13{\cdot}8}{17{\cdot}8})$

$= 4{\cdot}3$ above zero of gauge.

(*b*) *To transfer datum when the diurnal component of the tide is relatively large.*

On the day when the maximum range of the tide occurs, obtain at the two gauges, simultaneous observations of hourly heights for a period of 24 hours.

From the harmonic constants at the established gauge, as given in Admiralty Tide Tables or as computed from the analysis of a period of at least 15 days observations at the established gauge, compute the relative values of g′ and H for M_2 and O_1 from the formulae :

g′ of M_2 = g of M_2 − g of S_2 and H′ of M_2 = H of $M_2 \div$ H of S_2

g′ of O_1 = g of O_1 − g of K_1 and H′ of O_1 = H of $O_1 \div$ H of K_1

Using these relative values of g′ and H′, analyse the observations at both gauges (as instructed in Admiralty Tide Tables, Part III, Section III) ; thus obtaining :

at the established gauge, Z′ (the height of M.S.L. above chart datum) and the harmonic constants, M_2, S_2, K_1 and O_1.

at the new gauge, z′ (the height of M.S.L. above the zero of the gauge) and the harmonic constants, m_2, s_2, k_1 and o_1.

From the Admiralty Tide Tables, or from the previous analysis, obtain Z_o at the established gauge.

Then the height of chart datum, d, above the zero of the new gauge will be

$$z' + (Z_o - Z') - Z_o \times \left(\frac{\text{H of } m_2 + s_2 + k_1 + o_1}{\text{H of } M_2 + S_2 + K_1 + O_1}\right)$$

(*Note.*—In this method certain assumptions are made, notably in the relationship between the constitutents M_2 and S_2, and between K_1 and O_1. These are fully explained in Admiralty Tide Tables, Part III, Section III.)

Example.—To transfer datum from an established gauge for which the following constants are given in Admiralty Tide Tables :

M_2	S_2	K_1	O_1	Z_o
224°, 0·9	269°, 0·4	352°, 1·7	306°, 1·0	3·4

From these constants the following relative values are computed :

$M_2 : g'=315°, H'=2·2$; ($S_2 : 000°, 1·0$; $K_1 : 000°, 1·0$) ;
$O_1 : g'=314°, H'=0·6$

Using these relative values, analysis of the observations at the two gauges gives the following constants :

Established gauge :	M_2	S_2	K_1	O_1	Z'
	230°, 1·1	275°, 0·5	345°, 1·5	299°, 0·9	4·3
New gauge :	m_2	s_2	k_1	o_1	z'
	220°, 0·9	265°, 0·4	343°, 1·3	296°, 0·8	6·0

Then $d=6·0+(3·4-4·3)-3·4\times\left(\frac{0·9+0·4+1·3+0·8}{1·1+0·5+1·5+0·9}\right)$

$=2·2$ feet above zero of gauge.

Note.—Harmonic constants derived from 24 hours observations are seldom likely to be in complete agreement with those determined from a longer period of observation.

Establishment of a new Datum : Preservation of an Established Datum. Where there is no established chart datum and it is not possible to transfer datum from a previously established one by either of the methods previously described, a new datum must be selected and established.

On modern Admiralty Charts datum is not a tidal level directly related to any particular astronomical conditions, but is one below which the tide seldom falls. It is established by reference to some fixed mark ashore.

The method of selecting a suitable datum will depend upon the character of the tide, which must be determined from the harmonic analysis of observations extending over a period of not less than 15 days. If such a period of observation is not possible, then the harmonic constants may be analysed from two series of 24-hourly observations, separated by a suitable interval so that the assumptions may be eliminated, as instructed in Admiralty Tide Tables, Part III. The relative values of g' and H' will have to be deduced from those given, in Admiralty Tide Tables, Part II, for a port or ports in the vicinity.

(*a*) If the tide is mainly semidiurnal, that is, if the harmonic constants from the analysis show that H of M_2+S_2 is greater than 2H of (K_1+O_1), then datum is found from :
($Z_o \pm$seasonal correction) $-$ 1·1 (H of M_2+S_2).
The seasonal correction, if any, will be that given in Admiralty Tide Tables, Part II, for some port in the vicinity which is similarly situated either on the open coast or inside a river mouth or estuary.

(*b*) If the diurnal component of the tide is relatively large, that is, if the harmonic constants from the analysis show that 2H of (K_1+O_1) is greater than H of M_2+S_2, then datum is found from :
($Z_o \pm$seasonal correction) $-$ (H of $M_2+S_2+K_1+O_1$), the seasonal correction, if any, being found as in (*a*).

The above formulæ give chart datum relative to the zero of the gauge. To establish the datum for future reference, the level of chart datum on the gauge must be referred to bench-marks ashore. When land survey bench-marks are available, chart datum should be referred to them, preferably two of them.

If there are no such marks, then special bench-marks, preferably two of them, should be cut on masonry or solid rock in the most stable and permanent positions that can be found on land. On certain coasts there are no suitable features on which bench-marks can be cut and in those cases cement blocks should be set up on the most stable ground and bench-marks cut in them.

Tide Gauges and Tide Poles.—At many ports automatic tide gauges are maintained in continuous operation. In these machines the vertical movements of the water are transmitted to an arm carrying a pencil which traces a curve, representing the tidal movement, on a paper-mounted drum rotated by clockwork. The paper is usually marked with time and linear scales from which the recorded height of the tide at any time can be read. Where automatic tide gauges are in existence their records may generally be used with confidence by the hydrographic surveyor. A comparison with a few measurements from a fixed mark to water level at various states of the tide and a check on the time-keeping mechanism will serve to test the accuracy of the gauge's performance and it is only necessary to decide what uniform correction (if any) to readings is required to make them referable to chart datum.

Several attempts have been made to construct automatic tide gauges which can be used under the conditions normally met with in the course of a survey, and at the same time can be stowed easily on board ship when not required. Although good results have been obtained with some of them, their size and cost have been a barrier to their employment generally, and the surveys on which their use would be justified are not very many.

Two varieties of tide pole are in general use, an iron screw pole and a wooden telescopic pole. The former consists of an iron tube about 10 feet long with a screw-thread extension at its base so that it can be screwed a foot or so into the sea bed when this consists of sand, clay or mud. This pole is intended for use in open positions where there is no structure to which it can be lashed and it must be supported by wire guys attached to three or more anchors so that it will remain upright. A wooden pole inserted in the top serves as an extension when the tidal range is large. The wooden telescopic pole is of rectangular section and is made in three pieces ; when extended, it is 31½ feet in length. This pole is generally the most useful one when it is possible to lash it to a structure, *e.g.*, the end of a pier, but it may be erected without much difficulty in open water with suitable anchors and guys. As tide poles often have to be read from some distance, it is essential to paint the markings on them in the most easily distinguishable manner. This is best done by painting the whole of each foot alternately black and white. It is recommended that the figures representing the feet should be exactly 6 inches high and should be painted, black on the white part of the pole, and white on the black, with their bases resting on the exact foot mark designated by the figure ; their tops then indicate exactly 0·5 of a foot. The feet should be subdivided by horizontal strokes at every 0·2 of a foot.

The selection of a suitable position in which to erect a tide pole is a matter of some importance. It is obvious that the zero of the pole must be below chart datum or else it may dry out at low water. On first visiting a place where tidal information is scanty, it is often difficult to estimate the level to which the water may fall at spring tides and a reasonable margin against errors of judgment must be allowed. In any case, it is advisable to erect the pole at about the time of a low water, for there is then less liability to error and the actual operation is more conveniently performed since men will in most cases be able to stand round the pole and support it whilst the guys are being secured. When the range of tide is great, it is often convenient to have two tide poles, one to record the readings between low water and (say) mean tide level and the other between mean tide level and high water. This is particularly the case when the only available position for the pole is off a shelving beach, since at high water a pole placed to register low water readings may be so far from the water's edge that the figures cannot be seen by an observer on shore. When two poles are used they must overlap a small amount in the vertical plane and their relative levels must be determined from a simultaneous reading of the water level on both poles.

The second consideration in the selection of a position for the tide pole,

the readings of which will be used for reducing soundings, is that it should be adjacent to the sounding area in order to ensure as far as possible that the recorded tidal movement at the former truly represents that taking place in the latter. In the opening paragraphs of this chapter attention was directed to the fact that differences in the range and time of the tide are frequently found to exist in places comparatively close together. It is clear, therefore, that a single tide pole will serve only a small area ; when the sounding has been carried a few miles along the coast, it should be assumed that there *may* be a change in the range and (or) time of the tide, and another pole should be erected near the new sounding area for the continuation of the work, datum being transferred by one of the methods previously outlined. If it is only possible to erect tide poles at a considerable distance apart, the tidal movement at intermediate points can be obtained by interpolation between simultaneous readings. When the sounding area is a long way off shore usually the best that can be done is to erect a tide pole at the nearest point on the coast. The readings in such cases will form a *basis* for reducing the soundings ; the assumptions involved and the methods of obtaining the required reductions from the observations are discussed under the heading " Tidal Observations."

The third requirement for a tide pole position is shelter. Accurate readings cannot be obtained if there is much scend and there is, of course, more likelihood of the pole being carried away either by the weather or by passing vessels if it is in an exposed position. Nevertheless, the position must be fully open to the tide. For instance, the pole must not be erected inside the bar of a river (for the sake of smooth water) for reducing soundings taken outside the bar.

Very often the most suitable position for the tide pole is on a pier ; it can be lashed firmly to the structure and should be in no danger of accidental disturbance. If no position such as this can be found, the pole may be erected in any small bay which offers good shelter. A lee can often be found behind rocky ledges and it is sometimes feasible to add a few boulders to augment the protection from wind and sea. A pole that has to be erected in an exposed position should be marked by a flag or buoy to warn craft to keep clear.

When a tide pole has been erected, the *first* step should be to connect it, by levelling, with a fixed bench-mark on land (a temporary one will do) so that, in the event of damage or interference causing a change in the position of the pole, the datum level can be recovered. Thereafter, periodic tests should be carried out to make sure that there have been no undetected movements of the pole, such as may easily take place if it has to be erected in the open sea. When the pole is lashed to a permanent structure, a mark opposite any particular reading will reveal any movement which may have taken place ; if the pole is in the open, it is often possible to make the check by noting the reading at which the line of sight to the horizon cuts the pole when the eye is placed in a certain position on shore.

TIDAL OBSERVATIONS.

These provide, (*a*) The information required for reducing the soundings.
(*b*) The data for tidal prediction and investigation.

Observations for Reduction of Soundings.—For the reduction of soundings the tide pole should be read (to the nearest 0·1 of a foot) at half-hourly intervals whilst sounding is actually in progress. If, however, the tidal range exceeds 20 feet, readings should be obtained at 10-minute intervals.

The results should be plotted on squared paper as shown in Fig. 186, any convenient scale of times and heights being used. The scale of tide pole readings is on the left edge of the diagram and the scale of time along the bottom. A fair curve is drawn through the plotted points (shown with small circles) to eliminate small errors and mistakes of observation. In this example, for instance, it is clear that the reading at 1430, shown with a double circle, is a mistake, probably read off a foot wrong, and this is neglected when drawing the

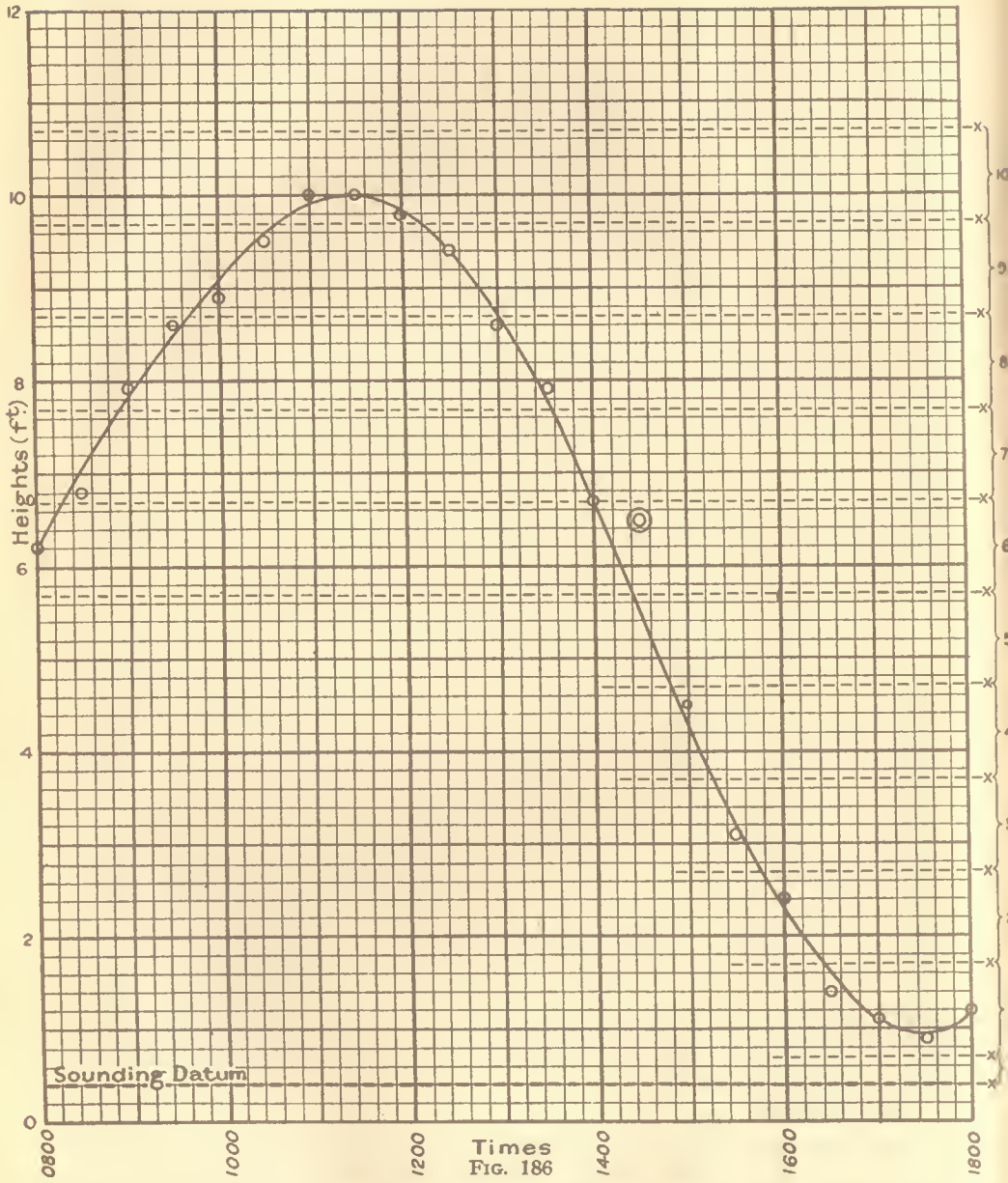


FIG. 186

curve. The times and heights of high and low water should be read off the curve, since this is more accurate than estimating them directly from the pole.

The general principle observed for the reduction of soundings in Admiralty surveys is that soundings are never increased by more than one quarter of the unit employed (see p. 227); therefore the tidal reduction changes to the next higher foot when the decimal of the height of tide above datum is 0·3. (When the unit is fathoms, this principle entails that 19 fathoms 4 feet reduces to 19 fathoms, and 19 fathoms 5 feet to 20 fathoms, etc.)

Suppose that the level of sounding (*i.e.*, chart) datum corresponds to 0·4 ft. on the pole, then, on the right-hand side of the diagram, points are marked (with crosses) at 0·7, 1·7, 2·7 ft., etc. (*i.e.*, 0·3, 1·3, 2·3 *above datum*). Horizontal

(pecked) lines, drawn through these points to cut the tidal curve, indicate the values of the reduction and the times at which it changes.

A table, which will provide the necessary data for the reduction of soundings, may be made out in the form : From 0800 till 0817, reduction = 6 ft.

,, 0818 ,, 0853, ,, = 7 ,,

,, 0854 ,, 0940 ,, = 8 ,, etc.

Except in special cases, such as when checking dredged depths, sounding in enclosed harbour basins, etc., it is the practice in Admiralty surveys to disregard fractions of a foot both in taking the soundings and in applying the reductions.

Hydrographic surveys often extend to considerable distances from the coast or may have no connection with the land at all, and the problem of obtaining tidal observations for the reduction of soundings is not so simple as it is in coastal or harbour work. There is no reliable information regarding tidal range in mid-ocean ; off a coast facing the ocean the range is likely to decrease progressively, but this may not be so in enclosed areas. In depths of over 20 fathoms, unless close to the land, tidal movements may be generally neglected without impairing the accuracy of the chart to any serious extent, or, alternatively, a small arbitrary correction, *e.g.*, 1 fathom, may be applied to all soundings. In shallower water, however, tidal movements must be taken into account.

Anything in the way of continuous observations at off-shore stations, whilst sounding is in progress, is usually impracticable, but useful data regarding tidal movements may sometimes be acquired by one or more of the following methods :

(*a*) Successful tidal observations have been made in the past with apparatus which could be used from the ship at anchor or which could be left to work by itself ; the principle of operation being to measure the change of pressure on or near the sea bed as the tide rises and falls. Unfortunately the apparatus was somewhat difficult to operate and was expensive, so that it has not been generally adopted. Up to date, however, this appears to be the most accurate means of obtaining tidal observations at sea.

(*b*) If in the area there are shoal patches with a fathom or less water, it may be possible to erect a tide pole on one of them, or on the edge of a drying bank (the mast of a wreck situated in the area may make an ideal support for the pole). If it is possible to read the pole at any time, then the established datum at a place on the coast in the vicinity should be transferred to this new pole by means of comparative observations, as already described. It may, however, be feasible to read the pole only under one of the following conditions:

(i) To make all the observations necessary for transferring an established datum ; but, during sounding, to observe only at the times of high and low water.

(ii) To make all the observations necessary for transferring an established datum, but to observe at no other time.

(iii) To observe only the times and heights of high and low water occurring during daylight.

(i) In this case the tidal curve for reducing soundings must be drawn through intermediate heights calculated from the times and heights of high and low water with the aid of Table I in Admiralty Tide Tables. This will give a good approximation, except in cases where there is marked distortion of the tidal curve due to shallow water effects.

Provided that the high and low water observations start before, and finish after, the period of sounding, this method can also be used with tides in which the diurnal component is relatively large ; but it must not be used when the diurnal component is so large that only one high and one low water occur in 24 hours.

No very high standard of accuracy can be claimed, but, when observations

are necessarily limited to a few readings at about the times of high and low water, it is probably the best that can be done in offshore positions.

(ii) In this case the ratio of the ranges, $\frac{r'}{R'}$; the mean tide levels, m′ and M′ ; and the mean time interval between the tides at the pole in the sounding area and that at the established datum (all calculated from the observations for transferring datum) must be used for computing the tides in the sounding area from tide pole readings at the established datum.

Example.—In a survey of the Kentish Knock Sand (lying about 20 miles off Margate) a permanent tide pole was erected at Margate, and a temporary one on the Kentish Knock Sand.

Observations for transferring the datum from Margate to Kentish Knock gave :

At Margate,	At Kentish Knock,
D = 2·3 ft. on the pole	d = 10·6 ft. on the pole
R′ = 14·3 ft.	r′ = 11·1 ft.
M′ = 10·35 ft. above zero of pole (or 8·05 ft. above chart datum)	m′ = 6·25 ft. above chart datum (or 16·85 ft. above zero of pole)

Mean time difference : Kentish Knock 5 minutes earlier than Margate.

Then, ratio of ranges, $\frac{r'}{R'} = 0{\cdot}78$.

The height of the tide at Margate referred to M′, when the reading on that pole is x feet = x—10·35 ft.

and, the corresponding height of tide at Kentish Knock
= m′ (above chart datum) +0·78 (x—10·35)
this height being reached 5 minutes before the Margate pole reads x feet.

Heights of tide at Kentish Knock computed by the above formula from tide pole readings at Margate :

At Margate			At Kentish Knock		
	Reading of	Height above		Height above	Height above
Time	Tide pole	M.S.L.	Time	M.S.L.	chart datum
1000	17·3	+7·0	0955	+5·4	11·7
1100	16·4	+6·1	1055	+4·7	11·0
1200	14·0	+3·7	1155	+2·8	9·1
1300	10·4	0·1	1255	0	6·3
1400	6·7	—3·6	1355	—2·8	3·4
1500	3·9	—6·4	1455	—5·0	1·2
1600	2·4	—7·9	1555	—6·2	0

These computed tidal heights at Kentish Knock are plotted as previously described, and the sounding reduction read off the fair curve.

This method gives a good approximation if the diurnal component of the tide is relatively small, and if the tidal curve is not very distorted by shallow water effects.

(iii) In this case, if the diurnal component of the tide is not very large, the datum will have to be transferred (though with some loss of accuracy) using only daylight observations of the times and heights of high and low water. To determine the sounding reductions, one of the methods described in (i) and (ii), above, must be employed, depending on whether the new pole can be read while sounding is in progress or not.

If no observations can be made ashore while sounding is in progress, and if there is no reason to believe that the tidal movement in the sounding area is of a type different from that on the adjacent coast, tidal movements should be predicted, by the methods given in Admiralty Tide Tables, Part III, for the nearest coast station listed in the Tide Tables, Part II (if the nearest coast station happens to be a standard port, predictions of high and low water are, of course, taken direct from Part I). The sounding reductions are then calculated

from these predictions combined with such observations as are available in the sounding area, by the appropriate method described above.

(*c*) If the establishment of a tide gauge in the sounding area is quite impossible, some indication of the tidal movement could be obtained by sounding, at various states of the tide, from the ship or boat anchored in a position where the bottom is level. These soundings combined with predicted tides at the nearest coast station could supply rough data for calculating the sounding reductions. The need for resorting to such methods is, however, exceedingly unlikely.

Tidal Observations for Analysis and Investigation.—The methods of analysing tidal observations for the computation of harmonic and non-harmonic constants are fully described in Admiralty Tide Tables, Part III. A continuous series of readings of the water level at hourly intervals is required for the purpose of analysis, and in finding the harmonic constants a period of 29 days is regarded as a unit ; when a longer series is available, the constants are calculated independently from each unit of 29 days. When observations cannot be continued as long as 29 days, the constants can be calculated from a 15-day series and finally, if circumstances preclude a 15-day period, approximate constants can be found from two series of 24 hours. Tidal readings for the purpose of analysis should be recorded to the nearest 0·1 of a foot, and the greatest care should be taken to ensure accuracy in all readings, especially when 24-hours series are being observed ; with longer series occasional errors of reading will not affect the results appreciably. The time used for the records should be the zone time applicable to the region, as given on the Admiralty Time Zone Chart.

Mean Sea Level.—The determination of mean sea level is of great practical importance and scientific interest. It is, however, subject to seasonal variations and cannot accurately be determined from less than a year's observations. (It should be noted that when the harmonic constants are computed, mean sea level on the central day of the series is obtained automatically. If the seasonal changes in mean sea level for a port in the vicinity are given in Admiralty Tide Tables, then the correction appropriate to the central day of the series should be applied to the computed height of M.S.L. (A_o or Z_o in harmonic notation). This may only be done if both ports are on the open coast ; it is not valid if one of the ports is in a river.)

Tidal Prediction.—Methods of predicting tidal movements from the data in Parts I and II of the Admiralty Tide Tables are fully described in Part III of that publication and need not be repeated here. Predicted heights should not, as a rule, be used for reducing soundings in a regular survey, but they are required for sweeping (see Chapter XI) and must sometimes be used for sounding if tidal observations cannot be made (see above).

Tidal Streams

Observations to determine the flow of tidal streams and currents are a necessary part of a hydrographic survey. The accumulation of really comprehensive data regarding horizontal water movement in any particular area is hardly practicable within the time normally available, since an extensive series of observations would be required at a very large number of stations. These horizontal movements of the water often vary considerably at places only a short distance apart, and the rate (and possibly the direction) of the stream is not generally constant at all depths below the surface in any one position. In consequence, although observations over short periods should never be neglected, complete observations, as described on page 322, must be limited to a few selected positions where the results will be of most value to the mariner and will provide data for approximate interpolation at other points in the area.

Apparatus and Methods Employed

For the measurement of tidal streams and currents various contrivances have been evolved to meet different requirements; they may, however, be divided into two main categories.

(*a*) *Drifting Logships*, which may be subdivided into:

(i) Those which are allowed to drift freely and whose positions are fixed, during the period of the observation, by an accompanying boat, or from stations ashore.

(ii) Those attached to a marked log line and which are streamed and observed from a vessel at anchor.

(*b*) *Current Meters*, which are lowered from a ship or boat at anchor, or are attached to a buoy, and are fitted with mechanism to register the flow of the stream during a certain period.

Types of all the above can be used for observing both surface and sub-surface currents; and one type in category (*a*) can be used for obtaining an average direction and rate between the surface and any desired depth.

Drifting Logships.—With the exception of those used for observing purely surface currents, the same types of logship are used whether they are allowed to drift freely, as in (*a*) (i), or are attached to a marked log line, as in (*a*) (ii). A ship (or boat) at anchor in a current or tidal stream usually yaws considerably and, with any wind "sails" towards and away from her anchor, so that the results of observations made from her will be burdened with certain indeterminate errors unless some accurate means can be found for plotting her movements relative to the ground while the observations are in progress. Consequently, whenever possible, the freely drifting logship should be used in preference to one streamed from an anchored vessel, especially for observations in harbour approaches, narrow channels and rivers. Generally speaking, observations from anchored vessels should be confined to deep water, or wide and unrestricted channels, where the whole body of water is moving in a more or less uniform direction at a more or less uniform rate.

Pole Logships. The most important observation from the navigational point of view (since it gives, more nearly than any other, the set actually experienced by a ship), and the one that should always be given priority, is that which measures the *average* direction and rate of flow between the surface and any depth below the surface, down to 30 feet or so (according to the depth of water and the average draught of the vessels using the waterway in which the observations are being made). For this, a pole logship is used, consisting of a light spar, 2½ to 3 inches thick, either square or round in section, but which must be of a uniform thickness throughout its entire length. The spar, cut to the required length, is weighted to float vertically with its upper end just showing above water. The weights must be of the same thickness as, and should form part of, the spar itself, so that a true average of the sets at all depths, down to the bottom of the spar, may be obtained. The spar should be surmounted by a small white flag, or multi-surface reflector, either of which show up clearly at night in the beam of an Aldis lamp or powerful torch.

Surface Logships. Any small buoy or fisherman's pellet will serve as a freely drifting logship for measuring surface currents, provided its draught does not exceed one foot and its freeboard is kept to the minimum (to avoid windage) consonant with it being plainly visible by day and, in the beam of a torch, at night.

Iroquois Logship. For obtaining the direction and rate of flow at any depth below the surface, from 6 to about 60 feet, the "Iroquois" logship can be used. Its construction can be seen in Fig. 187. The buoyancy is adjusted by means of the cork E until the buoy just supports the canvas vanes but no more.

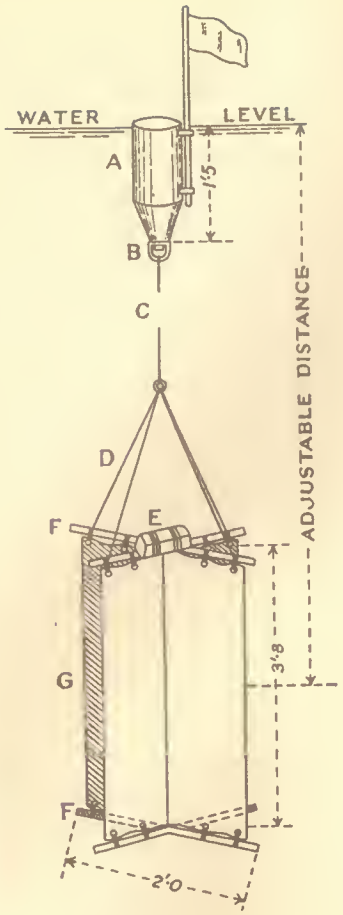


FIG. 187

A, cylindrical iron buoy, carrying a small flag or reflector.
B, swivel for attaching distance and log lines.
C, distance line of $\frac{1}{2}''$ S.S.F.W. ; this can be made up in various lengths according to the depth at which the observations are required.
D, slings of $\frac{1}{2}''$ S.S.F.W.
E, cork for adjusting buoyancy.
F, cross-pieces of iron flat, $1'' \times \frac{1}{4}''$.
G, canvas vanes seized to cross-pieces.

Methods of observing freely drifting logships. The methods are the same for all types. A small buoy should be anchored in the position at which the observations are required (though if there are sensitive cross transits intersecting the position the buoy may be dispensed with during daylight observations). The logship is carried in a boat, and at regular hourly intervals, it is streamed close up-stream of the buoy. When drifting freely its position is fixed from the boat and the time noted, after which the boat backs clear and then drifts along with the logship. The logship is allowed to drift for a short period, of about one or two minutes, according to the rate of the drift, and is then approached and its position fixed *before* it is picked up (or touched in any way). It is useful, for checking the terminal fixes, as well as for spotting vagaries in the direction or rate of the flow, for the boat to approach the logship closely and fix its position two or three times during its drift, though the greatest care

must be taken to avoid violent engine movements when close to the logship for fear of affecting its free drift. Fixes must be made with the utmost accuracy and the times of fixes must be recorded to seconds. For night observations the objects used for fixing must be illuminated by lamps, or by searchlight from the ship.

When observations are being made fairly close to the shore, such as in the entrance to a harbour, it may be preferred to fix the positions of the drifting logship from a conveniently placed shore station, by bearing (observed with theodolite, sextant or compass) and distance (observed with a range-finder). For night observations the logship can be illuminated by the accompanying boat.

Triangular Logship. For observations of surface currents from an anchored vessel, the logship of traditional design is commonly used, and although it appears always to underlog slightly, it is probably as simple and effective for its purpose as any that can be devised. It consists of a segment of stout wood with an angle of about 120° and a radius of about 2 ft. Sufficient sheet lead is nailed on near its circumference to make it float with its apex only just above water, so that it will not be appreciably affected by the wind. For use from the ship or a boat at anchor, a log line of small size (usually cod-line) and about 100 yards in length is required. One end of this is secured near the apex of the logship, and ordinary bottle corks are attached at intervals of not more than 15 ft., so that it will float on the surface as the logship drifts away. Distinctive markings should be inserted in the line at every 10 ft., starting with a zero mark at a point about 10 fathoms from the logship end.

Methods of observing from an anchored vessel. The log line having been rove through a fairlead aft, the logship is lowered into the water and allowed to drift away. At the moment the zero mark passes through the fairlead, the time is noted and the direction of the outboard part of the line is observed with a prismatic compass, or preferably a gyro repeater mounted on the taffrail. After an even number of minutes, depending on the strength of the stream, the amount of line which has run out is read and its direction is again observed. If the logship itself is visible at the beginning and end of the run, it is better to observe bearings directly to it and it may also be possible to obtain its distance directly by range-finder or by an angle of depression below the horizon. The marking of the log line at 10-ft. intervals simplifies any calculations of the tidal stream rate, since each 10 ft. of line run out per minute is equivalent to a velocity of 0·1 of a knot.

The determination of the direction and rate of flow from observations of this type is not so simple as would, at first, appear ; nor, as a rule, can any high degree of accuracy be expected, since the movements of the ship about her anchor during the observation, can, in most cases, be determined only approximately. Sometimes it may be possible to obtain station pointer fixes by sextant angles to shore objects at the beginning and end of the run, and in such cases the movement of the logship can be ascertained from a plot on a mooring board with all required accuracy. If, however, a land fix is not obtainable, no entirely satisfactory means have been evolved for detecting under all conditions the true movements of the observing position and hence those of the logship.

A method, which has been used with success in moderate depths, is to attach a small-sized wire, marked in fathoms, to a sinker and to lower it to the bottom, from the stern, at the beginning of the observation, paying out the wire but keeping it hand taut while the run is in progress. The sinker being stationary, the movement of the stern and hence that of the logship can be found, on a mooring board, by plotting the bearing of the wire and the amount run out at the end of the run multiplied by the cotangent of its angle of depression.

An alternative method is to fix the position of the stern by bearing and distance from a buoy which has been tautly moored with heavy anchors just

outside the swinging radius of the ship. The buoy must have a large reserve of buoyancy to avoid being towed under in a strong stream and every precaution must be taken to restrict its movement to the absolute minimum. It must be lighted at night or illuminated by a searchlight.

The ship should be fixed by bearing from the standard compass and range-finder distance of the buoy (*a*) when the logship is let go, (*b*) when the strayline has run out, and (*c*) at the end of the observation.

The observation spot itself can be fixed by noting the direction of the ship's head and applying the known distance along this direction at each observation.

The following example will illustrate the correction of observations. This is best done by plotting on squared paper or a mooring board.

Distance from standard compass to observation spot is 90 feet. Length of strayline is 50 feet.

(*a*) Logship let go. Ship's head 359°. Buoy 090°—480 feet.

(*b*) Strayline out. Ship's head 018°. Buoy 080°—470 feet. Direction of logship from observation spot 279°.

(*c*) End of observation (2 minutes after (*b*)). Ship's head 009°. Buoy 075°—370 feet. Direction of logship from observation spot 250°. Amount of line out 200 feet (+50 feet of strayline).

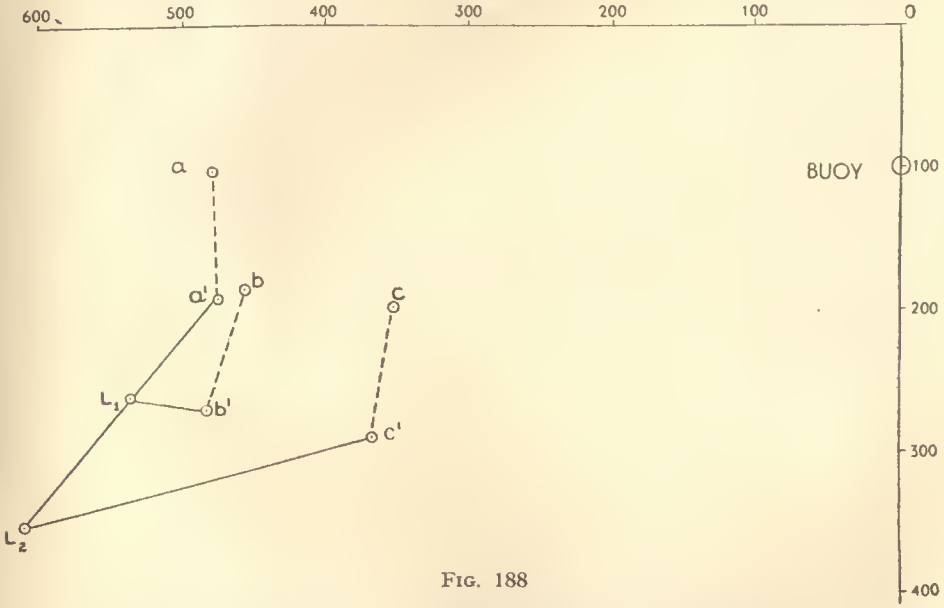


FIG. 188

In Fig. 188, *a*, *b*, *c* are the positions of the standard compass, *a'*, *b'*, *c'* are the corresponding positions of the observation spot. L_1 is the position of the logship when the strayline has run out and L_2 its position at the end of the observation.

From the plot, L_1L_2=220°, 117 feet ; *i.e.*, the stream is 220°, 0·6 knots.

Correction by this method can only be considered as approximate since the buoy is certain to be moving slightly during the observations and wind or surface stream will affect the buoyed log line and hence the logship. *a'*, L_1, L_2 should in theory plot as a straight line but, for the reasons given above, it is unlikely that they will do so.

The methods of making the observations with the pole logship and Iroquois logship from an anchored vessel are the same as those described above. In the case of the pole logship the log line is secured to the upper end of the spar; with the Iroquois logship it is secured to the swivel (B in Fig. 187).

Current Meters.—Various instruments have been designed for the purpose of making direct measurements from a stationary position of the rate and direction of the horizontal movement of the water at different depths. All such instruments rely on a magnetic compass for obtaining the direction measurements, but two entirely different principles have been invoked for obtaining the velocity measurements :

(1) From the number of revolutions made in a given time by a small propeller which is revolved by the flow of the water, *e.g.*, the Ekman Meter.

(2) From the angle of tilt taken up by a heavy disc which is constrained to the vertical by gravity, *e.g.*, the Wollaston Meter.

Ekman Current Meter.—This instrument is designed for use from a ship or boat at anchor where the stream does not attain a rate of more than 3 to 3½ knots. It is illustrated in Plate P.

The instrument is lowered and hoisted by a wire (½" F.S.W.), A, which is marked at every fathom, or may be led over a measuring sheave, and is fitted with an eye splice to take over a lug on the spindle B which passes right through the body of the instrument so that the latter can revolve freely. A sinker, consisting of a fish-shaped lead about 25 lbs. in weight, is attached to the link C so that the supporting wire does not grow out very far from the vertical in the stream. The action of the water on the vane D keeps the instrument fore and aft the stream. A very light propeller E is freely revolved by the flow of the water past the instrument and turns a train of sprocket wheels in the compartment F, which serve the double purpose of :

(1) Registering the number of revolutions of the propeller on the dials shown on the face of the compartment.

(2) Freeing a number of small bronze balls previously loaded into the magazine G, and allowing them to fall one by one, after every so many revolutions of the propeller, into the compass box H.

The compass needle is in the form of a sloping trough and the balls, falling into the upper end of the trough over the compass pivot, roll down and fall into compartments of the circular box J. A double set of compartments, arranged concentrically, is fitted in J and each compartment embraces an angle of 20°.

It is necessary to arrange for the recording gear to be started *after* the instrument is lowered to the required depth and stopped before it is hauled up : this is effected as follows :

Before being lowered, the instrument is " cocked " by closing the two doors K K, which lock automatically, and pushing the lever L towards the wire. The propeller will then be free but the closed doors prevent the water flowing through and turning it. After being lowered to the required depth a messenger N is fitted on the wire and dropped. Sliding down the wire, the messenger strikes the lever M, the doors are opened by the action of a spring and the water can now pass through to revolve the propeller. To stop the recording gear, a second messenger is fitted on the wire and let go. This strikes the first messenger, transmits the blow to the lever M and now allows L to move to the left. A toe actuated by L stops the propeller and the instrument may then be hauled up.

A position should be selected for operating the meter where the tidal stream will be affected as little as possible by the hull of the ship. At shallow depths it is best to suspend it from the end of the lower boom provided the ship is not rolling too much ; in depths greater than the draught of the ship, it may be operated from the stern. The procedure in use is as follows :

(1) Having assembled all parts and attached the suspending wire and sinker, load the magazine with balls, set the dials to zero and cock the instrument.

(2) Lower to the required depth and wait about a minute for the compass to settle down. If the wire is seen to grow out much, its inclination should be estimated and lowering continued until the correct depth is reached.

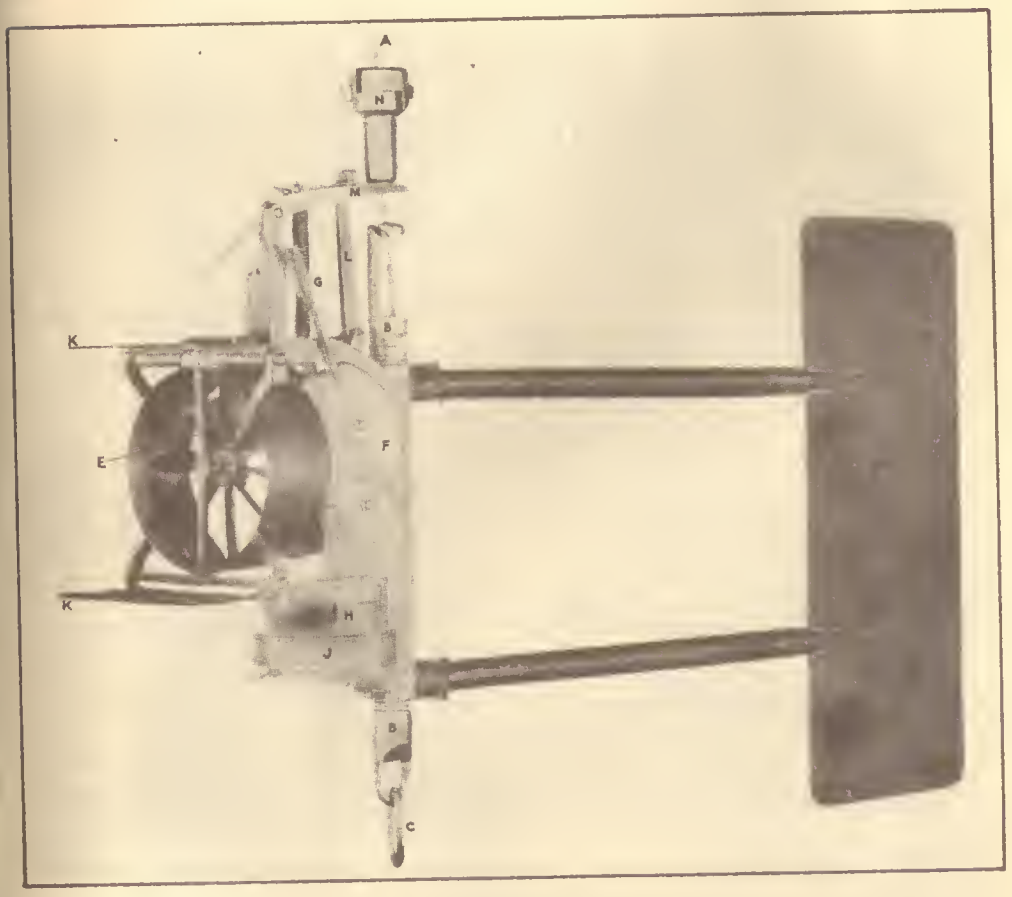


PLATE P

(3) Release the first messenger and start a stop-watch at the moment of its impact on the meter. This impact can be felt in moderate depths by placing a finger on the wire. At deep settings, however, this may not be possible, and the watch should be started when the first messenger is released and stopped when the second is released ; it can be assumed that they fall in equal times.

(4) Allow the meter to run for about two minutes in strong tidal streams, for four or five if the stream is weak.

(5) Release the second messenger and note the time of impact (or release) as before.

(6) Haul up the instrument, note dial readings and remove box J to count the balls in the compartments.

Calculation of the results is effected as follows :

(*a*) The rate of the stream is calculated from the number of revolutions per minute by means of a curve or formula derived from calibration tests and supplied with the instrument.

(*b*) The direction *towards* which the stream flows is found from a count of the balls in each compartment of the box J. If, for example, the balls are found to have fallen thus :

Compartment ..	010°–030°	020°–040°	030°–050°
Number	2	5	3

The mean direction is .. $\dfrac{2 \times 020 + 5 \times 030 + 3 \times 040}{10}$

$= 031°$.

Observations obtained by current meters of this type are subject to some inaccuracy due to the movements of the ship and the meter itself. No useful results are achieved by attempting to correct the observed readings for the ship movements (which may be approximately estimated), whilst no means even exist for ascertaining the movements of the meter. By repeating the observations several times, errors from these causes may be largely eliminated.

Wollaston Current Meter.—This instrument is much larger and heavier than the Ekman meter and is suitable for measuring currents up to six knots. Its external appearance is shown in plate Q.

The instrument is lowered and hoisted by a wire pendant (1″ F.S.W.) secured to a ball bearing shackle A which allows the rest of the apparatus to rotate freely about an approximately vertical axis. Below A, a vane B, attached to the rod C, serves to steer the meter into the current, and the main frame D is shackled to the lower end of C. The main frame carries two heavy streamlined sinkers EE and bearings F for the drum G which contains the recording mechanism. Two hangers on the drum carry the pressure bar H to which are attached, by means of rods, the pressure plates JJ and counterbalance weights KK. Two sizes of pressure plate are supplied, the larger for use for currents up to about 3 knots, the smaller for currents up to about 6 knots. The action of the water on the pressure plates turns the drum G through an angle dependent on the strength of the current.

A detailed description of the recording mechanism, with instructions for assembly, running and maintenance, is contained in a booklet supplied with the instrument by the makers (Messrs. Hughes & Son).

Briefly, the nature and operation of this mechanism is as follows :

The end plates of the drum are removable to permit access to the recorder, but are fitted to make watertight joints. Bars secured on the inside of the cover plates carry bearings for the recorder gear which is mounted on a base plate, suspended between these inner trunnions, so that angular movements of the outer drum in the fore-and-aft line of the instrument are not directly communicated to the remainder of the mechanism. The base plate carries a central clock-driven drum on which a blank paper chart is secured ; this rotates every 24 hours and runs for 8 days, or a fast running clock can be supplied to rotate the drum once in an hour and run for 30 hours. The latter clock can be started and stopped by mechanism accessible through a screw plug at the top of the outer casing, but the standard or slow clock is not provided with starting and stopping mechanism.

The *speed* record is made on the paper by a Ditmar pen mounted on a carriage which is drawn, between vertical guide rods, by a fine cord attached to a lever which, in its turn, is connected to one of the bars on the inside of the end plates. Since the recorder is slung on trunnions, it is not tilted by a rotation of the outer drum, but the rotation is transmitted to the pen which is given a vertical movement corresponding to the angle of tilt. A damping system is provided to eliminate the effects of any oscillation caused by waves and surging of the meter.

The *direction* record is made on the same paper by means of a cylinder, mounted, between pivots, on a carrier attached to the base plate and containing two parallel magnets. The outer surface of the cylinder carries a raised spiral

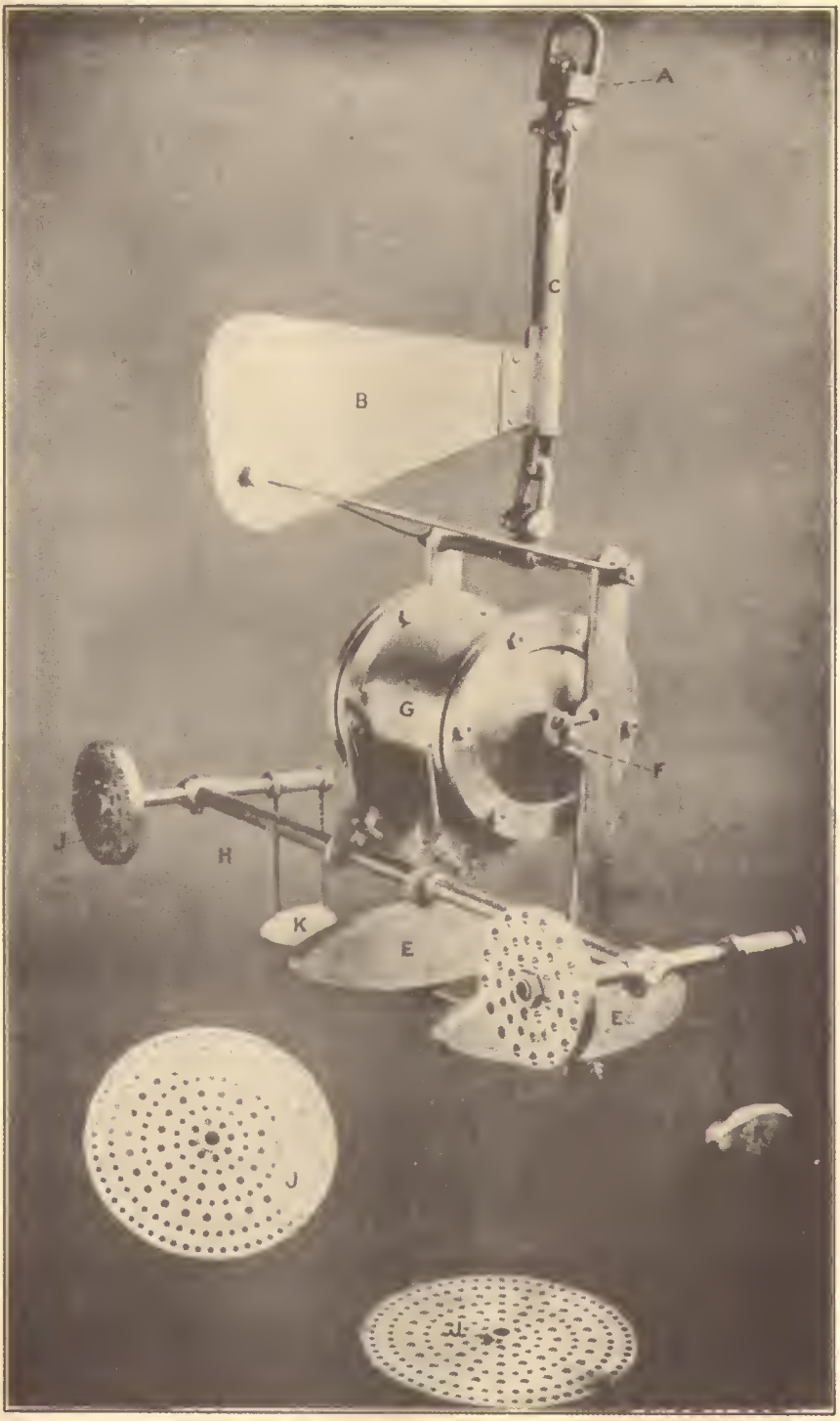
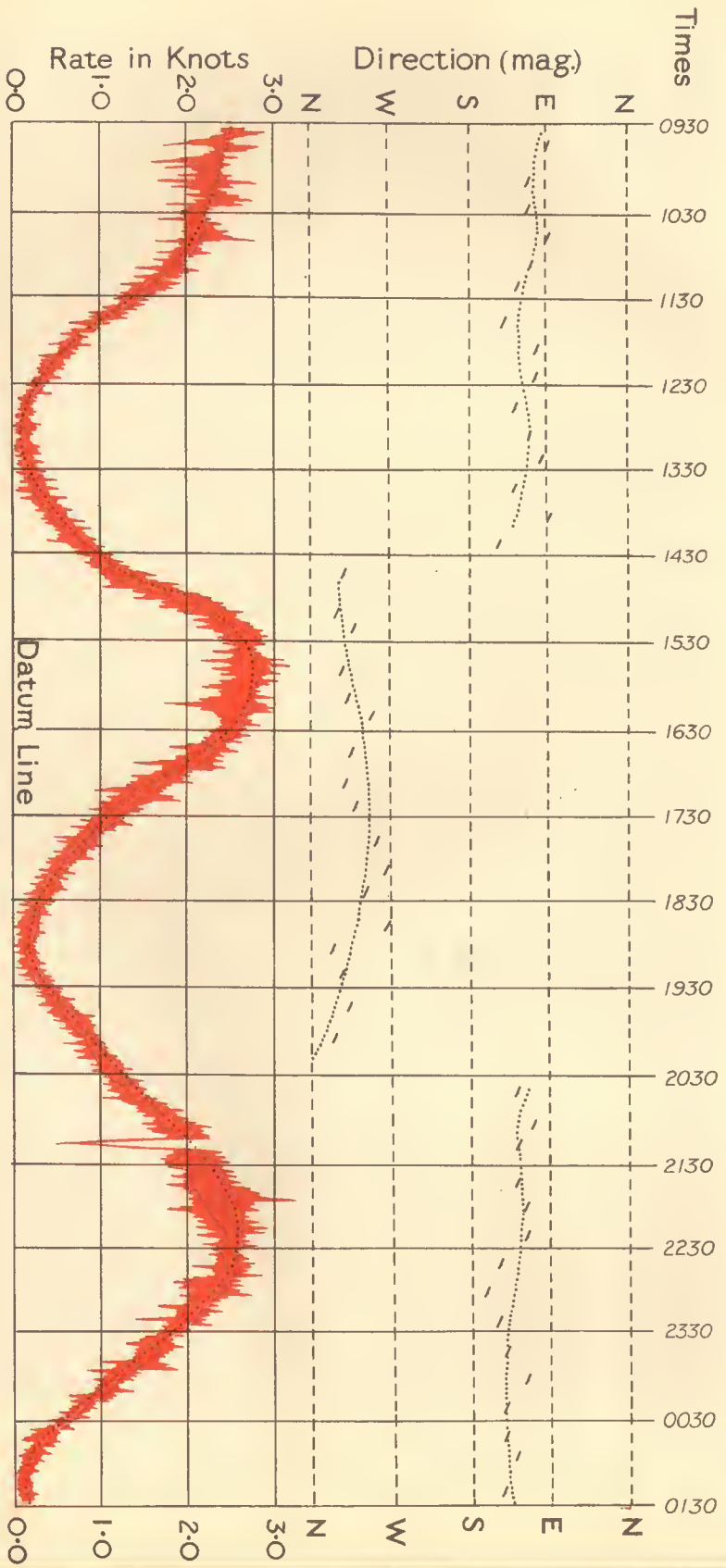


PLATE Q


Rate in Knots
0·0
1·0
2·0
3·0
Direction (mag.)
N
W
S
E
N
Times
0930
1030
1130
1230
1330
1430
1530
1630
1730
1830
1930
2030
2130
2230
2330
0030
0130
Datum Line

covered with inked ribbon and is not normally in contact with the clock drum. The cylinder carrier, however, is itself pivoted and is provided with a pawl engaging with teeth on the base of the clock drum. As the latter revolves the pawl is alternately lifted and allowed to fall into a space between two teeth ; when it falls, the cylinder with its ribbon is propelled towards the clock drum by a spring and leaves a mark on the paper from that portion of the spiral which makes contact. The motion is arranged to withdraw the cylinder again as soon as the mark has been registered. The height of the mark on the paper above the clock drum base indicates the direction towards which the current is flowing.

Calibration of each instrument is carried out before it is put in service and two sets of glass scales (one for each pair of pressure plates) graduated for the particular instrument, are supplied for reading off the speed and direction of the current from the record. A datum-marking device is embodied in the instrument and the datum line is marked on the scales so that they can be correctly placed on the paper.

A portion of a typical record is shown in Fig. 189. The speed markings are shown in red and the direction is indicated by a series of dashes at 20-minute intervals. Fair curves, as shown by the dotted lines, should be drawn and the rate and direction are then read off by means of the glass scale appropriate to the pressure plates in use. For convenience, a scale of speed, direction and time has been added in the figure.

The Wollaston current meter may be used from the ship at anchor in the manner previously described, but, since a continuous record is made, it is unnecessary to haul it up repeatedly as must be done with the Ekman meter. The results are subject to errors of uncertain magnitude due to movements of the ship, and at shallow settings the compass may be affected by the proximity of the hull. Owing to its weight (about 1¾ cwt.) the meter is difficult to operate from small boats.

A continuous series of current observations may also be obtained by leaving the meter secured to a buoy anchored in a sheltered position ; in open waters it is probable that the motion of the buoy would prevent the recording apparatus working satisfactorily. If a buoy is used it is necessary to have an arrangement to prevent the meter fouling the buoy moorings when the stream changes. This may be effected by securing the float, which must be used to support the meter, at the centre of a wire span connecting two buoys anchored well apart, or, if only one buoy is used, the float and supporting wire can be kept clear by securing it at one end of a distance piece, *e.g.*, a 30-foot pole, which has its other end attached to the buoy.

Roberts Radio Current Meter.—This machine has been developed by the U.S. Coast and Geodetic Survey. It consists of a floating raft which carries a radio transmitter, beneath which is suspended, at the required depth, the meter. At regular intervals a signal is emitted from the transmitter which is received by either the parent vessel or a shore station. By the use of several such meters, whose transmitters are operating at different frequencies, a number of observation spots may be occupied simultaneously and the parent vessel is not immobilised during the observations.

The construction of the meter is, briefly, as follows (see Plate R) :

The housing of the meter B is of streamlined shape and is suspended from a yoke K so as to be steady and stable in all directions. The horizontal and vertical fins C and D keep the meter head to stream and, to minimise yawing, a wire is led from the impeller guard F to the float moorings.

The impeller A has four blades and is magnetically connected to the driving element ; there is no mechanical connection between the two. The arrangement consists of two identical soft iron armatures, one on the impeller at N and one on the spindle of the meter-drive mechanism at M. Each armature has six

"alnico" studs mounted on its face near the periphery and the magnetic attraction between the armatures ensures that the armature at M rotates in sympathy with that at N. There can, therefore, be no leakage of water into the body of the meter. "Alnico" is an alloy of aluminium, nickel and cobalt and is capable of holding a very high magnetic charge. The deviation imposed on the compass needles by this arrangement is stated to be no more than one degree.

The compass system consists of two "alnico" needles, Q, so spaced as to

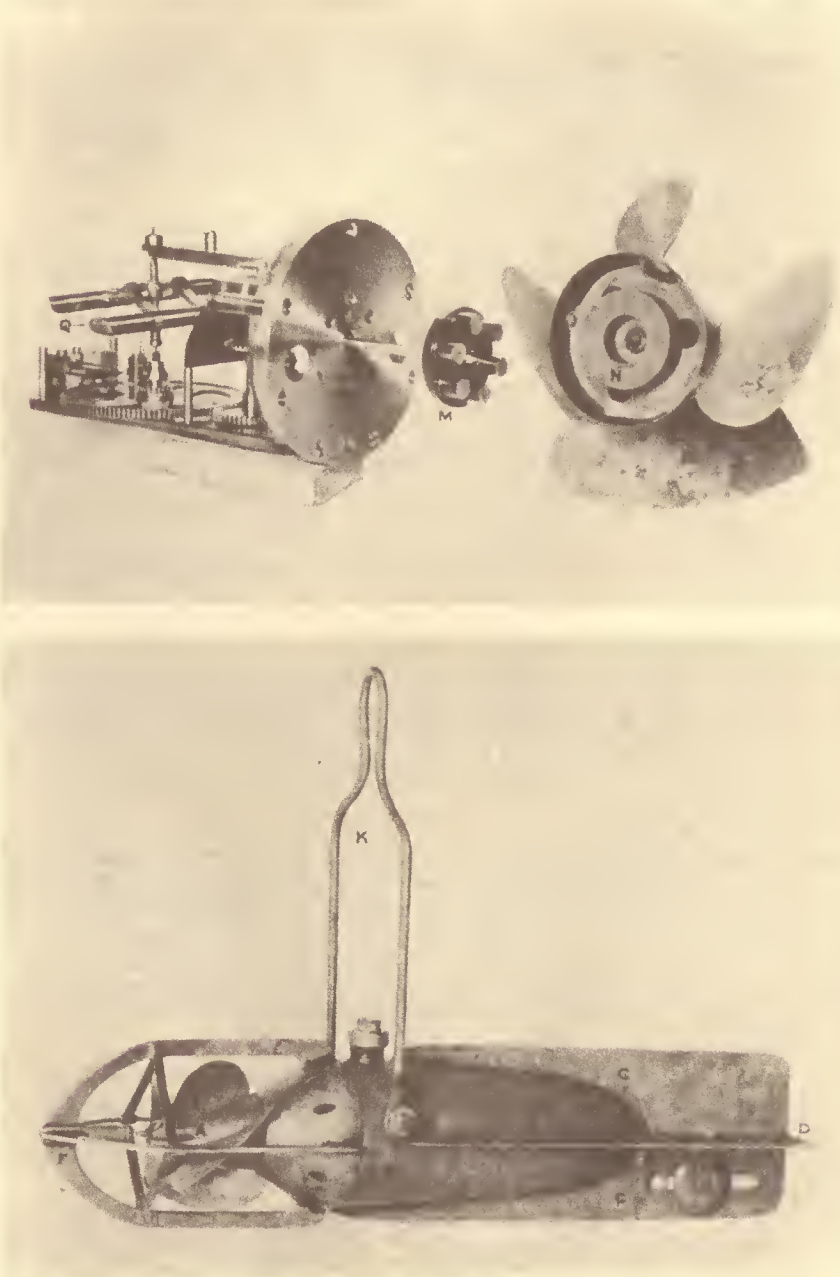


PLATE R

have equal moments of inertia about the N–S and E–W horizontal axes. They are mounted between bearings in gimbals.

The operation of the meter is as follows :

The meter aligns itself to the current and the impeller transmits its rotation magnetically to the gearing inside the meter body.

Two sets of contacts are provided within the meter ; one set is always orientated in the magnetic meridian by the compass needles, the other is orientated in the fore-and-aft line of the meter and hence in the line of the stream. A revolving contactor driven from the gearing can make contact with both sets of contacts.

Velocity signals are emitted regularly as the rotating contactor passes the compass contacts with each revolution. Direction contacts are made, every second cycle only, when the contactor passes the other set of contacts.

The signals are transmitted to the float and thence by radio to the receiving station where they operate one stylus of a chronograph, the other stylus being operated by a clock.

It will be seen therefore that the Velocity (" V ") signals can be distinguished on the chronograph tape from the Direction (" D ") signals from the fact that the latter only appear every other cycle. It will also be seen that as the direction of the stream changes so the position of the " D " signal will alter on the tape, relative to the positions of the " V " signals on either side of it.

Fig. 190 shows an example of three tapes where the stream velocity is the same on each but the direction is different.

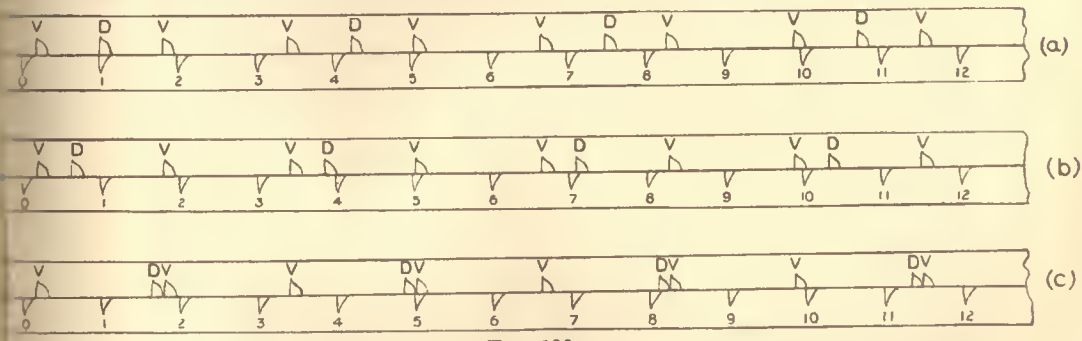


FIG. 190

The ticks of the clock are at the bottom of the line common to each stylus and are shorter than the " V " or " D " marks since the meter contacts remain closed for a longer period than those of the clock.

It will be noted that on tape (*a*) the " D " marks are exactly half-way between the " V " marks and thus indicate a stream direction of 180°. On tape (*b*) they are a little less than one-third the distance from the preceding " V " mark to the following one—approximately 100°. On tape (*c*) they are very close to the " V " mark which follows them indicating, approximately, a stream of 340°.

The meter is still under development. Present claims for it are that it will indicate velocities correctly down to 0·2 knots and directions to an accuracy of 10°

TIDAL STREAM AND CURRENT OBSERVATIONS REQUIRED

(*a*) The principal requirement is for observations that will give the tidal stream and current that would be experienced by a ship drawing 30 feet (or in shallower water, by a ship of correspondingly less draught). These observations should be made with the pole logship, as previously described, preference always being given to observations with a freely drifting logship.

(*b*) Whenever possible, the direction and rate near the bottom, preferably within a fathom of it when weather conditions allow, should always be obtained at tidal stream stations occupied by the ship at anchor.

(*c*) In intricate channels or at the meeting place of two or more waterways, the streams in the lower layers may differ very considerably from those in the upper layers, both in direction and rate. When the drift of a pole logship appears to differ from the surface flow, it should be investigated by observing the streams in the different layers (observations on the surface, at 3 fathoms and at 5 fathoms will usually suffice). Each set of observations must be analysed separately.

A series of observations of any of the above types consists of 25 or preferably 26 consecutive observations at hourly intervals. A half-series consists of 13 consecutive observations at hourly intervals, or, if the observations are interrupted, of a complete series of which not more than 11 consecutive observations have been omitted. A series or half-series cannot be made up from broken observations.

Although definite results can be obtained from a half-series when diurnal inequality is small, a complete series is necessary when the diurnal inequality is appreciable.

As the subsequent investigation and analysis of the tidal streams must generally be based on a short series of observations, it is of the greatest importance that each observation should be as accurate as possible.

In Home Waters, where the tides and tidal streams are predominantly semi-diurnal the observations should be made at or near the occurrence of spring tides and, when circumstances permit, a further set of observations should be made at or near neap tides. Observations abroad should invariably be made in two complete series, separated by a period of about 7 days, as instructed in Section III of Admiralty Tide Tables, Part III.

All observations should, of course, include the direction of the stream as well as its rate.

Currents

Horizontal movement of the water is the resultant of tidal stream and current, and it is this resultant which is recorded by the observations which have been described in this chapter. Currents may be permanent, seasonal or temporary, and their direction and rate, both surface and sub-surface, are of much interest and great importance. The distance off shore, and the depth, to which the tidal streams extend are generally unknown ; it is probable that in shallow enclosed seas, such as the North Sea, the tidal streams extend over their whole area down to the sea bed ; whereas off coasts where the bottom falls steeply to oceanic depths, the tidal streams are probably felt only a short distance away from the shore, and outside this distance all horizontal water movement is due to current.

Except for observations from a ship or boat at anchor (themselves far from being entirely reliable, see p. 310) all methods of observing currents must be considered to give only approximate results, subject to unknown errors. The method of assuming that the difference between the true and dead reckoning positions is due to a steady current lasting through the period between absolute observations for position is unsound, being based on approximate data only ; while that of observing the surface current from a boat riding to a weighted sea anchor, of large superficial area and veered to a great depth, only gives the difference between the surface current and the current at the depth of the sea anchor. Observations of the drift of a boat away from an anchored buoy (anchored with a sinker and sounding wire if the depth is great) are probably more reliable, though there can never be certainty that the buoy has not dragged.

Reliable conclusions about currents can only be drawn from a great number of observations (each of which is, by itself, of little value) spread over long periods of time. No opportunity, therefore, should be lost of obtaining such observations, both surface and sub-surface, by any means which ingenuity may suggest.

Analysis of Tidal Stream and Current Observations

Detailed instructions for the analysis and prediction of tidal streams and currents are contained in Admiralty Tide Tables, Part III. In general, the methods used are similar to those for analysing and predicting vertical movement of the water, but as the measurements on which they depend are less accurate and of shorter duration, the results are not so precise.

CHAPTER XVI

ASTRONOMICAL OBSERVATIONS

Observations for Position.

The location of a point on the earth's surface is defined in terms of latitude and longitude, *i.e.*, by its geographical co-ordinates which must be determined by astronomical observations at the point itself or derived through the triangulation from observations at some other point.

The figure of the earth is very nearly that of an oblate spheroid which can be regarded as formed by the revolution of an ellipse about its minor axis.

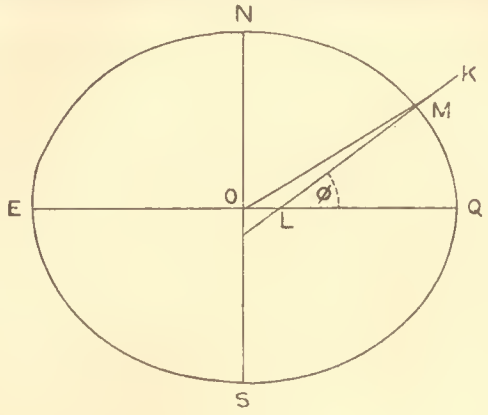


FIG. 191

NQSE, Fig. 191, is a plane section through the poles, N and S, of the earth and E, Q are the extremities of the equatorial axis which is about 23 sea miles longer than the polar axis NS. The *geographical latitude* (ϕ) of any place M on the meridian NQS is the angle at which a normal KML to the ellipse at M cuts the plane of the equator, *i.e.*, the angle MLQ. The *geocentric* latitude of M, *i.e.*, the angle MOQ, is only of minor importance to the hydrographic surveyor. The direction KM, in the absence of local deflection, coincides with that of the plumb line or vertical at M and the latitude of that point can be found by measuring the zenith distance of a heavenly body of known declination when it crosses the meridian NQS.

Longitude is measured from the meridian of Greenwich Observatory.

NQSE, Fig. 192, represents a hemisphere, N and S being the poles and EQ the equator. NGS is the prime meridian through Greenwich, G. If NMS is another meridian through any point M, the longitude (ω) of M is the angle GNM=GSM which may be measured in time or arc. The time at which a heavenly body crosses the meridian of Greenwich can be found from the data in a Nautical Almanac ; if the time of its passage across the meridian of M is observed, the difference will be the longitude of M.

Astronomical observations are an important element in any original survey and call for the highest standard of accuracy compatible with the use of portable instruments. The navigator may only be able to determine his position to the nearest mile or so, but he is entitled to expect that the geographical positions of points on the chart, as scaled off from the graduation, are free from *any* appreciable error. From the point of view of the map-maker and cartographer

an important reason for accuracy is that the relative location of individual and possibly overlapping surveys must often be effected solely by reference to

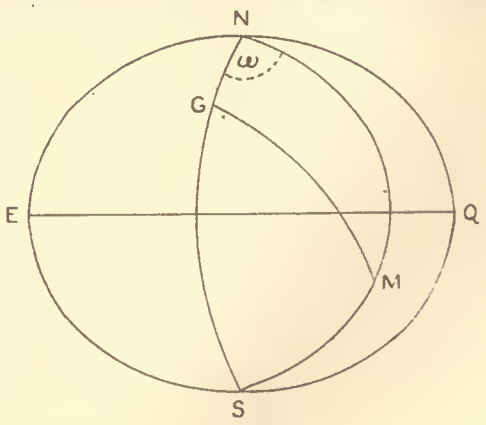


FIG. 192

the geographical co-ordinates of points fixed in the two surveys.

Observation Spot.—Normally in a hydrographic survey geographical co-ordinates are determined by means of astronomical observations with an astrolabe, theodolite, or sextant and artificial horizon, at some point on the land known as the observation spot. In choosing a suitable site for an observation spot, the following considerations should be borne in mind :

(1) The observation spot should be sited where it is possible to leave permanent marks, surface and sub-surface, for future reference.

(2) It must be in such a position that it can be directly connected by means of angular observations, to the main triangulation so that the geographical co-ordinates of other points of the survey may be readily calculated.

(3) It should be reasonably adjacent to a landing place so that transport by land of bulky and delicate instruments is reduced to the minimum.

(4) The ground should be firm, preferably rock, so that the observing instruments can be set up on a perfectly stable foundation.

(5) The position should be reasonably well sheltered from the wind which is not only liable to shake the instruments, but also fills the air with dust. A place exposed to a moderate breeze, however, may be preferable to no wind at all if mosquitoes and similar pests are numerous.

(6) It should be far enough from the shore to be free from earth tremors caused by the surf.

Accuracy of Astronomical Observations.

With modern instruments and under good conditions the results of a series of observations should give a position apparently correct to within about 1″·5 of latitude and 2″·5 of longitude. This standard of accuracy, however, is based on the assumption that the direction of the plumb line at the place of observation is the true vertical, or, to express it in a slightly different way, that the surface of a pool of mercury is truly " level " or that the bubble in a spirit level occupies the " highest " point in its tube. Careful observations have shown that in many parts of the world small deflections of the plumb line from the vertical exist ; normally they are not likely to exceed a fractional part of a second of arc and are therefore of negligible proportions, but cases have been found where the deflection amounts to several seconds. The vicinity of large mountain masses should be suspect and the surveyor should, when

possible, avoid observing in such positions, but he has no *certain* means of ascertaining from the superficial character of the ground whether deflection of significant proportions is likely to exist and he can only draw a conclusion from a comparison of the results of observations made at different points in the survey. The triangulation networks of hydrographic surveys rarely extend over distances of more than 200 to 300 miles measured in a direct line and, as scale and azimuth can be checked without much difficulty at various stages of the work, the accumulated error in the relative geodetic positions of two points near the extremes of the survey should be of very small proportions compared to the possible errors of astronomical observations for latitude and longitude at those two points. In comparing the results of the astronomical observations, therefore, at different points in a hydrographic survey with those obtained by triangulation, it is usual to assume that all discrepancy may be safely assigned to the observations themselves; if it is more than about 3″ in latitude and 5″ in longitude (*i.e.*, if it exceeds the discrepancy which may be expected if the normal errors in astronomical observations of 1½″ and 2½″ are in opposite senses at the two observation spots), deflection of the plumb line may be suspected at one or both places.

Time

In all astronomical observations for geographical position or azimuth the Greenwich Mean Time of each observation (or mean of a series of observations) must be determined. Since an error of 1 sec. of time may result in an error of 15″ of longtitude (*i.e.*, about 1,500 feet at the equator or 750 feet at latitude 60°), great accuracy in the determination is required. Before the introduction of the electric telegraph and radio signalling, the problem was one of considerable difficulty. The times of physical phenomena, such as eclipses, lunar distances, occultations, etc., could not be observed in the field with the degree of precision required by the surveyor, and recourse was had to the transport of a large number of chronometers from some place of reference of known or accepted longitude to the scene of operations. At the place of reference the errors of all the chronometers were carefully determined and on passage to and return from the survey area their steadiness of running was noted by means of daily comparisons. On arrival back at the place of reference the chronometer errors were again determined and it was assumed that in the interval their rates had been constant or had varied in accordance with empirical laws connecting changes of rate and temperature. Neglecting those chronometers which showed anomalous variations indicative of irregular running, the G.M.T. at the time of the observations was deduced from a mean of the errors of the remainder. Recent observations have shown that longitudes determined by this method were often several seconds (of time) in error, and this is hardly surprising in view of the length of voyages in those days.

Chronometer errors in all modern surveys are determined from comparisons with radio time signals and, since a large number of these are sent out daily from stations in various parts of the world, it is usually possible to obtain chronometer errors within a few hours of the time at which the observations are made. During *this* period, *i.e.*, in the interval between comparisons obtained before and after the observations, the surveyor must rely on the chronometer to maintain a steady rate, and for this reason time-keeping instruments of the highest class must still be used.

Chronometers.

The present-day equipment of a surveying ship consists of three or four chronometers, one of which is fitted to operate a chronograph, and a few chronometer watches and deck watches. Instructions for the care and treatment of chronometers and watches are issued in pamphlet form by the Royal

Observatory, Greenwich, and are contained in most works on Navigation, and the only notes considered necessary here are as follows :

(1) Chronometers should be wound daily at the same hour, whether they are of the two-day or eight-day type.

(2) Errors by radio time signals should be determined at intervals of not more than a few days in order that the steadiness of rate may be ascertained.

(3) Comparisons between the chronometers should be made daily at the same hour.

The ordinary marine chronometer beats every half-second of mean time and the method of comparing two chronometers is as follows :

Call one of the chronometers (usually that with the smallest and steadiest rate) A, open the glass lid so that its beats may be easily heard and start counting from 0 when the second hand reaches (say) the 55 second mark. Continuing to count the beats of A by ear, look at the second hand of chronometer B which should have its glass lid shut so that its beats cannot be heard. At the count of ten, *i.e.*, the even minute on A, note the time shown by B, estimating to the nearest one-fifth of a second. If the lid of B is now slightly lifted so that its beats can be heard at the same time as those of A, they will be easily distinguished by slight differences of audibility and tone, the interval between them can be estimated to one-tenth of a second and the previous comparison adjusted to this order of accuracy.

Comparisons between mean and sidereal time chronometers can be made with greater precision than between two mean time chronometers by means of coincidences. At intervals of about three minutes the beats will synchronise and it is only necessary to record simultaneous readings by the ordinary method of comparison at those times. The moment of synchronism can be estimated to within a few seconds and the accuracy of the comparison should be of the order of one-twentieth of a second, which is about the minimum interval between beats recognisable by ear.

(4) When a chronometer is required for observations ashore, it should, if possible, be landed at least 24 hours before it is actually required for timing the observations and it should be kept ashore afterwards until a reliable error has been obtained by time signal. However much care is taken in transport from ship to shore and back, it is possible that its rate will be affected and it should be given ample time to settle down and accommodate itself to the new conditions of temperature and humidity which may be experienced. When transporting a chronometer over short distances, *e.g.*, from the ship to the shore and back, it should be carried in an improvised sling with the face horizontal and the gimbals locked. It is unnecessary to stop the mechanism, but great care must be taken not to give the clock any sudden jerks or twists. Some chronometers, which are fitted for operating chronographs, are provided with a starting-and-stopping device. This should only be used when transport has to be effected over a considerable distance since the clock can hardly be expected to recover a steady rate for some days after restarting.

Chronograph.—This instrument is designed to enable times to be recorded with great accuracy. A typical form is shown in Plates S and T.

The body of the chronograph A contains clockwork mechanism for drawing the tape B at a constant speed across the upper part of the instrument. A starting-and-stopping switch C and a two-speed gear switch D are fitted, together with a fine speed adjustment operated by the governor knob E. For most purposes it is convenient to run the tape at a speed of about 1 inch per second. The tape is supplied in rolls ready for placing on the freely-revolving reel F which has a detachable side so that a roll can be slipped onto the spindle.

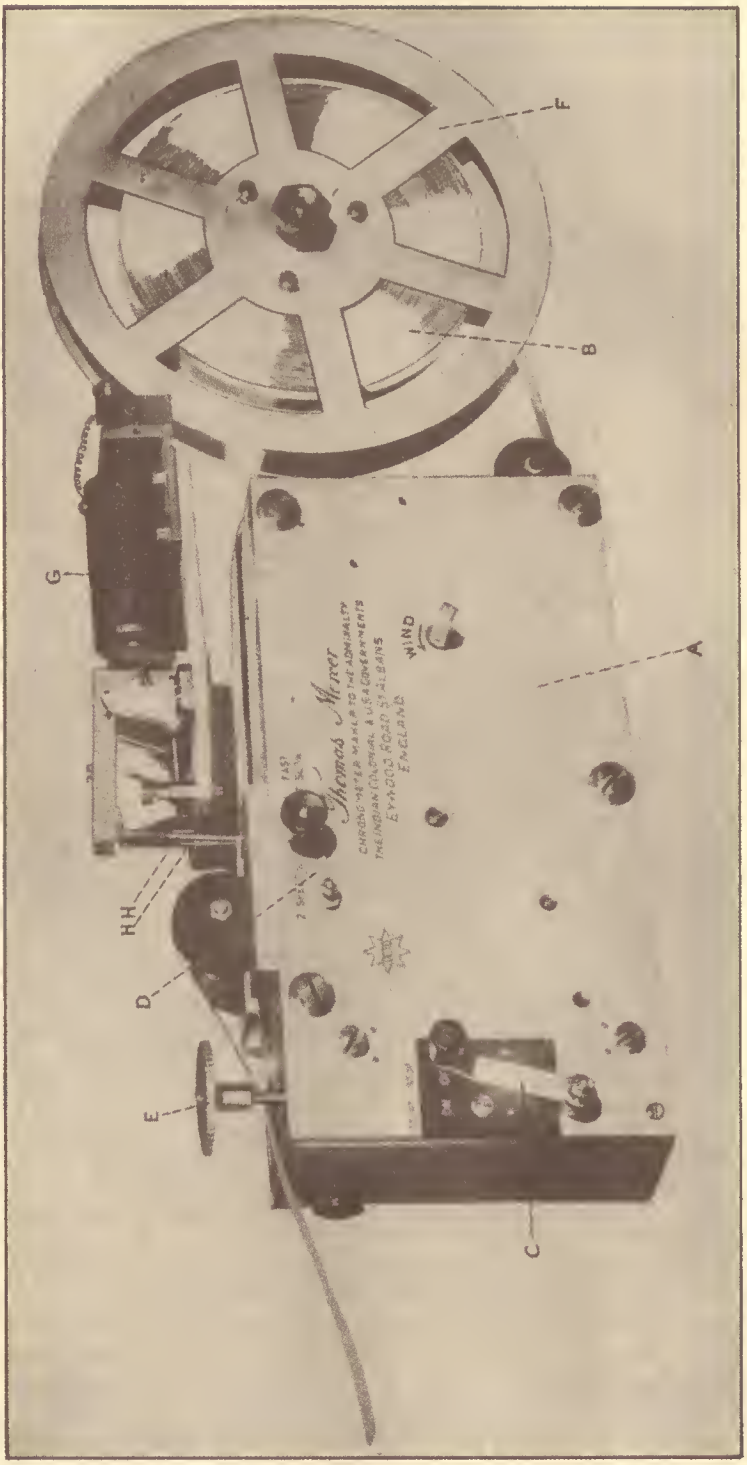

F
B
G
A
H H
D
E
C
WIND
Thomas Mercer
CHRONOMETER MAKER TO THE ADMIRALTY
THE INDIAN COLONIAL & U.S.A. GOVERNMENTS
ST. ALBANS
ENGLAND

PLATE S

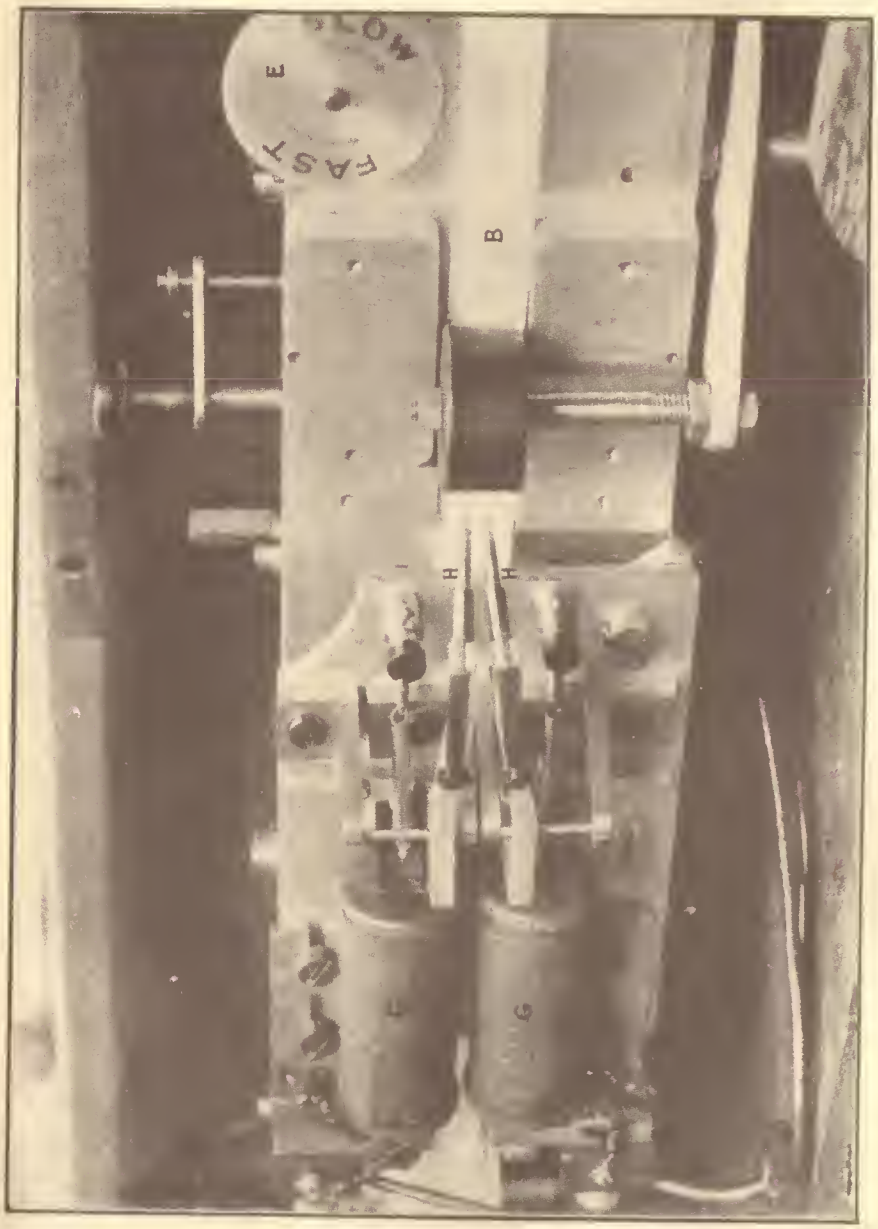


PLATE I

Two solenoids GG, fitted in a frame on top of the instrument, operate the steel pins HH and cause them to puncture the tape whenever an impulse of electrical current is passed through the windings. The motion of the pins is in the vertical plane, but their heads are fitted in spring-loaded pivots, so that the movement of the tape can draw the points clear as soon as a puncture has been made.

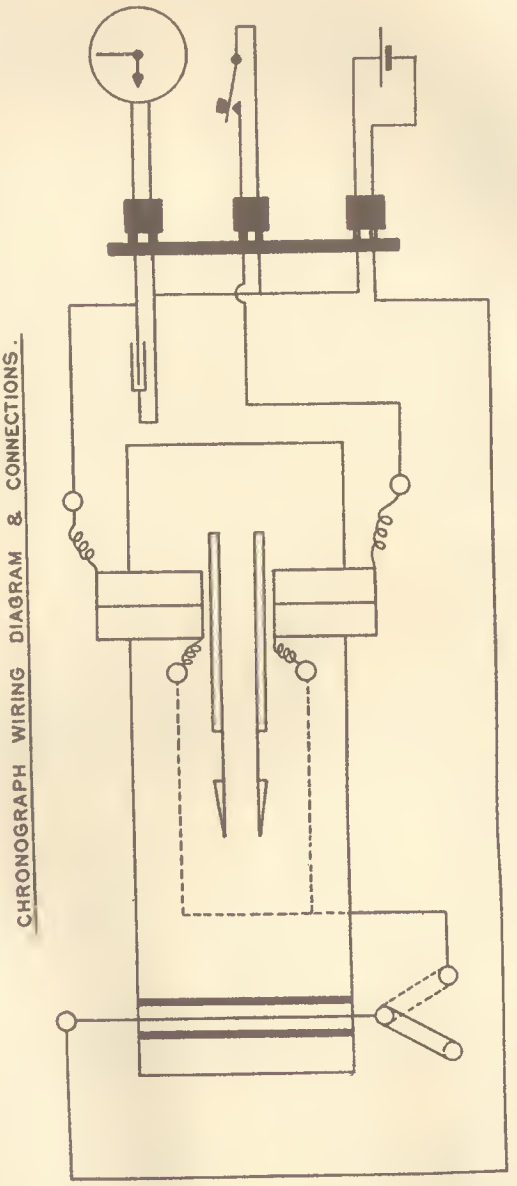


FIG. 193

Current is supplied by a 4½ volt battery and the remainder of the equipment consists of a chronometer and an incidence-marking key. The wiring diagram is shown in Fig. 193.

The chronometer is fitted with contacts so that at every other beat, *i.e.*, at each whole second, an impulse of current is passed through one of the solenoids and causes a puncture mark to be made in the tape. To enable the user to

distinguish the number of the second corresponding to the mark made by any particular beat of the chronometer, the make-and-break is so arranged that *one* half-second beat in each minute (usually at 59·5 seconds) also actuates the solenoid ; the puncture mark corresponding to the 60th second, *i.e.*, the exact minute can therefore be identified, and by counting from this point on the tape the number of the second corresponding to any other mark can be determined. A condenser is fitted to prevent sparking at the terminals. The incidence-marking key is fitted at the end of a wandering lead so that the observer can operate it from a distance. On being pressed, an impulse of current passes through the other solenoid and a record in the form of a puncture is left on the tape. An ordinary signalling key may be used as an incidence marker, but a special design of quick-action key has been evolved, and the use of this is perhaps preferable since its action is more nearly simultaneous with the pressure of the fingers.

If possible, two operators should be employed when the chronograph is used. The instruments must be placed on a good solid table and plenty of light is necessary ; a tent or hut should therefore be provided so that at night the lights are hidden from the observer outside whilst at all times there is freedom from wind and dust. One operator, seated in front of the chronograph, keeps it wound, starts and stops it, adjusts the speed and, if possible, marks and numbers in pencil the signals sent from the incidence key as the tape leads clear of the instrument. He also keeps the observer informed of the time of approach of each series of observations. The second operator sits to the left of the chronograph, winds the tape by hand onto a loose reel as it comes off the instrument and, if possible, records the clock time (hours and minutes) on the tape at each 60-second mark. The name of the heavenly body observed should be written on the tape at the beginning and end of the run.

The exact times of each observation are found by measurement from the puncture marks on the tape. This may be done with a protractor, but a form of incidence measurer similar to that shown in Fig. 194 is more convenient.

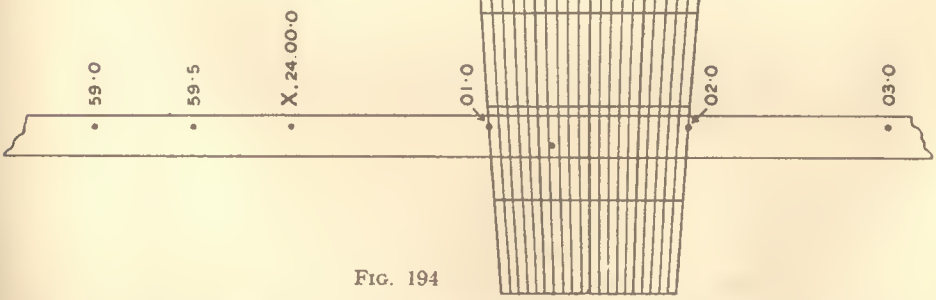


FIG. 194

The incidence measurer is made of celluloid and is graduated with fine lines, spreading fanwise, as shown in the figure. Lines perpendicular to the central line of the fan enable the axis of the measurer to be placed at right angles to the tape and it can then be moved up or down until the outer graduations coincide with the punctures corresponding to two adjacent second marks between which the observation mark has been recorded. In the figure the measurer is divided into 20 divisions, the chronometer marks at each second (and that at 59·5 seconds) are shown near the top of the tape, and an observation mark on the lower part. The chronometer time of the observation is seen to be Xh. 24m. 01·32 secs. For convenience, the measurer has been drawn in the figure

with a length of two inches only, but it should in practice be six inches or more in length so that the divergence of the lines between the top and bottom ends is not very great and will not cause any appreciable error in scaling off the position of a mark.

The chronograph method of recording times should be used for all observations on shore where a determination of longitude is required. In using the instrument it is assumed that :

(1) The puncture marks made on the tape synchronise with the aural reception of the beats of the chronometer ; this assumption is necessary because the method of determining chronometer errors by wireless time signals is essentially an aural one.

(2) There is no time lag between the closing of the incidence key and the register of the mark on the tape, or, if time lag exists, it may be regarded as equal to that in the chronometer circuit.

The surveyor has no method of verifying the accuracy of these assumptions, but the design of the instruments is such that errors from these causes are unlikely to exceed one twenty-fifth of a second and are certainly small compared to personal error, *i.e.*, the inability of the observer to co-ordinate exactly the pressure of his fingers on the incidence key with the occurrence of an event, *e.g.*, the passage of a star across the diaphragm wire of a telescope.

Observations for latitude and, under suitable conditions, for azimuth, require a less exacting standard of time-recording than that described, and this also applies to sights at sea. In these classes of work an accuracy of half of a second of time usually meets requirements and this can be obtained by visual observation of a chronometer watch or deck watch. The former should be used if there is a long interval between the times at which it is compared with a chronometer before and after the observations, whilst the latter may be used for observations on board ship. This system of time-recording is usually known as the " Stand-by-stop " method. The observer gives a warning " stand by " a few seconds before the observation and calls " stop " at the required instant ; an assistant then reads off the time by the watch by estimation to the nearest one-fifth of a second. The error of the watch at the moment of observation should be determined from that of a chronometer with which comparison is made both before and after the observations ; the rate of the watch in the interval being assumed constant. Making allowance for inaccuracy in the comparisons (this should not exceed one-fifth of a second), possible eccentricity of the second hand spindle in the watch dial, and inaccuracy in reading off at the " stop " signal, it should easily be possible to keep the total error within half a second, and the mean of a series of observations should be correctly timed within quarter of a second.

Radio Time Signals.

Full details of all radio time signals will be found in the Admiralty List of Radio Signals, Volume II, which is published annually. Supplements to both volumes are issued annually.

The systems of transmitting time signals may be divided into two main categories :

(1) **For the use of Surveyors.**—The *Modified Rhythmic* system which enables chronometer errors to be determined with an accuracy of two or three hundredths of a second. These signals should be used for obtaining chronometer errors required in connection with observations ashore whenever determination of longitude is one of the objects in view.

(2) **For the use of Mariners.**—The *International* (ONOGO) and *United States* systems which enable chronometer errors to be determined with

an accuracy of about one-fifth of a second. These signals may be used by the surveyor for obtaining chronometer errors required in connection with certain types of observations ashore when no great precision in time-recording is required, *e.g.*, latitude by circum-meridian altitudes, and with sextant observations at sea since these are subject to inherent inaccuracies which render any great nicety in time-recording unprofitable.

Modified Rhythmic System.—This comprises a series of 306 signals emitted in 300 seconds of mean time. In the series signals Nos. 1, 62, 123, 184, 245 and 306 are single dashes (—) of about 0·4 second duration and each dash is followed by 60 dots (·) of about 0·1 second duration. The first signal (beginning of the dash) is sent as nearly as possible at an exact minute of Greenwich Mean Time and the succeeding dashes are spaced precisely 60 seconds of mean time apart. If, for example, the first signal was sent at 17h. 55m. 00·045 sec. (*i.e.*, 0·045 seconds late), the 62nd signal would be at 17h. 56m. 00·045 sec., and so on with the remainder. The commencements of successive signals, whether dot or dash, are equally spaced out at intervals of $\frac{59}{61}$ (=0·9836) of a second of mean time. With a chronometer beating half seconds of mean time, it is clear that twice in each minute the radio signals will synchronise with the chronometer beats.

The most perfect method of obtaining the chronometer error would be to have a recording chronograph which marked the chronometer beats *and* the radio signals on the tape, but such equipment is in its present form too elaborate for field use. The method ordinarily employed is one of aural comparison and is carried out as follows :

(1) Note the chronometer time of the beat (whole or half second) immediately preceding the *first* dash of the series.

(2) Note the chronometer times of the coincidences (synchronisms of chronometer beat and signal), seconds or half-seconds only ; there will be ten of these in the complete series. It will be found that the times of coincidences can be estimated on the chronometer within two or three seconds ; they should be identified as precisely as possible, but it should be noted that a consistent error in judgment of two seconds in the time of each coincidence will cause an error of only 0·033 seconds in the final result.

(3) Subtract the seconds of (1) from (2) to obtain the " elapsed time."

(4) From Table XX (or the Admiralty List of Radio Signals, Vol. II) obtain the corrections corresponding to the elapsed times. (Correction=elapsed time (or elapsed time —30)×0·0164).

(5) Add the mean of the corrections to (1).

(6) The difference between the corrected value of (1) and the G.M.T. of the first dash of the series (as given in Admiralty List of Radio Signals, Vol. II) is the error of the chronometer.

An example is given on the next page.

Notes.—(i) When the commencement of the dash very nearly coincides with a beat of the chronometer, some difficulty may be found in deciding which beat actually preceded the dash. Any doubt can be dispelled by inspection, for, if the *first* coincidence after each dash is at a whole second, then the beat preceding the dash must have been a whole second, and *vice versa*.

(ii) As the first signal of the series comes rather unexpectedly, some time after the preparatory signal, it may be difficult to decide which chronometer beat preceded this dash, but this can be inferred from the beats preceding the later dashes as these begin exactly 60 seconds of mean time apart.

(iii) When one set of coincidences occurs very near the dash at the beginning of each minute, it is better to use the other set which will occur at about the

half minute for the comparisons (thus obtaining only five, instead of ten, elapsed times for the series).

(iv) It is assumed that the chronometer has no appreciable rate during the period of the signals.

Example of recording and computing a modified rhythmic time signal :
Rugby Time Signal beginning 17h. 55m. 00s. G.M.T. (assumed to have no error).
Chronometer time of beat preceding first dash=17h. 52m. 30·5s.....(1)

2 Chron. Time of coincidences	3 Elapsed Time ((2)—(1))	4 Correction (from Table XX) $\left(\begin{matrix}\text{(i) Col. (3)}\\ \text{(ii) Col. (3)} - 30\end{matrix} \times 0{\cdot}0164\right)$
(i) 44·5 secs.	14 secs.	0·230 secs.
(ii) 14	43·5	·221
(i) 47·5	17	·279
(ii) 19	48·5	·303
(i) 46·5	16	·262
(ii) 16	45·5	·254
(i) 48·5	18	·295
(ii) 17	46·5	·270
(i) 45·5	15	·246
(ii) 17	45·5	·270
		Mean=0·263 secs.
		(1) 17h. 52m. 30·5 secs.

Chronometer time of first dash 17h. 52m. 30·763s.
G.M.T. ,, ,, ,, 17h. 55m. 00s.

Hence, chronometer is 2m. 29·237s. slow on G.M.T.

Time signals on the Modified Rhythmic System are made twice daily from Rugby, Paris and from Bordeaux, and from certain other stations (*vide* Admiralty List of Radio signals). The errors of the 10h. and 18h. signals from Rugby are determined at the Royal Observatory, Greenwich, and are published monthly in Admiralty Notices to Mariners ; those of some other signals are published in the Bulletin Horaire.

Since Greenwich observatory is the origin of all longitudes, the surveyor should, if possible, use one of the signals referred to above when accurate chronometer errors are required. The equipment of a surveying vessel includes a portable radio receiving set, so that it is possible for time signals to be received at the observation spot, and the error of the chronometer, which is used for the observations, to be determined *in situ*. If, for example, the observations are made between the hours of 1300 and 1600 G.M.T., the chronometer error at 1000 and 1800 G.M.T. can be determined from the Rugby rhythmic signals at those hours and, assuming its rate to be constant in the interval, its error at any intervening time can be deduced. It is best not to " mix " the signals, *e.g.*, by determining the first error from Rugby and the second from Paris, particularly if the results of the observations are required before the errors of the time signals are received. This may mean a longer interval between determinations but the resulting clock rate is more likely to be accurate. The initial signals are usually correct as to timing within one-tenth of a second and the errors of successive daily or twice-daily signals from the *same* station do not often vary by more than one-fiftieth of a second so that two chronometer errors from those signals require corrections (if any) of practically the *same* amount. If, however, the signals are " mixed," it may happen that that used

for the first chronometer error was sent one-tenth of a second early whilst that used for the second error was one-tenth of a second late, and the deduced rate will be appreciably in error. With a constant error, however, the deduced rate will be correct and only a correction to the final longitude will be needed when errors of the time signals are received.

Radio waves travel at the speed of light (186,000 miles per second) and there is therefore in theory a small correction to apply for the interval between transmission and reception. This is usually neglected as far as observations and computations in the field are concerned.

International (ONOGO) and U.S. Systems.

Time signals on these systems are made from a large number of stations (see Admiralty List of Radio Signals, Vol. II) and enable chronometer errors to be determined with an accuracy of about one-fifth of a second. In nearly all cases transmission is effected automatically and is probably accurate to within one-tenth of a second, but, as no errors are published, the only safeguard is to compare chronometer errors obtained from two or more signals from different stations.

In most cases the series of signals occupies a period of 3 to 5 minutes, the actual time signal being certain dots or the beginning and (or) end of certain dashes, so arranged that several comparisons can be made during the period.

Astronomical Observations for Position Ashore—Principles and Methods

Astronomical observations in the field for the determination of geographical position involve measurement of the apparent altitudes of heavenly bodies. In some types of observing instrument the measured altitude is a fixed angle which need not be precisely evaluated, whilst in others the altitude is read off a graduated arc or circle. The apparent altitude is converted to the true altitude by the application of certain corrections and hence the true zenith distance of the observation spot is found. The time of the observation being recorded, the geographical position of the heavenly body at that instant is obtainable from data in a Nautical Almanac, and the observer's position is calculated by the ordinary formulæ of spherical trigonometry.

The methods of observation and the forms of computation used by the surveyor are designed to eliminate, as far as possible, all the errors to which such observations are liable. It is assumed that at any given altitude the corrections to the instrumental readings are the same. By balancing, or pairing, observations on opposite sides of the zenith, the resulting geographical coordinates are freed from errors due to using erroneous values for these corrections. This is the principle of equal altitudes. Suppose, for example, equal altitudes of a star east and west of the meridian are observed, any error in the assumed corrections to those altitudes will have no effect on the deduced time of the star's meridian passage and hence on the observer's longitude, for whatever error in angle tended to make the first observation too early in time will make the second an equal amount too late and the *middle* time will be correct.

The various corrections applicable to astronomical observations for position are fully described in works on navigation. It will only be necessary here to consider the possible errors of these corrections and those due to other causes.

In general, errors of observation may be divided into those which are constant or regular in character and those which are accidental or irregular :

(1) **Refraction.**

(*a*) *Constant :* The values of refraction (corrected for the height of the barometer and thermometer) taken from Table XII may be appreciably in error. Provided, however, the values are the same in all directions from the observer at any given altitude, any error in the adopted figure

will be eliminated by equal altitudes of heavenly bodies on opposite sides of the zenith.

(*b*) *Irregular :* When variation in the value of refraction exists in different directions during the period of observation, the results will be correspondingly in error. It is assumed that error from this cause is normally of very small proportions and may be rendered insignificant by meaning the results of several series of observations at different times.

(2) **Instrumental Errors.**

(*a*) *Constant :* These are errors in the assumed values of the prism angle in astrolabes, index and centring error in sextants, etc., and can be automatically eliminated by observing equal altitudes on opposite sides of the zenith.

(*b*) *Irregular :* These include changes in the instrument during observations due to temperature variations, earth tremors, etc., and errors due to want of sensitivity inherent in the construction and graduation of the particular instrument. Errors from these causes may be reduced to small proportions by repeating the measurements a sufficient number of times.

(3) **Personal Error.**

(*a*) *Constant :* This is the " Personal Equation " of the observer, *e.g.*, one person will constantly record the passage of a star over a cross wire a little too early, another a little too late. No system of observation will entirely eliminate errors from this cause, but it may be assumed that the mean of several observer's results is practically freed from them.

(*b*) *Irregular :* These depend on the ability of the observer to co-ordinate his actions with the impressions of his senses, *e.g.*, synchronising the pressure of his fingers on a chronograph key with his visual impression of the passage of a star across a diaphragm wire. They also depend to a lesser extent on his experience in using the instrument, but it is not safe to assume that, with the easily operated instruments now in use, experience counts for much. The accepted position must depend on a mathematical analysis of the results of each observer and an estimate of respective reliability must be based entirely on this.

(4) **Mistakes.**

These should be eliminated by proper safeguards in the methods of observation.

The Prismatic Astrolabe

This instrument is designed for equal altitude observations on land, and in the model adopted by the British Admiralty the observed altitude is 45° above the horizontal. The fundamental principle involved is that the measurement of a star's altitude at the moment of observation is made by means of the fixed angle between two faces of a prism and that such an angle is invariable. The prism employed is of pentagonal form and is shown in Fig. 195.

The prism consists of two parts, ABCDE and AEF, cemented together. The faces CD and AE are silvered to form reflecting surfaces, but, whilst CD is completely silvered, a circular area in the centre of AE is left bare so that some portion of light rays meeting the face EF may pass directly through the prism and emerge at the face BC which is parallel to EF. OO is the object glass of a telescope which is mounted in the same frame as the prism and may therefore be regarded as rigidly attached to the latter. MM is the surface of the mercury in a trough attached to the mounting.

When a parallel pencil of rays from a star reaches the instrument, a part of the beam is reflected from the mercury and passes straight through the prism into the telescope, and a part, meeting the face AB, undergoes a double reflection in the prism, and emerging by the path shown, also passes into the telescope.

The unsilvered area of the face ÀE is of such dimensions that the direct and reflected star images appear of equal brightness in the telescope. As the star moves in altitude, the star images will appear to move vertically in opposite directions in the field of view and, provided there is no lateral displacement, they will meet when the star reaches a certain altitude, dependent on the prism angles. These are so ground, in the model adopted by the Admiralty, that the altitude is within a few seconds of 45°.

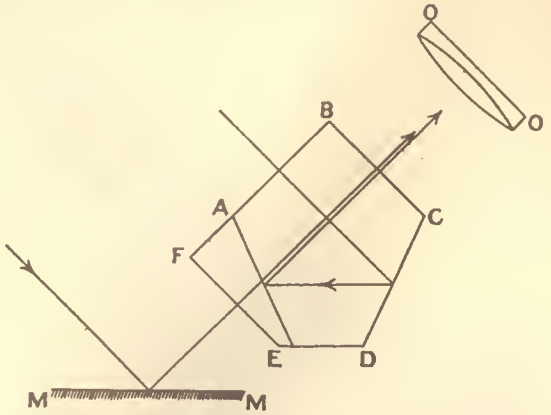


FIG. 195

In its simplest form an astrolabe consists of a telescope, a prism and a mercury trough in a mounting which can be approximately levelled and turned in azimuth. To such an instrument, however, there would be two objections :

(*a*) Only one observation of each star would be possible and accidental errors could not be reduced by the method normally employed with other instruments, *i.e.*, repetition of the measurement.

(*b*) It is difficult to estimate the exact instant of contact of the two star images since they appear fused for an appreciable interval of time.

To overcome these objections, the following fittings have been added :

(*a*) A series of weak deflecting prisms which can be interposed in turn in the path of the light beam before it reaches the mercury. Each prism deflects the beam in a vertical plane in such a way that a series of observations at altitudes differing by a few minutes of arc can be made. The mean of the series is 45° ± any error of the main pentagonal prism.

(*b*) A weak duplicating prism is fitted to cover one half of the entrant face AB of the main pentagonal so that the direct star image is split into two, separated laterally by about 3′ of arc. The mercury image can be made to pass between the two direct images, and the instant when the three are in line can be judged with far greater accuracy than that of the contact of two single images.

The standard 45° astrolabe adopted by the Admiralty is shown in Plates U and V.

The instrument is mounted on a small wooden tripod stand A which is of such a height that an observer seated on a stool finds the telescope eyepiece in a convenient position for his eye. The Telescope B can be unshipped for stowing away but its method of attachment to the body of the instrument is sufficiently rigid to prevent any appreciable movement. With the aid of the levelling screws C and the circular spirit level D, the instrument can be approximately levelled so that the upper part rotates about a nearly vertical axis. The main prism E is held in a metal casing which is pivoted on an axis near

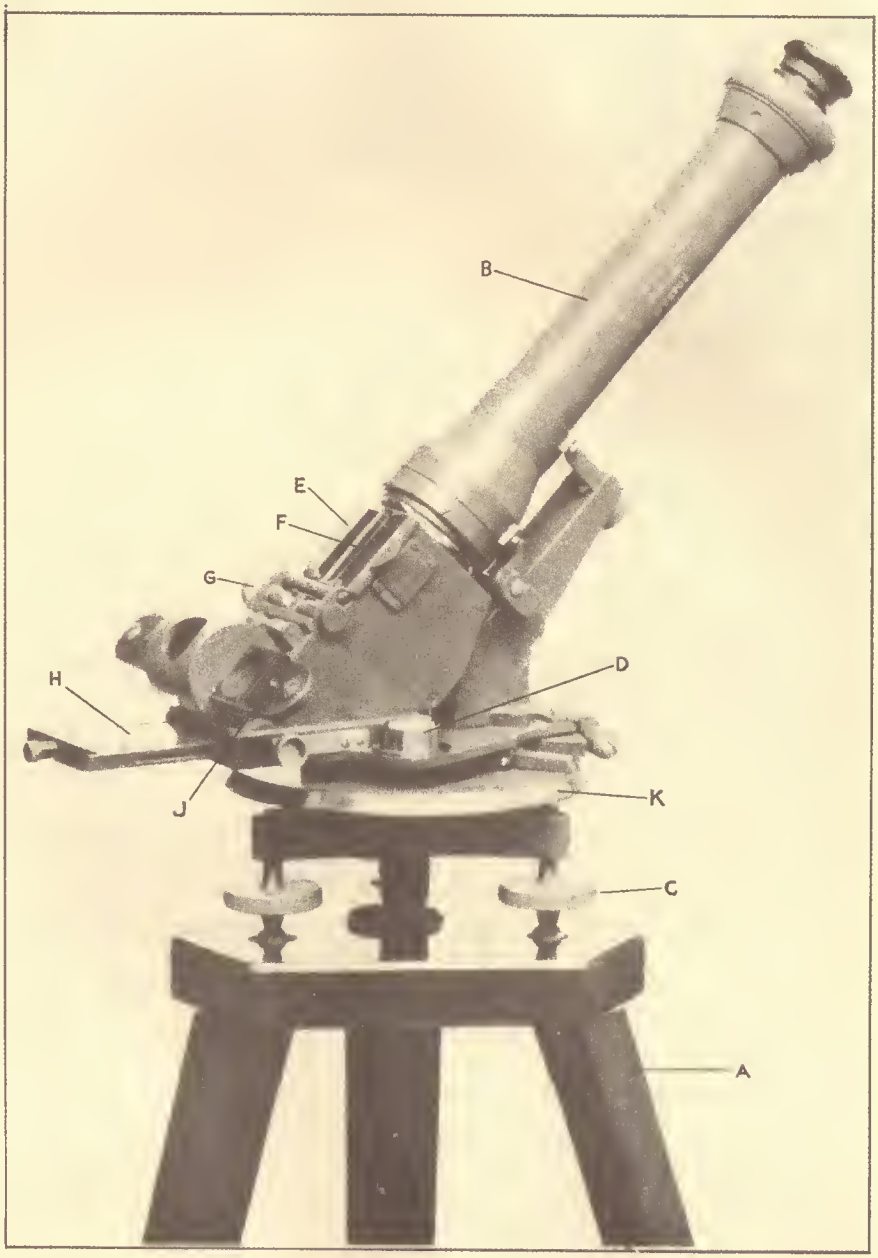


PLATE U

the lowest point so that it can be rocked slightly about this axis by means of the fine motion screw G. This adjustment is necessary before the first observation of each star to bring the duplicate images vertically over the single image, the bubble level by itself not being sufficiently sensitive to permit perfectly accurate levelling. The duplicating prism F is fitted in a holder to cover half the face of E and can be removed to permit access to the whole of the upper face of E. As moisture may collect between the lower face of F and the upper

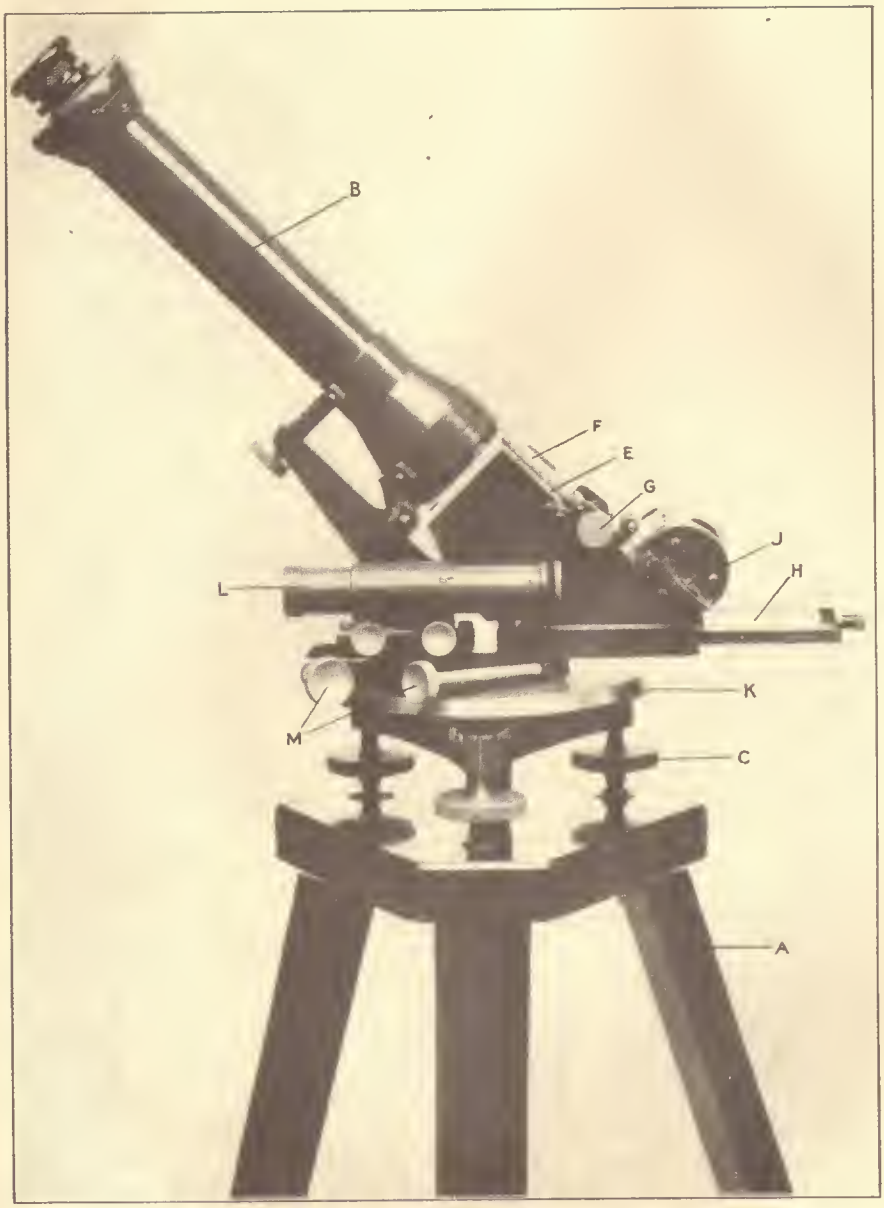


PLATE V

face of E, it is sometimes necessary to remove the small prism for cleaning purposes between sets of observations.

Three deflecting prisms of 3′, 9′ and 15′ deviation are mounted in the slide J which can be pushed horizontally into any one of three positions and engages with a spring detent when the prism is correctly placed in the path of the beam. In addition, the whole slide can be turned over through 180° in order to reverse the prisms. This arrangement permits a series of six observations of each star being made. The edges of the slide are marked R1, R2, R3 and S1, S2, S3 respectively, R1 and S1 referring to the 15′ prism, R2 and S2 to the

9′ prism, and R3 and S3 to the 3′ prism. For a rising star the correct sequence is :

R1, R2, R3 (reverse), S3, S2, S1.

For a setting star the sequence is :

S1, S2, S3 (reverse), R3, R2, R1.

The mercury pool is formed on the surface of an amalgamated copper plate H. The sides of the mercury trough are about 1 millimetre high and the depth in the pool should be regulated to this amount by sweeping the surface with a glass rod touching the sides. Any excess mercury is swept off into a surrounding trough and, if the rod is turned at the same time as the sweeping motion is made, any scum will be removed. Care must be taken not to touch the amalgamated plate with the fingers since grease will prevent the mercury amalgamating with the surface. If this should be accidentally touched it will probably be necessary to clean it with dilute acid. The mercury trough is fitted in a slide and in Plates U and V is shown in the " out " position where it can be filled and cleaned. When observing, it is slid into the body of the instrument where it is protected from wind and dust.

Attached to the stand is a friction-tight azimuth circle K divided into degrees. This is set with the aid of the tubular compass L so that the zero indicates true north. The compass can be removed when this has been done.

The main body, holding telescope, prism block, mercury trough, etc., turns on a vertical axis in the stand, and clamping and slow-motion screws M are provided so that the telescope may be set on any required bearing and, when the star has been picked up, it may be kept in the centre of the field by means of the slow motion screw.

A torch must be used at night for reading the azimuth circle setting. There is sufficient light from the sky to enable the observer to see the limits of the field of view in the telescope and, even if the cross-wires are not visible, he can estimate the centre of the field sufficiently accurately when he is working the azimuth slow motion to ensure that the star images meet at this point.

A larger model of the 45° astrolabe, designed to permit eight observations of each star, is also used, but owing to its weight and cost it has not been generally adopted

Programme of Observations with 45° Astrolabe.

Astrolabe observations involve the principle of equal altitudes and the method used to compute the results is such that latitude and longitude are found simultaneously. In order that this double determination may be made, it is useless to observe stars either very near the meridian or the prime vertical, and it is the practice to confine the observations to stars which reach an altitude of 45° in each quadrant on bearings between 25° and 65° from the meridian.

In preparing a programme of observations, the following points should be borne in mind :

(*a*) The six observations of each star occupy a period of about 1¼ to 2¼ minutes, and to allow time for the observer to shift his position and set the instrument on a new bearing, an interval of not less than 4 minutes should be allowed between the observations of successive stars.

(*b*) Good results are obtainable from observations of 12 stars, but 16 or 20 are preferable ; they should be arranged, as far as possible, so that " sets " of 4 stars, one in each quadrant, are observed before another set is started. This will give a well balanced programme and will ensure that, if the sky clouds over before the complete series has been observed, work will not be wasted (by having, say, no stars observed in the NE quadrant to balance those observed in the SW quadrant) ; it also ensures that errors due to fluctuating values in refraction will be reduced to the minimum.

(*c*) Sufficient stars are listed in "Apparent Places of Fundamental Stars (FK3)"* to enable 4 sets of 4 stars each to be observed in a period of three hours or so (under most conditions of time and place).

(*d*) It is best to confine the stars selected to those of about 4·3 magnitude and brighter. A number of lesser stars are listed in the almanacs and those down to the sixth magnitude can be seen in the telescope on a clear night, but they are so numerous in many parts of the heavens that it is easy to make a mistake in identification, and this may lead to a lot of difficulty.

(*e*) If the moon is of any size, avoid selecting stars within 15° of it as it may be impossible to see them in the telescope.

(*f*) It is generally useful to select a bright star for use as a trial observation at the beginning of the series. It will be easy to identify and pick up in the telescope, and a first adjustment of the prism (which may happen to be canted at a considerable angle) and, if necessary, of the azimuth circle for undetected error in the compass, can be made at leisure under the best conditions.

(*g*) Observations cannot be started until about the end of twilight.

(*h*) It is unnecessary to make use of the planets which involve troublesome corrections in the subsequent computations. Thickly grouped clusters of stars, such as the Pleiades, should also be avoided as mistakes of identification are probable.

The preparation of the programme involves finding the times to the nearest minute and azimuths to the nearest degree at which suitable stars will reach an altitude of 45° in each quadrant. There are various methods of doing this with the aid of altitude-azimuth tables, etc., but a diagram "Miscellaneous No. 5170," published by the Admiralty, is of invaluable assistance. Full instructions for using it are printed on the diagram and will only be briefly recapitulated here. They are as follows:

(1) Obtain an approximate latitude and longitude for the observation spot.

(2) Calculate the Sidereal Time of the limits of time between which it is intended to observe.

(3) From the diagram take off the limits of Hour Angle and Declination in each quadrant appropriate to the assumed latitude.

(4) Subtracting these hour angles from the sidereal time limits will give the limits of Right Ascension during the proposed period of observation.

(5) From FK3* (or the Nautical Almanac) make lists of all suitable stars in each quadrant within these limits of Right Ascension and Declination.

(6) Returning to the diagram, take off the hour angle and azimuth of each star, corresponding to its Declination and the assumed latitude.

(7) From the equation L.M.T. = R.A. + H.A. − R.A.M.S. ± 12h calculate the L.M.T. and, by applying the longitude, the G.M.T. at which each star reaches an altitude of 45°.

(8) Make a new list of the stars in their chronological sequence, picking them out in each quadrant at intervals of not less than 4 to 5 minutes so that a well-balanced series is finally written down. This list is for the use of the observer ashore and should show the stars' names, magnitudes, azimuths and times of observation. A watch set to the time used in the list, whether G.M.T. or L.M.T., should be taken ashore.

(9) In making out the final list, a few spare stars should be included, where

* "Apparent Places of Fundamental Stars (FK3)"—produced by international co-operation, and prepared for publication annually by H.M. Nautical Almanac Office (His Majesty's Stationery Office.)

long intervals occur, even if they are not required to complete a well-balanced set ; they may often be useful if passing clouds prevent some of the other observations being made.

Setting up and using the 45° Astrolabe.

Good observations cannot be made unless the observer is comfortable and able to give his whole attention to the business in hand. In addition to the instrument itself, the observer should be equipped with a stool for sitting on, a torch for reading the azimuth circle, a piece of cloth to put over the instrument and keep off dew when it is not actually in use, chamois leather for cleaning the prisms and eyepiece, a bottle of mercury, and a glass rod for cleaning the surface of the mercury pool. A barometer and thermometer are also required, and their readings should be recorded at hourly intervals.

The procedure on landing is as follows :

(1) Assemble and set up the astrolabe, preferably whilst it is still daylight, on a firm level piece of ground, pushing the tripod legs well in and making sure that there is plenty of room to move round it. A position reasonably well protected from wind and dust should be chosen and there must be an unrestricted view of the sky at the required altitude and between bearings of 25° and 65° in each quadrant. The chronograph operator must be within speaking distance of the observer so that he can pass information concerning the observations (times and bearings, etc.), but lights used by him must be hidden from the observer.

(2) Run the flex with chronograph key to the observing position.

(3) Level the instrument with the aid of the bubble level.

(4) Fill the mercury trough, clean the surface of the mercury and slide the trough back into the body of the instrument.

(5) Rotate the instrument until the compass needle is central and then set the azimuth ring, not forgetting variation, so that the index mark is on zero when the telescope is directed to true north.

(6) Focus the eyepiece by getting the cross-wires sharply defined against the sky.

(7) Rotate the instrument until the index mark shows that bearing of the first or trial star and clamp it. Set the slide to R1 or S1, as may be required for the first observation, rising or setting.

The observer should " stand by " at the telescope about three minutes before the predicted time of an observation. Assuming the instrument to be nearly level, the single and duplicate images should appear in the field of view within a short interval of each other. With a rising star the single image appears at the top of the field, the duplicate at the bottom, and *vice versa*. As soon as they are seen, the chronograph should be started. If the duplicate images only are seen at first, rotate the instrument a little to the left and right until the single image is found ; the duplicates may then be out of the field of view, but can be brought back by operating the rocking screw on the main prism holder. If the single image only is seen at first, operate the rocking screw until the duplicates are brought into the field. The chronograph key should now be held ready. The rocking screw should be turned until the single image is vertically above or below the centre of a line joining the duplicate images, and, as they approach each other, the slow motion azimuth screw should be turned to keep them as nearly as possible on the vertical cross wire and to make sure that they will be practically on it when they meet. A small further adjustment of the rocking screw may be necessary at this stage. As these movements disturb the mercury the instrument should not be touched during the five seconds previous to the meeting of the star images.

The observation is made by depressing the chronograph key at the instant the single image crosses the line of the duplicate images. As soon as this has happened, move the slide to R2 or S2, as the case may be, and stand by for the next observation which will be at an interval of about 15 to 25 seconds. During this interval it will probably be necessary to give the instrument some further movement in azimuth to keep the images on the vertical wire. The prism, however, should require no further adjustment.

Similarly, with the remaining observations until the set of six is completed, when the instrument can be set on the bearing of the next star and the slide put to R1 or S1, as necessary. If there is a long interval, it may be advisable to wipe over the prisms and cover the instrument temporarily with a cloth, whilst once in every two hours or so it is usually necessary to clean the mercury surface.

Computation of Position from Astrolabe Observations.

The method of obtaining latitude and longitude from astrolabe observations is based on Marc St. Hilaire's method of Position Lines, coupled with a least square approximation of best fit to a number of discordant results. The position line method is too well known to require an elaborate description here. Briefly, the approximate latitude and longitude of the observation spot must be known and adopted as an assumed position which, in the case of astrolabe sights, should preferably be not more than a mile in error ; and the zenith distance of each star, found from the solution of a spherical triangle in which co-latitude, polar distance and hour angle are known, is compared with the zenith distance found by observation. The difference between the calculated and observed zenith distances of each star is an intercept measured in minutes and (or) seconds of arc, and a position is found by plotting these intercepts. When the observed zenith distance is less than the calculated zenith distance, the intercept is termed *positive* or *towards* ; when greater, *negative* or *away*.

The mean value of the six altitudes observed with this type of astrolabe is within a few seconds of 45° ; it is unnecessary to know the angle exactly As the observations occupy some period of time—as much as $2\frac{1}{2}$ minutes when the bearing is 25° from the meridian—it is incorrect to assume that the mean altitude or zenith distance corresponds to the mean of the observed times since the rate of change of altitude is not uniform unless a star is in the prime vertical. A correction must therefore be applied in the process of computation either to the mean of the times or to the mean of the zenith distances. The simplest method of doing this is to work out each set using the mean of the times and to apply the correction to the intercept derived from the mean of the zenith distances.

The value of this correction can be expressed as :

$$(\frac{1}{2n} \Sigma (dz)^2) \times (\cot \alpha (\cot \alpha - \tan \phi_a \operatorname{cosec} \alpha))$$

where n = number of observations in the set.

dz = difference of each observation from the mean zenith distance, expressed in circular measure.

α = azimuth of the star measured from 0° to 360°.

ϕ_a = assumed latitude of the observation spot.

The correction may be written $C \times K$, where $C = \frac{1}{2n} \Sigma (dz)^2$ and is a constant for the instrument, and $K = \cot \alpha (\cot \alpha - \tan \phi_a \operatorname{cosec} \alpha)$, a factor which varies with each star observed and the latitude. The values of dz in the 45° astrolabe described are $7\frac{1}{2}'$, $4\frac{1}{2}'$, $1\frac{1}{2}'$ $-1\frac{1}{2}'$, $-4\frac{1}{2}'$, $-7\frac{1}{2}'$ or as nearly these angles as manufacturing accuracy allows. If these angles are expressed as radians and the formula computed, it will be found that the constant C for this particular

instrument is 0″·23. The value of K may be calculated from the above formula, tan ϕ_a being regarded as always positive, cot α and cosec α being given signs appropriate to the quadrant in which α lies ; but it may be more conveniently taken off a diagram No. 5072, published by the Hydrographic Department ; in this diagram the factor K is termed "A." C×K is the correction to each intercept.

If one observation of the set is missed, its opposite number should be removed and a new value C^1 of C must be found as follows :

Supposing the $7\frac{1}{2}'$ (*i.e.* first and last observations) are not used and that the instrumental constant for a whole set is 0″·23, then

$$C^1=\frac{(4\frac{1}{2})^2+(1\frac{1}{2})^2}{(7\frac{1}{2})^2+(4\frac{1}{2})^2+(1\frac{1}{2})^2}\times 0{\cdot}23\times\frac{6}{4}=0''{\cdot}10$$

In the spherical triangle connecting the assumed position, the elevated pole and the star's position at the time of observation, two sides, $90°-\phi_a$ and $90°\pm\delta$, and the included angle h are known, where

ϕ_a=the assumed latitude of the observation spot.
δ=the star's declination.
h=the hour angle.

The third side, zenith distance, can be found from the formula

cos z.d.=cos $(90°-\phi_a)$ cos $(90°\pm\delta)$+sin $(90°-\phi_a)$ sin $(90°-\delta)$ cos h
=sin ϕ_a sin δ+cos ϕ_a cos δ cos h.

If tan ϕ_a sec h=tan M, then the formula may be more conveniently written:

cos z.d.=cos $(\delta\sim M)$ sin ϕ_a cosec M.

In the computation the elements of right ascension and declination should be taken from "Apparent Places of Fundamental Stars"; the elements are given at 10-day intervals, and when interpolating it is necessary to follow the instructions given in the "Introduction" (it should be noted that the correction for the short period terms of nutation may amount to as much as 0·03s. in right ascension and 0″·19 in declination, though usually it is insignificant).

The working should be taken to hundredths of a second in time and arc, and seven-figure logarithms should be used throughout except for the azimuth calculations, where five-figure logs, giving results to the nearest minute, will suffice.

Example :

Date : 7th December, 1934.
Chronometer error by Rugby signal at 1000 G.M.T.=1h. 02m. 02·557s. slow.
,, ,, ,, ,, 1800 ,, =1h. 02m. 01·721s. ,,
(Radio Time signals assumed to be free from error.)
Hence, chronometer rate=0·1045s. gaining hourly.
Instrumental constant of astrolabe=0″·412.
Assumed position :—lat. (ϕ_a) = + 6° 18′ 16″·84.
long. (ω_a) = +99° 51′ 09″·60. (6h. 39m. 24·64s.)

	1.	2.	3.	4.
Quadrant ..	N.W.	N.E.	S.E.	S.W.
Star	γ_1 Androm.	a_2 Gemini	δ Can. Maj.	12 Eridani
Bar./Therm. ..	30·22 73°	30·20 72°	30·20 72°	30·20 73°
	h. m. s.	h. m. s.	h. m. s.	h. m. s.
R.A.	01 59 55·72	07 30 29·40	07 05 46·58	03 09 20·52
Dec. (δ) ..	+42° 01′ 24″·87	+32° 01′ 56″·49	−26° 17′ 09″·93	−29° 14′ 28″·91

	1.	2.	3.	4.
	h. m. s.	h. m. s.	h. m. s.	h. m. s.
Chron. T. ..	15 19 06·66	16 07 21·86	16 13 11·88	16 19 33·50
Error .. +	1 02 01·90	1 02 01·81	1 02 01·80	1 02 01·79
G.M.T. ..	16 21 08·56	17 09 23·67	17 15 13·68	17 21 35·29
Calcn. of R.A.M.S.				
S.T. at 0 hrs	5 00 18·229	5 00 18·229	5 00 18·229	5 00 18·229
Corrns. (1)	2 37·704	2 47·560	2 47·560	2 47·560
,, (2)	3·450	1·478	2·464	3·450
,, (3)	0·024	0·065	0·037	0·097
R.A.M.S. ±12 hrs	5 02 59·407	5 03 07·332	5 03 08·290	5 03 09·336
G.M.T ..	16 21 08·56	17 09 23·67	17 15 13·68	17 21 35·29
Long. (ω_s) +	6 39 24·64	6 39 24·64	6 39 24·64	6 39 24·64
L.M.T. ..	23 00 33·20	23 48 48·31	23 54 38·32	00 00 59·93
R.A.M.S. ±12 hrs	5 02 59·41	5 03 07·33	5 03 08·29	5 03 09·34
R.A.M. ..	4 03 32·61	28 51 55·64	28 57 46·61	5 04 09·27
R.A.* ..	1 59 55·72	7 30 29·40	7 05 46·58	3 09 20·52
H.A. (h) ..	2 03 36·89	21 21 26·24	21 52 00·03	1 54 48·75
L. tan ϕ_s ..	9·043 2981	9·043 2981	9·043 2981	9·043 2981
L. sec h ..	0·066 4967	0·113 4751	0·071 5789	0·056 9410
L. tan M ..	9·109 7948	9·156 7732	9·114 8770	9·100 2391
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
M .. +	7 20 14·15	8 09 53·05	7 25 21·63	7 10 45·38
δ	+42 01 24·87	+32 01 56·49	−26 17 09·93	−29 14 28·91
δ ∼ M ..	34 41 10·72	23 52 03·44	33 42 31·56	36 25 14·29
L cos (δ ∼ M)	9·915 0197	9·961 1755	9·920 0550	9·905 6232
L. sin ϕ_s ..	9·040 6635	9·040 6635	9·040 6635	9·040 6635
L. cosec M ..	0·893 7762	0·847 6514	0·888 7777	0·903 1793
L .cos z.d. ..	9·849 4594	9·849 4904	9·849 4962	9·849 4660
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Obsd. Alt. ..	45 00 00·00	45 00 00·00	45 00 00·00	45 00 00·00
Corrd. Refn. +	55·86	55·94	55·94	55·82
Obs. z.d. ..	45 00 55·86	45 00 55·94	45 00 55·94	45 00 55·82
Calc. z.d. ..	45 00 12·15	44 59 57·43	44 59 54·67	45 00 09·05
Intercept ..	− 43·71	− 58·51	− 61·27	− 46·77
Corrn. (C × K)	+ 0·87	+ 0·24	+ 0·43	+ 0·66
Corrd. Int'cept	− 42·84	− 58·27	− 60·84	− 46·11
L. cos δ ..	9·870 91	9·928 27	9·952 60	9·940 80
L. sin h ..	9·710 62	9·804 80	9·724 21	9·681 48
L. sec alt ..	0·150 40	0·150 40	0·150 40	0·150 40
L. sin α ..	9·731 93	9·883 47	9·827 21	9·772 68
Azimuth (α) ..	N 32° 39′ W	N 49° 53′ E	S 42° 12′ E	S 36° 20′ W

These results may be examined graphically, but in this example it will be noticed that there is a very wide spread—approximately 100″—between stars in opposite quadrants. This spread is far more than could be accounted for by an erroneous value of refraction and it must be mainly due to an angular error in the pentagonal prism ; probably the cement between the two internal

faces has become slightly defective and the entrant and exit faces of the reflected beam are no longer parallel. This will have *no* effect on the final result since this angular error can be assumed to be constant during the period of observation, but, for convenience in plotting, a uniform correction of +50″ is applied to all the intercepts which then become :

N.W. star + 7″·16
N.E. ,, − 8″·27
S.E. ,, −10″·84
S.W. ,, + 3″·89

The result of the plot is shown in Fig. 196.

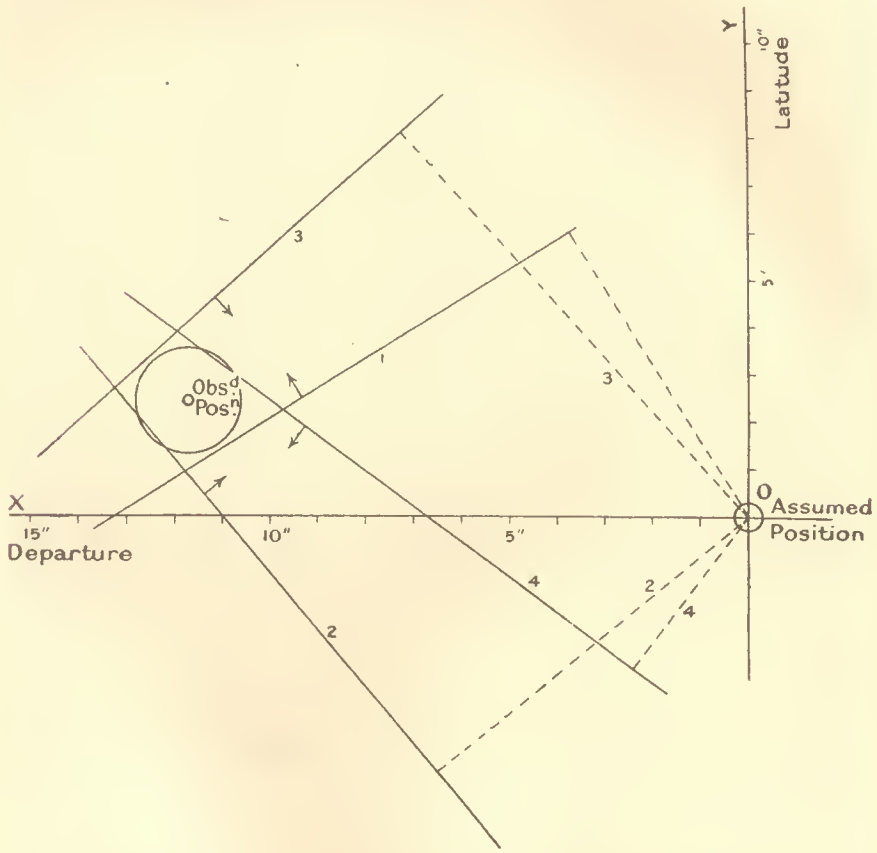


Fig. 196

OY is a meridian through the assumed position O, and OX is drawn perpendicular to OY ; these axes are divided on any suitable scale of latitude and departure respectively, say, ¼ of an inch to 1″ of arc. The bearings of the four stars are laid off (with pecked lines) from O and the position lines are drawn in the usual way. If all the observations were made with perfect accuracy and were subject only to the same errors due to incorrect values of refraction and prism angle, it would be possible to draw a circle to touch all the position lines. In practice this is seldom possible and it is necessary to draw a circle of best fit, as shown in the figure, such that its circumference falls as far outside one pair of opposing position lines as it does inside the other pair. The radius of this circle represents the mean error of the observations due to erroneously adopted values of refraction and prism angle, and the fit of the circumference on

the position lines provides a measure of the standard of accuracy of the four observations. The centre of the circle is the observed position and by measurement from the plot (Fig. 196) :

	D. Lat.	$= +\ 2''{\cdot}45$
	Departure	$= -\ 11''{\cdot}70$
	$\therefore$ D. Long.	$= -\ 11''{\cdot}77$

Assumed Position ..	$\phi_a = +6°\ 18'\ 16''{\cdot}84$	$\omega_a = +99°\ 51'\ 09''{\cdot}60$
Corrections	$+2''{\cdot}45$	$-\ 11''{\cdot}77$
Observed Position ..	$\Phi = +6°\ 18'\ 19''{\cdot}29$	$\Omega = +99°\ 50'\ 57''{\cdot}83$

Least Square Method of analysing Astrolabe Observations for Latitude and Longitude.

If only four observations, one in each quadrant, were obtained, they might well be examined by this graphical method and the resulting position could be taken off the plot. In practice, however, a series should consist of at least twelve and preferably sixteen or twenty sets of observations by any one observer and to eliminate, as far as possible, the personal equation, similar series should be obtained by two or three other observers. The accepted values of the latitude and longitude must then be derived from a consideration of the system of position lines given by the intercepts and azimuths of *all* the observed stars.

The method adopted for treating the system of position lines is the analytical or least square method ; in this, the position of the centre of the circle of best fit and its radius are made to satisfy the condition that the sum of the squares of the differences between its radius and the perpendiculars from its centre on to the position lines is the least possible. The co-ordinates of the centre of this circle (the axes being the meridian and the line at right angles to it through the assumed position) give the most probable values for the departure and difference of latitude from the assumed to the exact geographical position (being determined).

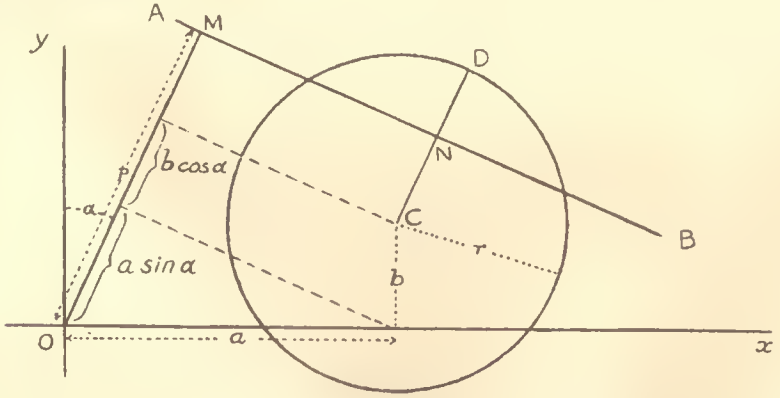


Fig. 197

Let AB in Fig. 197 be one of the position lines of the system obtained by one observer ; and let Ox, Oy be rectangular axes through O, the assumed position, Oy being the meridian. If C is the centre of the circle of best fit, and CND its radius perpendicular, at N, to AB, then the position and radius of this circle are to be determined so that the sum of the squares of the lengths DN is a minimum.

Let the azimuth of the star for AB, *i.e.*, the angle yOM, be α ; and the intercept, *i.e.*, the perpendicular OM from O to AB, be p. Then if a and b denote

the co-ordinates of the centre of the circle, the length of the perpendicular, CN, from the centre to AB is :

$$p - a \sin \alpha - b \cos \alpha ;$$

therefore, if r denotes the radius of the circle,

$$\mathrm{DN} = r \sim [p - a \sin \alpha - b \cos \alpha] ; \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Calculation of a, b and r :

We have therefore to find a, b and r from the condition :

$$\Sigma [r \sim [p - a \sin \alpha - b \cos \alpha]]^2 = \text{Minimum} \quad . \quad . \quad . \quad . \quad (2)$$

Differentiating this expression we have, for a minimum :

$$\left.\begin{aligned} \Sigma \sin \alpha \ [r \sim [p - a \sin \alpha - b \cos \alpha]] &= 0 \\ \Sigma \cos \alpha \ [r \sim [p - a \sin \alpha - b \cos \alpha]] &= 0 \\ \Sigma \ [r \sim [p - a \sin \alpha - b \cos \alpha]] &= 0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Then, if $s = \sin \alpha$; $c = \cos \alpha$; and n is the number of position lines :

$$a \Sigma s^2 + b \Sigma sc + r \Sigma s = \Sigma ps \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$a \Sigma sc + b \Sigma c^2 + r \Sigma c = \Sigma pc \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$a \Sigma s + b \Sigma c + rn = \Sigma p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

In obtaining the values of the expression Σs, Σc, Σsc, etc., it is necessary to have regard to the signs of sin α and cos α and also of p. The signs to be taken are :

sin α .. positive in the N.E. and S.E. quadrants,
cos α .. ,, ,, N.E. and N.W. ,,
p ..... ,, when towards,

and negative in other cases.

The values of a, b and r from equations (4), (5) and (6) are :

$$\left.\begin{aligned} a &= \frac{A_1 \ \Sigma p + B_1 \ \Sigma ps + C_1 \ \Sigma pc}{A_1 \ \Sigma s + B_1 \ \Sigma s^2 + C_1 \ \Sigma sc} \\ b &= \frac{A_2 \ \Sigma p + C_1 \ \Sigma ps + B_2 \ \Sigma pc}{A_1 \ \Sigma s + B_1 \ \Sigma s^2 + C_1 \ \Sigma sc} \\ r &= \frac{C_2 \ \Sigma p + A_1 \ \Sigma ps + A_2 \ \Sigma pc}{A_1 \ \Sigma s + B_1 \ \Sigma s^2 + C_1 \ \Sigma sc} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where $A_1 = \Sigma sc \ \Sigma c - \Sigma c^2 \ \Sigma s$; $A_2 = \Sigma sc \ \Sigma s - \Sigma s^2 \ \Sigma c$;
$B_1 = n \Sigma c^2 - (\Sigma c)^2$; $B_2 = n \Sigma s^2 - (\Sigma s)^2$;
$C_1 = \Sigma s \ \Sigma c - n \Sigma sc$; $C_2 = \Sigma s^2 \ \Sigma c^2 - (\Sigma sc)^2$;

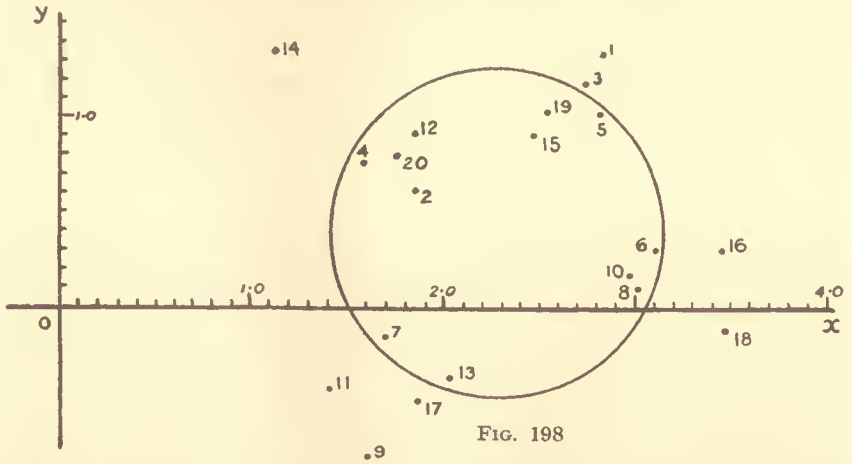


FIG. 198

The evaluation of a, b and r therefore requires the calculation of six expressions A_1, A_2, B_1, B_2, C_1 and C_2 involving α, in addition to Σs, Σc, Σsc, Σs_2, Σc_2, Σp, Σps, Σpc ; the latter are obtained in the tabulation of the data ; A_1, B_1, C_1 and B_2 are also required in the calculation of the weights.

$\text{Sin}^2\,\alpha$ (s^2) and $\cos^2\alpha$(c^2) are taken from tables of squares, and $\sin\alpha\cos\alpha$ (sc) is conveniently obtained, as $\frac{1}{2}\sin 2\alpha$, from the table of sines.

Calculation of Residuals.

Having tabulated the observational data and sin α, cos α, etc., and found the values of a, b and r, the residuals, v and their squares, v^2, are tabulated (the values of v^2 are required in the application of *Pierce's Criterion* and for the evaluation of the weights to be assigned to the values of the latitude and longitude). The residuals v are the lengths DN, and can either be taken from a diagram of the observations, or can be calculated from formula (1) on page 348.

If the former method is employed, the diagram should be drawn as shown in Fig. 198, on a scale sufficiently large for measurements to be made to the nearest 0·05 of a second. The centre of the " least square " circle and its radius are plotted with the values of a, b and r, calculated from the equations (7), and the individual observations are plotted as dots at the feet of the perpendiculars drawn from the centre of the circle on to the position lines. (See note on p. 360.)

Rejection of faulty observations.

If any of the residuals are large compared with the others it may be necessary to reject the corresponding observations. *Pierce's Criterion* is therefore applied. An observation should be rejected if the square of its residual, v^2, is greater than m times the sum of the squares of the residuals, *i.e.*, if v^2 is greater than $m\Sigma v^2$. Values of m are given in Table XXI.

It is to be noted that Pierce's Criterion must be applied with $m=1$, first and then, if any observations are rejected, it is to be repeated with $m=2$, and so on.

Rejected observations, if any, should be encircled in red in the table and on the diagram of observations.

If no observations are rejected the values of a, b and r are as already found ; but if any observations are rejected a, b and r must be recalculated from the observations retained, n now being the number of retained observations. Also the values of v and v^2 must be retabulated and Σv^2 re-determined.

Mean square errors of a and b.

The squares of the mean square errors of a and b should now be found, as they are required in the determination of the final accepted values of the latitude and longitude from the results of all the observers. If E_a, E_b, denote the mean square errors of a and b :

$$\left.\begin{aligned} E_a^2 &= \frac{n\Sigma c^2-(\Sigma c)^2}{A_1\Sigma s+B_1\Sigma s^2+C_1\Sigma sc}\times\frac{\Sigma v^2}{n-3}=\frac{B_1}{D}\times\frac{\Sigma v^2}{n-3}\\ E_b^2 &= \frac{n\Sigma s^2-(\Sigma s)^2}{A_1\Sigma s+B_1\Sigma s^2+C_1\Sigma sc}\times\frac{\Sigma v^2}{n-3}=\frac{B_2}{D}\times\frac{\Sigma v^2}{n-3}\end{aligned}\right\} \quad . \quad . \quad (8)$$

where $D=A_1\Sigma s+B_1\Sigma s^2+C_1\Sigma sc$, *i.e.*, the denominator of the expressions for a, b and r in equations (7).

Accepted position of one observer.

The values of a and b for the series of observations by the observer in question have now been finally determined ; and if ϕ_a, ω_a are the latitude and longitude of the assumed position, and Φ, Ω those of the exact geographical position : $\Phi=\phi_a\pm b$; $\Omega=\omega_a\pm a\sec\phi_a$, according to the signs of a and b.

Final accepted position of all observers.

The observations of each observer having been treated in the manner described in the preceding paragraphs, we have the values of the latitude and longitude ϕ_1, ω_1 ; ϕ_2, ω_2 ; ϕ_3, ω_3 ; etc., together with the squares of E_{a1}, E_{b1} ; E_{a2}, E_{b2} ; E_{a3}, E_{b3}, etc., for each series of observations. From these the final accepted values of the latitude and longitude are obtained in the usual way, *i.e.*, as the weighted means of ϕ_1, ϕ_2, ϕ_3, etc., and of ω_1, ω_2, ω_3, etc. It is

to be noted that the weights of ϕ_1, ϕ_2, ϕ_3, etc., are inversely as the squares of E_{b1}, E_{b2}, E_{b3}, etc., and those of ω_1, ω_2, ω_3, etc., inversely as the squares of E_{a1}, E_{a2}, E_{a3}, etc.

Thus if the final accepted values of the latitude and longitude from the observations of all observers are Φ and Ω we have :

$$\left.\begin{aligned} \Phi &= \frac{\phi_1 w_1 + \phi_2 w_2 + \phi_3 w_3 + \ldots}{w_1 + w_2 + w_3 + \ldots} \\ \Omega &= \frac{\omega_1 W_1 + \omega_2 W_2 + \omega_3 W_3 + \ldots}{W_1 + W_2 + W_3 + \ldots} \end{aligned}\right\} \quad \ldots\ldots \quad (9)$$

where w_1, w_2, w_3, etc., are the weights of ϕ_1, ϕ_2, ϕ_3, etc., and W_1, W_2, W_3, etc., those of ω_1, ω_2, ω_3, etc.

Probable error of final accepted position.

Finally the probable errors of the accepted values should be found from the usual formula. Thus if ε_1, ε_2, denote the probable errors of Φ and Ω :

$$\left.\begin{aligned} \varepsilon_1 &= \cdot 6745 \sqrt{\frac{w_1 u_1^2 + w_2 u_2^2 + \ldots}{(N-1)(w_1 + w_2 + \ldots)}} \\ \varepsilon_2 &= \cdot 6745 \sqrt{\frac{W_1 U_1^2 + W_2 U_2^2 + \ldots}{(N-1)(W_1 + W_2 + \ldots)}} \end{aligned}\right\} \quad \ldots\ldots \quad (10)$$

where N is the number of values of the latitude and longitude ; u_1, u_2, etc., the residuals of ϕ_1, ϕ_2, etc., from their weighted mean Φ and U_1, U_2, etc., those of ω_1 ω_2, etc., from their weighted mean Ω.

Example.

The following three series of observations were taken to determine the geographical position of the Power Station Chimney, Port Sudan. The second and third series do not contain sufficient observations to be very satisfactory, but the three series are given as an example of the weighting of the values of the latitude and longitude from the observations by different observers.

I

Obsⁿ. Spot : 0″·3 north of Power Stⁿ. Chʸ. OBSERVATIONS. OBSERVER A.

Ref. No.	Bearing (α)	Intercept (p)		sin α (s)		cos α (c)		sin α cos α (sc)		s^2	c^2	ps		pc	
		+	−	+	−	+	−	+	−			+	−	+	−
1	54° 31′ N.E.	3·71	—	0·8143	—	0·5805	—	0·4727	—	0·6631	0·3370	3·0210	—	2·1536	—
2	37° 01′ S.E.	6·81	—	0·6020	—	—	0·7985	—	0·4807	0·3624	0·6376	4·1000	—	—	5·4375
3	40° 04′ S.W.	—	1·05	—	0·6437	—	0·7653	0·4926	—	0·4143	0·5857	0·6759	—	0·8036	—
4	34° 14′ N.W.	—	6·71	—	0·5626	0·8268	—	—	0·4651	0·3165	0·6836	3·7747	—	—	5·5475
5	38° 00′ N.E.	2·10	—	0·6157	—	0·7880	—	0·4851	—	0·3791	0·6209	1·2929	—	1·6548	—
6	39° 24′ S.E.	7·05	—	0·6347	—	—	0·7727	—	0·4905	0·4028	0·5971	4·4749	—	—	5·4478
7	39° 24′ S.W.	—	2·35	—	0·6347	—	0·7727	0·4905	—	0·4028	0·5971	1·4916	—	1·8159	—
8	40° 06′ N.W.	—	5·24	—	0·6441	0·7649	—	—	0·4927	0·4149	0·5851	3·3752	—	—	4·0082
9	33° 41′ N.E.	2·48	—	0·5546	—	0·8321	—	0·4615	—	0·3076	0·6924	1·3754	—	2·0636	—
10	50° 27′ S.E.	7·57	—	0·7711	—	—	0·6368	—	0·4910	0·5946	0·4055	5·8370	—	—	4·8202
11	50° 26′ S.W.	—	0·57	—	0·7709	—	0·6370	0·4910	—	0·5943	0·4058	0·4394	—	0·3631	—
12	49° 48′ N.W.	—	5·95	—	0·7638	0·6455	—	—	0·4930	0·5834	0·4167	4·5447	—	—	3·8405
13	38° 00′ N.E.	3·63	—	0·6157	—	0·7880	—	0·4851	—	0·3791	0·6209	2·2348	—	2·8604	—
14	50° 27′ S.E.	4·60	—	0·7711	—	—	0·6368	—	0·4910	0·5946	0·4055	3·5469	—	—	2·9290
15	40° 04′ S.W.	0·63	—	—	0·6437	—	0·7653	0·4926	—	0·4143	0·5857	—	0·4055	—	0·4821
16	40° 06′ N.W.	—	8·89	—	0·6441	0·7649	—	—	0·4927	0·4149	0·5851	5·7262	—	—	6·8002
17	54° 31′ N.E.	7·53	—	0·8143	—	0·5805	—	0·4727	—	0·6631	0·3370	6·1316	—	4·3709	—
18	40° 15′ S.E.	8·72	—	0·6461	—	—	0·7632	—	0·4931	0·4174	0·5825	5·6343	—	—	6·6555
19	43° 11′ S.W.	—	4·18	—	0·6843	—	0·7292	0·4990	—	0·4683	0·5317	2·8605	—	3·0480	—
20	34° 14′ N.W.	—	8·32	—	0·5626	0·8268	—	—	0·4651	0·3165	0·6836	4·6804	—	—	6·8786
		54·83	43·26	6·8396	6·5545	7·3980	7·2775	4·8428	4·8549			65·2174	0·4055	19·1339	52·8471
$n=$ 20		Σp : +11·57		Σs : + 0·285		Σc : + 0·120		Σsc : − 0·012		Σs^2 : 9·104	Σc^2 : 10·896	Σps : + 64·81		Σpc : − 33·71	

$$A_1 = \Sigma sc\Sigma c - \Sigma c^2\Sigma s = -(0{\cdot}012\times 0{\cdot}120) - (10{\cdot}896\times 0{\cdot}285) = -3{\cdot}107$$
$$B_1 = n\Sigma c^2 - (\Sigma c)^2 = +(20\times 10{\cdot}896) - (0{\cdot}120)^2 = +217{\cdot}906$$
$$C_1 = \Sigma s\Sigma c - n\Sigma sc = +(0{\cdot}285\times 0{\cdot}120) + (20\times 0{\cdot}012) = +0{\cdot}274$$
$$A_2 = \Sigma sc\Sigma s - \Sigma s^2\Sigma c = -(0{\cdot}012\times 0{\cdot}285) - (9{\cdot}104\times 0{\cdot}120) = -1{\cdot}096$$
$$B_2 = n\Sigma s^2 - (\Sigma s)^2 = +(20\times 9{\cdot}104) - (0{\cdot}285)^2 = +181{\cdot}999$$
$$C_2 = \Sigma s^2\Sigma c^2 - (\Sigma sc)^2 = +(9{\cdot}104\times 10{\cdot}896) - (0{\cdot}012)^2 = +99{\cdot}197$$
$$A_1\Sigma p + B_1\Sigma ps + C_1\Sigma pc = -(3{\cdot}107\times 11{\cdot}57) + (217{\cdot}906\times 64{\cdot}81) - (0{\cdot}274\times 33{\cdot}71) = +14077$$
$$A_2\Sigma p + C_1\Sigma ps + B_2\Sigma pc = -(1{\cdot}096\times 11{\cdot}57) + (0{\cdot}274\times 64{\cdot}81) - (181{\cdot}999\times 33{\cdot}71) = -6130$$
$$C_2\Sigma p + A_1\Sigma ps + A_2\Sigma pc = +(99{\cdot}197\times 11{\cdot}57) - (3{\cdot}107\times 64{\cdot}81) + (1{\cdot}096\times 33{\cdot}71) = +983$$
$$A_1\Sigma s + B_1\Sigma s^2 + C_1\Sigma sc = -(3{\cdot}107\times 0{\cdot}285) + (217{\cdot}906\times 9{\cdot}104) - (0{\cdot}274\times 0{\cdot}012) = +1983$$

$14077 \div 1983 = +7{\cdot}099$ $\qquad$ $-6130 \div 1983 = -3{\cdot}091$ $\qquad$ $983 \div 1983 = 0{\cdot}496$

$a = +7{\cdot}099$ $(a \sec \phi_a = +7{\cdot}535)$ $\qquad$ $b = -3{\cdot}091$ $\qquad$ $r = 0{\cdot}496$

RESIDUALS.

Ref. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Residual (v)	−0·77	−0·43	+0·66	−0·66	−0·33	−0·34	−0·73	+1·20	+0·62	−0·37	+2·44	+0·97	+1·20	−3·34	+2·34	−2·45	+3·05	+1·28	−2·07	−2·27
. v^2	0·59	0·18	0·44	0·44	0·11	0·12	0·53	1·44	0·38	0·14	5·95	0·94	1·44	11·16	5·48	6·00	9·30	1·64	4·28	5·15

$\Sigma v^2 = 55{\cdot}7$

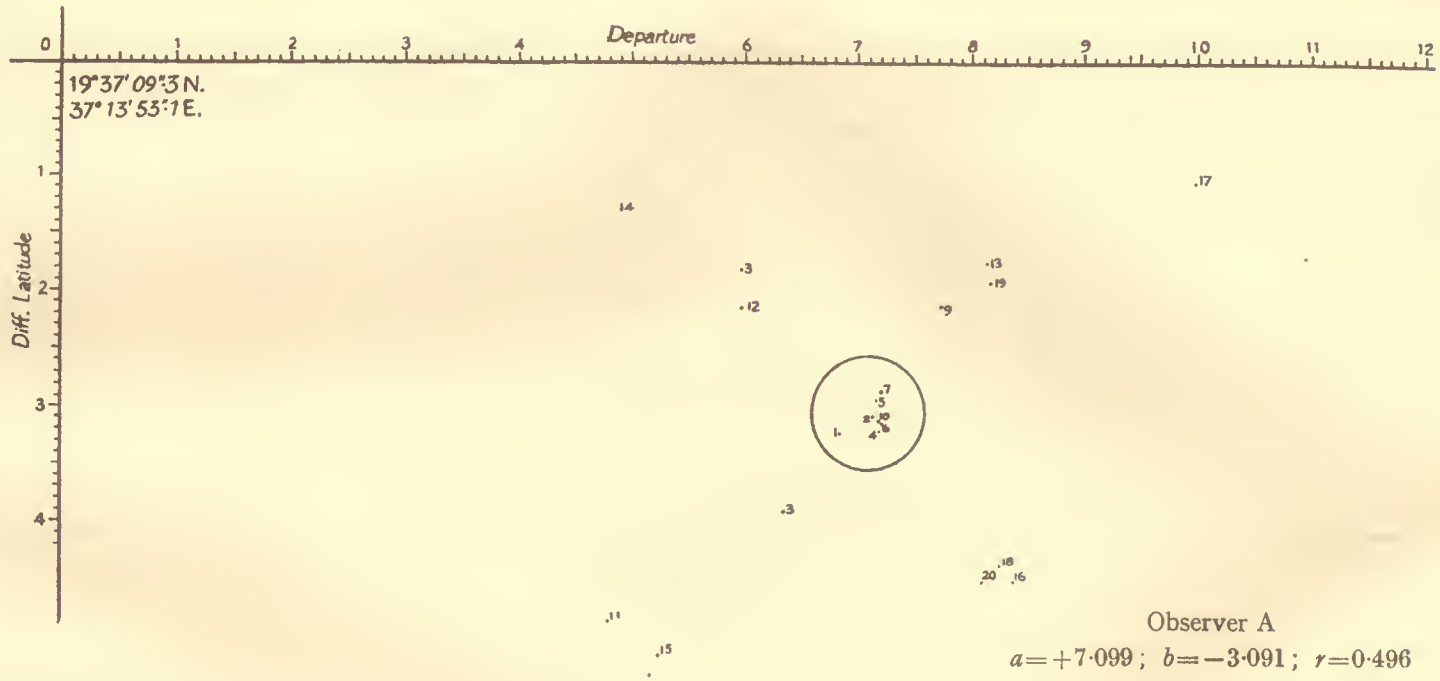
Pierce's Criterion Constant for $n=20$, $m=1$ is ·264; therefore for the rejection of an observation v^2 must be greater than $·264\times 55{\cdot}7$, *i.e.* $v^2 > 14{\cdot}7$. Since no value of v^2 exceeds 14·7 no observations are rejected and the values of a, b, and r are as found above.

$$E_a^2 = \frac{218}{1983}\times\frac{56}{17} = 0{\cdot}36 \quad \left(\text{Weight} = \frac{1}{0{\cdot}36} = 2{\cdot}8\right) \qquad E_b^2 = \frac{182}{1983}\times\frac{56}{17} = 0{\cdot}30 \quad \left(\text{Weight} = \frac{1}{0{\cdot}30} = 3{\cdot}3\right)$$

RESULT A

—	Latitude.	Longitude.
Assumed	19° 37′ 09″·3 N.	37° 13′ 53″·1 E.
Difference	−03″·09	+ 07″·54
Observed	19° 37′ 06″·21 N.	37° 14′ 00″·64 E.
Weight	3·3	2·8

DIAGRAM OF OBSERVATIONS—A



Observer A

$a = +7{\cdot}099$; $b = -3{\cdot}091$; $r = 0{\cdot}496$

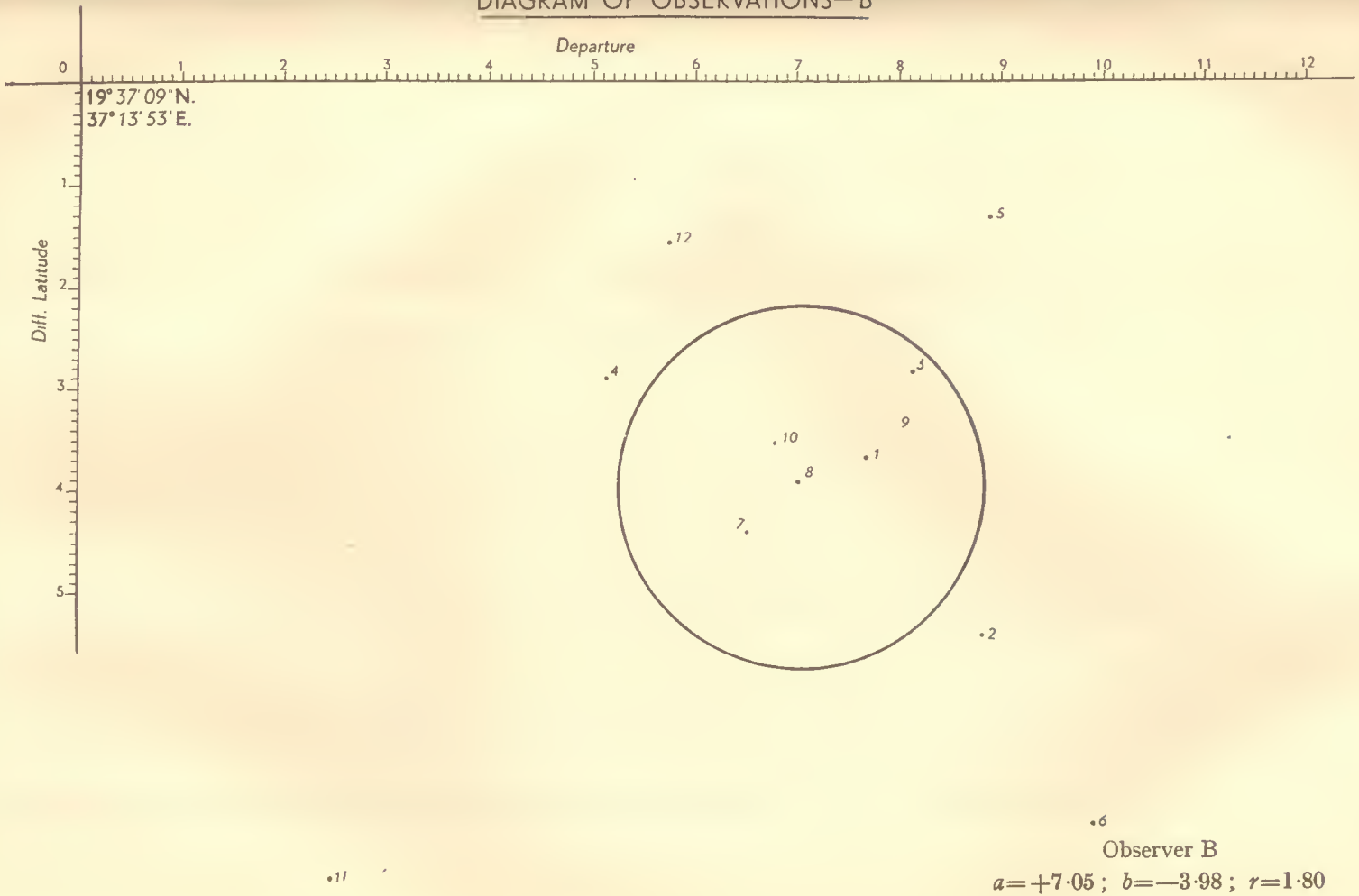
II.

Obsⁿ. Spot : 0″·3 north of Power Stⁿ. Chʸ. OBSERVATIONS. OBSERVER B.

Ref. No.	Bearing (α)	Intercept (p)		$\sin\alpha$ (s)		$\cos\alpha$ (c)		$\sin\alpha\cos\alpha$ (sc)		s^2	c^2	ps		pc	
		+	−	+	−	+	−	+	−			+	−	+	−
1	64° 53′ N.E.	5·4	—	0·9054	—	0·4245	—	0·3843	—	0·8197	0·1802	4·8893	—	2·2921	—
2	50° 25′ S.E.	10·3	—	0·7707	—	—	0·6372	—	0·4911	0·5940	0·4060	7·9382	—	—	6·5633
3	43° 25′ S.W.	—	3·5	—	0·6873	—	0·7264	0·4992	—	0·4724	0·5277	2·4055	—	2·5423	—
4	60° 06′ N.W.	—	5·9	—	0·8669	0·4985	—	—	0·4321	0·7515	0·2485	5·1147	—	—	2·9410
5	33° 40′ N.E.	3·9	—	0·5544	—	0·8323	—	0·4614	—	0·3074	0·6927	2·1620	—	3·2459	—
6	40° 21′ S.E.	12·0	—	0·6475	—	—	0·7621	—	0·4934	0·4193	0·5808	7·7694	—	—	9·1452
7	50° 25′ S.W.	—	2·2	—	0·7707	—	0·6372	0·4911	—	0·5940	0·4060	1·6955	—	1·4018	—
8	28° 36′ N.W.	—	6·8	—	0·4787	0·8780	—	—	0·4203	0·2292	0·7709	3·2552	—	—	5·9704
9	54° 23′ N.E.	4·5	—	0·8129	—	0·5824	—	0·4734	—	0·6608	0·3392	3·6582	—	2·6206	—
10	30° 41′ S.E.	6·5	—	0·5103	—	—	0·8600	—	0·4388	0·2604	0·7396	3·3169	—	—	5·5900
11	50° 25′ S.W.	3·2	—	—	0·7707	—	0·6372	0·4911	—	0·5940	0·4060	—	2·4662	—	2·0390
12	28° 37′ N.W.	—	4·1	—	0·4789	0·8778	—	—	0·4204	0·2293	0·7705	1·9637	—	—	3·5992
		45·8	22·5	4·2012	4·0532	4·0935	4·2601	2·8005	2·6961			44·1686	2·4662	12·1027	35·8481
		Σp : +23·3		Σs : +0·148		Σc : −0·167		Σsc : +0·104		Σs^2 : 5·932	Σc^2 : 6·068	Σps : +41·70		Σpc : −23·75	

$a = +7{\cdot}05$ ($a \sec \phi_a = 7{\cdot}48$) ; $b = -3{\cdot}98$; $r = 1{\cdot}80$.

DIAGRAM OF OBSERVATIONS—B



Observer B

$a = +7{\cdot}05$; $b = -3{\cdot}98$; $r = 1{\cdot}80$

RESIDUALS.

Ref. No.	1	2	3	4	5	6	7	8	9	10	11	12
Residual (v)	− 1·09	+ 0·53	− 3·35	+ 0·40	+ 1·50	+ 2·60	− 1·10	− 1·73	− 0·71	− 2·32	+ 4·30	+ 0·97
v^2	1·19	0·28	11·22	0·16	2·25	6·76	1·21	2·99	0·50	5·38	18·49	0·94

$\Sigma v^2 = 51{\cdot}4$

Pierce's Criterion Constant for $n=12$; $m=1$ is ·373 ; therefore for the rejection of an observation v^2 must be greater than $\cdot 373 \times 51{\cdot}4$, *i.e.* $v^2 > 19{\cdot}2$. Since no value of v^2 exceeds 19·2 no observations are rejected and the values of a, b and r are as found above.

$$E_a^2 = \frac{72{\cdot}8}{431{\cdot}9} \times \frac{51{\cdot}4}{9} = 0{\cdot}96 \qquad \left(\text{Weight} = \frac{1}{0{\cdot}96} = 1{\cdot}0\right)$$

$$E_b^2 = \frac{71{\cdot}2}{431{\cdot}9} \times \frac{51{\cdot}4}{9} = 0{\cdot}94 \qquad \left(\text{Weight} = \frac{1}{0{\cdot}94} = 1{\cdot}1\right)$$

RESULT B.

—	Latitude.	Longitude.
Assumed	19° 37′ 09″ N.	37° 13′ 53″ E.
Difference	− 03″·98	+ 07″·48
Observed	19° 37′ 05″·02 N.	37° 14′ 00″·48 E.
Weight	1·1	1·0

III.

Obsⁿ. Spot : 0″·3 north of Power Stⁿ. Chʸ. OBSERVATIONS. Observer C.

Ref. No.	Bearing (α)	Intercept (p)		$\sin\alpha$ (s)		$\cos\alpha$ (c)		$\sin\alpha\cos\alpha$ (sc)		s^2	c^2	ps		pc	
		+	−	+	−	+	−	+	−			+	−	+	−
1	38° 00′ N.E.	4·2	—	0·6157	—	0·7880	—	0·4851	—	0·3791	0·6209	2·5858	—	3·3096	—
2	50° 26′ S.E.	4·6	—	0·7709	—	—	0·6370	—	0·4910	0·5943	0·4058	3·5461	—	—	2·9301
3	54° 59′ S.W.	—	3·2	—	0·8190	—	0·5738	0·4699	—	0·6708	0·3292	2·6208	—	1·8362	
4	34° 14′ N.W.	—	8·6	—	0·5626	0·8267	—	—	0·4651	0·3165	0·6834	4·8380	—	—	7·1102
5	65° 22′ N.E.	6·7	—	0·9090	—	0·4168	—	0·3788	—	0·8263	0·1737	6·0901	—	2·7926	—
6	30° 42′ S.E.	8·7	—	0·5105	—	—	0·8598	—	0·4390	0·2606	0·7393	4·4417	—	—	7·4807
7	39° 24′ S.W.	0·0	—	—	0·6347	—	0·7727	0·4905	—	0·4028	0·5971	0·0	—	0·0	—
8	45° 20′ N.W.	—	10·1	—	0·7112	0·7030	—	—	0·4999	0·5058	0·4942	7·1832	—	—	7·1000
		24·2	21·9	2·8061	2·7275	2·7345	2·8433	1·8243	1·8950			31·3057	—	7·9384	24·6210
		Σp : +2·3		Σs : +0·079		Σc : −0·109		Σsc : −0·071		Σs^2 : 3·956	Σc^2 : 4·044	Σps : +31·31		Σpc : −16·68	

a=7·84 (a sec ϕ_a=8·32) ; b= −3·98 ; r= ·156.

RESIDUALS.

Ref. No.	1	2	3	4	5	6	7	8
Residual (v)	+2·34	−4·13	+0·78	−1·05	+1·08	+1·12	+1·72	−1·88
v^2	5·48	17·06	0·61	1·10	1·17	1·25	2·96	3·53

Σv^2=33·2

The constant for Pierce's Criterion for n=8 ; m=1 is 0·494 ; therefore for the rejection of an observation v^2 must be greater than 0·494×33·2, *i.e.*, $v^2 >$ 16·4. Observation No. 2 is therefore to be rejected.

The constant for $n=8$; $m=2$ is 0·341 ; thus for the rejection of further observations v^2 must be greater than $0{\cdot}341 \times 33{\cdot}2$, *i.e.*, $v^2 > 11{\cdot}3$. Therefore no further observations are rejected. Omitting Observation No. 2 the values of Σp, etc., for the seven observations are :—

Σp	Σs	Σc	Σsc	Σs^2	Σc^2	Σps	Σpc
−2·3	−0·692	+0·528	+0·420	3·362	3·638	+27·760	−13·752

Therefore $\underline{a = +9{\cdot}07}$ ($a \sec \phi_a = 9{\cdot}63$) ; $\underline{b = -4{\cdot}97}$; $\underline{r = +0{\cdot}94}$

The value of Σv^2 is found to be 6·4 ; and therefore

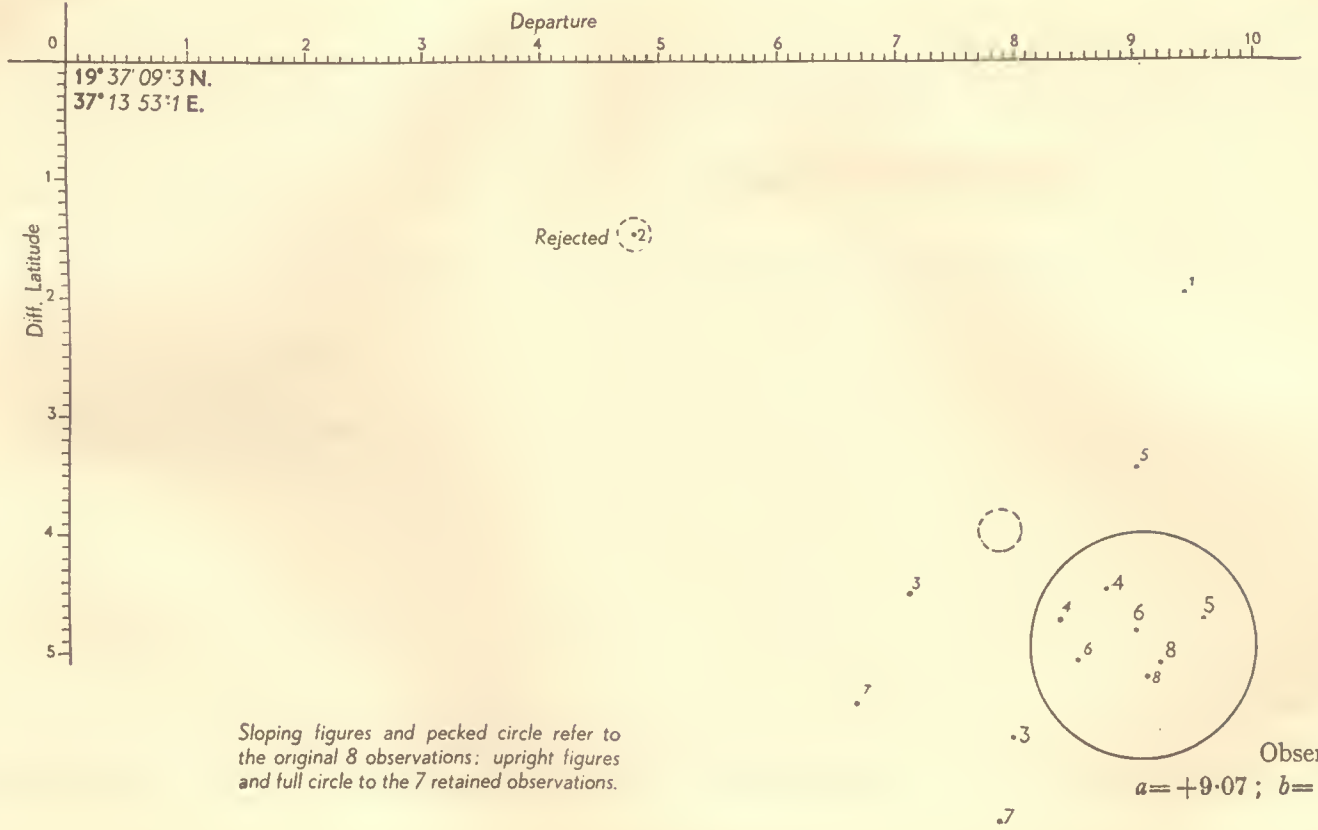
$$E_a^2 = \frac{25{\cdot}2}{79{\cdot}5} \times \frac{6{\cdot}4}{4} = 0{\cdot}51 \qquad (\text{Weight} = \frac{1}{0{\cdot}51} = 2{\cdot}0)$$

$$E_b^2 = \frac{23{\cdot}1}{79{\cdot}5} \times \frac{6{\cdot}4}{4} = 0{\cdot}46 \qquad (\text{Weight} = \frac{1}{0{\cdot}46} = 2{\cdot}2)$$

RESULT C.

—	Latitude.	Longitude.
Assumed	19° 37′ 09″·3 N.	37° 13′ 53″·1 E.
Difference	− 04″·97	+ 09″·63
Observed	19° 37′ 04″·33 N.	37° 14′ 02″·73 E.
Weight	2·2	2·0

DIAGRAM OF OBSERVATIONS—C



Sloping figures and pecked circle refer to the original 8 observations: upright figures and full circle to the 7 retained observations.

Observer C

$a = +9{\cdot}07$; $b = -4{\cdot}97$; $r = 0{\cdot}94$

IV—FINAL RESULT

Observation Spot .... 0″·3 North of Power Station Chimney

Observed Position	Observer A.		Observer B.		Observer C.	
	Latitude.	Longitude.	Latitude.	Longitude.	Latitude.	Longitude.
	19° 37′ 06″·21 N.	37° 14′ 00″·64 E.	19° 37′ 05″·02 N.	37° 14′ 00″·48 E.	19° 37′ 04″·33 N.	37° 14′ 02″·73 E.
Weights	3·3	2·8	1·1	1·0	2·2	2·0

Accepted Geographical Position of Observation Spot .. .. Lat. 19° 37′ 05″·38 N. Long. 37° 14′ 01″·33 E.
Reduction to Power Station Chimney .. .. 00″·30 00″·00

Accepted Geographical Position of Port Sudan Power Station Chimney .. .. Lat. 19° 37′ 05″·08 N. Long. 37° 14′ 01″·33E.

The probable error of the accepted latitude = 0″·41 and of longitude = 0″·48.

Note.

The values of v, the residuals, are the distances from the dots to the appropriate side of the circumference of the circle, measured either towards or through the centre of the circle; and the residual is positive if the point is to be moved away from the star, and *vice versa* if moved towards the star.

The appropriate side of the circle is determined by the following rule :—

If r is positive, all azimuths point away from the centre of the circle, and *vice versa* if r is negative.

If the residuals are calculated from the formula, attention must be paid to the signs of p, r, a, b, $\sin\alpha$, $\cos\alpha$, and a slide rule may be employed to obtain the values of $a \sin\alpha$ and $b \cos\alpha$ with quite enough accuracy for practical purposes.

Whichever method is employed, the accuracy of the residuals should be checked by the fact that their algebraic sum should be zero; and the general agreement of the sights should be indicated by a diagram, even if the first, or graphical, method of obtaining residuals is not employed.

Other Methods of Finding Geographical Position Ashore

The astrolabe is so simple to use and gives such good results that it should always be employed on land by the hydrographic surveyor in preference to other observing instruments for finding geographical position. If, however, an astrolabe is not available, it may be necessary to resort to either theodolite or sextant for this purpose.

A theodolite can be used in the same manner as an astrolabe for observations of stars at some pre-arranged altitude and satisfactory results have been obtained by this method.

It should, however, be noted that :

(1) Unless the circle readings are changed, only one shot is obtained at each star. Since a single observation is liable to be considerably in error, a set of observations should be made with a series of different circle settings, the mean of which is the required altitude. The circle settings of corresponding observations in the series cannot be made accurately enough to give such an exact mean as that obtained with an astrolabe.

(2) For a given telescope magnification, the apparent speed at which a star passes across the horizontal diaphragm wire is only half that at which the direct and reflected images meet in the astrolabe.

(3) The vertical is less precisely defined in the theodolite, by reference to the position of a bubble in a tube, than it is in the astrolabe, by reference to the surface of a mercury pool.

The method more usually adopted with instruments other than the astrolabe is to treat latitude and longitude separately, determining the former from observations of heavenly bodies on and near the meridian, and the latter from another set of observations of bodies east and west of the meridian, preferably when they are near the prime vertical. To preserve the principle of equal altitudes as nearly as it is possible to do so, the following procedure is adopted :

(1) For latitude, observations are confined to N. and S. stars which are so selected that they can be grouped in pairs in which the two stars do not differ in altitude more than 2° or 3°.

(During summer in high latitudes it may sometimes be necessary to make use of the sun for determining latitude, but as it is impossible to make corresponding observations on the opposite side of the zenith this practice should be avoided if possible.)

(2) For longitude, the observations are taken at such times that the mean altitude of each set (as deduced from the instrumental readings) is the same or nearly the same on either side of the meridian, whether a pair of stars or the sun is used.

In using a theodolite, the observations should be confined to heavenly bodies at altitudes of 20° to 70°. The lower limit is dictated by the uncertainty in the value of refraction below 20°, the upper by convenience.

Latitude Observations with a Theodolite.

Star programme.—In theory, latitude can be found from a single measurement of the altitude of a star or the sun when it is on the meridian, but for the reasons before stated it is much better to measure a series of altitudes, starting about 6 minutes before the body reaches the meridian and continuing for about 6 minutes after it has passed. The mean altitude of the series may be " reduced " to the meridian altitude by the method described later.

The observer's geographical position must be known within 5 miles or so ; the local sidereal time (L.S.T.) at which the observations may be started (say, the end of twilight) is then calculated from the formula L.S.T.=R.A.M.S.+H.A.M.S. (or, R.+L.M.T.), to the nearest minute. L.S.T. also=H.A.*+R.A.*, so it is obvious that the right ascension (R.A.) of any star on the meridian at the time is equal to this L.S.T., since its hour angle (H.A.) is 0 hrs. Starting

with this value, say R.A.(1), pick out from " Apparent Places of Fundamental Stars," and list all stars of magnitude 4·5 and brighter which will be at an altitude of between 20° and 70° when they cross the meridian, *i.e.*, those whose declinations satisfy the equation, alt.=90°− (lat.~dec.) (in the case of stars crossing the meridian below the pole, alt.=(lat.+dec.)−90°, and the R.A.= L.S.T.+12h.). Sufficient observations can usually be obtained in 5 or 6 hours, the list is, therefore, extended until stars whose R.A. is (1)+6h. are reached. North and south stars should be listed separately. Example :

Assumed Posn. : Lat.+6°20′, Long.−76°30′. L.S.T. of end of twilight=10h. 0m.

Hence, limits of dec. { N. stars, +26° 20′ to +76° 20′ ; S. ,, , −13° 40′ to −63° 40′ } Limits of R.A. } 10h. 0m. to 16h. 0m.

N. Stars.

Star.	Mag.	R.A.	Dec.	Z.D.	Alt.	Pair.
		h. m.	° ′	° ′	° ′	
μ Urs. Maj. ..	3·2	10 19	+41 49	35 29	54 31	
β Urs. Maj. ..	2·4	10 58	56 43	50 23	39 37	
α Urs. Maj. ..	1·9	11 00	62 05	55 45	34 15	
ψ Urs. Maj. ..	3·1	11 06	44 50	38 30	51 30	B
λ Drac.	4·1	11 28	69 40	63 20	26 40	
γ Urs. Maj. ..	2·5	11 51	54 02	47 42	42 18	A
δ Urs. Maj. ..	3·4	12 12	57 23	51 03	38 57	
κ Drac.	3·9	12 31	70 08	63 48	26 12	C
ε Urs. Maj. ..	1·7	12 51	56 18	49 58	40 02	
α Can. Ven. ..	2·9	12 53	38 39	32 19	57 41	
etc..						

S. Stars.

Star.	Mag.	R.A.	Dec.	Z.D.	Alt.	Pair.
		h. m.	° ′	° ′	° ′	
G Vel.	4·1	10 12	−41 49	48 09	41 51	A
G Car.	3·4	10 15	61 01	67 21	22 39	
α Antliae	4·4	10 24	30 45	37 05	52 55	
η Argus	Var.	10 43	59 22	65 42	24 18	C
ξ Hydrae	3·7	11 30	31 31	37 51	52 09	B
λ Cent.	3·3	11 33	62 41	69 01	20 59	
δ Cent.	2·9	12 05	50 23	56 43	33 17	
δ Crucis	3·1	12 12	58 24	64 44	25 16	
α Crucis	1·1	12 23	62 45	69 05	20 55	
γ Crucis	1·6	12 28	56 46	63 06	26 54	
γ Cent.	2·4	12 38	48 37	54 57	35 03	
β Crucis	1·5	12 44	59 21	65 41	24 19	
etc.						

From these lists suitable stars may be paired, as shown in the last column, noting that, to make a good pair, stars must not differ in altitude by more than 2° to 3°, must not be observed more than 2 hours apart (less if possible) and, as the observations of one star take 10 to 15 minutes, at least 15 minutes must be allowed between consecutive stars. A chronological list, for use in the field, is then prepared, thus :

N. or S.	Star.	Mag.	L.S.T.	Alt.
			h. m.	° ′
S	G Vel.	4·1	10 12	41 51
S	η Arg.	Var.	10 43	24 18
N	ψ Urs. Maj.	3·1	11 06	51 30
S	ξ Hydrae	3·7	11 30	52 09
N	γ Urs. Maj.	2·5	11 51	42 18
N	κ Drac.	3·9	12 31	26 12
	etc. .	etc.	etc.	etc.

A watch set to L.S.T. may be taken ashore or, if preferred, the times can be converted to L.M.T., or to the time kept by the chronometer watch in use. The preliminary lists should be kept handy during the observations in case an alteration in the programme is necessitated by clouds or some other cause.

Procedure in Observing.—When the theodolite has been set up and levelled over the observation spot, the lower plate should be clamped so that the horizontal circle reading is 360° when the telescope is in the true meridian the direction of which may be found by the compass fitted to the theodolite or by means of a known, or observed, bearing to some suitable mark.

Each observation consists in bringing the star on to the horizontal cross-wire (close to its intersection with the vertical wire) and calling " stop " when contact is made so that an assistant can note the time on a chronometer watch to the nearest 0·2 of a second. The vertical circle is then read by the observer and recorded by the assistant. If the bubble in the upper level is visible from the eye end of the telescope, as in the Tavistock theodolite, it must be centred carefully just prior to each observation. If the bubble is not visible from the eye end, its position in its tube must be read immediately after each observation, and the vertical circle readings must be corrected for any displacement by the methods described in Chapter III, pp. 52 and 53. Observations should be started about 7 minutes before the star reaches the meridian, and the " set " should consist of an even number, of about 12 to 16, face being changed half-way through. (Some observers prefer to change face after the first, and then after every other observation. In experienced hands, this method is probably slightly the better, but the rate of observing is slower and the danger of upsetting the level of the theodolite is greater.) Ideally, the number of observations taken before the transit should equal the number taken after.

The assistant should record the height of the barometer and thermometer for each set, in order that the correction for refraction may be adjusted later.

Computation of Latitude from Theodolite Observations of Circum-Meridian Stars.

Circum-meridian observations are confined to a period when the star is within a few minutes of meridian passage and an observed altitude can be corrected to the meridian altitude by the following formula—which is suitable for logarithmic or machine computation :

$$\text{Corrn.} = \cos\delta \times \cos\phi \sec \text{alt.} \times \frac{\text{vers h}}{\sin 1''}$$

Where δ = declination of the star
ϕ = latitude of observation spot.
h = hour angle.

This correction is known as the " Reduction to the Meridian " and is so called because it is subtractive from the observed zenith distance. The value of $\frac{\text{vers h}}{\sin 1''}$ is given in Table XIII for hour angles up to twenty-five minutes. It is not necessary to calculate the reduction for each individual observation, for, by taking out the value of $\frac{\text{vers h}}{\sin 1''}$ for the hour angle of each observation, a mean value of this quantity is obtained and can be used in the equation. The reduction so found can then be added to the mean of the observed altitudes. It is not strictly correct to assume that the hour angle of each observation is equal to the difference between the watch time of that observation and the calculated watch time of transit, but, by pairing observations on opposite sides of the zenith, any error in this assumption is eliminated.

In computing the results, the star's declination should be taken from the "Apparent Places of Fundamental Stars" or the standard edition of the Nautical Almanac, but its R.A. and R. (*i.e.*, R.A.M.S.$\pm$12h.) may be taken from the abridged edition of the Nautical Almanac if desired. An example is given overleaf.

Example of computing latitude from observations of circum-meridian stars.

Date, 8th March, 1921. Assumed Position, Lat. +18° 11′, Long. −76° 30′.
Theodolite, No. . . . Watch Error by Washington Radio signals.
Bar. 29·60. Therm. 88° F.

ξ ARGUS: *South*

At mer. passage (θ)		Altitude.	Watch Times.	Hour Angles (h)	Vers h / sin 1″
	h. m.	° ′ ″	h. m. s.	m. s.	
Approximation: L.S.T.=R.A.*	7 46	47 07 06	4 36 32	6 13	75·9
R	11 05	07 46	37 51	4 54	47·1
		08 05	38 58	3 47	28·1
L.M.T.	20 41	08 19	40 03	2 42	14·3
Long. (W)	5 06	08 33	41 20	1 25	3·9
		08 46	42 41	0 04	0·0
G.D. (9th)	01 47	08 41	43 50	1 05	2·3
		08 28	45 09	2 24	11·3
		08 10	46 31	3 46	27·9
		07 36	47 59	5 14	53·8
		07 07	49 01	6 16	77·1
		06 45	50 10	7 25	108·0

	h.	m.	s.		
R.A.*	7	46	00·2		
R.	11	04	51·3		
L.M.T. of θ	20	41	08·9		
Long. (W.)	5	06	00		
G.M.T. of θ	1	47	08·9		
Watch fast	2	55	36·4		
Watch Time of θ	4	42	45·0	12)95 22	12)449·7
					Mean 37·47

		°	′	″			
Mean Alt.		47	07	56·83			
Refn.	−		00	49·12	log cos δ	$\bar{1}$·958	44
					log cos φ	$\bar{1}$·977	75
True Alt.		47	07	07·71	log sec alt.	0·167	19
Redn.	+			47·54	log 37·47	1·573	68
Mer. Alt.		47	07	55·25	log redn.	1·677	06
Mer. Z.D.		42	52	04·75			
δ		−24	40	01·64	Redn. = 47″·54		
Lat. (φ) =	+	18	12	03·11 . . (1)			

Similarly, from observations of *o* Urs. Maj. : *North :*

Lat. (φ) = + 18° 11′ 26″·83 . . . (2)

Hence, latitude from mean of N. and S. stars is :

(Φ) = + 18° 11′ 44″·97

Note.—Should the observations for longitude disclose any large error in the assumed longitude, $\frac{\text{Vers h}}{\sin 1''}$ must be re-computed with the hour angles (h) corrected for the observed longitude (in time).

Longitude Observations with a Theodolite

Stars at approximately equal altitudes east and west of the meridian should be observed in preference to the sun, with limiting altitude of about 30° and 70°. Observations should be made when the motion in altitude is most rapid, *i.e.*, when the star is on or near the prime vertical.

Star programme.—A programme of stars suitable for observing for the determination of longitude should be prepared with the aid of Diagrams D and E (facing page 366), as shown in the following example, in which the assumed position of the observation spot is Lat. +31° 00′, Long. +32° 10′.

(1) Calculate the L.S.T. of the time at which it will be possible to start observing, say the end of twilight. In this case, let this L.S.T.= 10h. 0m.

(2) From Diagram D it can be seen that for latitude +31° the declination of stars which are on the prime vertical when their altitudes are 30° and 70° are about +15° and +29° respectively.

(3) From Diagram E it will be seen that stars with these declinations are on the prime vertical when their hour angles are 4h. 15m. (or 19h. 45m.) and 1h. 30m. (or 22h. 30m.) ; and, since R.A.*=L.S.T.−H.A.*, the R.A. of these stars (of declination +15° and +29°) which are on the prime vertical at 10h. L.S.T. are : *east* of the meridian, 14h. 15m. and 11h. 30m. ; *west* of the meridian, 5h. 45m. and 8h. 30m.

(4) From "Apparent Places of Fundamental Stars" pick out, and list separately, the east and west stars suitable for observation for a period of, say, 4 hours after 10h. L.S.T., that is, those with declinations between +15° and +29°, whose R.As lie, for east stars between 11h. 30m. and 18h. 15m. (*i.e.*, 14h. 15m.+4h.), and for west stars between 5h. 45m. and 12h. 30 m.

E. or W.	Star	Mag.	Dec. +	R.A.	H.A.	L.S.T.	Pair
			° ′	h. m.	h. m.	h. m.	
East	93 Leonis ..	4·5	20 31	11 45	20 36	8 21	
	β Comae ..	4·3	28 09	13 09	22 18	11 27	A
	τ Bootis ..	4·5	17 43	13 45	20 09	9 54	
	η Bootis ..	2·8	18 40	13 52	20 17	10 09	
	α Bootis ..	0·2	19 27	14 13	20 25	10 38	B
	α Cor. Bor...	2·3	26 54	15 32	21 48	13 20	
	β Serp. ..	3·7	15 35	15 44	19 51	11 35	
	κ Serp. ..	4·3	18 18	15 46	20 15	12 01	D
	γ Serp. ..	3·9	15 50	15 54	19 52	11 46	C
	ε Cor. Bor...	4·2	27 02	15 55	21 53	13 48	
	γ Herc. ..	3·8	19 17	16 20	20 23	12 43	
	β Herc. ..	2·8	21 36	16 28	20 43	13 11	
	δ Herc. ..	3·2	24 54	17 13	21 19	14 32	
	λ Herc. ..	4·5	26 09	17 29	21 40	15 09	
West	1 Gem. ..	4·3	23 16	6 01	2 58	8 59	
	μ Gem. ..	3·2	22 33	6 20	3 05	9 25	
	ν Gem. ..	4·1	20 15	6 26	3 27	9 53	
	γ Gem. ..	1·9	16 27	6 35	4 01	10 36	
	ε Gem. ..	3·2	25 11	6 41	2 37	9 18	
	ξ Gem. ..	4	20 39	7 01	3 24	10 25	B
	λ Gem. ..	3·6	16 38	7 15	3 59	11 14	C
	δ Gem. ..	3·5	22 05	7 17	3 10	10 27	
	β Gem. ..	1·2	28 09	7 42	1 43	9 25	
	δ Cancri ..	4·2	18 21	8 42	3 46	12 28	D
	ι Cancri ..	4·2	28 57	8 43	1 31	10 14	A
	ε Leonis ..	3·1	24 01	9 43	2 48	12 31	
	μ Leonis ..	4·1	26 15	9 50	2 18	12 08	
	η Leonis ..	3·6	17 01	10 04	3 56	14 00	
	ξ Leonis ..	3·6	23 41	10 14	2 53	13 07	
	δ Leonis ..	2·6	20 49	11 11	3 23	14 34	

The above lists could be expanded if desired. It will be noted that no stars with magnitudes duller than 4·5 have been included, and it will seldom be found necessary to do so. The hour angles are taken from Diagram E, with the assumed latitude and declination of the star as arguments ; the L.S.T. is found from the equation L.S.T.=H.A.+R.A. It can be seen that some of the listed stars are on the prime vertical before the L.S.T. is 10h. and must be discarded, others are on the prime vertical when the L.S.T. is greater than 14h. and will not be used unless necessary. Stars with approximately equal declinations and hence at approximately the same altitude when on the prime vertical can now be

paired as shown in the last column, and a chronological list for use ashore can be made as follows :

E. or W.	Star	Mag.	L.S.T.	Alt.	Pair
			h. m.	°	
West	ι Cancri	4·2	10 14	68½	A
West	ξ Gem.	4	10 25	42	B
East	α Bootis	0·2	10 38	39½	B
West	λ Gem.	3·6	11 14	33½	C
East	β Comae	4·3	11 27	66½	A
East	γ Serp.	3·9	11 46	31½	C
East	κ Serp.	4·3	12 01	37	D
West	δ Cancri	4·2	12 28	37½	D

The altitudes in this list are taken from Diagram D, with the assumed latitude and the star's declination as arguments. The minimum interval between consecutive stars in this example is 11 minutes which is quite sufficient with an instrument such as the Tavistock theodolite where all readings are made from the same position, but a rather longer interval should be allowed if two independent micrometers and the bubble position have to be read at each observation.

Procedure in observing.—As with latitude observations, the theodolite should be set up and levelled over the observation spot and the lower plate clamped so that the horizontal circle reading is 360° when the telescope is in the true meridian.

A set of observations should consist of an even number (say, 10 or 12), face being changed half-way through the set. (As with latitude observations, many observers prefer to change face after the first and then after every other observation, and be content with fewer observations, 4 to 8 as a rule.)

The rate of change of altitude of a heavenly body when on the prime vertical (15′ cos ϕ per min.) is 15′ per min. on the equator, 13′ per min. in latitude 30°, and 7½′ per min. in latitude 60° ; so, assuming that 12 observations take 6 to 7 minutes, for east stars an altitude about 1° less than that shown in the list should be set on the theodolite for the first observation of a set, and for west stars an altitude about 1° greater than that shown on the list. It is obvious that the azimuths will be 090° and 270°.

If the upper bubble cannot be seen from the eye end of the telescope and the level of the instrument adjusted (if necessary) before each observation, the bubble error must be read at each reading of the vertical circle. In order to upset the level of the instrument as little as possible by unintentional pressure on the slow motion screws, the cross-wires should be moved a little ahead of the star, which should then be allowed to make its own contact with the horizontal wire. The times should be recorded by chronograph, if possible.

As before, readings of barometer and thermometer are required for each set of observations.

Computation of Longitude from Theodolite Observations of pairs of stars on or near the Prime Vertical.—An assumed position of the observation spot must be used in the computation ; longitude may be approximate, but latitude should be correct within about 15 seconds of arc. This order of accuracy is quite sufficient if the declinations of the two stars of a pair do not differ by more than 1° or 2°, since any small error in the assumed latitude will affect both sets equally, making both hour angles (very nearly) the same amount too large or too small and not affecting the deduced time of meridian passage. In the computation it is assumed that the mean of the observed altitudes of each set corresponds to the mean of the observed times. This is strictly true only when the star is on the prime vertical, but since all observations are made so close to the prime

DIAGRAM FOR DETERMINING GRAPHICALLY THE ALTITUDE OF A HEAVENLY BODY WHEN ON THE PRIME VERTICAL

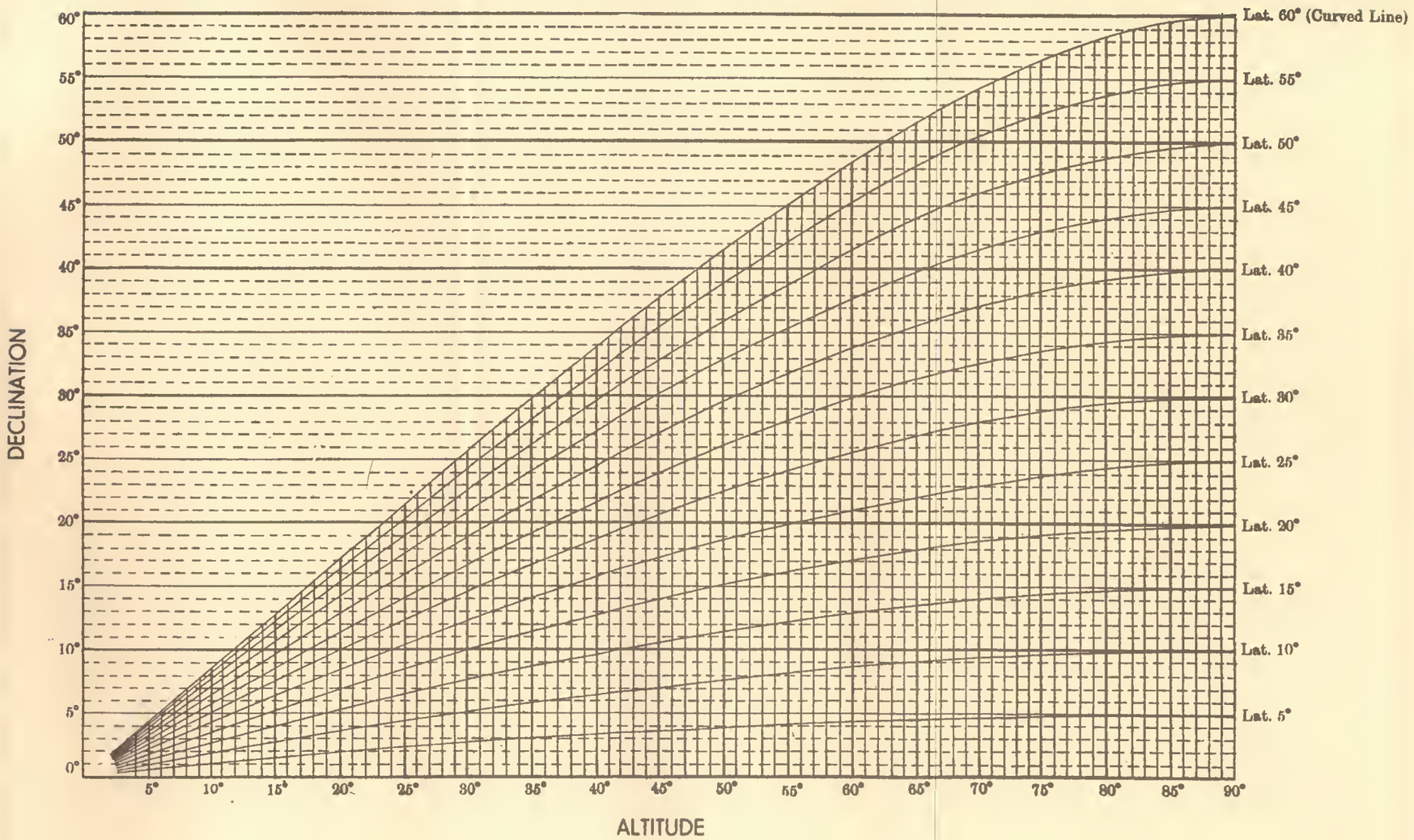


Diagram D

DIAGRAM FOR DETERMINING GRAPHICALLY THE HOUR ANGLE OF A HEAVENLY BODY WHEN ON THE PRIME VERTICAL

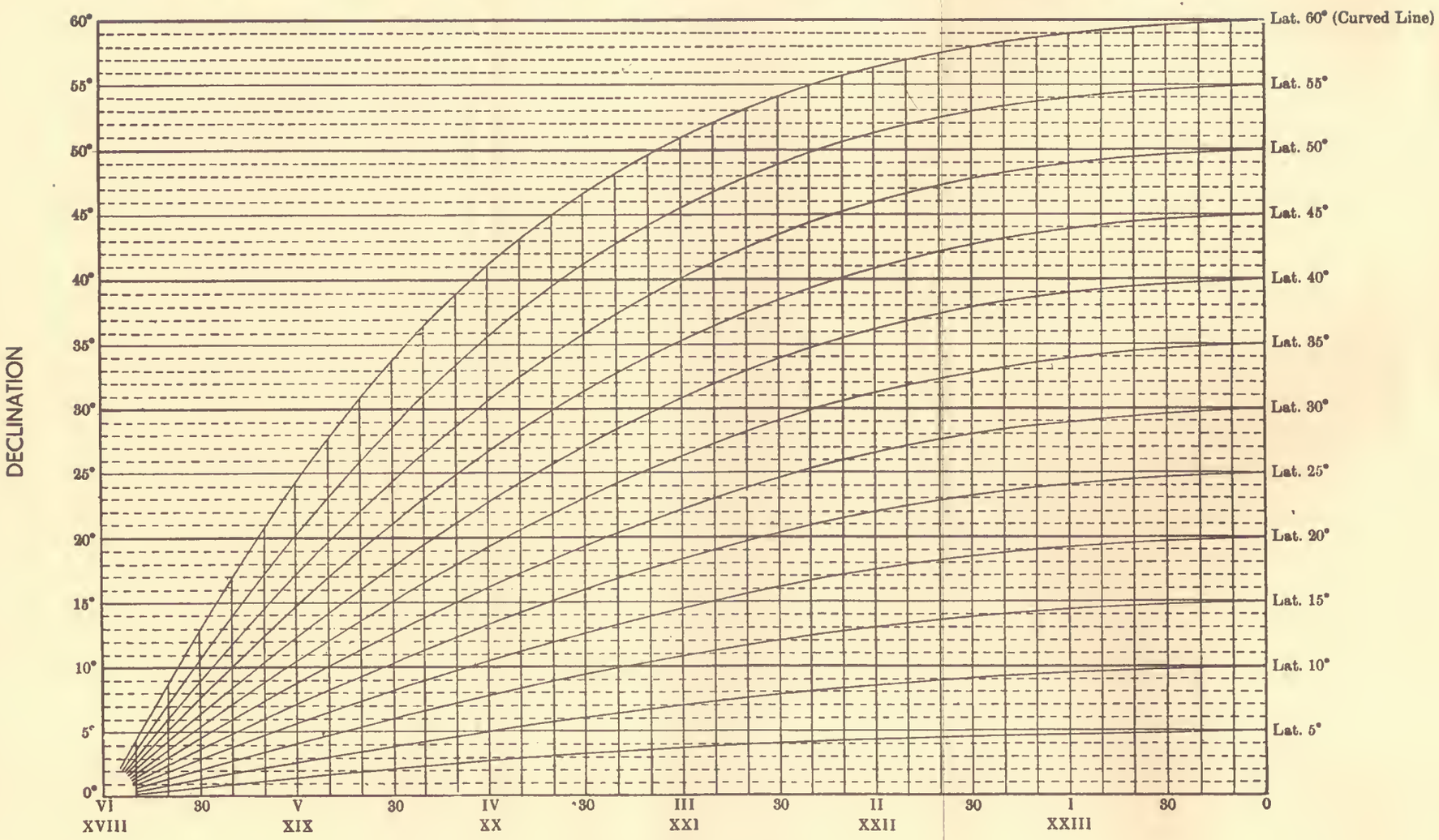


Diagram E

vertical, and since any error is cancelled by suitable pairing, the assumption is quite justified.

A convenient formula for the computation is :

$$\tan\frac{h}{2}=\sqrt{\cos s\times\sin(s-H)\times\operatorname{cosec}(s-\phi_a)\times\sec(s-p)}$$

where h=the hour angle, measured from the upper transit.

H=true mean altitude.

ϕ_a=assumed latitude, north or south.

p=polar distance (from the elevated pole).

$$s=\frac{H+\phi_a+p}{2}$$

Example :

Observations for longitude with Theodolite No.....

Assumed position : $\phi_a=+11°\ 25'\ 53''$ $\omega_a=+114°\ 20'$.

Chronograph error, by Cavite Radio signals : 0h. 10m. 16·44s. slow on G.M.T.

Barometer : 30·06. Thermometer : 88°.

α Ophiuchi—*East star*

	h.	m.	secs.
Mean time of obs.	14	39	31·11
Chron. slow ..		10	16·44
G.M.T.	14	49	47·55

	°	′	″
Dec. .. +	12	36	42·5
Polar distance, p,	77	23	17·5

	h.	m.	s.
R.A.	17	31	31·25
R (R.A.M.S.+12)	15	46	03·56

	°	′	″
Mean obs. alt.	41	34	13·00
Corrd. Refn.	—	1	0·7
True alt. H	41	33	12·3
ϕ_a	11	25	53
p	77	23	17·5
2)	130	22	22·8

s	65	11	11·4	L. cos	9·622 9038
s—H	23	37	59·1	L. sin	9·603 0123
s—ϕ_a	53	45	18·4	L. cosec	·093 3971
s—p	12	12	6·1	L. sec	·099 9233
				2)	9·329 2365
				L. $\tan\frac{h}{2}$	9·664 6182

$\frac{h}{2}=24°\ 47'\ 44''·68$

$=1h.\ 39m.\ 10·98s.$

	h.	m.	s.
h ..	3	18	21·96
R.A...	17	31	31·25
L.S.T.	14	13	09·29
R ..	15	46	03·56
L.M.T.	22	27	05·73
G.M.T.	14	49	47·55
Long. in time =	7	37	18·18
Similarly from observations of its pair, β Leonis—*West* Long.=	7	37	17·98

Mean—ω in time, 7h. 37m. 18·08s.
Ω in arc, +114° 19′ 31″·2

Computation of Longitude by Machine.—A suitable formula for machine working is : $\cos h=\dfrac{\sin H-\sin\delta\sin\phi_a}{\cos\delta\cos\phi_a}$

where, h =the hour angle measured from the upper transit.

H =true mean altitude.

ϕ_a =assumed latitude, north or south.

δ =declination.

An example is given on page 370.

Longitude by Equal Altitudes—Observations with Theodolite.

If a heavenly body, whose declination is constant, is observed east of the meridian and observed again, west of the meridian, when it has reached the same altitude, it is obvious that the mean of the times of these observations of equal altitude will give the exact time of the meridian passage, from which the longitude may be calculated.

In the case of stars, although the observation appears attractive, it is seldom practicable owing to the long interval which must elapse between the observations—with consequent risk of a change in refraction and of the sky clouding over—if the conditions are fulfilled of a rapid change in altitude coupled with the observations being made at not too high an altitude.

If, however, circumstances necessitate the sun being used for longitude observations, the method of equal altitudes should be adopted.

The actual observations are made in a similar way to star observations, except that it is the contact of a *limb* of the sun with the horizontal cross-wire which is observed (instead of the bisection of the *centre* of a star) and, of course, a coloured sun glass must be fitted to the eyepiece of the telescope. Eleven observations form a satisfactory set, and two sets should be observed, both a.m. and p.m., one on the lower limb and the other on the upper limb. The vertical circle should be set in advance, 10′ of arc being a convenient interval, and the sun allowed to make its own contact ; the level should be checked about ten seconds before each contact. A chronograph should be used for the times.

As the sun will have altered in declination between the a.m. and p.m. observations, the time of the meridian passage will not be the mean time of the observations. The correction to be applied to the mean time to obtain the correct time of apparent noon is :

$$\pm\frac{t\times \mathrm{d}\delta}{15}(\tan\phi\ \mathrm{cosec}\ t - \tan\delta\ \cot t) \quad . \quad . \quad . \quad . \quad (1)$$

where t = half the interval between the sets of observations.
$\mathrm{d}\delta$ = the change of declination in one hour.
δ = the declination of the sun when on the meridian.
ϕ = the latitude.

This correction is positive when the sun is moving away from the elevated pole, negative when it is approaching it.

Change of refraction between the observations will cause the apparent sun (or heavenly body) to descend to an equal altitude early or late according to whether the refraction is less in the afternoon or greater. The mean time of the observations may be corrected for this error, to give the time of the meridian passage, by $\pm\frac{\mathrm{r}}{30}\ \mathrm{cosec}\ \alpha\ \sec\phi$ seconds, where r is the change of refraction in seconds, α is the azimuth measured from the elevated pole, and ϕ is the latitude. The correction is positive if the refraction is less for the p.m. observations and vice versa.

(An ingenious adaptation of the equal altitude observation, in the case of stars, is to observe equal altitudes of two different stars, of approximately the same declination, one east and one west of the meridian, chosen so that only a short interval elapses between the times that they are at equal altitudes (and also moving quicly in altitude). The equation of equal altitudes ((1) above) is then applied to an imaginary star whose R.A. is the mean of the R.As of the two stars observed ; t, the half-interval between observations, is found, for use in equation (1), from the formula, $t=\frac{1}{2}((\mathrm{R}-\mathrm{R}')+(\mathrm{S}'-\mathrm{S}))$, where R and R′ are the R.As of star E and star W, and S and S′ are the sidereal times of observations of star E and star W respectively. However, since, except for a slight saving in computation, this method has no advantage over that of pairing stars, working out each set of observations separately, and then meaning the results (page 367), it will not be considered further here.)

As the computation of the equal altitude formulae takes very little less time than working out each set as an absolute determination of longitude, it is usually more satisfactory to compute by the formula given on page 367, and then mean the results of the a.m. and p.m. observations, as shown in the following example.

(Declination, etc., should be taken out from the standard edition of the Nautical Almanac, simple interpolation being sufficiently accurate.)

Observations for longitude with Theodolite No..... Date :—20th July, 1937.

Assumed position :— $\phi_A = +52° 30' 10''$, $\omega_A = -2° 32'$ (approx.).

At A.M. sights : Bar. 30·10, therm. 70° F.

At P.M. sights :—Bar. 30·00, therm. 78° F.

By Rugby Radio signal at 10 h. (20th) chron. fast 0h. 38m. 18·15s.

By Rugby Radio signal at 18h. (20th) chron. fast 0h. 38m. 18·40s.

	A.M. Sights		Sun's L.L. Obsd. Alt.	P.M. Sights	
	Chron. Times.	Diff.		Chron. Times.	Diff.
	h. m. s.	s.	° ′ ″	h. m. s.	s.
	9 35 05·15		40 10 00	16 14 07·54	
		16·05			16·05
	36 21·20		20 00	12 51·49	
		16·57			16·49
	37 37·77		30 00	11 35·00	
		16·27			15·96
	38 54·04		40 00	10 19·04	
		16·74			16·79
	40 10·78		50 00	09 02·25	
		16·14			16·01
	41 26·92		41 00 00	07 46·24	
		16·16			16·69
	42 43·08		10 00	06 29·55	
		16·44			16·26
	43 59·52		20 00	05 13·29	
		16·11			16·41
	45 15·63		30 00	03 56·88	
		16·38			16·39
	46 32·01		40 00	02 40·49	
		16·21			16·13
	47 48·22		50 00	01 24·36	
	11)455 54·32			11)85 26·13	
Means	9 41 26·76		41 00 00	16 07 46·01	
Chron. fast ..	0 38 18·12			0 38 18·32	
G.M.T.	9 03 08·64			15 29 27·69	
	° ′ ″			° ′ ″	
Obsd. Alt. ..	41 00 00			41 00 00	
Corrd. Refn. —	01 04·4			01 03·1	
S.D. .. +	15 46·0			15 46·0	
Prlx. .. +	06·7			06·7	
True Alt. ..	41 14 48·3			41 14 49·6	
Decn. at 0h. +	20 47 33·3			20 47 33·3	
Corrn. .. —	04 13·1			07 12·7	
Corrd. Decn. +	20 43 20·2			20 40 20·6	
	m. s.			m. s.	
Eq. of T at Oh. +	06 09·01 (to A.T.)			06 09·01 (to A.T.)	
Corrn. .. +	01·34			02·28	
Corrd. Eq. of T +	06 10·35			06 11·29	

	A.M. Sights			P.M. Sights	
	° ′ ″			° ′ ″	
H ..	41 14 48			41 14 49	
ϕ_a ..	52 30 10			52 30 10	
p ..	69 16 40			69 19 39	
	2)163 01 38			2)163 04 38	
s ..	81 30 49	L. cos	9·169 0112	81 32 19	L. cos 9·167 7393
s−H	40 16 01	L. sin	9·810 4675	40 17 30	L. sin 9·810 6887
$s-\phi_a$	29 00 39	L. cosec	10·314 2807	29 02 09	L. cosec 10·313 9391
s−p ..	12 14 09	L. sec	10·009 9794	12 12 40	L. sec 10·009 9388
Sum			39·303 7388		39·302 3059
½ sum			19·651 8694		19·651 1529
Deduct 10			10·000 0000		10·000 0000
L. tan $\frac{h}{2}$			9·651 8694		9·651 1529
		h. m. s.			h. m. s.
$\frac{h}{2}$ =		1 36 38·76			1 36 30·29
h =		3 13 17·52			3 13 00·58
L.A.T. = ...		8 46 42·48			15 13 00·58
Eq. of T. .. +		06 10·35			06 11·29
L.M.T.		8 52 52·83			15 19 11·87
G.M.T.		9 03 08·64			15 29 27·69
Long. (W.)		10 15·81			10 15·82

Meaning these results :—

ω (in time) — 0h. 10m. 15·82s. West
and Ω (in arc) — 2° 33′ 57″·3 West.

Computation of Longitude by Machine.—As stated on page 367, a suitable formula for machine working is :

$$\cos h = \frac{\sin H - \sin \delta \sin \phi_a}{\cos \delta \cos \phi_a}$$

where h = the hour angle, measured from the upper transit.
H = true mean altitude.
ϕ_a = assumed latitude, north or south.
δ = declination.

Example :

	A.M. Sights					P.M. Sights				
	°	′	″			°	′	″		
δ	20	43	20	sin cos	0·353 8376 0·935 3069	20	40	21	sin cos	0·353 0258 0·935 6136
ϕ_a	52	30	10	sin cos	0·793 3828 0·608 7230	52	30	10	sin cos	0·793 3828 0·608 7230
H	41	14	48	sin	0·659 3021	41	14	49	sin	0·659 3057
sin δ sin ϕ_a					−0·280 7287					−0·280 0846
					0·378 5734					0·379 2211
cos δ cos ϕ_a					0·569 3428					0·569 5295
cos h					0·664 9306					0·665 8498
	h = 48° 19′ 22″·8 = 3h. 13m. 17·52s.					48° 15′ 08″·8 = 3h. 13m. 00·59s.				

and thence as before.

Determination of the Value to be accepted from Theodolite Observations for Latitude and Longitude.

Each pair of circum-meridian stars, which are observed, gives one value for latitude and similarly each pair of E. and W. stars or corresponding sets of A.M. and P.M. suns give one value for longitude. Hence a number of values for each is obtained by each observer and the finally accepted position must be determined from a consideration of all of them. The method adopted is the analytical or least square method and is the same as that already described in connection with astrolabe observations, except that latitude and longitude are treated separately.

It is, however, necessary to stress the fact that for a reliable value of an observed quantity a considerable number of observations are necessary and it is of little use to apply the least square method unless the observations have been made by three or more observers, each obtaining at least ten values of the latitude and longitude to be determined. If there are fewer observations, it is impossible to " weight " results in any manner other than by an arbitrary procedure such as assuming the weights in each case to be proportional to the number of observations obtained by each observer.

As an example of the least square method has already been given earlier in this chapter (pages 350–360), the process as applied to theodolite observations for latitude and longitude is only summarised here.

(1) Tabulate under each observer his values (seconds only of arc or time) with their arithmetic mean, together with the residuals, v (*i.e.*, the differences between the individual values and the arithmetic mean), with their proper signs, and the squares of the residuals, v^2. (The sum of the residuals, Σv, should be zero.)

(2) For use in weighting the means, calculate the quantity E^2 for each observer from the formula :

$$E^2 = \frac{\Sigma v^2}{n\,(n-1)}$$

where n=the number of values.

(3) Apply Pierce's Criterion (see constants given in Table XXI and instructions on page 349) to see if any values should be rejected and, if this proves to be the case, recalculate v, v^2, Σv, Σv^2 and E^2 for the sets of observations concerned.

(4) Tabulate each observer's means (corrected as in (3), if necessary) and their weights which are inversely proportional to the respective values of E^2. Calling the means of latitude ϕ_1, ϕ_2, etc., and the corresponding weights w_1, w_2, etc. ; and of longitude ω_1, ω_2, etc., and W_1, W_2, etc., calculate the weighted means (*i.e.*, accepted results) from the formulae :

$$\Phi = \frac{\phi_1 w_1 + \phi_2 w_2 + \phi_3 w_3 +}{w_1 + w_2 + w_3 +} \quad \text{and} \quad \Omega = \frac{\omega_1 W_1 + \omega_2 W_2 + \omega_3 W_3 +}{W_1 + W_2 \pm W_3 +}$$

(5) The difference between each observer's means and the weighted means gives another set of residuals, u_1, u_2, etc., for latitude and U_1, U_2, etc. for longitude. Tabulate these together with the quantities w_1u_1, w_2u_2, etc., W_1U_1, W_2U_2, etc., u_1^2, u_2^2, etc., U_1^2, U_2^2, etc., and $w_1u_1^2$, $w_2u_2^2$, $W_1U_1^2$, $W_2U_2^2$, etc. Then the probable errors of the accepted results may be calculated from the formulae :

$$\text{P.E. of lat.} = 0{\cdot}6745 \sqrt{\frac{\Sigma w u^2}{(N-1) \times \Sigma w}} \text{ and,}$$

$$\text{P.E. of long.} = 0{\cdot}6745 \sqrt{\frac{\Sigma W U^2}{(N-1) \times \Sigma w}}$$

where N is the number of means.

ASTRONOMICAL OBSERVATIONS FOR TRUE BEARING, OR AZIMUTH, ASHORE

For the correct orientation of the survey, the direction of the meridian at one or more of the main stations is required and, if this information has not been included with any triangulation data which may have been supplied, it must be determined by astronomical observations. By applying angles observed during the triangulation, the true bearings, or azimuths, of all rays at that station can be found and, through the triangulation, the azimuths of the rays at any other station can be computed.

The azimuth of a heavenly body is the angle between the meridian plane of the observer and the vertical plane passing through the observer and the body. The astronomical observations for the determination of azimuth consist in measuring the horizontal angle between the R.O. and one or more heavenly bodies, at the same time observing the altitude of the body or, alternatively, recording the time at which it makes contact with the vertical diaphragm wire of the telescope. If the observer's geographical position is known, the azimuth, α, is computed from a solution of the spherical triangle in which :

(1) In the case of an altitude-azimuth three sides are known, *i.e.*, co-latitude and polar distance which are given and zenith distance which is deduced from the observed altitude.

(2) In the case of a time-azimuth, two sides and an included angle are known, *i.e.*, co-latitude and polar distance which are given, and hour angle which is deduced from the recorded time of the observation.

Two horizontal angles (one observed, the other computed) enable the azimuth of the R.O. to be determined, *vide* Fig. 199.

The computed azimuth is measured clockwise from north and the observed angle clockwise from the R.O.

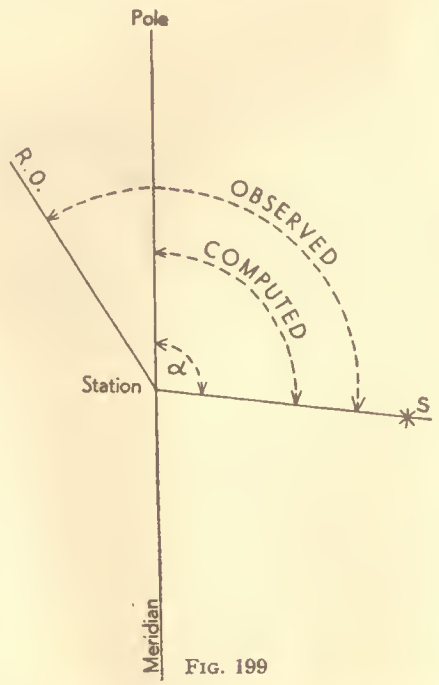


FIG. 199

To observe a true bearing on land, a theodolite is set up over one of the main stations and some well-defined terrestrial object is selected for use as the " Referring Object," or R.O., from which the actual measurements are made.

The R.O. should preferably be one of the other main stations, but if this cannot be arranged, *e.g.*, owing to the difficulty of putting a light on it for night observations, a temporary R.O. (not less than half a mile from the observer) may be used and the observed azimuth is subsequently connected to the triangulation by measuring the angles between this temporary mark and one or more of the main stations. In such cases the connection must be effected with as much or even greater care than that used in the measurement of any other angles in the main triangulation.

Stars should, when possible, be used in preference to the sun in any survey based on a regular triangulation. In older hydrographic surveys sun azimuths were the general rule, but they hardly furnish sufficiently accurate results for satisfactory use in modern work. In any case, star azimuths are easier to observe with a modern theodolite than sun azimuths and their only disadvantages are that the observer has to work at night and that a light must be kept at the R.O. during the period of observation.

Time azimuths are easier and quicker to observe than altitude azimuths and should normally be employed by the hydrographic surveyor who, it is assumed, will generally have available chronometers and (or) watches, of which the errors are known, and an assistant to record times for him. In an altitude azimuth *both* circles and the bubble position must be read at each observation and the theodolite telescope must be adjusted in azimuth and altitude so that a star makes simultaneous contact in the field or view on both cross-wires, or, in the case of the sun, two limbs make simultaneous contact on the vertical and horizontal wires respectively ; whereas, in the time azimuth, contact with the vertical wire only, and the reading of the horizontal circle are all that are required, apart from accurate time recording.

The most suitable time to observe the true bearing of a heavenly body is when its movement in azimuth is slowest, *i.e.*, when it is near the prime vertical or when a circumpolar star is at elongation. Errors in the time or altitude measurements will then have a minimum effect on the results. Since errors in horizontal angle, caused by dislevelment of the theodolite, increase rapidly with the altitude of the body observed, it is inadvisable to make observations for azimuth at altitudes greater than 30°. The observer's geographical position must be accurately known (in the case of altitude azimuths, latitude only need be accurate), but, to minimise the effects of possible errors in this " assumed position," stars should be " paired " with approximately the same altitudes and declinations (say, within 3° or 4°) on each side of the meridian in the same manner as observations for longitude.

A programme of observations should always be prepared and for stars east and west of the meridian Diagrams D and E (page 366) may be used. It is, however, unnecessary to make the observations when the body is exactly on the prime vertical, the only essential requirement is that it should be at a suitable altitude (see below) and *well* east or west of the meridian. Sufficient suitably paired stars can usually be found on a star globe, from which the L.S.T. (corresponding to the time the star is at the required altitude) and azimuth can be read off.

Time Azimuths by Stars.

Three or four stars on each side of the meridian should give a sufficiently accurate result and a programme of observations in their chronological sequence should be prepared, showing times (L.S.T. or G.M.T.) to the nearest minute, and altitudes and azimuths to the nearest degree. For time-azimuths the most suitable altitudes are from about 10° to 25° and azimuths should be between 075° and 105° or 255° and 285°. Within these limits the motion in azimuth is comparatively slow and times may be recorded with sufficient accuracy by the " stand-by stop " method on a chronometer watch, estimating to the nearest one-fifth of a second. An assistant to record the times and angles should be housed under cover where his lights will not interfere with the observer.

Procedure in Observing.—Before observing, the theodolite must be carefully levelled and accurately centred over the station mark ; if a false station has to be used, the distance and bearing to the true station must be measured with precision. This is particularly important if a temporary R.O. a comparatively short distance away has to be used, since an error of 1 inch in centring or measurement is equivalent to 6″ of arc at a distance of half a mile.

Having set up and levelled the theodolite over the station mark, the lower plate should be clamped with the R.O. reading about 360°, and, with the aid of a compass, the approximate reading of the true meridian should be noted so that the theodolite may be turned onto any required azimuth to find a star. Let the telescope be in the face left (F.L.) position. Then proceed as follows :

(1) Unclamp the upper plate and, swinging *left*, intersect the R.O. and book the horizontal circle reading.

(2) Continuing the swing left, bring the line of sight onto the approximate bearing of the first star and elevate the telescope to the required altitude. The star should come into the field of view two or three minutes before the calculated time at which it reaches that altitude.

(3) Giving the observer a " stand by," operate the upper plate and telescope tangent screws so that the vertical wire is brought a little in advance of the star and the horizontal wire a little above it, if apparently rising, and *vice versa* ; the star will thus meet the vertical wire near its intersection with the horizontal wire.

(4) At the moment the star makes contact on the vertical wire, call out " stop " so that the observer can record the time, and then read off and book the horizontal circle reading. If there is any doubt about the identification of the star, it is well to read the vertical circle also after this observation.

(5) Continuing to swing left, intersect the R.O. again. This reading should be the same as that in (1) within the limits of error to be expected with the type of instrument in use.

(6) Change face and repeat the entire sequence of operations (1) to (5), but swinging *right*. This completes the set of observations of the particular star.

The observations to the second star of the pair may be taken on the same zero, *i.e.*, with the R.O. reading about 360° or 180°, but, in order that other parts of the arc may be used, different zeros should be used for the other pairs of E and W stars in the manner detailed in Chapter III (pages 48–50) for the measurement of horizontal angles with a theodolite.

Computation of Time Azimuths.—The formula used for solving the spherical triangle by logarithmic computation is :

$$\text{Tan } \alpha = \tan h \times \cos M \times \operatorname{cosec} (M - \phi_a), \text{ where } \tan M = \tan \delta \times \sec h.$$

α=the horizontal angle between the *North* pole and the heavenly body.

h=the hour angle, measured from the upper transit.

ϕ_a=assumed latitude of the observer, $\frac{N}{S}$ lat. being $\frac{+ve.}{-ve}$

δ=declination of the heavenly body, $\frac{N}{S}$ dec. being $\frac{+ve.}{-ve}$

Care must be taken to apply the correct signs throughout. M may lie in one of the two quadrants ; it is immaterial which is taken.

When tan α is $+ve$, α lies between 0° and 90°, when $-ve$, α lies between 90° and 180°. The hour angle, h, is always regarded as $+ve$.

Since some considerable interval, depending on the observer's dexterity in using the theodolite, must elapse between the F.L. and F.R. observations of a star, it cannot be assumed that the mean of the observed times corresponds to the mean of the azimuths, and the two observations should be worked out independently if accurate results are required.

Example of computing time azimuth of stars.

Date—2nd March, 1933.

Near Europa Point lighthouse, observations to find true bearing of Ceuta lighthouse from Europa Point light.

True station reading 138° 09′, distant 42·5 feet.

Theodolite : Tavistock, No. 10.

Watch error by Radio signals : 5h. 20m. 16·9s. slow on G.M.T.

Position of Europa Point lighthouse : $\phi_a = +36° 06' 19''.7$

$\omega_a = -5° 20' 41''.1$

Star R.A.* Dec. (δ)*	East α Leonis 10h. 04m. 50·7s. + 12° 17′ 35″·5.				West γ Pegasus 0h. 09m. 46·5s. + 14° 48′ 41″·4.			
	F.L., S.L.		F.R., S.R.		F.R., S.R.		F.L., S.L.	
Clock time	h. m. s. 13 45 01·1		h. m. s. 13 50 10·5		h. m. s. 13 55 22·2		h. m. s. 14 00 39·3	
Error .. +	5 20 16·9		5 20 16·9		5 20 16·9		5 20 16·9	
G.M.T. .. −	19 05 18·0		19 10 27·4		19 15 39·1		19 20 56·2	
Long. (ω)	0 21 22·7		0 21 22·7		0 21 22·7		0 21 22·7	
S.M.T.	18 43 55·3		18 49 04·7		18 54 16·4		18 59 33·5	
R.A.M.S.±12 hrs ..	10 40 27·8		10 40 28·7		10 40 29·5		10 40 30·4	
L.S.T.	5 24 23·1		5 29 33·4		5 34 45·9		5 40 03·9	
R.A.	10 04 50·7		10 04 50·7		0 09 46·5		0 09 46·5	
H.A.	19 19 32·4		19 24 42·7		5 24 59·4		5 30 17·4	
h	4 40 27·6		4 35 17·3		5 24 59·4		5 30 17·4	
L. tan δ	9·338	2789+	9·338	2789+	9·422	3044+	9.422	3044+
L. sec h	10·468	3508+	10·442	1739+	10·817	6809+	10·888	4970+
L. tan M	9·806	6297+	9·780	4528+	10·239	9853+	10·310	8014+
	° ′ ″		° ′ ″		° ′ ″		° ′ ″	
M	+32 38 46·3		+31 05 52·4		+60 04 52·7		+63 56 49·6	
ϕ_a	+36 06 19·7		+36 06 19·7		+36 06 19·7		+36 06 19·7	
M−ϕ_a	− 3 27 33·4		− 5 00 27·3		+23 58 33·0		+27 50 29·9	
L. cos M	9·925	3212+	9·932	6189+	9·697	9008+	9·642	6629+
L. tan h	10·441	6528+	10·411	8055+	10·812	5938+	10·884	8376+
L. cosec (M−ϕ_a) ..	11·219	4011−	11·059	0475−	10·391	0984+	10·330	6558+
L. tan α	11·586	3751−	11·403	4719−	10·901	5930+	10·858	1563+
	° ′ ″		° ′ ″		° ′ ″		° ′ ″	
α	91 29 05·1		92 15 41·9		82 51 02·3		82 06 27·5	
Obsd. angle from Ceuta light ..	74 41 57·0		73 55 02·0		249 01 48·0		248 17 25·0	
True Bg. of Ceuta light	166 11 02·1		166 10 43·9		166 10 45·7		166 10 57·5	
Means	166° 10′ 53″·0				166° 10′ 51″·6			

Meaning results of E. and W. stars :

T.B. of Ceuta light from obsn. spot .. = 166° 10′ 52″·3

Correction for false station .. .. .. = − 01 13 ·5

T.B. of Ceuta light from Europa light .. .. = 166° 09′ 38″·8

Computation of Time Azimuth Observations by Machine.—A suitable formula for machine working is : $\cot \alpha = \dfrac{\cos \phi_a \tan \delta - \sin \phi_a \cos h}{\sin h}$

where α, h, ϕ_a and δ are as previously defined.

As far as the determination of h, the computation is the same as for logarithmic working ; then

	F.L., S.L.	F.R., S.R.	F.R., S.R.	F.L., S.L.
h	h. m. s. 4 40 27·6	h. m. s. 4 35 17·3	h. m. s. 5 24 59·4	h. m. s. 5 30 17·4
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
=	70 06 54·0	68 49 19·5	81 14 51·0	82 34 21·0
sin h	+ 0·940 3772	0·932 4631	0·988 3549	0·991 6092
cos h	+ 0·340 1334	0·361 2652	0·152 1665	0·129 2716
sin ϕ_a		+0·589 2736		
cos ϕ_a		+0·807 9336		
tan δ	+0·217 9108		+0·264 4261	
cos ϕ_a tan δ ..	0·176 0575	0·176 0575	0·213 6388	0·213 6388
sin ϕ_a cos h ..	0·200 4316	0·212 8840	0·089 6677	0·076 1763
Diff.	− 0·024 3741	− 0·036 8265	+ 0·123 9711	+ 0·137 4625
cot α	− 0·025 9196	− 0·039 4939	+ 0·125 4317	+ 0·138 6256
α	91° 29′ 05·1″	92° 15′ 42·0″	82° 51′ 02·3″	82° 06′ 27·5″

and thereafter as before.

Altitude Azimuths by Stars.

These observations are carried out in a similar manner to time azimuths, except that :

(1) Owing to uncertainty in the value of refraction at low altitudes, the minimum altitude should be about 20°.

(2) Since the exact altitude at the time of observation is required,

(*a*) The tangent screws of the theodolite must be operated so that the star makes contact on *both* cross-wires simultaneously. This is most conveniently effected by bringing the star onto the vertical wire and keeping it there with the slow motion tangent screw of the upper plate until it makes contact on the horizontal wire. Both horizontal and vertical circles are then read.

(*b*) If the bubble on the vertical arm can be seen from the telescope end, it must be centred before the observation ; if not, the bubble position must be read immediately after the observation and the altitude must be corrected for any displacement by the method given in Chapter III (pages 52 and 53).

Time need not be recorded.

The formula used in logarithmic computation is :

$$\tan^2 \frac{\alpha}{2} = \sec s \times \sin (s - H) \times \sin (s - \phi_a) \times \sec (s - p),$$

$$\text{where } s = \frac{H + \phi_a + p}{2} \text{ and}$$

α = the horizontal angle between the elevated pole and the heavenly body.
H = the true altitude.
ϕ_a = the assumed latitude of the observation spot, always regarded as *+ve*.
p = the polar distance of the body, measured from the elevated pole and always regarded as *+ve* : thus $p = 90° \pm \delta$.

A suitable formula for machine working is $\cos \alpha = \dfrac{\sin \delta - \sin \phi_a \sin H.}{\cos \phi_a \cos H}$

Azimuth by Circumpolar Stars at Elongation.

This is the easiest method of determining a true bearing, the computations are simple and the results very accurate. The disadvantages are that its use is restricted within certain limits of latitude, the number of suitable stars are few and the time of elongation must be previously calculated. Any star with declination greater than, and of the same name as, the observer's latitude will be at elongation twice during the period it is above the horizon and its altitude at those times is always greater than the latitude.

It was stated on page 373 that it is undesirable to make azimuth observations at altitudes greater than 30°, therefore, since the altitude at elongation must be greater than the latitude, the method is virtually restricted to latitudes of less than 30°. Near the equator, circumpolar stars are not visible at elongation, but good results by observations of Polaris have been obtained in as low a latitude as 3° north ; so the method may be said to be available between latitudes +3° and +30° and between −3° and −30°.

The nearer a star is to the pole, the slower its apparent motion, and close circumpolar stars should therefore be selected in preference to others. The apparent places of a number of close circumpolar stars (*i.e.*, those with declinations greater than 81°) are listed in " Apparent Places of Fundamental Stars " and these generally are the most convenient to use, but, as several of them are of very small magnitude, their altitudes at the time of elongation should be previously calculated in order that they may be found and identified in the field of view of the telescope. At low altitudes, allowance must be made for refraction. A further advantage of using these close circumpolar stars is that a small error in the assumed latitude has practically no effect on the result. As with other methods of observing, stars should be paired, if possible, on either side of the meridian, but good results should be obtained from observations of Polaris alone.

Selection of Circumpolar Stars.—This is best illustrated by an example. Let the assumed position of the observation spot be ϕ_a= +5° 57′ 54″, ω_a= +100° 38′ 15″, and that observations could be started at L.M.T.=19h. 0m. on 4th March, 1937. From the abridged nautical almanac, R=10h. 47m. approx., hence L.S.T.≒L.M.T.+R=5h. 47m.

Close circumpolar stars are at elongation about 6 hours before and after meridian passage ; therefore for east elongation a star must be found with R.A.=5h. 47m.+6h.=11h. 47m., or greater, and for west elongation one with R.A.=5h. 47m.−6h.=23h. 57m., or greater.

From " Apparent Places of Fundamental Stars " are taken :

	Star	*Mag.*	*R.A.*	*Dec.*
East :	6B Urs. Min.	6·3	12h. 15m.	+88° 03′
West :	α ,, ,,	2·1	01 41	+88 58

Others may be selected at later times. The azimuths of close circumpolar stars, such as the above, do not alter appreciably for a period of about 5 minutes before and after elongation. It will be sufficient, therefore, to calculate the time of elongation to the nearest minute from the formula :

$$\cos h = \tan \phi_a \times \tan p,$$

where h = the hour angle, or 24 hrs.−H.A.

ϕ_a = the assumed latitude.

p = the polar distance of the star. (*i.e.*, $90° - \delta$)

For the purposes of identification, the altitude may be calculated from the formula, $\sin H = \sin \phi_a \times \sec p$, a correction for refraction being applied, if necessary.

In the case of the two selected stars, therefore, the preliminary computation is as shown overleaf :

Star	East 6B Urs. Min. 12h. 15m. 00s. +88° 03′	West α Urs. Min. 1h. 40m. 46s. +88° 58′
L. tan ϕ_a	9·019 1	9·019 1
L. tan p	8·532 1	8·256 2
L. cos h	7·551 2	7·275 3
	h. m. s.	h. m. s.
h	5 59 11	5 59 34
H.A.	18 00 49	5 59 34
R.A.	12 15 00	1 40 46
L.S.T.	6 15 49	7 40 20
R.	10 47 27	10 47 41
L.M.T.	19 28 22	20 52 39
Long. (E.)	6 42 33	6 42 33
G.M.T. of elongation	12 45 49	14 10 06
L. sin ϕ_a	9·016 8	9·016 8
L. sec p	0·000 3	0·000 1
L. sin H	9·017 1	9·016 9
	° ′	° ′
H	5 58	5 58
Refn. .. .. +	09	09
Approx. Alt. at elongation	6 07	6 07

Procedure in Observing.—Allowing a period of about 10 minutes in which the observations may be made, the star 6B Urs. Min. should be observed between 12h. 41m. and 12h. 51m. G.M.T., and α Urs. Min. between 14h. 05m. and 14h. 15m. The first observations in each case should therefore be made on F.L., S.L. two or three minutes before the time of elongation, which will give time to change face and make a second set on F.R., S.R. about two or three minutes after elongation. As with other methods, the horizontal circle readings of the R.O. at the beginning and end of the swing and of the star should be booked. Times should also be approximately recorded to ensure that a limit of 5 minutes on either side of elongation is not exceeded.

The azimuth of the star is computed from the formula :

$$\text{Sin } \alpha = \sec \phi_a \times \sin p.$$

In very accurate work a small correction for the effect of diurnal aberration is applied, but as this only amounts to 0″·3 of arc, it is unnecessary to take it into account here.

True Bearing by Observations of the Sun.

The procedure in observing is similar to that described for stars, except that :

(1) A suitable dark eye-shade must be affixed to the telescope each time the sun is observed and removed when the R.O. is observed. Unless this is done very carefully, the level of the theodolite will be disturbed,

(2) With a time azimuth, the observation of the sun on each face must consist in bringing first one limb (L or R) into contact with the vertical wire of the diaphragm, and then the other limb. For the purposes of the computation, the *mean* of the two horizontal circle readings and the *mean* of the times are used.

(3) With an altitude azimuth the observation on each face must consist in making simultaneous contact of two limbs on the horizontal and

vertical wires first in one quadrant and then in the opposite quadrant, as shown in Fig. 200.

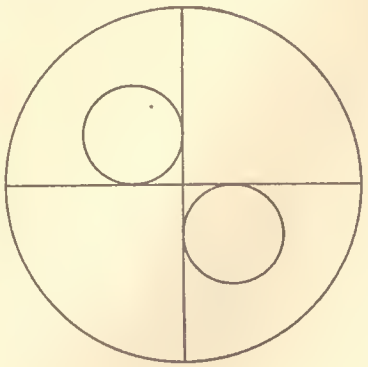


FIG. 200

This is most easily done by setting the horizontal wire in advance of the sun and keeping the vertical wire on the left or right limb, with the upper plate slow motion, until contact is made. The means of the vertical and horizontal circle readings are used in the computation. Times should be recorded to the nearest minute so that the sun's declination may be corrected to date.

When possible, two or four sets at about the same altitudes should be taken a.m. and p.m. and each set should be worked out independently. The most suitable altitudes are about 15° to 20° : below 15° altitude azimuths may be unreliable owing to uncertainty in the value of refraction and a mean of the upper and lower limb altitudes is appreciably in error owing to the effect of refraction on the semi-diameter of the sun. As the horizontal angle subtended by the sun's semi-diameter increases with altitude, the mean of the two horizontal angles of a " set " does not represent exactly the mean angle to the sun's centre. This small error can be practically eliminated by reversing the order in which the limbs are observed in alternate sets.

The computation is similar to that for E or W star azimuths, except that the H.A. sun (L.A.T.) is found from G.M.T. $\pm$long. $\pm$Eq. of T. The mean observed altitude requires a correction for refraction and parallax. The sun's declination and equation of time should be taken out of the standard edition of the Nautical Almanac.

HINTS ON THE USE OF A THEODOLITE FOR ASTRONOMICAL OBSERVATIONS

(1) For star observations diaphragms fitted with spider's webs are more satisfactory than glass diaphragms.

(2) Most open sights have a slight error. If it is intended to bring stars into the field of view by their means this error should be ascertained during daylight. To illuminate open sights a light should be held behind the observer's head.

(3) In observations for altitude, contact should not be made on the intersection of the cross-wires, but a little to one side. As far as possible the same position on the cross-wire should be used for each observation, in case the wire is not exactly horizontal.

(4) In all observations, when it is possible to do so, the sun or star should be left to make its own contact, thus minimising the danger of the level of the instrument being effected by unintentional pressure on the slow motion screws.

(5) Before any set of observations, the observer should verify that all slow motion screws are in the centre of their run, and thus obviate the danger of them being " two blocks " before the set is finished.

(6) A lamp, when used as the R.O. in star azimuth observations, should be placed in a box with a vertical slit, the width of which should vary between $\frac{1}{4}$ inch at half a mile to $\frac{1}{2}$ inch at one mile. As explained on page 374, the observations for connecting such a temporary R.O. with the main triangulation must be made with meticulous accuracy.

Sea Sights for Position

When a survey is made at any considerable distance from dry land, it is frequently impracticable to make connection with the shore by means of a beacon triangulation or taut wire measurements, and it is necessary to determine geographical position from astronomical observations at sea. The methods of using a sextant to find latitude and longitude at sea are fully described in works on Navigation and need not be discussed in detail here. It may, however, be remarked that the surveyor must aim at a higher standard of accuracy than that required for the ordinary purposes of navigation and for this reason rather more care and refinement are needed.

The most convenient form of computation for the surveyor's purpose is usually that employed for the Marc St. Hilaire position line method in which the observed position is obtained by plotting intercepts and position lines from an assumed position (the D.R.). Five-figure logarithms will always give results well within the limits of accuracy obtainable with sextant observations at sea and the abridged edition of the Nautical Almanac may be safely used for taking out the apparent places of the heavenly bodies. Time-recording is normally effected by the " Stand by-Stop " method with a deck watch on the bridge, an assistant noting times of the observations by estimation to the nearest one-fifth of a second. Since the rates of most deck watches are unreliable, watch error should always be determined from comparisons with a chronometer shortly before or after the observations.

For star sights the most suitable form of instrument is a stellar sextant of the type shown in Plate I (page 211), of which the centring error is known and which is fitted with a quick-reading micrometer head. The latter is a particularly valuable adjunct since it allows a series of observations to be made very rapidly, an important consideration when there may only be a short period available, and, in addition, no artificial illumination is needed during twilight for reading off the angles. Sextants of this type are only designed to read directly by micrometer or vernier to minutes of arc, and by estimation to, say, 20″, whereas most observing sextants can be read to 10″. The greater theoretical accuracy of the latter, however, is almost certainly outweighed in the case of star sights by the advantages which a stellar sextant has in its large aperture telescope, large mirrors and rapid reading. For sun observations, of course, a good observing sextant fitted with shades is essential.

Surveys out of sight of land may be divided into :

1. Surveys in which a temporary fixed control in the form of anchored beacons is established.
2. Surveys of deep water areas in which no fixed control can be provided.

In the former, the geographical position of one or more of the beacons should be determined by observations of stars during twilight from the ship at anchor or hove to close to the beacon which it is required to fix. When a true bearing has also been observed, the geographical position of any other point in the survey can be calculated through the triangulation.

In the latter, the positions of the soundings must be determined with the aid of any astronomical observations which can be usefully obtained and must

depend to some extent on dead reckoning between successive sights. The observations in these cases must include sun, moon and venus sights during the day as well as stars at twilight.

Twilight Stars.—The most accurate method of determining geographical position at sea is by observations of stars and planets during morning and evening twilight. At these times there is, under good conditions of weather, a short period, varying from perhaps only ten minutes to half an hour or more, during which stars of the second magnitude and brighter can be reflected in a sextant and at the same time the horizon is clearly defined. With a cloudless sky it is possible to select stars and planets on different sides of the zenith and, by combining the observations, eliminate or reduce to small proportions the constant errors to which measurements of this type are subject. An ideal set of observations would consist of four stars at about the same altitude and differing in azimuth by about 90° from each other. If the observations were entirely free from error, the four position lines would all meet in a point—the observed position—but normally they form a quadrilateral such as that shown in Fig. 201.

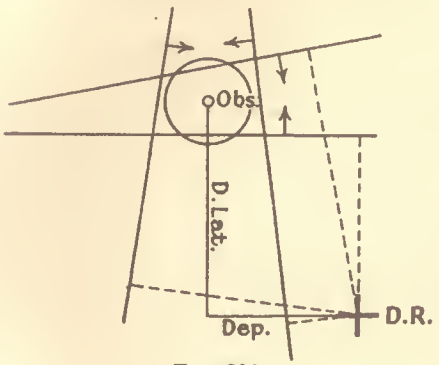


FIG. 201

The " centre " of the quadrilateral, *i.e.*, a point equidistant from each pair of position lines, is the observed position and the radius of the circle drawn to overlap one pair by the same amount as it falls inside the other pair represents the mean error of the four observations. Arrows, as shown in the diagram, pointing from the position lines towards the respective stars should always be added to the figure and should all point in the same relative direction, *i.e.*, inwards towards the centre of the circle or outwards away from it. If they do not do so, that is, if one pair of arrows points inward and the other outward, one or more of the observations must be inaccurate or else a mistake in computation may be suspected. When the ship is not stationary during the period of observation, due allowance for course and distance steamed must be made by using a different D.R. in each computation or, more conveniently, by transferring the position lines in the plot for " run." Observations of three stars only on suitable bearings will in theory give a result free from constant errors, the observed position being at the centre of gravity of the triangle formed by the position lines, but there is no check on the accuracy of the observations or computations. When more than four stars are observed, a circle with circumference equidistant from all the position lines cannot normally be drawn ; the observed position can, however, be derived well enough from an inspection of the plot. The accuracy of sea sights hardly merits a mathematical analysis such as that used with astrolabe observations.

The observations of each star or planet should consist of a set of three or, if time admits, five altitudes, and for the purposes of the computation the mean altitude is regarded as corresponding to the mean of the times. Azimuths may, if desired, be taken out with sufficient accuracy from a diagram such as

Weir's or from tables. Four stars on suitable bearings should give a good result, but if time permits seven or eight should be observed. Stars should be paired, as far as possible, both in altitude and azimuth. Under good conditions an observer should obtain a position correct within less than a mile from a set of four stars at twilight, but a more reliable position is obtained when the results of three or four observers, each taking about eight stars, are meaned. The simplest method is for each observer to obtain a mean position ; these mean positions may be given weights, if desired (allotted according to the skill of the observer, and after inspection of his plotted position lines) and they are then meaned for the final result. Good results have also been obtained by plotting all position lines on one piece of paper and choosing the best position by inspection ; if this method is adopted, the position lines of the different observers should be plotted in different colours to facilitate pairing. When practicable, observations should be obtained in the same position both a.m. and p.m.

Identification of the stars is greatly simplified if a rough bearing is obtained at the time of observation ; then with the help of a star globe or the publication " What Star is it ? " there should be little difficulty.

Fixing by Day.—Fairly satisfactory geographical positions for beacons out of sight of land can be obtained, under favourable conditions, from a series of sun sights ; for longitude, the sun should be observed a.m. and p.m. at approximately equal altitudes when its altitude is over 30° ; and for latitude, a meridian altitude should be observed ; (in low latitudes, observations of short equal altitudes may be added with profit). The chief cause of unreliability in positions thus obtained is the lack of any check, by " pairing," of the latitude ; and a disadvantage, often, is the upsetting of the surveying programme through having to visit the beacon at such frequent intervals.

When a ship is employed on sounding or oceanographical work in circumstances that preclude the use of any fixed form of control, it is necessary to fix as often as possible by astronomical observations during daylight as well as by making use of twilight stars. Such observations cannot normally be " balanced " by corresponding ones on the opposite side of the zenith, and owing to the uncertainty in the value of refraction it must be accepted that any one position line may be as much as 2′ or 3′ in error. Sun sights should be obtained, if possible, at about hourly intervals during the day and the moon at such periods as it is visible should also be observed. Venus, when not within 20° of the sun, can be seen in a clear sky ; if its approximate altitude at any time is calculated and set on the sextant, there should be little difficulty in finding it and the observation is often of great value, especially if its R.A. differs from that of the sun by about three hours since a latitude sight can then be obtained at about 0900 or 1500. Equal altitudes of the sun E and W of the meridian give the most reliable longitude by day, allowance being made for the run. Latitude at apparent noon may be determined by circum-meridian altitudes of the sun, but careful observations of its altitude, when on the meridian, by two or three observers working independently will probably give at least as good a result as a set of circum-meridian altitudes by *one* observer. When the sun is within 20° of the zenith at apparent noon, the supplement of the altitude can also be measured by bringing the sun down to the horizon on the opposite side of the zenith. This is rather an awkward observation but meaning altitudes thus obtained with direct altitudes eliminates the errors due to dip and terrestrial refraction.

Fixing by Stars at Night.—Positions can frequently be obtained on moonlight nights from observations of stars, but a false horizon is very common at such periods and such observations should usually be regarded as only an approximate check on the ship's position but valuable as an indication whether an unsuspected current is setting her far off the required course.

Observations for True Bearing at Sea

Observations for true bearing at sea are required when a survey depends on a beacon triangulation which is not connected to any point on the land. The true bearing or azimuth of one or more of the sides, *i.e.* from one beacon to another, must be determined in order that geographical positions and rectangular co-ordinates may be calculated and the orientation of the survey as a whole may be established. Azimuth observations at sea may also be required to connect a position with a distant land object, such as a well-defined peak or a lighthouse, and in such cases may be used in conjunction with a taut wire distance or an angle to another fixed point on the land to determine the geographical co-ordinates of that position.

Since a theodolite cannot be used on board ship, recourse must be had to a sextant for making the observations. With this instrument it is only possible to make a direct measurement of the angular distance between the R.O. and a heavenly body whereas the *horizontal* angle between them is required for computing the true bearing. If, however, the altitudes above the horizon of the R.O. and the heavenly body are measured at the time the angular distance is observed, the horizontal angle between them may be found; the precise formula for this is:

$$\tan^2 \tfrac{1}{2}\alpha = \frac{\sin \tfrac{1}{2}(C+H-h) \sin \tfrac{1}{2}(C-H+h)}{\cos \tfrac{1}{2}(C+H+h) \cos \tfrac{1}{2}(C-H-h)}$$

where α = horizontal angle
C = angular measurement from heavenly body to R.O.
H = altitude of heavenly body corrected for Dip
h = altitude of R.O. corrected for Dip

When computing the horizontal angles observed altitudes must *not* be corrected for refraction but they must be corrected for dip and, in the case of a R.O. on or nearer than the horizon, the altitude becomes a minus quantity, i.e. the zenith distance is greater than 90°. Attention to the signs in the above formula is then necessary: alternatively the half-log haversine formula may be used to calculate the horizontal angle, thus avoiding any possibility of error over the signs.

When the observed angular distance is near 90° the altitude of the elevated object has least effect on the resulting horizontal angle (see Table XIX). The R.O. and the heavenly body should therefore be chosen so that the angular distance between them is, if possible, not far from 90°.

Measurement of the angular distance is rather more easily made with a sextant if the higher object is *reflected, i.e.*, when the heavenly body lies to the right of the R.O. When, therefore, a choice is available, this disposition should be selected. For observations of stars, a sounding or stellar sextant (with a known centring error) is the most convenient, but for sun observations an ordinary observing sextant is needed since shades in front of one or other of the mirrors will be required. The heavenly body should be at an altitude of about 10° to 20°; below 10° appreciable errors due to uncertainty in the value of refraction may arise, whilst above 20° small errors in the observed angles have a considerable effect on the results.

The sun is generally used for azimuth observations at sea and gives sufficiently accurate results, but a bright star or planet observed during twilight is sometimes better if the R.O. can be seen at that time. For instance, if the only suitable R.O. is nearly east or west of the observer, the angular distance to the sun, when at a convenient altitude, will not usually be anywhere near 90°, whereas this condition may be easily fulfilled by selecting a star to the north or south. Again, an azimuth is sometimes required to a distant shore light on a structure which cannot be seen by day, and a star or planet must then be used for the observations.

Since the apparent altitude of the heavenly body is required in the formula for correction of the observed angular distance, its measurement should be a

part of the observations, and it is usual to compute the result as an altitude-azimuth from the formula :

$\tan^2\frac{\alpha}{2} = \sec s \times \sin(s-H) \times \sin(s-\phi_a) \times \sec(s-p)$, where $s=\frac{H+\phi_a+p}{2}$

when computing by logarithms.

For machine working, $\cos\alpha = \frac{\sin\delta - \sin\phi_a \sin H}{\cos\phi_a \cos H}$ is a suitable formula.

It may occasionally be necessary to dispense with the altitude measurements, *e.g.*, if the observations can only be made after dark when the horizon cannot be seen, and to compute the results as a time-azimuth, but the apparent altitude must then be deduced for use in correcting the angular distance and this is not so satisfactory as a direct measurement.

Two or three sets of observations should give a result correct within 1′ of arc, an order of accuracy sufficient for a beacon survey, and it may be possible occasionally to pair stars east and west of the meridian. Times need only be recorded to the nearest second on a deck watch and the apparent places of the sun and stars may be taken from the abridged Nautical Almanac. It will also be sufficient to take the computations to the nearest decimal of a minute of arc and to use five-figure logarithms throughout. The correction for parallax is very small and scarcely affects the final result.

Procedure in Observing.—Suppose that at a moored beacon, A, the true bearing of another beacon, B (the R.O.), is required and that in the evening the sun at an altitude of about 15° subtends a suitable angle, *i.e.*, about 90° from the R.O. The ship should heave to or anchor close to A and the distance and bearing of the latter should be measured at the moment of observation so that any necessary correction for "false station" may be subsequently applied. It can often be arranged that, at the time the angular distance between the R.O. and the heavenly body is measured, the observer on the bridge is on the alignment of A and B and in consequence no correction for false station is required.

To determine the azimuth, the angle between the R.O. and the sun's centre is required. This point cannot be exactly judged and the observer therefore measures with a sextant first the angle from the R.O. to the nearer limb and then to the further limb, or *vice versa*, an assistant recording the times of each observation. The mean of the angles and times are used in the computation, and the sun's apparent altitude at this mean time is also required. If two observers are available, the first may measure the angular distances and be able to synchronise his observations with those of a second observer measuring the altitudes, and the mean altitude will correspond to the mean time. If only one observer is available, he should measure first the altitude, then the angular distances, and finally the altitude again. In this case the altitude, at the mean time of the angular distance measurements, is deduced.

It will be appreciated that if the R.O. is a land object and is elevated above the horizon, its altitude is required also, in order that the observed angular distance may be converted to true horizontal angle ; but this may be observed at leisure either before or after the other observations.

Example of Computation.—16th Dec., 1932. Observations of the sun from the ship hove to near Lode beacon to determine the true bearing of Mine beacon from Lode. No correction for false station necessary. Height of eye 33·5 feet. One observer.

Assumed position : $\phi_a = +9°\ 08'{\cdot}0$, $\omega_a = -13°\ 47'{\cdot}6$;
Observing Sextant No. 182 : C.E., Nil ; I.E., $-0'{\cdot}4$;
Watch error 1m. 54s. slow on G.M.T.
Barometer : 30·6. Thermometer : 60° F.

App. watch time of observations	=	17h. 42·5m.
Watch error	+	1·9
G.D. of observations	=	17h. 44·4m. on 16th Dec.

Altitude Measurements.

Before.			After.		
Watch Times.	Obs. Alt. ☉		Watch Times.	Obs. Alt. ☉	
h. m. s.	° ′		h. m. s.	° ′	
17 39 15	11 38·9		17 45 00	10 22·4	
39 47	31·6		45 29	16·2	
40 16	26·1		46 01	08·8	
3)138	3)96·6		3)90	3)47·4	
17 39 46	11 32·2	Means	17 45 30	10 15·8	

Angular Distance Measurements.

Watch Times.	Observation.	Observed Angles.
h. m. s.		° ′
17 42 10	Mine — ☾	96 02.2
42 54	Mine — ☽	96 38·0
2)64		2)40·2
17 42 32	Means	96 20·1

From abridged N.A. : $\delta = -23° 20'·4$
S.D. $= 16'·3$
At watch time 17h. 39m. 46s. obsd. alt. ☉ $= 11° 32'·2$
,, ,, ,, 17h. 45m. 30s. ,, ,, ☉ $= 10° 15'·8$
Hence, at watch time 17h. 42m. 32s. :

$$\text{Sextant alt. of } ☉ = 10° 15'·8 + (11° 32·2 - 10° 15'·8) \times \frac{178}{344}$$

$$= 10° 55'·3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Computation of horizontal angular distance (A) :

Mean obs. ang. dist.	96° 20′·1				
C.E.+I.E. ..	— 0·4				
Mean True ang. dist.	96 19·7 (*C*)				
Alt. ☉ (from (1)) ..	10° 55′·3	½(*C*+*H*+*h*)	53° 39′·75	L. sec	0·22728
C.E.+I.E. ..	— 0·4	½(*C*+*H*—*h*)	53 45·45	L. sin	9·90661
Dip	— 5·7	½(*C*—*H*+*h*)	42 34·25	L. sin	9·83027
S.D.	+ 16·3	½(*C*—*H*—*h*)	42 39·95	L. sec	0·13352
Corrected alt. ☉ ..	11 05·5 (*H*)				2)0·09768
True alt. R.O. (on horizon) ..	=	$\frac{A}{2}$	= 48 12·9	L. tan	0·04884
Dip	= 0° 05′·7 (*h*)				
C+*H*+*h*	=107° 05′·7	*A* (Hor. ang. dist.)	=96° 25′·8 .. (2)		

Computation of azimuth (α) :

Corrected alt. ☉ ((*H*), above)		11° 05′·5		
Refraction	—	4·9		
Parallax	+	0·1		
True alt. ☉ (H)		11° 00′·7		
Latitude (φa)		9 08·0		
Polar distance (p)		113 20·4		
		2)133 29·1		
s		66° 44′·5	L. sec	0·403 54
s—H		55 43·8	L. sin	9·917 18
s—φa		57 36·5	L. sin	9·926 55
s—p	(←)	46 35·9	L. sec	0·162 98
				2)0·410 25
$\frac{\alpha}{2}$		58° 03′·3	L. tan	0·205 12
α	=	N116° 06′·6W		
Hence azimuth of ☉	=	243° 53′·4		
Horizontal angular distance (from (2))	=	96 25·8		
True Bearing Lode-Mine	=	147° 27′·6		

If preferred, the azimuth may be computed by the Half-log Haversine formula in the form :

$$\text{Hav } \alpha = \text{cosec c cosec z } \sqrt{\text{hav } [p+(c \sim z)] \text{ hav } [p-(c \sim z)]}$$

where α = azimuth of the heavenly body.
c = co-latitude = 90° − latitude.
z = zenith distance.
p = polar distance = 90° ± declination.

The working of the previous example will then be as follows :—

c =	80° 52′·0	log cosec 0·00554
z =	78 59 ·3	log cosec 0·00807
c∼z =	01 52 ·7	
p =	113 20 ·4	
p+(c∼z) =	115 13 ·1	½ log hav 4·92655
p−(c∼z) =	111 27 ·7	½ log hav 4·91719
α =	N116 06·5W	log hav 9·85735

Other Methods of Obtaining Position at Sea

Although a little out of place in a chapter on astronomical observations, other methods of obtaining position at sea may conveniently be mentioned here.

Astronomical Observations combined with Compass Bearings.—The methods of combining astronomical observations with compass bearings of distant land objects are described in books on navigation.

Taut Wire.—Taut wire measuring gear is described in Chapter II, and a method of obtaining position at sea by its use is given on page 21. This is one of the most satisfactory means of obtaining a position when out of sight of land, and, under good conditions, may be considered accurate to about 1:500. Its use, however, is virtually restricted to a distance of about 130 miles from a point of departure of known position (since each drum contains a bare 140 miles of wire) ; and, for an absolute position, at least two such points of departure, suitably placed, are required.

Radio Aids to Navigation.—These have been developed with great rapidity in recent years, and it is now possible to fix out of sight of land with much greater accuracy than heretofore. Several systems, briefly described below, are still (1947) in the experimental stage, and it is too early to say whether they will prove sufficiently accurate and reliable to be used extensively by the hydrographic surveyor.

Radio Bearings.—The accuracy of this long-established " aid " cannot be considered greater than about 2°, equivalent to an error in position of 1¾ miles at a distance of only 50 miles from the radiobeacon or D/F station (and, under certain conditions, the error may greatly exceed this). Its use for surveying, therefore, must never be regarded as more than a temporary expedient.

Radar.—Ranges up to a maximum of about 25 to 30 miles can be obtained with modern types of radar to an accuracy of about 10 yards, *i.e.*, an error of 1 : 5000 to 1 : 6000. As ranges can be taken only of objects which are above the horizon, the use of radar for obtaining position is restricted to those areas where an optical range-finder, of similar accuracy, could be used if conditions of visibility allowed.

Radar, however, may be put to many other uses during the survey, some of which are discussed in the following chapter. It may be particularly useful for fixing the ship when sounding in poor visibility ; position being found from

two or three nearly simultaneous ranges (due allowance being made for the ship's course and speed) ; or a form of " fixed angle " sounding (see page 208) can be carried out by keeping one object at a constant range and " cutting off " with the range of a second object.

Loran, Decca and Gee.—Special latticed charts are required for use with these " aids," on which are printed various sets of interlacing lines of position of hyperbolic form. The different sets are printed in different colours and all lines in each set are numbered for identification. Though different methods are used and different principles involved in the three systems, all record the identification numbers of the lines of position on which the ship is situated at the time of the " fix " ; thus, 45 on one dial and 30 on another would indicate that the ship was situated, say, on number 45 green line and on number 30 red line, the position of the intersection of these two lines can be seen on the latticed chart.

As in station pointer fixing, the accuracy of the position depends chiefly on the relative positions of the ship and the transmitting stations at the time of the " fix."

Consol.—This is a system of a direction-finding type where the lines of position are great circles radiating from the transmitting station. Two or more stations are necessary to give an absolute position. A M/F receiver is tuned in turn to each station and records coded signals, from which the bearings are deduced. This system appears more suitable for long ranges than the " hyperbolic " systems mentioned above.

CHAPTER XVII

SURVEYING PROBLEMS

Surveying out of Sight of Land.

Many hydrographic surveys are undertaken either partly or wholly out of sight of land. Where the depths are such as to indicate possible dangers to shipping, such surveys must be made in as much detail as that required in coastal work and a framework in the form of a floating triangulation must be provided to enable a thorough examination of the ground to be made under conditions which exclude plottable error. Where, however, the depth of water is great and it is in consequence unlikely that any dangers exist, a less detailed examination will suffice and such work does not as a rule justify time spent on a regular triangulation which will in any case be hardly practicable when the general depths are over 50 fathoms.

In discussing surveys out of sight of land, it is convenient to divide them into four main types.

(1) **Surveys connected to the Land.**—When shore objects cannot be used for fixing, the normal method of providing the necessary framework for a detailed survey is by means of a floating triangulation consisting of beacons anchored, or, if the scale is large, moored, in such positions that a station pointer fix is obtainable anywhere within the limits of the survey area. Subject to the general depths being neither too great nor too small, the beacons can usually be disposed to form a series of well-conditioned polygons and (or) quadrilaterals (*vide* pages 82–86). It is always desirable to connect such a floating triangulation to fixed points on the land if this can be done without undue expenditure of labour and time, and within 30 or 40 miles of the coast it should generally be possible to effect this by one or more of the following methods :—

(*a*) A floating triangulation, in which the inshore beacons are fixed by angles to and from shore stations and from which an extension is made by means of other beacons to connect with those in the survey area. Examples of such triangulations are given on pages 84 and 85 which would enable soundings to be fixed up to about 35 miles off shore.

(*b*) Taut wire measurements from two or more points near the land to one or more of the beacons of the floating triangulation, as shown in Fig. 9, page 21. In this method the scale of the survey must be determined by a further taut wire run between two of the beacons or by one of the other methods of base measurement described in Chapter II. A true bearing must also be found by astronomical observations (pages 383–6).

(*c*) Beacons laid in transit from fixed points ashore, their distances from the land being measured by taut wire. (See page 86 and Fig. 44a.)

(*d*) Sub-aqueous sound ranging ; an alternative to taut wire. (See page 25.)

(*e*) Radar ranges, another alternative to taut wire, provided the distances are within optical range. (See pages 26 and 386.)

(*f*) Ship stations, which can be used in a variety of ways in lieu of anchored beacons.

An example of method (*f*) is shown in Fig. 202, in which the area enclosed by a pecked line is to be surveyed. A and B are two trig. points on the land, L is a light-vessel, and S is a station where a ship is anchored temporarily for observing the angles necessary to connect the beacons a, b, c of the floating triangulation to A and B. The two points on shore are not visited, but sextant angles to A, B, etc., are observed at L, by an assistant stationed on the light-

vessel, and at S. Angles at a, b and c are also observed at the same time, provided enough assistants are available and can be stationed at those beacons. When a light-vessel and the ship are used in this way it is very desirable that, if

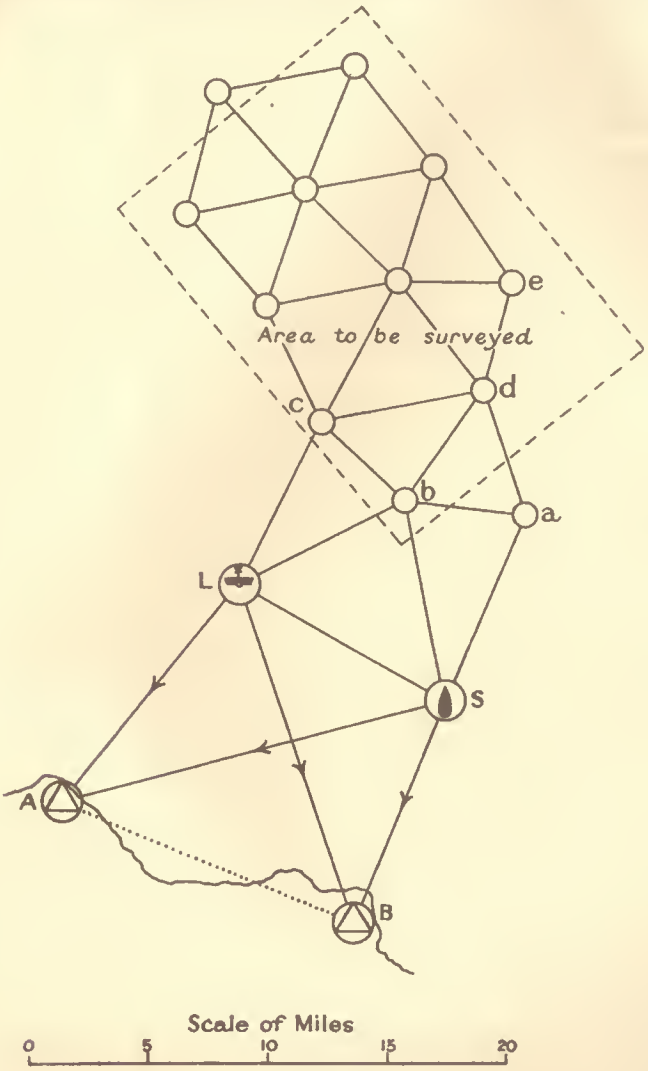


FIG. 202

possible, all observations to and from them should be made simultaneously so that large errors, due to movements round their anchors, are not introduced, and a system of signals should be arranged so that this may be done.

The computation of the sides in the figure ABSL is similar to that of the " inaccessible base," of which an example is given on pages 22–24.

From the side SL the remainder of the floating triangulation may be computed in the usual way and rectangular co-ordinates of the beacons can be calculated for plotting purposes. The use of the ship and light-vessel allows a simple and rapid connection to be made between the land stations and the nearest beacons which in the example are over 15 miles away ; if the connection had to be made by intermediate beacons, several would be required and they would afterwards serve no useful purpose in the survey.

Numerous examples could be cited of the use of the ship and light-vessels in a similar manner. The latter can be seen at a very long distance on a clear night from an elevated position and the necessary angles may be observed during the hours of darkness, thereby often providing an invaluable link between the land and an off-shore survey. Stranded wrecks and temporary marks erected on sandbanks can frequently be used and, being stationary, usually provide a more reliable connection than light-vessels. Navigational buoys may also be used for the same purpose.

(2) **Detached Surveys.**—When detailed surveys are required at such distances from the land that a connection cannot be made without an undue expenditure of time and labour the work is based on a fully-floating triangulation (*vide* Chapter IV), beacons being anchored or moored in more or less regular formation to provide a control framework and for use as sounding marks in the area. At one or more of the beacons astronomical observations for geographical position and true bearing are obtained, and the scale of the survey is determined by a measurement of the distance between any two of the beacons. This measurement may be made :

(*a*) By taut wire or sound-ranging : these are the most accurate means available and should be used for any extensive or large-scale work.
(As previously explained, radar ranging for surveying is still (1947) in the experimental stage, but, within the limits of optical range, it is, probably, at least as accurate as taut wire for determining the scale.)

(*b*) By the mast-head angle method (*vide* pages 15, 16) : this should give sufficiently accurate results for surveys of small extent on scales not larger than 1/50,000.

(*c*) By log, the distance between the two beacons being run in both directions and the results meaned : an accuracy of about 1/100 should be obtainable by this method and may be sufficient for the survey of a small area on a scale of 1/100,000. If the measurement is afterwards made by more accurate methods, it will only involve a uniform correction to the logarithms of the sides of the triangulation.

(3) **Less Detailed Surveys.**—In areas where it can be anticipated with reasonable confidence that no dangers to navigation exist, a less detailed examination than those visualised in the preceding paragraphs will often fulfil requirements. Such surveys are often needed to demarcate the limits of a bank in mid-ocean with sufficient soundings to indicate its contours with tolerable accuracy. In these cases the only essential requirement is that the ground should be examined in a fairly methodical manner, and time spent on a regular floating triangulation is not justified.

If possible, one beacon at least should be anchored in the area to provide a fixed datum point from which the ship and (or) boats can work. In good weather beacons have often been successfully anchored for short periods in depths of more than 400 fathoms, using small-sized mooring wire such as Kelvin sounding wire and, if necessary, increasing the buoyancy of the beacon with additional drums and securing floats at intervals on the wire.

A method of making a small survey using a single beacon is shown in Fig. 203, in which the approximate limits of a bank are indicated by a dotted line. A beacon is anchored in the position shown, and lines of soundings (shown pecked) radiating approximately from it, are run to cover the area. Within a mile or so of the latter, fixing is effected by compass bearing and range-finder distance ; elsewhere fixing depends provisionally on a bearing and log distance from the beacon, but each time it is approached a check on the intervening run is obtained and the fixes may be adjusted for errors due to an unsuspected set. Every care must be taken to ensure that the log gives accurate results or that its errors are determined and applied, since there is no other check on the

actual distance steamed except by means of the revolutions. If the soundings indicate shoal depths in any part, these positions must be more closely examined by anchoring dan buoys and running additional lines from them. In calm weather motor-boats, fitted with a suitable type of echo sounding gear, may be used to assist the ship, a boat being stationed at a distance of, say, $\frac{1}{2}$ to $\frac{3}{4}$ of a mile on each beam and fixing (relative to the ship) by means of masthead angles and compass bearings. The geographical position of the beacon is determined by astronomical observations, preferably of stars at twilight.

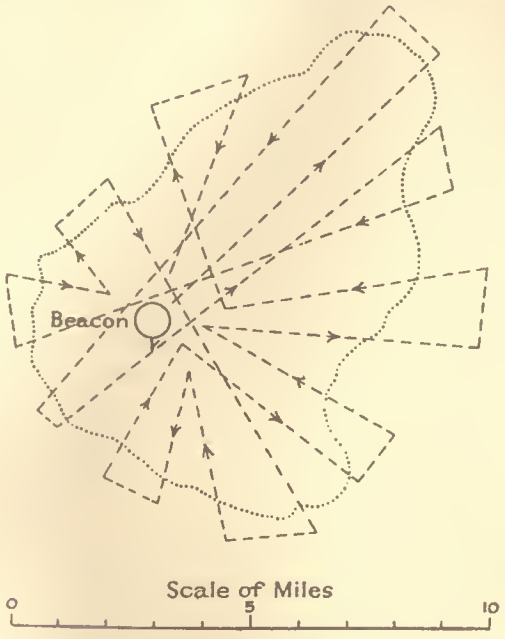


FIG. 203

(4) **Deep Sea Soundings.**—When the depth of water precludes the use of any beacons, soundings out of sight of land must be fixed entirely by astronomical observations. In such surveys instructions are usually issued as to the exact requirements, *e.g.*, if the ship is fitted with a deep-water echo set, where the lines of soundings are to be run, or, if with a deep sea sounding machine only, at what intervals the soundings are to be obtained. In other cases the location of the soundings is left to the discretion of the surveyor who will be required to make a sufficiently detailed examination of the survey area to indicate the contours of the bottom with tolerable accuracy, taking into consideration the resources and time at his disposal.

The ship should, if possible, be in a position to start sounding soon after daylight so that a good fix by A.M. twilight stars can be used as a starting-off point. Thereafter the position at any time must depend on dead reckoning combined with such other astronomical observations as are obtainable. Since any single observation may give a result as much as 2′ or 3′ in error, it will usually be necessary to apply arbitrary corrections designed to give the most harmonious fit consistent with the various individual sights and dead reckoning. The latter must be kept as accurately as possible, but strong currents are not infrequent, even in mid-ocean, and a considerable error in the reckoning may accumulate in the course of a few hours. Generally speaking, therefore, when an accepted position has to be determined from a scrutiny of all available results, the greater part of any discrepancy between the D.R. and the observed position must be assigned to the former.

Running Surveys.

A running survey is one in which the greater part of the work is done from the ship steaming along the coast, fixing chiefly by dead reckoning and astronomical observations, and at the same time observing angles and bearings to selected and prominent coastal features. A line of soundings is run on the course made good and any additional information of a hydrographical nature, which can be obtained whilst under way, is collected.

In the earlier years of hydrography, running surveys formed an important and considerable part of the surveyor's work. The results were not always very accurate, but a rough and incomplete chart of a large area was often a more urgent requirement than detailed surveys which required a much longer time to execute. In modern times the occasions when a running survey is necessary are comparatively rare, but there are still a few parts of the world where they may be of value and they are often useful as a method of exploring the ground so that a suitable scheme of triangulation for a detailed survey may be devised.

Running surveys may vary considerably in character. In the least accurate form, the ship will perhaps be able to steam only once along the coast and, with no initial data to work on, the coastline and more conspicuous coastal features will have to be plotted by bearings and angles from a series of positions dependent for their accuracy on dead reckoning and a few astronomical observations. The amount of detail will be small, the accuracy not very great, and the whole work will generally depend on a compromise between a number of discordant results. At the other extreme, a running survey may be required of a portion of the coast which is inaccessible for landing but where some, at least, of the work can be based on trigonometrical points which have been previously fixed and plotted. In these cases the ship will probably be able to steam along the coast more than once, stopping as necessary to elaborate the detail and, in fact, gathering enough material to form a tolerably comprehensive and accurate chart of the area. The procedure then differs little from that described on page 252 under the heading, " Shooting up the coastline from Boats," and need not be further discussed here.

Suppose a rapid running survey, in its most rudimentary form, *i.e.* with no previous data to work on, is required of a considerable extent of coast where only the general trend is known. A suitable scale for the work must first be selected ; good results can hardly be obtained on a scale larger than 1/50,000 ($1\frac{1}{2}$ ins. to a mile) and usually a scale of 1/100,000 will be the best. A blank field board should then be prepared on Mercator's projection with scales of latitude and longitude, corresponding to the selected scale of the survey, and one or more lines representing true meridians marked on it so that differences of latitude and longitude, courses, distances, and bearings may be quickly laid off. As the accuracy of the survey will depend largely on patent log distances, compass courses and bearings, the errors of the log and compass should be carefully determined and applied in plotting the observations.

To get the best results, several observers should be available on the bridge and the work must be carefully organised so that each has a definite duty to perform. For instance, two may be employed solely in taking angles and bearings of the shore subjects, one drawing sketches of the coast as the ship steams along, one taking astronomical observations and one plotting, whilst an assistant keeps a complete record in a deck book of all observations, log readings, engine revolutions, courses steered, times, etc.

The method adopted is to steam at a moderate and constant speed approximately parallel to the coast, with as few alterations of course as possible. The ship's position is provisionally plotted every few minutes from the courses steered and log readings, due allowance being made for the effects of wind and any known current. If a taut wire apparatus can be used, the counter should be read whenever a fix is required since distances can then be taken off with much greater accuracy than by log readings. As the ship proceeds, angles

and bearings are taken to any recognisable points on the coastline and to conspicuous objects inland, the times of each observation being noted. In this connection angles or bearings to distant hills are particularly useful since, after adjustment of the plotted track to accord with astronomical observations, they may be re-plotted first and, if the rays intersect satisfactorily, serve as objects for station pointer fixes from which the nearer detail can be laid down. Sufficient angles and bearings must be obtained to plot the positions of off-lying islets, rocks, etc., and of all noticeable promontories on the coast. A few observations to points in the intervening bays should also be obtained, if possible, but generally the latter will have to be put in by eye (unless radar is fitted, which will enable much of it to be plotted with accuracy). In taking the observations, it is usually preferable to select one of the more distant objects as an R.O. or zero ; the bearing of this is observed and protracted on the field board and the observations to other points may then be measured with a sextant from the R.O. and plotted with a station pointer. The heights of hill features can be calculated from vertical sextant angles ; rapid sketches, made at different positions, will be of material assistance in correcting and adding to the detail as it is plotted.

All angles and bearings should be plotted as soon as they are observed in order that mistakes of identification and measurement do not go undetected and so that the detail may be provisionally drawn in whilst it is still in sight. Exact intersection of three or more rays to an object cannot usually be expected

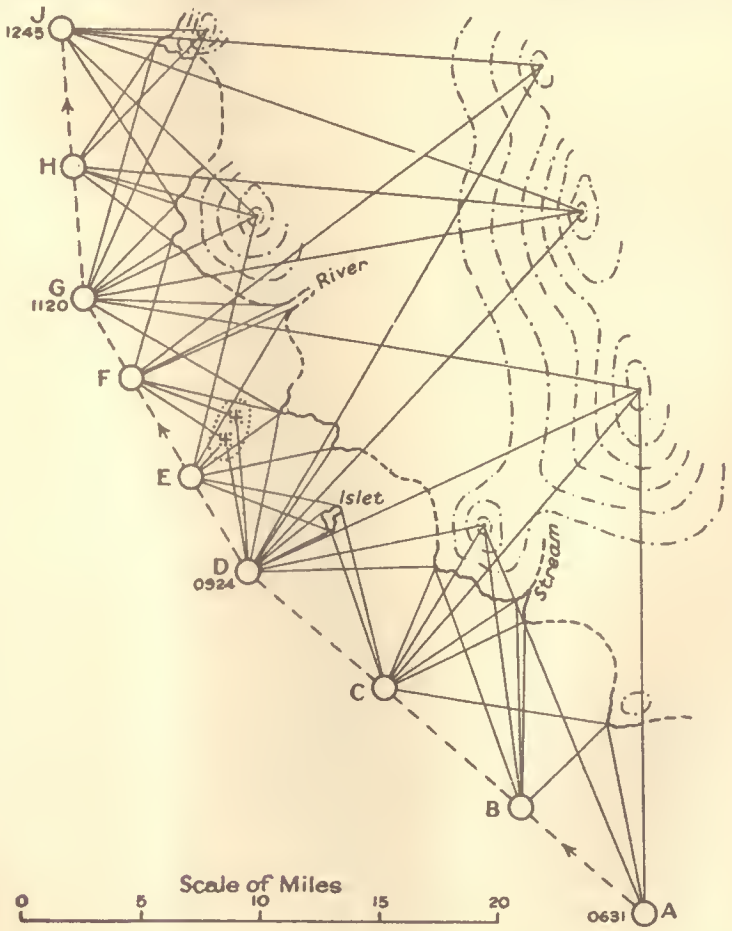


FIG. 204

but the " triangle of error " should never be so large that there is difficulty in choosing an accepted position. When the results of the astronomical observations are available it will generally be necessary to adjust the ship's position to accord with them and this will involve replotting all the observations.

Example of a Running Survey :

The ship starts from A (Fig. 204), a position which is fixed by observations of stars at morning twilight, to make a running survey of about 40 miles of coastline trending N.N.W. A straight course is steered as far as D, where the position is again determined by sights of the sun and Venus (on the meridian), and the rays shown are observed at A, B, C and D. Course is then altered towards G, further rays being observed at E, F and G, and thence towards J, where a latitude by ex-meridian altitudes of the sun is obtained. The positions of B, C, D, etc., relative to A, are provisionally plotted by dead reckoning, due allowance being made for any known or suspected currents, and a preliminary plot of all rays is made from these provisional positions, and the coastline, off-lying islet and rocks, the range of hills inland and any other conspicuous detail is drawn in lightly. The positions of D and J are then replotted relative to A to accord with the differences of latitude and longitude obtained by astronomical observation, the intermediate stations B, C, etc., are adjusted to accord with the new positions of D and J, the rays are replotted and the detail drawn in afresh.

Adjustment of Running Survey Plot to harmonise with Astronomical Observations.—One method is illustrated in Fig. 205.

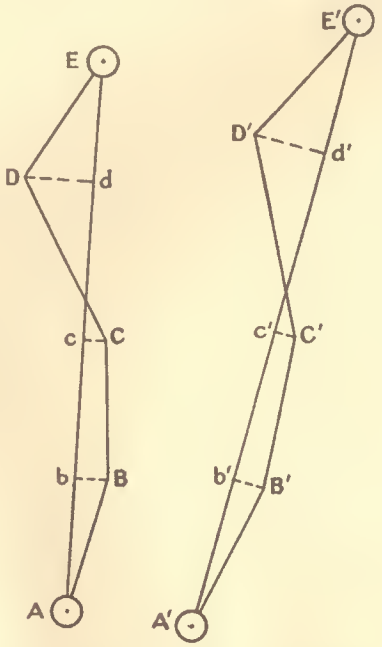


Fig. 205

The ship's course, provisionally plotted from the dead reckoning, follows the line ABCDE. Astronomical observations are obtained at A and E and show by differences of latitude and departure that the line AE is truly represented in bearing and length by A′ E′. Join AE and A′ E′ and drop perpendiculars from the points B, C and D onto AE to meet it in *b*, *c*, and *d*. Set a pair of proportional dividers to the relative lengths of AE and A′ E′ and transfer the points *b*, *c*, *d* on AE to their corresponding positions *b′*, *c′*, *d′* on A′ E′. Draw perpendiculars from *b′ c′ d′* and with the proportional dividers cut off the

lengths *b'* B', *c'* C', *d'* D' corresponding to *b*B, *c*C, *d*D. Then A' B' C' D' E' represents the course made good by the ship.

An alternative method is shown in Fig 137b, page 252.

More detailed and accurate running surveys than that described may be made if ample time is available. For instance, boats working close inshore to sketch in the smaller detail may be employed in conjunction with the ship and a line of beacons can be anchored offshore to be fixed relatively by compass bearings and taut wire measurements and to serve as a kind of rudimentary triangulation from which to work. If air photographs of the area are obtained much extra detail can be inserted and information, previously doubtful, can be either confirmed or rejected. In every case the surveyor will be faced with the problem of making a satisfactory compromise between discordant observations and the best results can only be obtained after long experience in this type of survey.

Correction of Sextant Angles for Altitude of Objects Observed.—All angular observations in Running Surveys (and most of those in Sketch Surveys ; see later), are made with a sextant and are, therefore, angular distances and not (unless both the objects observed are at the same height above sea level as the observer) the true horizontal angles required for plotting. This can be seen,

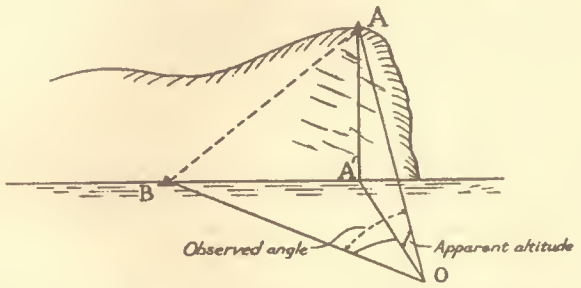


FIG. 206

in Fig. 206, where an angle is observed at O between a conspicuous summit, A, and a rock, B, at sea level. The angle observed is AOB, the angular distance, whereas the angle required is A'OB, A' being the point at sea level vertically below A. In the case illustrated, where one of the objects is on the level of the observer and the other elevated, the true horizontal angle can be found from the formula :

cos. horizontal angle=cos. angular distance × sec. apparent altitude.

Corrections (computed from this formula) to be applied to the observed angular distance to obtain the true horizontal angle, are given in Table XIX It will be seen that they are considerable and that large errors may be introduced in the plot if they are neglected. However, since cos. 90° is O, the horizontal angle will be equal to the angular distance when they are both 90°, whatever the altitude of the elevated object may be, and this should be remembered when choosing the R.O. from which to measure the angle to an elevated object. If an elevated object is required as a zero, it is frequently possible to pick out some distinctive mark on the shore (say a tree slightly taller than its fellows) exactly 90° distant from the true zero and use it as a false R.O. ; the angle required is then either the sum, or the difference, between 90° and the angle observed to the false R.O. Well-defined summits are frequently the most convenient objects to use as a R.O. (being probably the most easily distinguished mark for the observers and also a suitable zero for plotting the rays later) ; but provided the altitude of such a R.O. is not too great, or the coastline too featureless, it will nearly always be possible to pick out an object at sea level vertically below the summit and observe to that instead of to the summit itself, and so avoid tiresome corrections.

When both the observed objects are elevated, the true horizontal angle is

obtained by solving the spherical triangle in which the three sides—the angular distance and the co-altitudes of the observed objects—are known, and the angle opposite one of them (*i.e.*, that opposite the angular distance) is the required true horizontal angle. In this case the observed angle is *not* corrected automatically by being made a right angle.

Sketch Surveys.

There is no clearly-marked distinction between a running survey and a sketch survey, but generally speaking it can be said that the fomer is chiefly the product of observations made under way whilst the latter is a survey of a more deliberate character and the observations are mainly made from the ship or a boat at anchor. Naturally, a higher standard of accuracy is obtainable when the observations are made at leisure from a stationary position, and a sketch survey of a moderate area on a scale of 1/25,000 (about 3 inches to a mile) or smaller should be practically free from plottable error.

In comparing a sketch survey with regular surveys, such as those which have been considered in the earlier chapters, the following are the chief points of difference :

(1) Any triangulation is of the most rudimentary form and there is no rigid framework of trigonometrical points whose positions may be defined by co-ordinates. In consequence of this, it is almost certain that errors of plottable magnitude will accumulate if the survey is extended over any great distance or area.

(2) The accuracy of the survey depends almost entirely on the plot, without mathematical checks on the observations, and fixation is effected by reference to natural objects, such as hill-summits, trees, tangents of the land, etc. ; in some cases these may not be very well defined but must be used for lack of more suitable marks.

(3) Compass bearings are largely used as a means of determining direction.

(4) Distances by vertical sextant angle to objects of known height, by range-finder and by log are often required.

(5) The topography is mostly or entirely sketched in from ship and boat stations, and is not very detailed. Unimportant parts of the coastline may be drawn in largely by eye.

(6) No systematic exploration, by means of regularly spaced lines of soundings, of the water area is made, and the existence or non-existence of dangers must depend chiefly on a visual examination of the area. The amount of sounding, which is carried out, naturally depends on the time available and in most cases a few zig-zag lines and soundings taken "on passage," *i.e.*, when the ship or boat happens to be moving from one position to another, must suffice. It should generally be possible to erect a temporary tide pole and secure sufficient readings for the approximate reduction of soundings, but it may not be feasible to make a long enough series to establish a really satisfactory datum.

(7) In the absence of other data, geographical positions may have to depend on sights taken from the ship at anchor. Azimuths may have to be observed astronomically or may depend on gyro or magnetic bearings.

A sketch survey may be required :

(*a*) When time and circumstances do not permit a more regular and detailed survey.

(*b*) In areas which owing to their general lack of depth are of little importance to navigation.

(*c*) As exploratory surveys which will serve to determine the general form of the coast and the character of the area, and which will, if necessary, enable a suitable scheme of triangulation to be evolved, should a detailed survey be ultimately required.

(*d*) As a preliminary measure to discover whether a place is worth surveying in detail.

Sketch surveys may be undertaken by the ship or boats, or both may work in conjunction. In exposed waters the ship should usually be employed, having advantages over boats in the comparative steadiness of the observing position, good range of visibility, accuracy of bearings and possession of a range-finder. In many cases, however, the work is more conveniently done from boats which are at liberty to go almost anywhere without danger of grounding, can anchor and weigh in a few moments and can land parties for observations on shore, if required. It frequently happens, also, that the ship herself at anchor will provide a useful mark from which the boats can work.

To carry out a sketch survey it is essential that the general principles of surveying should be thoroughly understood, but it remains true that the whole operation is chiefly one of knack and the results depend on the surveyor's ability to devote his time to the essential observations without wasting it on petty detail. The procedure must be varied widely in different circumstances, but, as a rule, a sketch survey might be described as a species of traverse. In some cases, data in the form of one or more fixed trigonometrical points, which may be plotted on a field board, will be available and the sketch survey will be based on them. In others, a suitable natural scale, *e.g.* 1/50,000, must be decided on before the work is commenced and a field board prepared with corresponding scales of miles and cables and a straight line to represent the true meridian. A prick hole *a* is then marked on the board to represent the first station of the traverse ; this will be the point first occupied by the ship or a boat at anchor, or, in the case of a very simple survey, *e.g.*, the coastline of a small island, by the vessel hove to. At *a* the true bearing by gyro compass of one of the more distant objects in the survey is observed and plotted, and sextant angles are measured between this object and any easily recognisable detail which can be seen, particular care being taken to include well-defined hill summits and similar objects which it is anticipated will be visible when the vessel moves on to a second station, *b*. Range-finder distances are also taken to any neighbouring objects and vertical sextant angles to summits whose heights may subsequently be useful. All rays should be plotted before the vessel moves from one station to the next.

A provisional position further along the coast is now selected for the next station, *b*, and the vessel weighs and proceeds in that direction, sounding as she goes. Every effort should be directed to *making good* a straight course between *a* and *b*, and this can very often be done by selecting a position for *b* so that a transit, of which the bearing has been observed from *a*, can be steered. In any case, even if no transit is available, a line of constant bearing on one of the objects observed from *a* should, if possible, be maintained. Having steamed a suitable distance to a position where rays to some of the objects observed from *a* will make a good cut with those already plotted, the vessel anchors again at the second station of the traverse, *b*, and bearing and angles are measured as before. The station *b* must now be plotted. The distance *a–b* may be measured by taut wire, or less accurately by log. The position of *b* can therefore be plotted and the accuracy of the plot should be checked by the cut-offs on *a–b* of rays to any of the objects which were fixed by bearing and range-finder distance from *a*. If the ship remains at *a* and a boat is sent to carry out the survey, the distance *a–b* may be measured by means of a masthead angle and checked as before by cut-offs from objects previously fixed. Several objects should now have two rays plotted to them,

i.e., those from *a* and *b*, and may be regarded as provisionally fixed. The vessel then proceeds to a third station, *c*, situated on a line of known bearing from *b*, and the position of *c* can be plotted by the rays to the provisionally fixed objects. Assuming satisfactory resection at *c* is secured, the positions of those objects can be accepted. The work is carried forward in a similar way to the next and subsequent stations of the traverse and the various objects, which have been fixed, will provide a rudimentary form of triangulation and the points may be used for the purpose of station pointer fixes to enable additional detail to be plotted. It is clear that at all stages, and particularly in the early part of the work, great care is necessary in plotting to prevent the accumulation of error.

Radar.—It is obvious that radar can be of great assistance in a sketch survey. Its function in the surveying operations is precisely the same as that of an optical range-finder, but its accuracy, long range and speed of working make it infinitely preferable to the optical instrument.

Air Photographs.—Much time will be saved (which can be utilised for taking more soundings) if air photographs of the area can be obtained ; at their worst they can be of use for the discovery of shoals and the insertion of coastal detail, while at their best they can furnish all the land detail, including a graphical triangulation. *Vide* Chapter XIV.

Example of Sketch Surveying.— A sketch survey is required of the anchorage lettered " x " in Fig. 207.



FIG. 207

The figure is a reproduction of a small portion of a published chart and it is clear that the detail is very scanty and probably inaccurate so that none of it can be accepted as a basis for making a new survey. The ship therefore anchors to the south-eastward of the charted anchorage and the procedure is illustrated in Fig. 208, where the positions " X " and *a* correspond approximately to the points so marked in Fig. 207.

A scale of 1 inch to a nautical mile is selected as being suitable and the position, *a*, is marked near the south-east corner of a field board. A careful bearing of the islet G is observed and plotted, and from this angles are measured to the summits B, C, D, E and K, and also to the islet A. Range-finder distances are obtained to A and B and these are plotted on the board. At this and all subsequent stations, tangents are observed and plotted to all points of land, rays to any easily-recognisable objects on the coastline, and vertical sextant

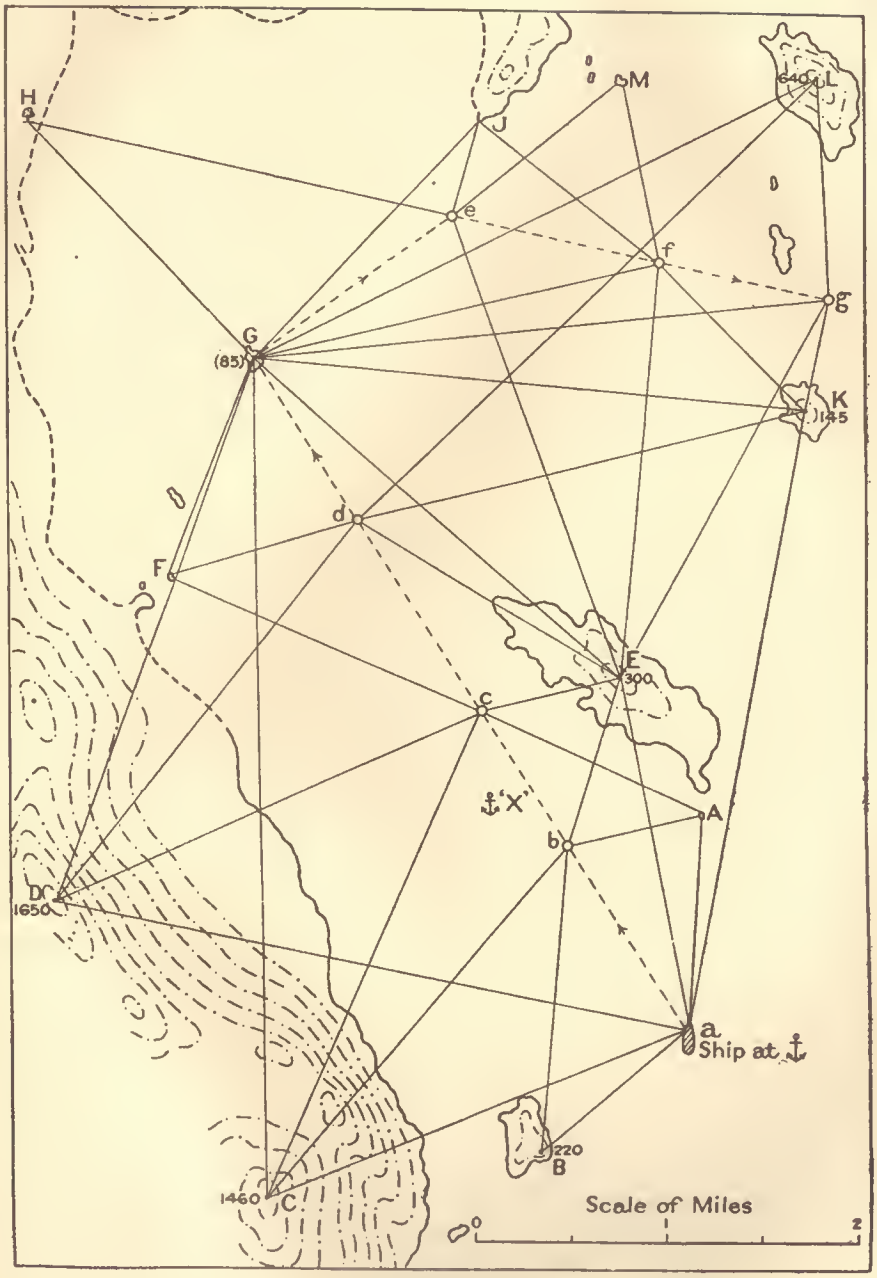


FIG. 208

angles are measured to the summits so that their heights may be calculated in due course. For the sake of clarity, only the more important rays are shown in the figure.

A boat leaves the ship and, sounding as she goes, runs a transit on G, until, arriving at *b*, she anchors. Here a masthead angle to the ship is observed so that the distance *a*–*b* is found and *b* can be pricked through on the line *a*–G. Bearings or angles to A and B are then observed and, when plotted, should resect exactly at *b*, thereby confirming the correctness of the length *a*–*b* and the accuracy of the positions A and B. Other rays from *b* may then be plotted

and provisional positions will be obtained for C and E. Continuing to the third station *c* the boat anchors again on the line *a*–G and fixes by resection from E, A and C. Similarly at *d*, where the boat fixes by resection from D, G and E. It is decided to land at G and angles are observed to the various points shown in the figure, the position being fixed by resection from E, *a*, C and D.

Leaving G, the boat selects a new transit on the islet M which has been observed from G and the next station is made at *e* which is fixed by resection from G and E. The height of D has been calculated from vertical sextant angles at *a*, *b*, *c* and *d* and a check on the position of *e* is afforded by a vertical angle to D from that point. Leaving *e*, a stern transit on the conspicuous tree H is run and the stations *f* and *g* are occupied and fixed by resection as before.

The points A, B, C, etc., will now form a framework providing station pointer fixes over most of the area and the detail can be supplemented by coasting along the shore and round the islands, shooting up the smaller promontories and drawing in the coastline. Soundings are taken on passage from one station to the next and a few zig-zag lines may be run to fill in the spaces.

Survey of a River.

River surveys are often required in addition to hydrographic work in open waters, and the methods employed must be varied to suit the character and importance of the waterway. In those parts of a river which are capable of being used by ocean-going vessels, nothing less than a detailed survey, based on a regular triangulation or an accurate traverse, will fulfil requirements. Where the river is broad, there is usually no great difficulty in providing sufficient trigonometrical points on the banks to form a series of well-conditioned figures by which the triangulation may be carried as far as necessary. The same also applies in narrower waters if a few points well back from the banks can be seen from the river and incorporated in the triangulation, but, when the river is shut in by trees and houses on both banks so that nothing can be seen beyond them, the surveyor is generally presented with an awkward problem. This is particularly the case in rivers fringed by mangroves since most, if not all, angles will have to be observed with a sextant and with short rays it will be difficult to prevent error accumulating.

When a navigable river is only a hundred yards or so in breadth and is shut in by trees, etc., it is hardly feasible to carry a regular triangulation, consisting, say, of a series of quadrilaterals, throughout its length since the figures, to be reasonably well-conditioned, would have to be absurdly small, especially at any marked bends. In practice, therefore, extension by single triangles must often be used in conjunction with traverse methods if the latter can be applied.

Example of a Survey of a River (see Fig. 209) :

A and B are two fixed trigonometrical points at the mouth of a narrow river and *a*, *b*, *c*, etc. are a series of marks on the banks by which the triangulation is extended up the river. The first figure AB*ab* is a quadrilateral in which all the angles have been observed, but thereafter the extension is carried on by means of single triangles *abc*, *bcd*, etc., and it is sometimes necessary to work *back* a short distance before the triangulation can be carried forward. For instance, having arrived at the triangle *fgh* and the side *gh*, a mark at *j* is fixed by the triangle *ghj* so that the triangulation may be carried up-river round the bend. When the river narrows down to 150 yards or less, it will probably be more practicable to work ahead by traverse rather than by triangulation, distances being measured by sextant and 10-foot pole or tachymetry (*vide* Chapter VI).

In a triangulation or traverse under these conditions some error is bound to accumulate, but with care it should be possible to prevent it exceeding plottable dimensions by one or more of the following means :

(1) All triangles should be " closed " by the observation of all three angles. If a sextant is being used and the sides are only a few hundred yards in length, closing errors of 2′ or 3′ must be anticipated.

(2) Co-ordinates of the marks should be calculated and used for plotting.

(3) If possible, the secondary triangulation up the river should be closed onto a point, *e.g.*, a hill summit, fixed in the main triangulation scheme. When this can be done, the accumulation of any large errors will be avoided.

(4) In straight reaches of the river, crossing rays, such as B*j* and *jl* in Fig. 209, should be observed. When plotted, they will give a useful check on errors of *direction*.

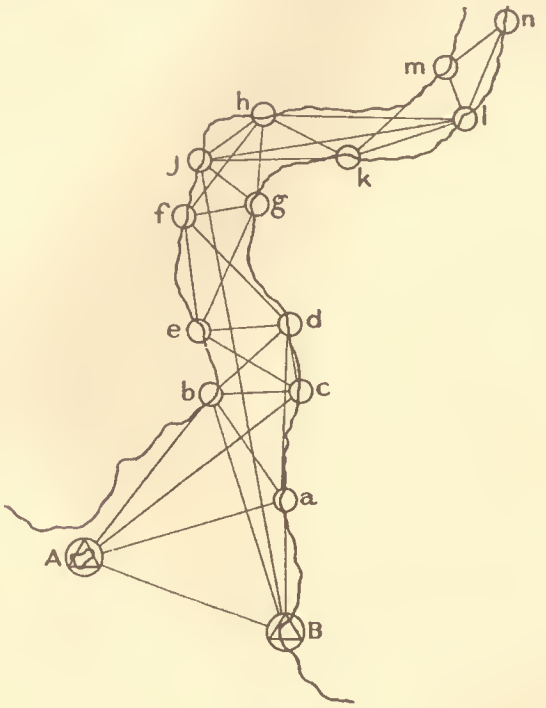


FIG. 209

If a minor triangulation of this type can be " closed " onto a previously fixed terminal point, it is probable that some appreciable error will be found to have accumulated and the differences of co-ordinates between the initial and terminal points, as calculated through the minor triangulation system, will not quite agree with the accepted values. They must be made to do so, and this will necessitate an adjustment to the positions of the various intermediate points. Elaborate methods of distributing the errors are not usually required and the simplest way of doing this is to regard the minor triangulation as a form of traverse between the points already fixed.

Supposing in the example shown in Fig. 209 the terminal station *n* is a point which has been previously fixed relative to A and B : the differences of the co-ordinates of A and *n* are therefore known. The minor triangulation system may be considered as a traverse in which the legs are A–*a*, *a*–*b*, *b*–*c*, etc., and the co-ordinates of *a*, *b*, *c*, etc. can be adjusted by the methods given in Chapter VI, pages 145–6. If the corrected sides and angles are required, they may be calculated from the adjusted co-ordinates.

Rivers, which are only navigable by small craft, do not usually justify anything in the nature of a regular triangulation and a running survey by one or more motor-boats will fulfil requirements. The normal method of doing this is as follows :

The boat starts from a position near the river entrance where a good fix can be obtained and proceeds up the middle of the river, sounding on passage. A patent log is fitted aft, and distance run at any time is read off, due allowance being made for any known current. Preferably the boat should anchor occasionally and obtain readings with a current log so that the proper corrections can be applied to the log distances. Courses steered are taken from the boat's compass and the times of all alterations of course are noted, as well as the log readings. Observations from the boat are made with a prismatic compass to conspicuous objects, tributary entrances, etc., and a small range-finder may be used for measuring the breadth of the river. A complete record of times, alterations of course, soundings, log readings, etc., must be kept and all observations should be plotted as the boat proceeds. Where elevated points are seen near the river banks, it may be advisable to land and see if a fix can be obtained by sextant angles to points already fixed on the coast and to shoot in bends and reaches which are in view.

If possible, a good fix at the farthest point up river should be obtained and the intervening work can then be squared in to agree with this position. Errors are likely to accumulate chiefly when rounding bends and efforts should be directed to making sharp and definite alterations of course at these points in preference to gradual turns which cannot be exactly plotted. On the return passage additional detail may be added and courses and distances should again be noted so that the earlier work can be checked and adjusted. Should it be impossible to connect with the main triangulation when the survey has been carried 20 miles or more up a river, astronomical observations at the farthest point may afford the best form of control over the intervening work. As with sketch surveys, air photographs, if obtainable, will be of the greatest value.

Searching for Reported Dangers.

Searches for reported dangers (sometimes called " vigias ") must often be carried out by surveying vessels and may occupy a considerable time. Experience shows that many reports are based on erroneous observations or deductions. For example, the lead has not struck bottom when it appears or was supposed to have done so ; discoloured water has been assumed to indicate a shoal whereas it is actually occasioned by a concentration of small organic or inorganic matter ; and floating ice covered with detritus has been mistaken for an island. Nevertheless, it is often very difficult to *disprove* the existence of a reported danger and the only effective method is to examine the area by systematic sounding or sweeping. In the vicinity of reefs and shoals strong currents are not uncommon and reported positions may in consequence be some miles in error. The area over which the search is made must therefore be large in almost all cases, an ample margin being allowed for this uncertainty.

Searches in Shallow Water.—Where the general depths are less than 100 fathoms, dangers may sometimes exist which, although of small area, rise almost vertically from the sea bed. In such cases no indication of their existence will be found unless the area is closely sounded on a fairly large scale and, to ensure that a thorough examination is made, a regular floating triangulation (*vide* Chapter IV) is essential. If time and circumstances permit, the area should also be completely swept to a depth of, say, 10 or 12 fathoms with the Oropesa sweep gear. The risk of the surveying ship herself striking the shoal must not be overlooked, and, when the report indicates with reasonable certainty that the danger is such as to imperil the safety of a shallow-draught

vessel, it is advisable to start operations at some distance from the reported position, using always the inner leg of the sweep so that the ship herself never passes over an unswept area. For instance, suppose a critical examination of a report indicates that the position may be as much as two miles in error and it is decided to examine an area 5 miles square centred on that position. If a land fix cannot be obtained, sufficient beacons are anchored outside the limits of the area to provide a fix at any point within it, angles are observed and the beacons are plotted in the ordinary way to provide a floating triangulation. Starting then at, say, the north-east corner of the square with the starboard leg streamed, the ship steers to the south-east corner, thence to the south-west corner, and so on. Having completed the circuit, the second is made on a course laid about 1 cable inside the first, and the ship continues to work towards the reported position. Sounding is carried on simultaneously with the sweeping, and the danger, if found, should be examined by the boats.

Searches in Deep Water.—Submarine banks rising from oceanic depths must necessarily stand on bases of considerable area and the approximate slope of the bottom, which may be expected where these exist, is shown in Fig. 210.

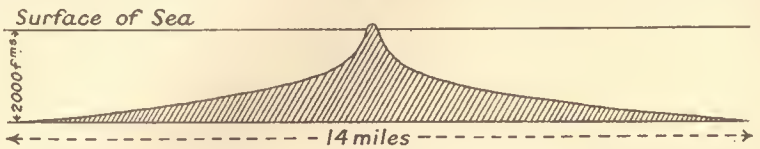


FIG. 210

It will be seen that as the depth decreases the slope rapidly becomes steeper ; in less than 500 fathoms slopes up to as much as 60° have been found, but below 1,000 fathoms it is unlikely to exceed 15°. Taking these facts into consideration, therefore, it is clear that the density of soundings taken to prove or disprove the existence of a bank or shoal in any given area should be chiefly dependent on the average depths prevailing there. Assuming, for instance, that a bank of the form shown in Fig. 210, sloping evenly in all directions, rises almost to the surface from a general depth of 2,000 fathoms, it will be seen that soundings (or lines of sounding) spaced something less than 14 miles apart should give an indication of its existence. Once the indication has been found, the direction of the up-slope can be followed by means of additional soundings until the shoalest part is reached and this can be closely examined, if necessary, with the aid of a floating triangulation. In this connection it should be noted that in coral waters the edge of a bank is a very likely spot for the growth of a shoal, and this part must, therefore, be closely searched.

Precise rules for the density of soundings required to search a deep water area effectively cannot be given but, making allowances for the fact that the slope of the bank may not be the same in all directions and that an indication of shoaling must be definite enough not to be confounded with mere unevenness in the bottom, the following procedure should normally be safe :

In depths of	3,000	fms.,	soundings	10	miles	apart.
,,	,,	,,	2,000	,,	,,	7	,,	,,
,,	,,	,,	1,500	,,	,,	3½	,,	,,
,,	,,	,,	1,000	,,	,,	2	,,	,,
,,	,,	,,	500	,,	,,	½	,,	,,

If a deep-water echo sounding equipment is fitted, the distances between adjacent lines of sounding may be slightly greater than the above.

Although the surveyor has to rely chiefly on soundings for finding reported dangers, the importance of visual indications should not be forgotten. The edges of a bank are frequently marked by tide rips, companies of sea-birds may be seen following shoals of fish which are often found in such areas, and

the ocean swell becomes steeper and may break in depths of less than 15 fathoms. Again, in clear water and under favourable conditions, a bank may be visible at a considerable distance, especially from aloft (or from an aircraft), the water over it appearing a lighter green or blue than the surrounding ocean.

When a search is to be made for a reported danger in mid-ocean, it is usually best to arrive at the required position shortly after daylight so that, if possible, a good fix by A.M. twilight stars is obtained shortly before. A field board should be prepared and it is convenient to graduate it on Mercator's projection ; a suitable scale for the work is about $\frac{1}{4}$ or $\frac{1}{2}$ inch to the mile when the general depths are great, but in shallower water a scale of 1/50,000 or even 1/25,000 may be preferable. If the danger is found, a more detailed examination can be made on a larger scale. Having decided on the extent of the area to be searched and the distance apart of the soundings (if echo sounding gear can be used, the distance between the lines), the work is carried out in the manner laid down at the beginning of this chapter. Sounding may be continued during the night if it is anticipated that no large error in the reckoning will accrue, and this may be preferable to lying to near a reported reef. If a lighted beacon can be temporarily anchored in the area, it will be of great assistance whether sounding is continued or not, since it may be used as a fixed datum point from which to work, or the ship may lie to in its vicinity without fear of being carried many miles away by wind and current during the hours of darkness.

Searches with the aid of Aircraft.—Under suitable conditions of visibility aircraft can be of the greatest assistance when large areas are being searched for reported dangers.

Such a search, carried out in a large area of deep water in which many " P.D." and " E.D. " shoals existed, may be described briefly. Three ships assisted by two aircraft working from a shore base were employed. The ships anchored (or, when the water was too deep, hove to near a beacon) in pre-arranged positions, usually in line and about 30 miles apart (see Fig. 211), fixing

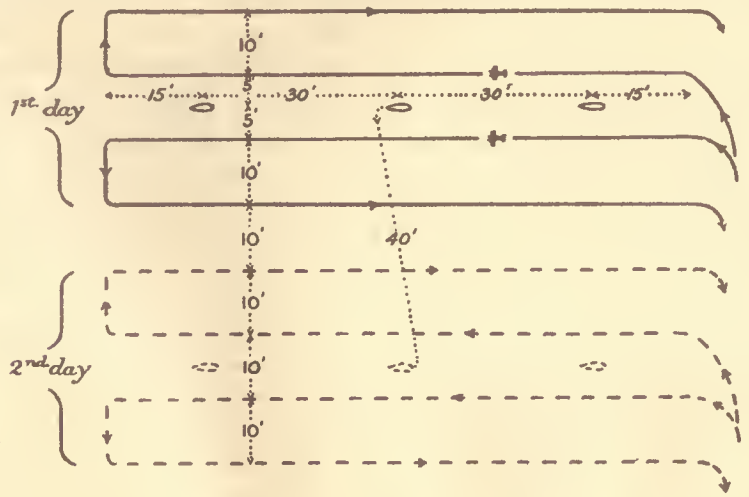


FIG. 211

themselves by star sights. The aircraft flights were planned on the assumption that any shoal, close enough to the surface to be dangerous, could be seen by the observers at a minimum distance of 10 miles (this proved to be a gross under-estimate). The aircraft started their runs about 15 miles from the end ship and, navigating independently, flew parallel courses about 5 miles on either side of the centre line of the anchored ships. About 15 miles beyond the

third ship each aircraft turned outwards for 10 miles and returned on parallel courses. During their flight they fixed themselves by bearings of the anchored ships (who made smoke to assist recognition) and of previously fixed shoals; they were also fixed independently, at frequent intervals, by the ships while they were in sight (*i.e.*, usually when within 20 miles). On completion of the runs, the aircraft reported any shoal or suspicious discoloration of the water they had sighted, and supplied copies of their logs to the controlling ship, and then returned to their base.

The tracks of the aircraft were plotted in the controlling ship from their own logs combined with the bearings of them taken by the ships, and thus, comparatively accurate positions of all reported shoals were determined. The ships then moved 40 miles at right angles to their line and anchored ready for the next flight. Reported shoals were examined and fixed astronomically, by the methods previously described, and lines of sounding were run by the ships while on their way to their new stations. As a further check on positions, taut wire distances between certain shoals were run as opportunity occurred.

The search described above was made before the days of deep-sea echo sounding or radar, and the aircraft were not fitted with surveying cameras nor had they the efficient forms of radio communication carried by modern aircraft. The description should suffice, however, to show the endless possibilities of such an operation now these modern aids are available and also the very careful organisation necessary for its conduct.

Searches by ships sounding in formation.—Successful searches have been carried out recently by two or more ships sounding in formation. The ships in line abreast kept station by radar 5 miles (or some convenient distance) apart and sounded day and night. Positions were determined astronomically and, when possible, by bearing and distance (radar or visual) from previously fixed "datum" beacons. Using this method an area, 200 by 100 miles, has been sounded out, on a scale of 1 : 150,000, in three weeks.

Use of Dry Proofs.

The surveyor sometimes has to re-examine an area shown on a published chart and, if the scale of that chart is suitable, it is often unjustifiable to spend time in making a regular triangulation such as that needed in a new survey.

The process of printing (from an engraved plate) the charts supplied for ordinary navigational purposes involves damping the paper and some distortion results on the sheet drying. In consequence angles and distances taken off the published chart will rarely be found to agree precisely with their observed values, especially if the sheet is large. For this reason, a "dry proof," *i.e.*, an impression pulled onto a dry sheet of paper, is supplied for the surveyor's use. The impression is fainter than a damp-pulled copy and unsuitable for ordinary use, but it should be an exact facsimile of the plate and thus free from distortion. Soundings and other detail appearing on that part of the dry proof which is to be re-surveyed can be stopped out so that the new work can be placed directly on the chart.

When a dry proof is provided for a survey, it is first necessary to pick out on the sheet a number of well-defined objects which will serve as marks for fixing the new detail, *e.g.*, lighthouses, beacons, church spires, windmills, etc. On arrival at the scene of operations, as many as possible of these objects must be identified on the ground. Some will perhaps have disappeared, others may be hidden from seaward and, in the case of the remainder, it is essential to make sure that they are correctly identified and charted. This can usually be done well enough by heaving to or anchoring off shore and observing angles or bearings to all selected marks which are visible. If the rays are then plotted on a piece of tracing paper, it should be possible to place the tracing on the dry proof in such a position that they all intersect the charted positions. If any throw off, mistakes of observation or identification must be assumed. Sometimes,

where few objects can be identified, it is necessary to send observers to one or more of them whence angles can be observed between other visible marks and checked with the corresponding angles measured on the dry proof.

In the ideal case, enough suitably-disposed marks can be identified to provide a station pointer fix anywhere in the area to be re-surveyed. The work could be plotted directly on the dry proof, but it is more convenient to prick the points through onto a field board and use the latter. When the survey is completed, the work can be transferred from the board to the dry proof by means of a tracing.

More often, however, it will be necessary to supplement the identified points with other marks which will help to provide a fix in the area to be re-surveyed, and various cases will now be considered.

(1) When *one* point only can be identified with certainty, the surveyor must :

(*a*) Measure a base.

(*b*) Provide some form of triangulation framework in which the identified point is included as one of the stations.

(*c*) Observe a true bearing of one of the sides of the triangulation. The geographical position used in computing the true bearing should be derived from that of the identified point, the latitude and longitude of which is taken from the graduation of the proof.

The method employed for the base measurement and the scheme of triangulation must depend on the character and extent of the survey. In nearly all cases when a dry proof is supplied for the work the area involved is small, so very exact and laborious methods serve no useful purpose. A base measured by masthead angle or by taut wire between anchored beacons will generally fulfil requirements and a few natural or artificial marks on the land and (or) beacons will suffice for a triangulation in which most of the angles can be measured with a sextant. The triangulation framework should be plotted on a plotting sheet or a field board by means of co-ordinates or from a " long side " and a true meridian through the identified point must be drawn on the plot and on the dry proof, thus enabling a tracing of the new work to be correctly fitted on. The scale of the plot must be that of the dry proof, *i.e.* the " natural scale " given in the title, or that measured off the graduation if no natural scale is stated.

(2) When *two* points only can be identified with certainty, they will provide a side from which a simple form of triangulation may be extended. They should if possible be occupied in order that rays may be observed to such other shore marks or beacons as are necessary to provide station pointer fixes in the area to be surveyed ; additional marks can be shot up from ship and boat stations fixed by angles to these points. If the two identified points are well separated, the side joining them may be used as a " long side " in the plot and computation is reduced to a minimum. If they are too close together to serve in this way, the side joining them must regarded as a " Base " and the other sides of the triangulation must be computed in order that an accurate plot may be made. In this case the length of the base is accepted from a measurement of the dry proof. *Example :*

A re-survey on a dry proof is required of the area shown in Fig. 212. Two points, A and B, seen on the larger island are identified on the dry proof and an intermediate mark, C, is erected. The summit of the small island, D, is a well-defined point and angles are observed to it from A, B and C. The points A and B are pricked through onto a field board and D is plotted, using the observed angles ABD and BAD. The mark C is then plotted from A, B and D, the angles at the latter being inferred, and A, B and C will provide a fix in the outer part of the survey area. For the inshore work the marks *a*, *b* and *c* are erected and are fixed by rays from ship stations dependent on angles to A, B and C.

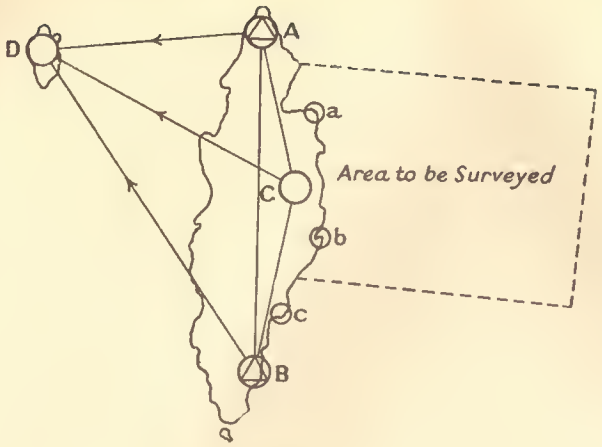


FIG. 212

(3) When several points can be identified with certainty but are not suitably disposed to afford station-pointer fixes in all parts of the area to be surveyed, it is only necessary to supplement them sufficiently to make good this defect. Additional marks, consisting of natural or artificial objects on land and anchored beacons at sea, may be fixed by intersection or resection from the identified points or may be shot up from ship and boat stations. Plotting can conveniently be done on a Whatman or field board onto which the identified points are pricked through.

When a dry proof is provided for the survey, the new work is normally inked in on the sheet, an area having been stopped out to enable this to be done. The surface of a dry proof, however, is not very suitable for fine pen work and it is often preferable to make a tracing (paper, not cloth) of the new work when it is to be forwarded into office. In this case it is, of course, essential to include on the tracing enough old detail from the dry proof to enable the former to be accurately fitted on to the latter.

When no dry proof is available for a re-survey, it is sometimes necessary to use a navigational chart instead. The procedure is the same as that described above, but, owing to a chart's liability to distortion, variations in scale must be anticipated, particularly if the proof is from a copperplate, and the surveyor must be prepared to accept minor discrepancies.

The method used for printing any chart may be discovered by examination of the letter or letters just inside the extreme south-east corner of the sheet, C indicating a print from the copper, Z or S (sometimes followed by small letters) from zinc or stone, etc.

CORRECTION OF PUBLISHED CHARTS.

In the first instance, charts are compiled chiefly from regular surveys and data for major amendments and additions is furnished from the same source. Nevertheless they can only be kept up to date by incorporating on them information received from mariners and others who, at various times, have opportunities of verifying the accuracy and completeness of the detail portrayed. In addition, therefore, to the normal programme of work the surveyor should be prepared to gather material for the correction of charts and nautical publications whilst on passage and at other times, since the special equipment of instruments carried in a surveying ship allows more accurate observation (and consequently more valuable results) than those obtainable in other vessels.

Matter for the correction of published charts must be transmitted in such form that the fullest possible use can be made of the observations, and it is

essential that sufficient data should be given to enable the new material to be plotted in its correct position relative to other charted features. If the information refers to a place out of sight of land, a position by latitude and longitude must be given, with a statement of the astronomical observations, radio bearings or other methods which were used to determine that position. In all other cases a position should be defined by reference to clearly-marked and identifiable land objects already shown on the published chart and the method of fixing must be fully stated. When suitable objects on the land are available for a station pointer fix, sextant angles should be used for fixing and for recording a position, but, as a safeguard against clerical and other errors, the true bearing of one, at least, of the objects used should also be observed and recorded. If a position is defined by bearings and distances, they should be measured, when possible, from two or more charted objects on the land.

Information for the correction of published charts is normally forwarded into office on a Hydrographic Note (form H.395) and this should be accompanied by a tracing (paper) unless it refers only to alterations in the characteristics of a light or the non-existence of a charted object. The tracing must show sufficient of the old material appearing on the chart to enable it to be fitted on accurately.

The procedure required in work of this description must be varied so widely in different circumstances that rules, to meet all cases, cannot be laid down. Often the principal difficulty is that of determining which objects are correctly shown on the chart, and can be used to give a reliable fix from which the new detail can be plotted, and which are not. This is particularly the case with some of the older charts which were based largely on running surveys. Good judgment is necessary in selecting for the fix those objects which *appear* to be correctly charted and are identified with certainty, but in cases of doubt it is a good rule to plot all the observations taken at any position on a piece of tracing paper. When this is placed over the chart, a position by re-section can be accepted where it is found that the *majority* of the rays intersect the charted positions of the observed points. Station pointer fixes should be obtained where possible, but more often the position will have to depend on compass bearings, a bearing and a vertical angle, or other methods ordinarily used in navigation.

Note.—Considerable caution is necessary if tangents of the land are used since the charted tangent may be below the horizon and the observed bearing or angle will consequently be incorrect.

When a satisfactory fix has been obtained, as much as possible of the detail appearing on the chart should be shot up from this position by means of angles and bearings ; and relevant matter in the Admiralty Pilots should be checked by personal observation. Information obtained by hearsay is often unreliable and is of little value for the correction of charts.

CHAPTER XVIII

THE FAIR CHART

The work in the field is plotted on field boards or Whatman boards from which tracings are made (pages 185–7). In the case of a very small survey where the detail is contained in an area represented by a few square inches only on the paper, a tracing is often a sufficiently permanent record. In all other cases the detail is transferred from the tracings and redrawn on good quality linen-backed paper and this forms the permanent record of the survey and is called the "Fair Chart." Suitable paper for this purpose consists of sheets measuring 53 by 31 ins., known as "Antiquarian," and sheets measuring 40 by 26¾ ins., known as "Double Elephant"; if an Antiquarian sheet is not big enough, paper of the necessary size may be cut off a continuous roll, measuring 6 ft. by 6 yds.

Importance of Neatness and Accuracy.—The fair chart is a permanent record, is the basis from which the navigational charts are produced, and becomes a document of historical interest. No pains, therefore, should be spared to ensure that all the detail shown on it is accurate, clear and legible. The highest standard of draughtsmanship is the gift of a few only, but, with the exercise of care and patience almost anyone can produce creditable results in which at least the essential features of accuracy and legibility are retained. Some draughtsmen are able to work quickly and naturally whilst others find any form of drawing a slow and laborious process, but none can do satisfactory work if they are hurried. Ample time should therefore be allocated for drawing fair charts and the comfort of the draughtsman in the matter of suitable drawing tables and other necessary equipment should be studied.

Care of the Paper.—The paper used has a specially prepared surface and every care should be taken to preserve it in good condition. The drier the paper, the easier it is to work on, particularly when colour washes are about to be applied. The sheet should always be covered when not in use and in humid climates it may be found advisable to run over the surface with a hot iron, a piece of thin paper being placed between the iron and the sheet, before work is first started. The use of india-rubber or any erasing agent destroys the surface of the paper and for this reason every effort should be made to avoid mistakes in the pen-work; pencil lines should be as light as possible so that they may be removed by the gentle application of a soft rubber. Whilst the drawing is in progress, it is almost inevitable that the paper becomes slightly soiled, but there is no reason for any excessive amount of dirt to accumulate if care is taken and the sheet is always kept covered except over the small area on which work is actually being done at any time.

The use of a pricker or beam compass for drawing lines on the fair sheet should be avoided altogether, since the surface will be damaged and the ink will "run" in the scratches where the penwork passes over them. For this reason most, if not all, the graduation should be done on the plotting sheet (see page 179) except, perhaps, the final subdivision, and transferred to the fair sheet by means of the pricking-through tracing, as described overleaf. If care is exercised the final subdivision can be done with chord dividers without damaging the surface.

With careful treatment mounted paper should suffer very little distortion. It is best kept flat during the whole time that drawing work is in progress, and, if it has to be moved, it should not be tightly rolled. Normally a soft

rubber only should be used for cleaning the surface, but, when the drawing is complete, a final clean-up with bread-crumbs improves the appearance of the sheet. It is a good rule to keep water, inks and mixed-up paints on a side table.

Paints, Brushes and Inks.—It is customary to wash over the land area and certain other portions of a fair chart with light washes of water-colour paints. The approximate amount of paint required to cover any particular area should be estimated and at least twice this amount should be mixed up ready for use before operations are started. Some paints dissolve less readily than others, cobalt blue, for example, being notoriously bad. The mixed paint can be placed in a glass bottle, kept corked up until required for use ; this will ensure that, when required later, it will be of the right consistency. If the paint is too sedimentary, it should be strained through fine clean linen ; the addition of ox gall, which acts as a lubricant, makes the application of a wash very much easier. The paint should be well stirred before and during use to ensure that an even consistency is maintained.

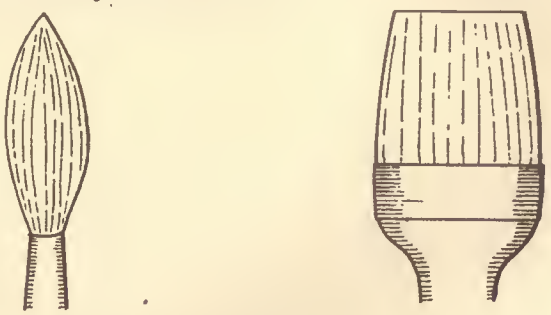


Fig. 213

Sable hair brushes should be used in preference to camel hair or other varieties as they hold the paint better and retain their shape better when charged. For washes a large brush should be used, never a smaller size than " Goose Quill " or roughly as shown in Fig. 213.

All penwork on the fair chart should be done with the best waterproof ink obtainable. *Black* Indian ink of good quality is quite waterproof and consequently is not affected by colour washes, but other coloured inks are not fully waterproof and they should therefore be used, if possible, *after* the colour washes have been put on and have dried. Chinese, or stick ink, is not waterproof and should not be used. Some of the coloured inks, *e.g.*, yellow, are unsuitable for chart work and it is often better to use a strong watercolour solution.

Procedure in drawing a Fair Chart.—A sheet of paper of the required size, with an ample margin, should be cut off and laid out to flatten under weights several days before any work is placed on it. If the surface appears to be damp or hairy, application of a hot iron may improve it. In estimating the area of paper required, it must not be forgotten that in addition to the detail of the survey, room within the borders of the graduation must be left for a title, memoir and scales.

At this stage the position should be as follows : On the plotting sheet there should be shown the positions of all main and secondary stations, sounding marks, fitting-on points for the fair tracings, beacons, the sheet corners and a skeleton subdivision of the graduation. On the fair coastline tracings and the fair sounding tracings there should be shown the fitting-on points and all the detail of the survey.

The next step is to make a pricking-through tracing (paper, not cloth) ; this is carefully flattened down on the plotting sheet and all the points shown

on the latter are pricked through and circled in ink. The tracing is then put over the blank sheet of paper which will be the fair chart, and the points are pricked through onto it ; for the time being selected positions should be very lightly marked in pencil only. Some of them will eventually be circled in coloured ink, but this should not be done until after the colour washes have been applied.

If preferred, the points may be pricked through direct from the plotting sheet laid over the blank sheet, but great care must be taken to keep the pricker upright.

For transferring the work from the fair tracings to the fair chart transfer paper is necessary ; this can be made by rubbing red or blue chalk or graphite on to a sheet of thin paper, but the manufactured article is preferable.

The high water line, shown on the fair tracings by a continuous black line (except on a mangrove shore), is first transferred to the fair chart. Each coastline tracing in turn is carefully fitted on by its fitting-on points and, a piece of transfer paper being inserted beneath it, the point of a hard and well-sharpened pencil is run along the high water line and the trace recorded on the fair sheet. This should be inked in at once. The low water line and the one, three and six fathom lines, which are limiting lines for colour washes on the sea, should now be transferred from the sounding tracings in a similar manner, but for the moment they should only be marked on the fair sheet in very light pencil. Subsequently they are inked in in red, but the colour is liable to run if this is done before the colour washes are put on.

Application of Colour Washes.—The sheet is now ready for the application of colour washes and this is a process which calls for considerable skill. Colour washes greatly enhance the appearance of a chart and render certain features more distinguishable, but they have the drawback of being liable to cause considerable distortion of the paper unless great care is taken. For this reason very large areas of paper should not be washed overall ; in such cases a wash round the limits of the area, its border softened towards the centre, is preferable. Heavy washes must be avoided as they obscure the detail and it is always better to err on the side of lightness since too pale a wash can always be recoated if necessary and darkened to a suitable tint, whereas too heavy a wash tends to become opaque and cannot be satisfactorily lightened.

The land area is tinted with a pale wash of burnt sienna if it is in general of barren or desert character, but areas of cultivation or land of generally cultivated character, trees, etc., are tinted with Hooker's green, No. 2. The extent of the land area to be tinted must be decided before the wash is applied ; exact limits inland are usually unnecessary but the colour border should be softened down to form a fairly even boundary and blank spaces must be left for the title and memoir, if they are to be inserted on that part of the chart. The area between the low water line and the 1-fathom line is tinted with a dark wash of cobalt blue, between the 1-fathom and 3-fathom lines with a lighter wash of the same colour, and a ribbon of dark blue follows the 6-fathom line, its border softened towards the shallower water.

The paper should be bone dry and scrupulously clean before the washes are applied and in this connection it must be remembered that it is almost impossible to remove pencil marks, etc., once they have been washed over. If it can be arranged, it is preferable to apply the wash on a rainy day as the paint does not dry so quickly ; in very dry weather a little gin added to the water will retard the drying process.

The brush should be thoroughly wetted and charged with colour. The wash should then be laid on in the required area, starting from the top and left side of the paper and working evenly downwards to the bottom and right with as few brush strokes as possible. Before recharging the brush sufficient liquid should be left on the paper to unite with the fresh colour with an equal

consistency. A well of colour should be dispersed as soon as possible, otherwise the edges will dry and leave an irregular line of darker colour.

If a side of the area to be coloured has to have a softened border, a " white line " about $\frac{1}{2}$ in. wide should first be made with clear water. Before the water has time to dry the wash should be laid on and merged into the " white line " with a broad brush, wetted but with the water shaken out. Speed is essential in this process and, if large areas are to be coloured, it is advisable to employ an assistant to keep the paint stirred and to merge the wash into the " white line." If small patches in a large area are left out in the process of applying the wash, no attempt should be made to fill them in at once, but the wash should be continued over the main body of the area as rapidly as possible the quicker the work the more evenly will the colour lie. Any necessary fillings should be done afterwards with a fine brush when the paper is dry, the edges being softened with another brush.

If the wash, when put on the paper, has a blotchy appearance, no attempt should be made to rectify this whilst the colour is still wet. When it is bone dry, the uneven area can be sponged lightly with a soft Turkey sponge which should be rinsed out in a bowl of clean water after each stroke. This treatment needs care in order not to disturb the surface of the paper. If drastic sponging is needed the paper should be allowed to dry between spongings. If a further application of colour is required, the paper should always be allowed to dry first.

The whole of the area between the 3-fathom and H.W. lines should be colour-washed at the same time with light cobalt blue ; when dry, the area within the 1-fathom line should be re-done with wash of the same strength.

Soundings and Topography.—After the colour washes have dried the soundings and other features may be transferred from the respective tracings. The outline of all the topographical detail, *e.g.*, hill contours, buildings, roads, etc., is traced through from the coastline tracings with a hard pencil and the position of every sounding is " dotted " through so that it can be inked in in its correct place. In representing topographical and other features, use is made of many of the conventional signs and abbreviations, which are given on Admiralty Chart No. 5011. The soundings are the most important detail appearing on the fair chart and every one of them must be clear and legible. All soundings should be written horizontally in sloping figures at a uniform slope of about 15°. If preferred, upright figures may be used but, for the effect to be good, the draughtsmanship must be above the average. It is a help if light pencil lines, horizontal and at the uniform slope (or vertical) are drawn to guide the eye.

Normally all soundings should be written the same size, but it is permissible to reduce them a little over shoal areas where it may be desirable to show an extra large number, but they must never be so congested as to impair their legibility. Black Indian ink is normally used for all soundings, but if for any reason it is desired to differentiate between " lead line " and " echo " soundings, the colour burnt sienna may be used for the latter and a note regarding its use inserted on the fair chart. Soundings on Admiralty charts are inserted in fathoms and feet below 11 fathoms and in fathoms only at greater depths. Abbreviations indicating the quality of the bottom should be freely interspersed amongst the soundings. Drying heights are always shown in feet.

Great care must be taken that small rocks are shown distinctly ; those which do not cover at high water should have their heights marked against them ; those which cover and uncover periodically should be marked as drying so-many-feet, whilst those which are awash or have less than a fathom over them at low water should be shown by the conventional symbols.

Colours used on Fair Charts.—The colours to be used in the representation of topographical detail and other features and symbols on the fair chart are set out in the following table.

Objects to be represented.	Colours to be used.	Remarks as to application of colour.
Arbitrary or assumed points	Prussian green ..	For connecting overlapping sheets ; circles 0·2 in. diameter.
Breakers	Red	——
Bridges, stone or iron ..	Red	——
Bridges, wood	Black	——
Cables	Red	——
Coral (drying reefs) ..	Light red, burnt sienna	Burnt sienna pale wash, burnt sienna and light red penwork for shading.
Cultivation	Prussian and Hooker's green, No. 2	Hooker's green pale wash, prussian green for penwork.
Dredged area	Neutral tint or Payne's grey	Very pale wash, breaking blue water tint if necessary. Limits of area shown with red pecked line.
Eddies	Red	——
Fathom lines	Red	Continuous firm line for 1, 3 and 6 fathom lines ; above these the recognised symbols.
Fixed marks used during survey	Red .. / Black ..	Red circles of 0·1 in. diam. Very small black circles also used when the fixed marks are spires, beacons, etc.
Floating beacons ..	Blue	Circles of 0·2 diam., with anchor or anchors added. (Triangle enclosed when appropriate, see "Triangulation Stations.")
Fishing stakes	Red	——
Grass land	Prussian green ..	——
Graticule intersections..	Brown	Circles of 0·1 in. diameter with lines 0·1 in. from circumference in a N. S. E. and W. direction.
Gravel	—	See "Stones."
Heights above H.W. ..	Red	Upright figures, no brackets.
Heights above L.W. ..	Black	Upright or sloping figures, underlined in black.
H.W. line	Black	Continuous firm line.
Hill contours	Grey, neutral or black	Can be shaded by horizontal shading the same colour but much lighter ; or stippled with light washes.
Hill summits	Red	A dot large enough to be distinguishable.
Houses and buildings ..	Red	Pale red wash shaded with dark red ; small isolated buildings dark red only.
Kelp	Indian yellow ..	——
Lakes	Prussian blue ..	Pale wash, black outline.
Land tint	Burnt sienna, Hooker's green, No. 2	Very pale washes only.
Leading lines, etc. ..	Red	Firm, continuous line as far as it is to be used, but dotted where prolonged to the objects.
Letters for triangulation stations	Red or blue ..	Red for shore stations, blue for floating stations.
Limits of dredged or swept areas	Red	Light pecked line.
L.W. line, sand or stones, etc.	Red	Continuous firm line.
L.W. line, mud ..	Black	Pecked line.
Mangroves	Prussian green ..	Pale washes and dark penwork ; edges of mangroves to be shown in black pecked lines.
Marsh	—	See "Swampy Ground."
Meadows	—	See "Grass Land."
Mud (low water) ..	Payne's grey ..	Regular dotting. (See also "L.W. line, mud.")
Observation spot ..	Red	Cross thus +.
Piers, stone or iron ..	Red	——
Piers, wood	Black	——
Railways	Black	Continuous firm line with very short lines at right angles.

Objects to be represented.	Colours to be used.	Remarks as to application of colour.
Reefs (drying rocky ledges)	Raw sienna ..	Pale wash and black penwork for shading, (also for rocks.)
Rivers	Indigo	Stream by a line, where it is broader a double line, and may be washed with prussian blue.
Roads	Burnt sienna ..	Pale wash and dark double lines for shading.
Rocks (low water) ..	—	See " Reefs,"
Sand (high water) ..	Black	Regular dotting.
Sand (low water) ..	Indian yellow ..	Regular dotting. (See also " L.W. line, sand.")
Sandhills	Black	——
Seaweed	Indian yellow ..	——
Shingle	—	See " Stones."
Soundings	Black	Sloping or upright figures.
Stones (below H.W. line)	Burnt sienna ..	Pale washes to represent stones, etc., shaded with darker colour. (See also " L.W. line, stones.")
Streams	—	See " Rivers."
Swampy ground ..	Prussian blue ..	——
Summits	Red	See " Hill summits."
Swept areas	—	See " Dredged Areas."
Tidal stream reference ..	Red	Diamond-shaped symbol with distinguishing letter.
Tide rips	Red	——
Tracks over bars, etc. ..	Red	Pecked lines, when the tracks have been run.
Trees	Hooker's green, No. 2	Very pale wash to represent trees, shaded with dark colour.
Triangulation stations: Main	Red	Circles of 0·2 in. diam. surrounding a triangle.
Secondary or semigraphically co-ordinated	Red	Circles of 0·15 in. diam. (No triangle.)
Floating	Blue	Circles of 0·2 in. diam. with triangle, anchors as for " Floating Beacons "
True meridian	Red	——
Water tint	Cobalt blue ..	Dark from L.W. line to 1-fathom line, light between 1 and 3 fathoms ; ribbon of dark tint at 6-fathom line.
Writing	Black	*All* descriptive writing and names in black.

Representation of Hills.—Heights.—The shape of hill features should be represented on the fair chart by means of contours or form lines. The appearance of the chart is much improved and an impression of relief is given if some form of shading is used. The light is always assumed to be coming from the north-west and shading may be effected by thickening up the contours on the south-eastern sides or by applying a light wash of Payne's grey or neutral tint on the slopes which would be in shadow.

(The method and principle of drawing form lines by means of horizontal hachures, as shown on many of the published navigational charts, are given on pages 449 and 450, Plate Z.)

Heights (in feet) should be inserted against every distinct summit and, where the ground level cannot be observed, the note " Tops of Trees " should be added. (See page 266.) In the case of beacons, lighthouses, chimneys, etc., the height given on the fair chart should be that of the *top* of the object above *mean high water springs.*

Names and Styles of Lettering.—Distinct names are necessary for many of the features shown on a chart, *e.g.*, villages, mountains, rivers, points, islands, shoals, rocks, etc., and descriptive matter is required in connection with many of the symbols, *e.g.*, the colour of a buoy, characteristics of a light, etc. The

Names and Styles of Lettering.

Double Stone Ornamented

ABCDEFGH

Double Stone Shaded

ABCDEFGHIJK

Single Stone Shaded

ABCDEFGHIJK

Single Stone Open

ABCDEFGHIJK

Roman Ornamented

ABCDEFGHIJKLMN

Roman Shaded

ABCDEFGHIJKLMN

Roman Clarendon & Clarendon Print

ABCDEFGHIJKLMNOPQRSTUVWXYZ

abcdefghijklmnopqrstuvwxyz

Roman Black

ABCDEFGHIJKLMNOPQRSTUVWXYZ

Roman Black Sloping

ABCDEFGHIJKLMNOPQRSTUVWXYZ

Roman Numerals

I II III IV V VI VII VIII IX X XI XII

Egyptian Capitals

ABCDEFGHIJKLMNOPQRSTUVWXYZ

ABCDEFGHIJKLMNOPQRSTUVWXYZ

Egyptian Print

abcdefghijklmnopqrstuvwxyz

abcdefghijklmnopqrstuvwxyz

Egyptian Stump

abcdefghijklmnopqrstuvwxyz

Print

abcdefghijklmnopqrstuvwxyz

abcdefghijklmnopqrstuvwxyz

Stump

abcdefghijklmnopqrstuvwxyz

Hair-line Print & Stump

abcdefghijklmnopqrstuvwxyz

abcdefghijklmnopqrstuvwxyz

Numerals

0123456789½⅓¾ 0123456789½⅓¾

O_1 O_2 O_3 O_4 O_5 l_1 l_2 l_3 l_4 l_5 etc.

writing of the names, etc., is the process of " Lettering," and nothing more enhances the final appearance of the fair chart than care devoted to this work and adherence to a uniform system by which the various styles of lettering are restricted to particular purposes.

The styles of lettering which should be used on fair charts are shown on Plate W and are specified below.

Single Stone Open ; for titles of fair charts. All in capitals.

(*Note.*—" Double Stone " and the ornamented and shaded letters shown on Plate W have recently been discarded, having been found unsuitable for photo-lithography.)

Roman Clarendon ; for titles of tracings, when used instead of a fair chart. All in capitals. (" Times Roman," not shown on Plate W, may also be used for this purpose.)

Roman Clarendon and Clarendon Print ; for all conspicuous objects and lists of conspicuous objects.

Roman Black ; for all principal names on land, on large scale fair charts. All in capitals.

Roman Black Sloping ; for names of important seas, gulfs, bays, rivers, estuaries, straits, lagoons, or other parts of the sea. All in capitals.

Egyptian Capitals ; for names of important peaks, mountain ranges, etc., and railways on large scale charts.

Sloping Egyptian Capitals ; for names of large reefs or patches of shoal water on large scale charts.

Egyptian Print ; for names of mountain ranges, hills, peaks, etc., railways and beacons.

Sloping Egyptian Print ; for names of reefs, patches of shoal water, etc.

Egyptian Stump ; in lieu of Egyptian print on small scale charts.

Print ; for names on land of secondary importance, each name commencing with a Roman Black.

Sloping Print ; for names on sea and rivers of secondary importance, each name to commence with a Roman Black Sloping.

Stump ; for all unimportant names and writing, each name commencing with a Roman Black Sloping.

Hair-line Print and Stump ; for notes, descriptive words, natures of the bottom, minor names and all writing in connection with work of a sketchy or approximate character either on land or water.

It will be observed from the above that the general principle employed in regard to names is that names of land features are written in upright characters, and names of water features are sloping. In cases of doubt the character of the lettering may give some indication as to which object is referred to.

A high standard of lettering can only be attained by long and constant practice, but very fair results are always possible if care is taken. The pen must be kept free from dirt and a fine nib, such as Gillott's crow quill, is generally used. Horizontal and vertical (or sloping) lines to guide the eye are practically essential and these should be drawn very lightly in pencil or traced through onto the fair sheet with transfer paper.

The beginner will often find some difficulty with the dimensions and spacing of lettering, especially the larger ones required in the title, and the following hints will be found useful.

The breadth of a letter is measured between verticals drawn through its extremities, *i.e.*, through the extreme edges of the serifs (the " caps and boots "), *vide* Fig. 214. If the height of the lettering required be 5 units, then the breadth of the letters should be as follows :

A B C D E F G H J K L N O Q R T U V X Y Z	5 units.
I	2 units.
M W	7 units.
P	4½ units (R=P+½).
S	4 units.
Width of thick strokes, vertical and diagonal, and the thickest part of the curve of each letter	1 unit.
Serifs	½ unit, each side.
Distance between words	7 units.

The letter O should be circular outside and should be slightly larger than the other letters, projecting above and below the horizontal guiding lines about the thickness of a line.

With " Stone " open and shaded lettering the dimensions given refer to the inside letter before the enclosing margin and shading have been put in. " Egyptian " lettering, having no serifs, should have the breadth of the letters adjusted accordingly.

The distance between letters should normally be 2 units for " Stone," open and shaded lettering, but may be decreased for " Roman " and " Egyptian." Certain combinations of letters, however, can be moved closer together ; no rule can very well be laid down for this since it is dependent upon optical illusion, but generally speaking it applies to letters at least one of whose strokes are not vertical, *i.e.*

AW CO OU RV KO

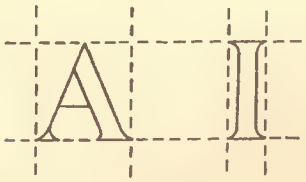


Fig. 214



Fig. 215

Difficulty is sometimes experienced in writing names and words in the water area of the chart when it has been closely sounded. Normally soundings should never be expunged or left out to make room for names, etc., but this rule may be relaxed occasionally in the case of one or two unimportant soundings. It is clear that the writing must be so placed that there is no doubt as to which object it refers to, and, by following a curve instead of the horizontal or by spreading out or closing up the letters (without, however, making the spacing unequal), it is generally possible to make use of spaces between the soundings. In other cases the letters should be broken at the soundings in the manner shown in Fig. 215.

Over the land there is generally little difficulty in writing names, descriptive words, etc., in such positions that they do not obscure detail of importance.

Names adjacent to the coast should be written on a curve, the principle being that the name starts roughly at right angles to the coast and curves towards the horizontal. Correct and incorrect curvature are shown on Plates X and Y.

Graduation, Borders and Scales.—All fair charts are drawn on the true meridian, *i.e.*, the upper and lower borders are parallels of latitude, and the left and right borders are meridians, and they must be graduated by the methods given in Chapter VIII ; but, as explained on page 409, to avoid unnecessary scratches on the fair chart, the whole process (except, perhaps, the final subdivision of the border into decimals of a minute) should be done on the plotting sheet and pricked through to the fair chart with the other points of detail. (See page 411.)

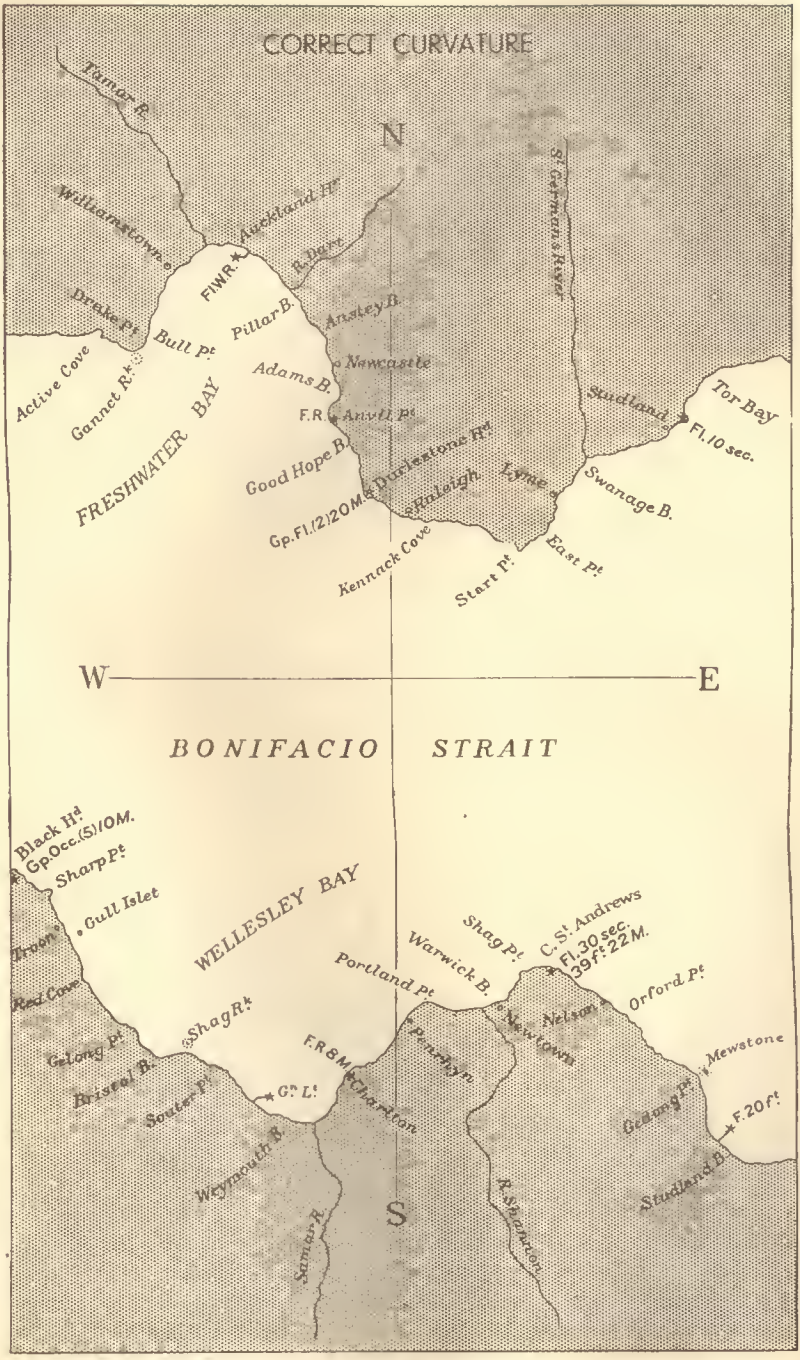

CORRECT CURVATURE
Tamar R.
N
Williamstown
Auckland Hr
R. Dart
St Germans River
FIWR
Drake Pt
Bull Pt
Pillar B.
Ansley B.
Active Cove
Gannet Rk
Adams B.
Newcastle
FRESHWATER BAY
F.R.
Anvil Pt
Studland
Tor Bay
Fl.10 sec.
Good Hope B.
Durlestone Rk
Raleigh
Lyme
Swanage B.
Gp.Fl.(2)20M.
Kennack Cove
Start Pt.
East Pt.
W
E
BONIFACIO STRAIT
Black Hd
Gp.Occ.(5)10M.
Sharp Pt
Gull Islet
WELLESLEY BAY
Shag Pt.
C. St Andrews
Fl.30 sec.
39 f: 22 M.
Warwick B.
Portland Pt
Orford Pt
Red Cove
Shag Rk
Newtown
Nelson
Geelong Pt
Bristol B.
F.R.&M.
Penrhyn
Mewstone
F.20 f:
Souter Pt
Gn L:
Charlton
Weymouth B.
Studland B.
S
Somer R.
R. Spencer

PLATE X

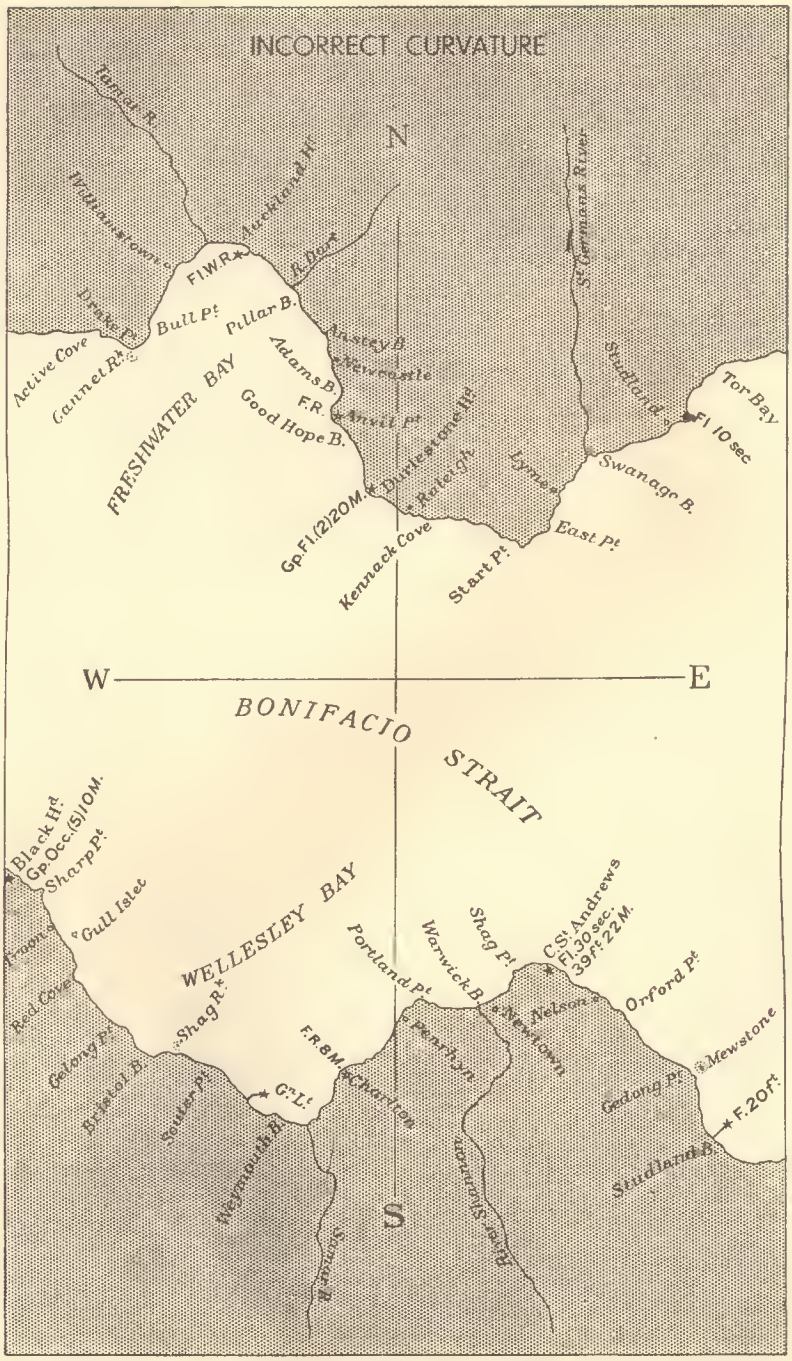

INCORRECT CURVATURE
N
W
E
S
BONIFACIO STRAIT
FRESHWATER BAY
WELLESLEY BAY
Tongue R.
Williamstown
Auckland Hr.
F.W.R.
R. Dora
St. Germans River
Drake Pt.
Bull Pt.
Pillar B.
Active Cove
Gannet Rk.
Adams B.
Amsterdam B.
Newcastle
F.R.
Anvil Pt.
Good Hope B.
Dirleseane Hd.
Raleigh
Lyme
Studland
Tor Bay
Fl 10 sec
Swanage B.
Gp. Fl.(2)20M.
Kennack Cove
Start Pt.
East Pt.
Black Hd.
Gp Occ.(5)10M.
Sharp Pt.
Gull Islet
Red Cove
Shag R.
Portland Pt.
Warwick B.
Shag Pt.
C. St Andrews
Fl.30sec.
39ft. 22M.
Orford Pt.
Nelson B.
Newtown
Penrhyn
F.R.&M.
Charlton
Geelong Pt.
Bristol B.
Sutter Pt.
Weymouth B.
G.L.
Mewstone
F.20ft.
Studland B.
River Shannon
Tamar R.

PLATE Y

The limits of the sheet are represented by the inner borders of the graduation and these lines should be drawn as finely as possible. The east and west borders (the bounding meridians) can be drawn as straight lines ; but the north and south borders, being parallels, may be appreciably curved and, if so, must be represented by a series of straight lines joining subdividing points of the graduation, sufficient in number to prevent the parallels departing appreciably from the true forms of the curves.

The styles to be used for the graduation of the borders and for the scales are shown on Admiralty Chart No. Misc. 19. (It should be noted that charts of surveys out of siglit of land, when drawn on a scale of less than 2 inches to the sea mile (1 : 36,500) should be graduated on Mercator's projection.)

To nullify the effects of distortion of the paper (due to the application of colour washes and other causes) it is essential that a close graticule should be given but, since a large number of parallels and meridians, if drawn from border to border, would confuse the detail on the fair chart, only the intersections should be shown (in brown). These intersections, which should be pricked in before any colour wash is applied, should not be more than 6 inches apart, north and south or east and west.

No work (except the figures of the graduation and, possibly, the name of the surveyor who drew the chart) should appear outside the inner bodrer of the graduation.

Natural Scale.—The natural scale of the fair chart should be determined by measuring the diagonals between the corners of the graduation. The two measurements should agree and the scale is found by comparing them with the true distance calculated from differences of latitude and longitude. The scale is expressed in the form $\frac{1}{25{,}000}$, $\frac{1}{100{,}000}$, etc. (It should be noted that on maps the scale is usually expressed in the form 1 : 25,000, 1 : 100,000, etc.)

Title and Memoir.—The title of the fair chart should contain the following information :

(1) General locality and name of the survey.
(2) Names of the surveyors.
(3) Name of the ship.
(4) Date of the survey.
(5) The geographical position of one of the main trigonometrical stations shown on the chart.
(6) Unit and datum of heights.
(7) Unit of soundings.
(8) Datum of soundings (full details should be given).
(9) Natural Scale.
(10) List of objects conspicuous to the navigator.

(*Caution.*—This list should be kept to a minimum, bearing in mind that objects which are conspicuous to the surveyor, after a long stay in the area, are not necessarily so to a newcomer. Objects that are only conspicuous under certain conditions of light, or from one direction only, should not normally be included in this list, nor should objects of an obviously impermanent nature.)

A memoir should be added to the fair chart, containing the following information :

(1) How the chart was constructed, *e.g.* from a regular or irregular survey, or sketch survey ; and on what the triangulation depends, *i.e.* base measurement, ordnance survey data, etc.
(2) References to triangulation data, or original documents appertaining to the triangulation.
(3) The calculated geographical positions of a few selected main stations, together with the position from which they are derived.

(4) References to returns, documents, etc., relating to data from which the geographical positions are derived.

(5) Methods employed for plotting and graduation.

(6) Methods by which the topography has been sketched in or, if from other sources, the authority in full detail.

(7) The use made of air photographs.

(8) A concise statement regarding tidal observations obtained, including positions of tide poles and duration of observations at each, and whether tidal constants have been computed.

(9) A concise statement regarding tidal stream and current observations obtained.

(10) A statement of the positions at which special current observations (not included under (9)) have been made.

(11) Any additional remarks considered necessary, such as the general nature of the bottom.

Specimen Fair Chart.—A pocket at the end of this book contains a reproduction of a fair chart, consisting of a small coastal plan; this indicates the appearance of a completed drawing. A reproduction of the published chart based on this fair chart is also included.

Overlaps.—The size of the plotting table and of the instruments available put a limit to the size of a fair chart and, as large sheets are in any case inconvenient to draw, they should not exceed 6 feet by 3½ feet. When the area of the survey is more than can be included on a single fair chart, the work should be drawn on two or more sheets, care being taken to provide satisfactory means of joining up the various pieces of work. Each sheet should overlap its neighbour and three or four points, either main trigonometrical stations or arbitrary positions, should be common to both sheets. In addition, the work itself, whether consisting of soundings or topography, should be extended on each sheet sufficiently far to overlap a small amount.

If possible, the junctions between different sheets should be arranged in such positions that they do not come at important features of the survey, *e.g.*, in the centre of a narrow channel, on a leading line, etc.

Tracings.—The standard of draughtsmanship required on tracings which are designed to take the place of fair charts is the same as that for a fair chart itself. Such tracings should always be made on tracing *paper* or Kodatrace, and their size should not exceed 40 inches by 27 inches, but they need not embrace all the points used in the survey. A title, memoir and scales should be included and, normally, the tracings should be graduated. If they are not graduated a true meridian must be drawn and data for plotting sufficient points should be given on the tracing (unless such data is known to be available in the Hydrographic Department); this is particularly important in the case of large tracings, where distortion may be expected.

Colour washes should never be used on tracings. If it is desired to add colours, the tracings may be very lightly chalked; only the conventional colours given on pages 413 and 414 should be used.

If a binding machine is available, tracings should be bound with tape before transmission. The packing of tracings needs great care: in particular, they should never be rolled too tightly.

Tracing of objects controlling the survey.—To facilitate the re-plotting of any portion of the survey (should this prove necessary in the Hydrographic Department), a tracing showing *every* fixed mark used in the survey and also the nicknames by which they are referred in the sounding and field books, should always be forwarded with the fair chart.

CHAPTER XIX

SAILING DIRECTIONS AND ADMIRALTY PILOTS

General.—Though a good chart is the primary guide to a navigator, there is a large amount of matter necessary for his information that cannot be shown on it, and therefore detailed sailing directions should always be rendered with the fair chart. Notes for these should be kept during the course of the survey, and the directions themselves should be compiled immediately after the conclusion of the work in the field, when every essential point is fresh in the memory.

It is impossible to write from notes, at a distance of time, so accurately or graphically as when those notes are assisted by a fresh recollection of events and places.

While the sailing directions are, of course, written as information supplementary to the fair chart of the survey they should also, if possible, be applicable to the existing published chart, so, should any object be referred to, which does not appear on the published chart, its position must be determined by a bearing and distance from some prominent object common to both charts. It may well arise that the fair chart differs completely from the old one, which may have been a most sketchy survey, in which case the sailing directions can, obviously, be read only with the former.

The sailing directions should be carefully checked with the fair sheet before they are transmitted.

Generally speaking sailing directions should not contain information which can be easily gathered from the study of the chart. Thus, as a rule, bearings and distances from one small point to another should not be given, as they distract attention from more important facts; but the general direction or trend of the coast should be recorded and, where there is anchorage, bearings of charted objects useful in approaching and picking up the anchorage should be inserted.

Courses and bearings.—Courses and bearings should invariably be given as TRUE and in degrees from 000° (North) to 359°.

Geographical sequence.—Care should be taken in writing sailing directions for incorporation into an existing book, to follow the same geographical sequence as is pursued in the book itself, *i.e.*, to work from east to west, or from north to south, as may be already adopted in the book.

Correction of Admiralty Pilots.—In writing sailing directions that will partly add to and partly supersede those already published in an Admiralty Pilot, the whole of the relevant part of the book should be rewritten, retaining those paragraphs that hold good, and interpolating the fresh matter.

Order of writing.—It is not possible to lay down invariable rules as to the order in which the different remarks comprised in sailing directions should be dealt with, but the following rules should be observed as far as possible:

(*a*) General remarks, stating the order that will be pursued in treating the subject, with a general description of the district, divided into sections; geographical, geological, hydrographical, and meteorological.

(*b*) Description in detail of the coast, dangers, harbours and anchorages should then follow, including in each chapter a conveniently large

portion of the coast, say, from one prominent headland to another, or a certain group of islands, completing each section before commencing another.

(*c*) Lastly should come the sailing directions for each portion of coast or passage, after all the places and objects have been separately described. In like manner, any general sailing directions for making passages over long distances should be dealt with separately. Directions for harbours or anchorages should, as a rule, be placed immediately after their description.

(*d*) In large channels, after a general description, the details should be confined to one reach at a time, each shore being described separately, and in the same direction, which should, as far as possible, be that followed by a vessel arriving from seaward, and from Europe.

(*e*) When both sides of a channel are clearly visible from a vessel passing through they should be described together and not alternately, as stated above for a large channel.

General description of coast.—In describing a portion of coast, some such order as the following should be observed :

(*a*) The appearance or aspect of the coast on making the land, describing prominent objects, as, on a bold coast, the headlands, peaks, etc., with their form, colour and height ; or, on a flat coast, the conspicuous natural objects, buildings, beacons, etc.

(*b*) Description of the outlying dangers and islands, and off-shore soundings, with the nature of the bottom. Directions for clearing the off-lying dangers.

(*c*) Information as to pilots ; their cruising ground ; any special regulations or signals ; the possibility of obtaining tugs, etc.

(*d*) In case of stress of weather, the best anchorage or the nearest harbour of refuge to run for ; or, in extreme cases of underwater damage, etc., the best place to lay the ship aground.

(*e*) A description of the coast, with the character of the inshore dangers, headlands, points, bays, anchorages, islands, rivers, etc., and especially lights, buoys, and beacons. The amount of detail given in this description will, of necessity, vary with the nature of the coast and other considerations, but, as the Admiralty Pilots are the only works which contain descriptions of buoys and beacons, full details of them should always be given. (See **Detailed description of coast,** opposite.)

(*f*) Harbours and anchorages require the closest description, and it should always be clearly stated whether they afford good, fair, or indifferent shelter ; the exact position of the recommended anchorage should be stated and whether the holding ground is good or bad, a full description of the quality of the bottom, should be given, such as a layer of mud over sand ; also whether anchorage can be taken up for long or short periods, and whether it is suitable for a number of vessels in company or for single vessels only.

If any particular natural conditions, such as squalls, sudden shifts of wind, etc., tend to make an anchorage inconvenient or unsafe, it should be stated whether such conditions can be forecast by observation of such local peculiarities as excessive humidity or dryness, high peaks becoming obscured by cloud, unusual swell or motion of the water, remarkable clearness of the atmosphere, etc.

(*g*) Currents and tidal streams should be grouped together in separate paragraphs ; also the time of the turn of the tidal stream with reference to the time of high water, and the duration of slack water.

The tidal streams in the neighbourhood of the standard ports of reference given in the tide tables should be referred to those ports; elsewhere to the time of local high water. All references must be made to high water, before or after, and never to low water. The terms "flood" and "ebb" stream should never be used excepting in a port or river, and then only when the turn of the stream is within an hour of high water or low water; but the stream should be named by the direction towards which it is running, as "north-going stream," "south-west-going stream," etc., or in rivers and estuaries, "in-going" and "out-going."

Admiralty Tide Tables, Part III (shortly to be replaced by "Admiralty Handbook of Tidal Prediction and Analysis for Seamen"), should be consulted in connection with all tidal stream and current observations. (See also Chapter XV.)

Detailed description of coast.—The detailed information required for (*e*), above, should comprise the following:

Lifeboat and life-saving stations.

Dangers: Their nature; the depth over them at chart datum, or at mean low water springs when chart datum is unknown; whether visible; if breaking, at what period of tide; whether any part dries, and how much. Whenever banks or rocks dry, information concerning them should take the form of stating the number of feet they dry. The term "awash" should only be used when the danger is awash at chart datum (or at mean low water springs, when chart datum is unknown). When it is awash at any other state of tide it should be given as "dries so many feet." Marks for clearing them by day or by night. The bearings of leading marks and clearing marks should be given in degrees, and for such clearing marks it should be stated that, for instance, the mark leads eastward or westward of the danger, and not clears to the eastward or westward.

Bars: The best time for crossing; bar signals.

Points and headlands: Character of; whether cliffy, wooded, sandy, etc.

Hills, mountains: Height and appearance of elevations, distinguishing between mountains and hills. As a general rule, except in a very mountainous country, all may be considered as mountains over 1,000 feet in height. Mention whether summits are usually clouded or not, and whether snow lies on them, and if so, at what season. When the height given is that of the tops of trees, it should be so stated.

Islands: Height; whether wooded, cultivated, bare, etc.

Landing places: Should be specially mentioned when the coast is open or exposed to swell.

Rivers: When rivers of any importance are mentioned, give, if possible, their source, length, general course, points to which the tide flows, and to which they are navigable both by ships and by boats. The rate of flow before and after rains should also be given.

In the case of large rivers; a table, giving distances from the bar or the outer light or buoy, the least depth to be passed at chart datum, (or at mean low water springs when chart datum is unknown), and the rate and time of turn of tidal streams at various places on the river, will be useful.

The size of streams and whether fit for drinking or boiler use; the latter information is only required in uncivilized parts of the world where ports at which water can be obtained are a considerable distance apart.

Approaches to Ports, etc. : In all descriptions of tidal harbours, rivers or ports, the least depth (referred to chart datum or to M.L.W.S. if chart datum is unknown) a vessel must pass over to reach her destination and the position of the bar or shallow water referred to, should be stated. The distance of the port from the sea must also be given.

Lights : Every detail that can be useful to the mariner should be given ; character, visibility, arcs of visibility and obscuration, limits of coloured sectors, details of fog signal, position, appearance, colour and height of the light structure, the elevation of the light above high water (and when placed high, information as to whether it is specially liable to be obscured by clouds or mist) ; description and position of lighthouse buildings, fog signal buildings, and signal stations ; also the means of communication with nearest village, port, etc. For light-vessels ; the appearance, number of masts, type of daymark, height of the light and its character and visibility, must be given ; also the degree of trust that can be placed in the maintenance of their positions.

Photographs or sketches of lighthouses, taken from seaward, are especially valuable ; the bearing and distance of the lighthouse from the view-point must always be given, but need not be repeated in the letterpress. (See also **Views,** page 426, and **Photographs,** page 428.)

Directions for harbours.—Sailing directions for a harbour, written only for a vessel entering, will often be sufficient, but in certain cases it may be desirable that directions should also be given for leaving a port.

Descriptions of seaports.—In describing seaports, give sufficient general statistics to enable a stranger to judge of their importance, probable abundance of supplies, etc. ; see above, *Approaches to Ports,* for depths in approaches.

(*a*) Draught of the largest ship that can enter, in case of tidal or shallow harbours, at mean low water springs, and depths alongside any piers.

(*b*) Facility for, and manner of, fuelling, with nature of shelter afforded by the harbour or anchorage. Number of lighters available. Depth alongside fuelling wharves and piers at mean low water springs ; or, still better, the maximum length and draught of vessels which can lie alongside at mean low water springs.

(*c*) How coal and fuel oil are obtained, and whether from a near or distant locality.

(*d*) Number and tonnage of arrivals annually.

(*e*) Chief exports and imports. Any special port regulations, bye-laws and Custom-house regulations ; holidays, etc.

(*f*) Dock accommodation, both wet and dry, with full particulars. Patent slip. Gridiron.

(*g*) Facilities for repairs to a vessel and her machinery. Capacity of largest crane or sheers.

(*h*) Quarantine regulations. Hospitals or home for sailors, with number of beds. Shipping office for seamen.

(*i*) Prevailing diseases, if of a virulent character (as on the coast of Africa) ; seasons at which they may be expected and brief precautions for guarding against them, diet, clothing, etc.

(*j*) National prejudices in cases where serious offence may be given unintentionally.

(*k*) Population.

(*l*) Means of communication by ship, rail, road, air, telegraph, telephone and radio.

(*m*) History and antiquities should be mentioned only briefly, but a reference should be made to works in which further information may be found; political facts may be briefly stated, but all political opinions should be avoided.

(*n*) Time signals, harbour signals, and storm warnings, when established, should be described, and the places from which they are displayed should be clearly indicated.

Units of measurement.—Throughout observe uniformity in the various terms used in expressing capacity, distance, height and length, respectively. Heights should be given in feet above the level of high water springs; depths in fathoms or feet below chart datum (or below mean low water springs where datum is not known). The height in feet above chart datum (or mean low water springs) should be given for banks or rocks that dry at certain times of tide. Distances, when less than a mile, should be given in cables or yards; when parts of a mile or of a cable are mentioned, they should be as vulgar fractions, not as decimals, which may lead to error.

Nomenclature.—Names require much care. In all cases when the native name is obtainable, it only should be used, except when a European name has been attached so long to a place that it is generally recognised, in which case both may be given. In the British Isles and the Irish Free State the Ordnance Survey maps are the authority for names on the land. The only exceptions to this general ruling are the Gaelic terms " Beag " meaning " little "; " Beinn " meaning " mountain," and " Mor " or " Mhor " meaning " great "; which should always be spelt " Beg," " Ben," and " More," respectively. Nomenclature given in triangulation data issued by the Ordnance Survey is not always in agreement, and should not be accepted.

For the transliteration of foreign names, the rules given in " Alphabets of Foreign Languages transcribed into English according to the R.G.S. II System " (Gleichen-Reynolds) should be followed.

The surveyor should always be on the look-out in all parts of the world to correct instances in which names appearing on the Admiralty charts, etc., are not in accordance with that system. In all cases where names appearing in the published Admiralty Pilots are altered in the sailing directions accompanying fair charts, the authorities for such alterations should be stated.

Geographical terms.—Care should be taken that the geographical terms used for natural formations are used as correctly as possible as regards new names, but as regards old ones the rule laid down under " Nomenclature " should be adhered to, *i.e.*, to beware of meddling with names already established. A few common terms, often used incorrectly, are defined below:

A *bay* is a comparatively slight indentation in the coastline in distinction to a gulf, loch, firth, etc.

A *channel* is usually a narrow and deep passage, sometimes dredged, which may lead into a harbour or through a sound or strait.

Coast is the term used with reference to the land, whilst the term *shore* is used with reference to the sea; for instance, the west *coast* of India forms the eastern *shore* of the Arabian sea.

A *creek* may be either of fresh or salt water, but is tidal throughout its whole course.

A *head* (*or headland*) is a comparatively high promontory, with either a cliffy or steep face, whereas a *point* is a comparatively low and sharp promontory. The word *cape* should be restricted to promontories facing the open sea.

Mountains, *hills* and *knolls* are comparative terms of heights varying with the general configuration of the country, but *peaks* are essentially hills or mountains whose summits are comparatively sharp.

Overfalls, rips, or eddies are the effect of the current or tidal stream passing over an irregular bottom, whereas breakers are the effect of the state of the sea.

A "*reef*" is an area of rocks, or coral, detached or otherwise, of considerable extent which dries or nearly does so, in places.

A *river* is a flow of fresh water, which, excepting in its lower reaches, is always in the same direction ; but the term *stream* should never be used except in connection with the streams caused by the tides.

A *roadstead* is an open anchorage which may or may not be protected by shoals, whereas *harbours* and *ports* are those protected by the formation of the land or by artificial works.

The expression "*shoal*" should be limited to a detached area, the depths over which constitute a danger. A detached area, the depths on which do not constitute a danger, should be called a *bank*. A shoal area connected to the shore is not properly a *shoal*, but rather a *spit*, *sand* or *bank*.

Shoals and *banks* may be of any material.

A *sound* or *strait* is the name given to a passage having an outlet at either end, a sound is usually wider than a strait.

Verification of information in Admiralty Pilots whilst on passage.—Much of the information in the current editions of the Admiralty Pilots regarding landmarks and conspicuous objects, on or near the coast, is word for word the same as that given in the early editions, and it is, in many cases, not known whether such objects are still visible or conspicuous.

Every opportunity should therefore be taken to verify such information whilst on passage, paying particular attention to leading lines and clearing marks. Any discrepancies observed should be reported by Hydrographic Note.

Views.—Views of the coast are a valuable adjunct to the written matter appearing in the Admiralty Pilots. Their purpose is to help the seaman recognise different parts of a strange coast and enable him to identify objects by which he can fix his position on first sighting land. In their object, therefore, they differ fundamentally from views which are drawn by the surveyor for topographical purposes (*vide* Figs. 143 and 143A, page 265) and the same view-points are rarely suitable for both.

Views for inclusion in Admiralty Pilots should, as a rule, be drawn from view-points distant about four to five miles offshore in such positions that they will be of use to the seaman when approaching a coast under conditions of moderate visibility. Views from points nearer to the coast, *e.g.* from an inshore anchorage, are seldom of any value, though an exception to this rule is the case of leading marks. Views of the latter should be drawn from a point a short distance outside the position where it is first necessary for a vessel to get onto the leading line.

In addition to views of the above type, sketches of beacons, lighthouses and other navigational aids, accurately drawn on a reasonably large scale, showing their appearance from the normal direction of approach, can usefully be made for inclusion in the Pilots.

Views should be drawn from the ship or a boat at anchor ; the former is usually the more convenient in that a steadier platform is provided and the height of eye will correspond more nearly to that of the seaman who will subsequently make use of them. Views must be drawn accurately to scale and few people are capable of doing this without the help of angular measurements. Unless the land is very high, it is always advisable to exaggerate the vertical scale in comparison with the horizontal, a proportion of 3 to 2 being generally satisfactory. Selecting one object as a zero, horizontal sextant angles should be measured to all other readily distinguishable features and their elevations should also be measured above the sea or shore horizon. Using scales of, say,



Tamara I. Lighthouse bearing 355° distant 5¾ miles. Coral I. Roume I. Kassa I. Blanche I.

Iles de Los from the southward.



Paul I. Shot I. Pikaluyak I. Pillar I. Cap I. Hillsbury I.

Left tangent of Pikaluyak I. in line with right tangent of Pillar I., bearing 281° leads through the narrows into Strathcona Run.

Approaches to Nain.

PLATE A*

0·2 inch to 1° horizontally and 0·3 inch to 1° vertically, these objects should first be plotted in their correct relative positions and the remaining detail can then be drawn in by eye. Only by adopting this procedure can it be certain that all essential features are correctly placed with regard to each other, both vertically and horizontally.

Views should first be drawn in pencil, using squared paper for the purpose. They should then be traced off and transferred to plain unbacked drawing paper in a form suitable for photographic reproduction, before being forwarded into office with the sailing directions ; *i.e.*, they should be pen drawings in Indian ink for the outlines and detail, in which very fine lines must be avoided. Shading to indicate differences of distance, form of hills, etc., may be either by penwork or a sepia wash. The writing indicating leading marks, hills, or other special features should be written in red above or below the view ; pencil lines may be drawn to the objects for the purpose of identification where necessary. Views must never be folded.

The observed angles to the various objects need not be recorded on the views, but it is essential that the position of the view-point should be clearly stated either in the form of a true bearing and distance to a prominent feature appearing in the view or by means of sextant angles to objects which provide a station pointer fix.

Two examples of views are shown in Plate A*, the upper a distant view of the coast and the lower a view of a leading line which leads clear of dangers in the approach to a harbour.

Photographs.—These may be used to supplement views drawn by hand or sometimes in lieu of them. Photographs have an advantage over views in that they can be obtained instantaneously and without anchoring the ship, but in most cases they are hardly a satisfactory substitute for the reasons that :

(*a*) The vertical and horizontal scale is the same with the result that low-lying coastal features may be hardly recognisable.

(*b*) Objects of importance, which can be stressed in a sketch, may be indistinguishable.

(*c*) Touching up to remedy (*b*) requires great skill to be effective.

Photographs should not be mounted, but should be attached to cartridge paper by means of slits cut in the paper and the necessary names, etc., should be written on the latter. Like "Views" they must not be folded for transmission.

CHAPTER XX

MAGNETIC OBSERVATIONS

Magnetic observations are required in different parts of the world to determine Variation, *i.e.* the angle between the true and magnetic meridians, and for scientific purposes they are also needed for the determination of the earth's magnetic force and dip. Since terrestrial magnetism is subject to secular change, repeated observations in the same places are desirable at intervals of a few years.

The spot chosen for magnetic observations on shore should be on ground as free as possible from magnetic matter. Volcanic and crystalline rocks, such as basalts and granites, should be avoided, and the safest ground is alluvial or sedimentary rock such as shales and slates ; coral is usually free from magnetic material. Observations, however, are also useful at places where local attraction, due to the nature of the ground, is suspected ; by comparison with the variation found elsewhere in that neighbourhood, the extent of such local attraction may be measured.

The observation spot should always be marked in such a way that its exact position can be recovered in later years, and the surveyor will usually find it convenient to observe at one of the main stations of a triangulation. Such a position has advantages over others in that true bearings (or azimuths) of other visible stations can be calculated directly through the triangulation and will be available for comparison with compass bearings. If some other position has to be used, a true bearing must be obtained by astronomical observations or the position must be connected by theodolite angles to stations of the triangulation so that a true bearing can be deduced.

VARIATION. (Declination of the Compass Needle).

Observations for variation, either ashore or afloat, or both, may be considered an essential part of any hydrographic survey.

Observations on Shore.—*Tubular Compasses.*—Tubular compasses are supplied for attachment to certain types of level and theodolite, and have now almost entirely superseded landing compasses of the " card " type. In this form of compass the needle is pivoted at the centre of a closed cylindrical tube ; at one end of the tube (that corresponding to the north-seeking end of the needle) a needle-locking device, operated by a milled head, is fitted, and at the other a glass diaphragm and a magnifying eyepiece. The attachment of the compass tube is so designed that its axis is parallel to that of the telescope in the instrument. When the latter is turned into the magnetic meridian, the south end of the needle, which is shaped like an inverted V, comes into view in the eyepiece and accurate setting may be made by observing its intersection on the diaphragm wires.

Light is admitted into the tube through a small window on its upper side and is reflected by a prism to the eyepiece end. A moveable weight, friction tight on the north end of the needle, can be adjusted, if necessary, to make the needle horizontal and thus bring the south end central in the field of view. The pivot is very delicate and the needle should always be locked except when the compass is actually in use on the instrument.

Method of Observing with Tubular Compass.

Observations with this type of instrument present little difficulty, but

great care must be taken to remove to a good distance any objects made of or containing iron or other magnetic material. Having set up and levelled the instrument over the selected position, it should be turned in azimuth until the compass needle is central, the final intersection on the diaphragm wires being made with the slow motion screw; the horizontal circle reading of the magnetic meridian is then read and recorded. The mean of several readings should be taken, the instrument being turned sufficiently to throw the needle off centre first to one side and then the other between each reading. The telescope is then turned until intersection is made on an object whose true bearing is known and the horizontal circle is again read and recorded.

This procedure is then repeated on 3 further 'zeroes', the setting of the horizontal circle being altered by 90° between each series. It is necessary to use 4 positions around the horizontal circle to eliminate the effects of any residual magnetism in the theodolite. All observations are made on the same face and if a bracket is supplied with the compass it should invariably be used.

The calibration correction shown in the lid of the theodolite box is then applied to the mean of the readings of the magnetic meridian on all 4 zeroes. From the circle readings of the R.O. the reading of the true meridian can be deduced, the difference between this and the corrected reading of the magnetic meridian being the variation.

Only theodolites whose compass calibration certificate shows them to have been calibrated on 4 zeroes at a Magnetic Observatory should be used for magnetic observations. Should no such instrument be available the theodolite to be used should first be tested by measuring the angle between a reference object and the magnetic meridian on 4 zeroes, 90° apart. If a difference of reading of more than 5 from the mean is recorded the theodolite should not be used for magnetic observations.

Using a compass calibrated on 4 zeroes results should be accurate to within 2′ of arc. Remarks as to whether the compass used was calibrated on 1 or 4 zeroes, or not at all, should accompany all reports of variation obtained.

Card-type Landing Compass. Method of Observing.

If no tubular compass is available, it may be necessary to measure variation with a compass of the card type. In this case, the system of observation must be such as to eliminate errors of graduation and centring, and compass bearings of a number of objects distributed as equally as possible round the horizon should be taken for comparison with the true bearings of those objects and a mean figure must be adopted for the variation. Spare cards, caps and pivots are sometimes supplied with the compass and in this case the observations should be repeated with as many different combinations of these as possible. Instruments of this type should be tested at a magnetic observatory for the determination of card error, *i.e.* lack of parallelism between the axes of the magnets and the zero line of the graduation.

Observations at sea.—These are needed to supplement observations on shore and may often be the means of indicating whether the latter have been influenced by an unknown source of local magnetic attraction. The ship's standard compass must be used for the observations which consist in taking bearings of a heavenly body or a distant land object with the ship headed on eight or sixteen equidistant points. If possible, the " swing " should be made in a depth of 50 fathoms or more so that the compass is not likely to be influenced by magnetic material in the ocean bed, and the ship should be steamed round both to starboard and to port, the mean of the two results being accepted. The ship should be on an even keel and should be steadied for at least a minute on each point to allow transient induced magnetism to disappear before the bearings are observed.

If the sun or other heavenly body is used for the observations, it should not be more than a few degrees above the horizon so that it can be viewed directly over the azimuth mirror and the time of each observation should be recorded. Then, if the observer's geographical position is known (to the nearest mile or so), the true bearings of the heavenly body may be taken out from azimuth tables with sufficient accuracy. A distant land object may be used instead of a heavenly body if the swing can be made in a position where a station pointer fix is obtainable by angles to surveying marks. In this case the ship should be fixed at each observation, the fixes plotted on a field board or the plotting sheet, and the true bearings of the object used measured off from each fix.

Assuming coefficient A is zero, as it should be in most ships, the mean of all differences between the true and compass bearings is the variation. If coefficient A has any known value, this must be applied to the mean.

Unifilar Magnetometer

This instrument, designed for use on shore, is employed to measure H, the horizontal component of the earth's magnetism; it can also be used to determine Variation (known to scientists as Declination of the compass needle).

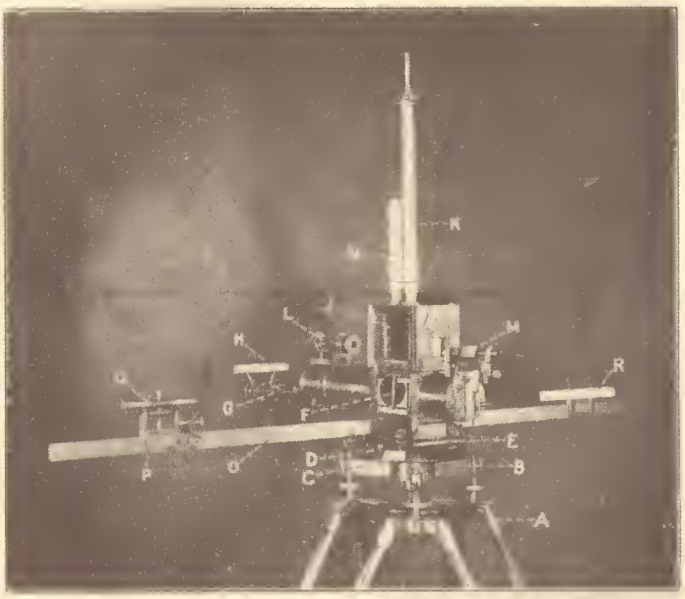


Plate B*

The general appearance of the unifilar magnetometer is shown in Plate B*.

The instrument is mounted on a tripod A on the head of which the footscrews of the tribrach B rest. The lower plate C is secured to the tribrach, has a horizontal engraved circle similar to that of a theodolite, and carries the axis of the upper plate D to which the body of the instrument is secured. Clamping and slow motion screws are fitted to the upper plate so that the instrument can be pointed in any required direction and horizontal angles are read by means of verniers with an accuracy of about 10″. Levelling of the plates is effected with the aid of the bubble tube E.

The body of the instrument consists of a hollow tube having a box-shaped compartment F at its centre, and at one end of this tube a telescope G, carrying an ivory scale H, can be attached. A suspension tube, similar to that illustrated but shorter, can be mounted on top of the compartment F and from the torsion head at the top of this tube a mirror magnet can be suspended to hang in F. The mirror magnet (not shown) is tubular in shape and carries beneath it a

mirror which enables the observer to view the scale H. This mirror is fixed at right angles to the axis of the magnet and the latter should therefore be parallel to the collimation axis of the telescope when the diaphragm cross-wires are seen to intersect the zero or centre mark on the scale.

A second box-shaped compartment J, carrying the suspension tube K, can be mounted on top of F to enable a collimator magnet to be suspended in J where it is viewed by another telescope L. The collimator magnet is a hollow cylinder in one end of which (usually the north end) is inserted a cell holding a lens and in the other a cell containing a fine scale on glass. The azimuth mirror M can be adjusted so that, with the axis of the instrument in the magnetic meridian, light is reflected through the magnet, and the scale, which is at the principal focus of the lens, is read by the telescope focussed for infinity. Damping devices are fitted in F and J to control the oscillations of the magnets, and a thermometer N is attached for temperature readings.

A brass bar O, graduated from its centre in centimetres, can be mounted on the upper plate and supports a carrier P, in which the collimator magnet Q is mounted for certain observations, and a thermometer R. The bar, when in position, is at right angles to the telescope axes.

The torsion heads are designed to enable the respective magnets to be raised or lowered to their correct positions in the compartments F and J and to allow the suspension to be turned through any required angle. The movable part of the torsion head carries an index and the fixed part is graduated so that an angle of twist can be measured. The suspension usually consists of a very fine gauge wire (about 0·03 mm. in diameter), but a tungsten filament is now sometimes used.

Other items of the equipment consist of non-magnetic plummets (of equal weight to the collimator and mirror magnets), which can be attached to the suspension for removing torsion, and an inertia cylinder which is used for finding the moment of inertia of the collimator magnet (above which it can be fitted). A sighting tube for use on the carrier P is also provided.

The instrument generally must be treated with great care and, in particular, the magnets and the inertia bar. The magnets are fitted with stirrups and should only be handled by these ; if they are dropped or mishandled, their magnetic moments may be altered and the accuracy of the observations will be vitiated. After use, they should be wiped over very gently with chamois leather or soft cambric. The inertia bar should be kept wrapped up in tissue paper when not in use. The suspensions also require very careful treatment if breakages are to be avoided.

To find H (the horizontal component of the earth's magnetism).

The observation is divided into two parts :

(1) The time of a vibration of the collimator magnet is accurately obtained.

(2) The deflection of the mirror magnet is observed when acting under the influence of the collimator magnet placed on the bar at specified distances from it.

In addition, the effect of 90° torsion on the scale reading is observed.

From (1), $m \times H$ is obtained
,, (2), $\frac{m}{H}$,, ,,

} Where m = the magnetic moment of the magnet.

Part (1) of the observation is repeated after part (2) so that a mean value of $m \times H$ can be found at a time co-incident with that at which $\frac{m}{H}$ is obtained.

Part (1)*—The Vibration Experiment.*

The instrument must be completely protected from sun and wind and should be set up in a hut or tent erected on a site as free as possible from magnetic

matter (see page 429). All magnetic objects, keys, knives, etc., must be removed to a safe distance. Having placed the circle on the tripod, level the instrument, attach the telescope L and the box J with the long suspension tube K and thermometer N. Attach the plummet, equal in weight to the collimator magnet, to the suspension and take out torsion by sighting it through the telescope and turning the torsion head until the swing of the plummet is equalised on each side of the vertical cross-wire. Then remove the plummet, suspend the collimator magnet in its place and rotate the instrument until it is approximately in the magnetic meridian, which will be the case when the centre division of the scale is seen to oscillate equally on either side of the vertical cross-wire. The collimator magnet must be adjusted to the correct height so that its axis is in line with that of the telescope by operating the rack and pinion gear at the head of the suspension tube. The easiest way of telling when the magnet is at the right height is by unshipping the eyepiece of the telescope and looking through the tube.

Focus the telescope and close up the box J. Then release the damping device carefully and start the magnet swinging by means of a small piece of iron such as a screw-driver or steel pen. Centralise the swing of the magnet carefully by means of the horizontal plate tangent screw so that it swings an equal number of divisions on either side of the vertical cross-wire. The amplitude of the swing should be adjusted to about 30–40 divisions on either side of the centre line, *i.e.*, equivalent to about 45′–60′ of arc, and then 120 semi-arc vibrations should be timed. If a loud-ticking chronometer is available, timing can be done single-handed; if only a watch is available, an assistant is required. To save the trouble of counting during the whole period of the 120 vibrations which occupy a considerable time, the method usually employed is as follows:

With the scale moving apparently to the right (or left), note the time (by estimation to 1/10th of a second) of the centre division passing the vertical cross-wire; this is the zero observation. Note the time of nine more swings after this with the scale moving in the *same* direction, numbering these 2, 4, 6, etc., to correspond to the numbers of the *semi-arc* vibrations. From the elapsed time between the zero observation and that numbered 18, the approximate period of one semi-arc vibration is found and the chronometer time (to the nearest half second) of observation No. 100 is computed. This will enable the observer to identify vibration No. 100 and to time a similar series of ten swings, Nos. 100-118, as before. A chronograph, if available, may be used with advantage for this observation.

It should be noted that the time of a vibration will be very nearly the same each day at the same place, but will vary in different parts of the world. (At Abinger Magnetic Observatory the time of the double swing is about 8½ seconds.) Temperatures should be noted at the beginning and end of the experiment.

Part (2)—*The Deflection Experiment.*

Remove the collimator magnet and box and attach the shorter suspension tube. Suspend the plummet equal in weight to the mirror magnet and take out torsion as before. Attach the telescope G fitted with the ivory scale and the graduated bar with its thermometer and magnet carrier. Suspend the mirror magnet in the box F and adjust it to the correct height by sighting along the sighting tube resting on the magnet carrier. Close the doors of F and remove all magnetic substances, including the collimator magnet, to a safe distance. Looking through the telescope G, damp down the oscillations of the mirror magnet to (at the most) three divisions on the scale and by means of the horizontal plate tangent screw turn the instrument until the centre scale division is seen to oscillate equally on either side of the vertical cross-wire of the telescope: read the horizontal circle.

The deflection of the mirror magnet is now measured when the collimator

magnet is placed in the carrier at distances of 22·5, 30·0 and 40·0 cms. on the graduated bar. Placing the collimator magnet in the carrier at the first position, turn the instrument until the centre scale division is again seen to oscillate equally on either side of the vertical cross-wire, read the horizontal circle and temperature. Move the carrier to the next position and repeat the observation. It is advisable to hold the mirror magnet steady with the damping device before making any movement of the collimator magnet or instrument. A complete programme with the collimator magnet in position involves twelve observations which may conveniently be taken as follows :

(i) Collimator magnet on *east* side of bar, north pole of magnet *east*, at distances of 22·5, 30, 40 cms.

(ii) Collimator magnet on *east* side of bar, north pole of magnet *west*, at distances of 40, 30, 22·5 cms.

(iii) and (iv) Similarly with magnet on *west* side of bar.

Temperatures should be taken at every reading. Finally, the first observation should be repeated, *i.e.* with the collimator magnet removed altogether.

This completes the deflection experiment and the vibration experiment should now be repeated.

Observation for Torsion of Suspension.

The torsion effect of the collimator magnet suspension must now be observed as follows. Suspend the collimator magnet as in experiment (1), bring it as nearly as possible to rest with the scale roughly centralised in the field of view and read off the scale division : call this *a*. Turn the torsion head 90° clockwise and read off the scale division again : call this *b*. Turn the head back 90°, read again and call it *c*. Similarly obtain readings *d* and *e* with the head turned 90° anti-clockwise and back to the central position. Then the mean of $a \sim b$ and $b \sim c$ will give the effect of 90° torsion in a clockwise direction and the mean of $c \sim d$ and $d \sim e$ the effect of 90° torsion in an anti-clockwise direction. During the torsion experiment it is not necessary for the magnet to be entirely at rest ; if it is not oscillating more than three divisions, the mean reading can be estimated with great accuracy. The value of a scale division is normally determined at a magnetic observatory and should be stated on the certificate supplied with the magnetometer so that the torsion effect in scale divisions can be converted at once into minutes of arc.

Computation.

The computation of the results is greatly facilitated if use is made of form 162, reproduced on pages 436 and 437 (and obtainable from the Royal Observatory, Greenwich). Every magnetometer should have a certificate in which the following data, determined at a magnetic observatory, is recorded :

(*a*) Angular values of the scale divisions.
(*b*) Temperature co-efficients and corrections.
(*c*) Induction co-efficients.
(*d*) Moment of inertia of the collimator magnet.
(*e*) True values of the distances graduated on the bar.

The observed readings are recorded in the upper half of the form and an approximate value of Log $\frac{m}{H}$ is first calculated. From this the values of A_1, A_2, A_3 are found (in the lower right-hand part of the form) and P and Q are obtained, enabling an accurate value of Log $\frac{m}{H}$ to be found. The value of T^2 is found in the lower left-hand part of the form and hence the value of Log $(m \times H)$. Thus m and H can be computed. The method of computation and the meaning of most of the symbols is evident from a perusal of form 162, but the following notes may be of assistance :

(1) The " Effect of 90° of Torsion " must be entered as minutes of arc, *i.e.* the mean effect in scale divisions multiplied by the angular value of one division.

(2) The term $1+\tau$ is the magnetic force equivalent to the torsion, *i.e.*, if d'=effect of 90° torsion in minutes of arc, $1+\tau=1+\frac{d'}{90°-d'}$

(3) The values of c, the temperature corrections, should be given in the certificate. (See (*b*) above.)

(4) K is the moment of inertia of the collimator magnet, including its fittings ; the value of K or π^2 K should be given in the certificate. (See (*d*) above.)

Moment of Inertia.—The moment of inertia of the collimator magnet is usually determined at a magnetic observatory, but it may be found in the field, if necessary, with the aid of the inertia cylinder which is inserted in the stirrup above the collimator magnet. With the magnet suspended from the torsion head, a series of vibrations, say 50, is timed as in the vibration experiment, and a further series is timed with the inertia cylinder removed. The method of computing the moment of inertia is shown on the back of form 162.

To find Variation (Declination of the Compass Needle).

Set up the magnetometer as for the vibration experiment, taking special care to remove torsion with the aid of the plummet before suspending the collimator magnet. Remove the azimuth mirror and with the collimator magnet lowered clear of the line of sight, focus the telescope, and intersect a distant object whose azimuth is known ; read the horizontal circle. Raise the magnet and turn the instrument until it is in the magnetic meridian, *i.e.*, with the centre scale division on the vertical cross-wire or oscillating equally a small amount either side of it ; read the horizontal circle. The focus of the telescope must not be altered at any time during the observation. Then reverse the magnet in its stirrup and, after bringing the centre scale division into co-incidence with the vertical cross-wire, read the horizontal circle again. Lower the magnet and take another reading on the distant object. Then carry out the torsion experiment as before. The variation (declination) is found from the difference between the mean of the circle readings of the distant object and of the magnetic meridian, applying a correction equal to half the value of the torsion effect, if it is appreciable. The observations should be repeated A.M. and P.M., and a mean result accepted.

Barrow's Dip Circle

This instrument is used primarily for the measurement of Magnetic Dip, but it can also be used for determining the total force. Its general appearance is shown in Plate C*.

The instrument is mounted on a tripod, and footscrews and a bubble tube are provided for levelling. The tribrach carries a horizontal circle on which is mounted the body of the instrument. This consists of a box-shaped compartment in which the dipping needle is mounted, and a vertical circle read by two verniers. Attached to the arms carrying the verniers are two microscopes fitted with cross-wires which can be aligned with the ends of the dipping needle, the near side of the box being made of clear glass to permit this being done. Two dipping needles, numbered 1 and 2 and with their ends marked A and B, are used for the dip measurements ; these are temporarily magnetised for the observations by being placed in the wooden holder, shown on top of the box in Plate C*, and stroked with bar magnets. Two other needles, numbered 3 and 4, *permanently* magnetised, are provided for total force observations ; these should on no account be stroked with the bar magnets. No. 3 is used in

OBSERVATIONS OF VIBRATION OF DEFLECTING MAGNET

Before Deflection Observations.	After Deflection Observations.
Semiarc of Vibration. At Commencement, α= At End .. .. α'=	Semiarc of Vibration. At Commencement, α = At End .. .. α'=
Temperature (t_1) °.	Temperature (t_3) °.

$\frac{t_1+t_3}{2}$= °. (Argument for *c*.)

	Time Noted. h m s	Means of Groups. h m s	Time Noted. h m s	Means of Groups. h m s
0				
2				
4				
6				
8		. . ,		. . ,
10				
12				
14				
16				
18				
100				
102				
104				
106				
108		. . ,		. . ,
110				
112				
114				
116				
118				
	Difference ÷ 100 =	.	Difference ÷ 100 =	.
	Time of Vibration M.S. secs. =	.	Time of Vibration M.S. secs. =	.

Effect of 90° Torsion.

Torsion Circle. °	Scale Reading. d	Effect. d
=	. .	= .
=	.	= .
=	.	= .
=	.	= .
=	.	= .
	Mean =	. =
		s

Daily gaining Rate of Chronometer = s = − ,
1 + Correction for daily Rate = + ,
Correction for Semiarc of Vibration = − ,
Sum = $\left(1-\frac{s}{86400}-\frac{\alpha\,\alpha'}{16}\right)$ = ,

Log. μ = 0 , 8 6 1 5 3
Log. $\frac{m}{H}$ = ,
Log. $\left(\mu \div \frac{m}{H}\right)$ = ,
$\mu \div \frac{m}{H}$ = ,

Effect of 90° of Torsion = ,
1 + τ = + ,
− c = − ,
. . + $\mu \div \frac{m}{H}$ = + ,
,

Mean of Time Vibration (T_0) = ,
Log. T_0 = ,
. . . Its Log. = ,
(Sum)
Log. T_1 = ,
× 2
Log. T_1^2 = ,
. . . . Its Log. = ,
Log. T^2 = ,
Log. π^2 K = ,
Log. $\frac{\pi^2 K}{T^2}$ = Log. m H = ,
Log. H = ,
Sum = Log. m^2 = ,
Log. m = ,
m = ,

Computation of P and Q.

Pα = 2664 Log. $\frac{A_1}{A_2}$ = , ; Pβ = 4737 Log. $\frac{A_2}{A_3}$ = +
Pα − Pβ = − ,
(Pα − Pβ) × 1·286 = − ,
P = Difference = − ,
(Pα − Pβ) × 740·6 = −
$\frac{P^2}{2}$ = +
Q = Sum = −

OBSERVATIONS OF DEFLECTION OF DEFLECTED MAGNET

G.M.T. of Observation. 19

Deflecting Magnet.				Distant 22·5 cm.	Distant 30·0 cm.	Distant 40·0 cm.
Reference.	Position	Direction of North-seeking Pole.	Temperature.	Circle Reading.	Circle Reading.	Circle Reading.
			°	° ′ ″	° ′ ″	° ′ ″
a	E h m (·)	E	,	· ·	· ·	· ·
b	W h m (·)	E	,	· ·	· ·	· ·
c	E	W	,	· ·	· ·	· ·
d	W	W	,	· ·	· ·	· ·
e	$= \frac{a+b}{2}$		,	· ·	· ·	· ·
f	$= \frac{c+d}{2}$		,	· ·	· ·	· ·
	$e - f$		,	· ·	· ·	· ·
u	$= \frac{e-f}{2}$		,	u_1 · ·	u_2 · ·	u_3 · ·
Mean Temperature (t_2). Argument for *c*			· ,			

Circle Reading, Magnet undeflected : Before experiment .. ° ′ ″

After experiment .. · ·

	r_1=22·5 cm.	r_2=30·0 cm.	r_3=40·0 cm.
1 + Induction Correction $= 1 + \frac{2\mu}{r^3} =$	1,00128	1,00054	1,00023
Temperature Correction $= + c =$	,	,	,
$1 + \frac{2\mu}{r^3} + c =$	,	,	,
Log. $\frac{1}{2} r^3 =$	,	,	,
Log. sin $u =$	,	,	,
Approx. Log. $\frac{m}{H} =$	,	,	,
+ Log. $\left(1 + \frac{2\mu}{r^3} + c\right) =$	,	,	,
Log. A =	,	,	,
− Log. $\left(1 + \frac{P}{r^2} + \frac{Q}{r^4}\right) =$	,	,	,
Log. $\frac{m}{H}$ =	,	,	,

Mean=Adopted Log. $\frac{m}{H}$ = ,

Log. m H = ,

Log. H^2 = ,

Log. H = ,

H, in C.G.S. Measure= ,

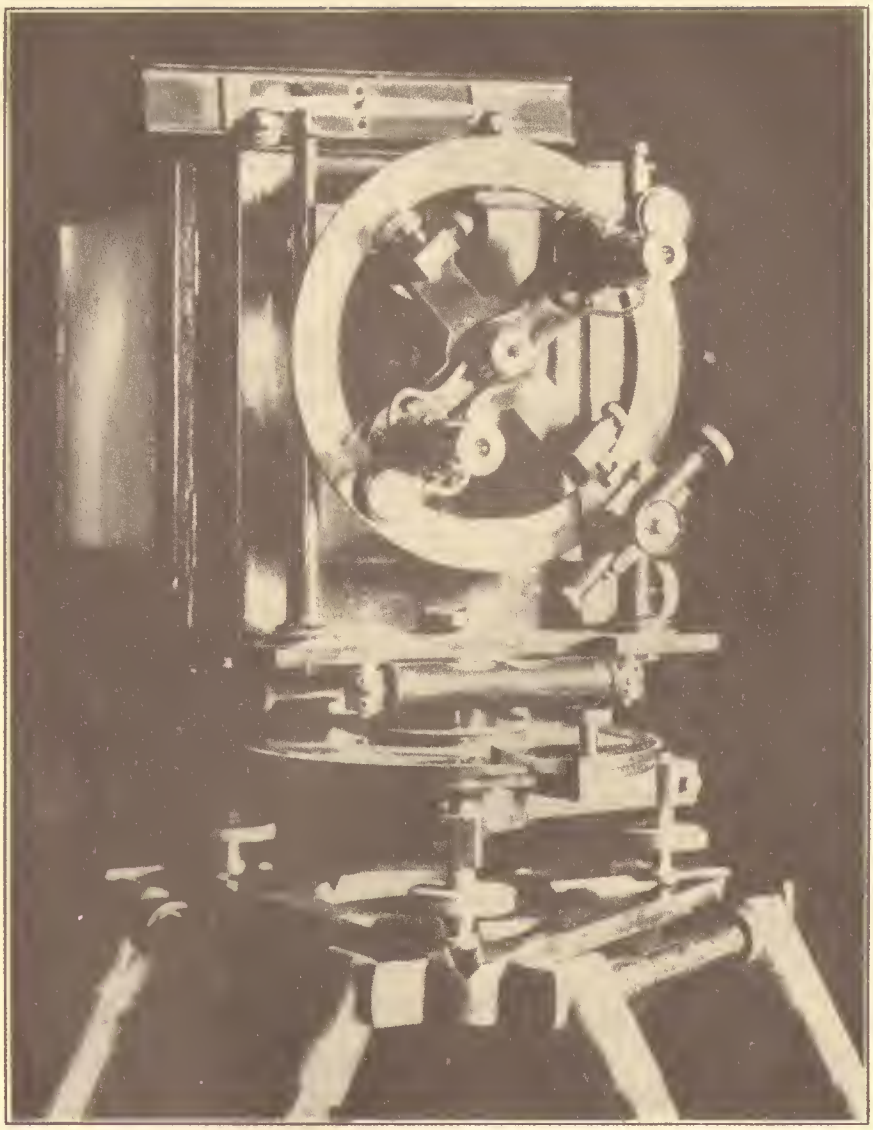


PLATE C*

the box on the same support as the two dipping needles and No 4 can be mounted in the frame carrying the microscopes but at right angles to them.

To find the Dip.—Set up and level the instrument in a sheltered position and remove all magnetic material to a safe distance. Take one of the dipping needles, magnetise it in the holder ; clean its pivots and the agate supports by pushing into or wiping with pith or cork. Place the needle on the supports with the inscribed letters facing the microscopes and lower it onto the knife-edges. Set the verniers of the vertical circle to read 90° and turn the instrument until one point of the needle is intersected by the microscope cross-wires and read the horizontal circle ; then turn the instrument (if necessary) and make intersection of the other point of the needle with the second microscope, and read the horizontal circle again. Turn the instrument through 180° and repeat the process. Then the mean of the four readings will be the division of the

horizontal circle to which the vernier should be set in order that the plane of the vertical circle should be *at right angles* to the magnetic meridian. Turn the instrument 90° so that it is in the magnetic meridian, set the cross-wires coincident with first one needle and then the other and note the vertical circle readings. *Caution.*—The bar magnets provided for magnetising and reversing the polarity must be well away from the instrument when the observations are being made.

To eliminate errors, readings are repeated with every possible combination, *i.e.*, with the instrument facing east and west and the needle facing east and west, and the series is repeated with the polarity of the needle reversed so that the other end is dipping. The complete series involves 16 vertical circle readings of each needle and the mean is the Dip.

Stops are provided on the horizontal circle to facilitate turning the instrument onto any required setting. Care should be taken to see that the needle does not stick at any time and it should be made to oscillate slightly by tapping on the roof of the box.

To find Total Force.—Having completed the dip observations and without moving the instrument, place No. 3 needle on the agate supports and attach No. 4 needle to the supports between the microscopes. Intersect the ends of No. 3 and obtain the vertical circle readings ; the mean will be the inclination of No. 3 to the horizon. Repeat this observation with the north end of No. 4 turned in the opposite direction by the revolution of the movable arms which carry the microscopes ; half the difference of the readings in the two positions is the angle of deflection u'.

Remove No. 3 needle, substitute No. 4 (which is loaded at one end) on the agate supports and measure its inclination to the horizon, η, in the four positions of the needle and circle. The deviation of this needle from the position due to the earth's magnetic force alone is $u=\theta-\eta$, θ being the angle of dip and η being positive when measured on the same side of the horizontal line with θ, and negative in the contrary case.

The value of the force is given by the formula :

$$R = A\sqrt{\frac{\cos\eta}{\sin u \sin u'}} \quad \text{where } A = \frac{H}{\cos\theta}\sqrt{\frac{\sin u \sin u'}{\cos\eta}}$$ as observed at a base station, at which H (the horizontal component) has been determined with the unifilar magnetometer, and the dip has also been observed. The base station where A is determined should not be too far from the observation spot.

CHAPTER XXI

OCEANOGRAPHICAL OBSERVATIONS

Nature of Observations.

Oceanographical observations are of practical value and scientific interest and often form a part of the hydrographic surveyor's work. The object of such observations is to determine various properties of sea water in different parts of the ocean, the most important being :

(1) The temperature.
(2) The salinity.
(3) The hydrogen-ion concentration.

It is obvious that the temperature observations must be made *in situ* and special types of thermometer, on which readings can be made to 0·02° Centigrade, are provided for the purpose. Salinity and hydrogen-ion concentration may also be measured at the time a water sample is obtained if special apparatus (which will not be described here) is provided, but it is more usual to bottle off water samples and send them for investigation to a fully equipped laboratory on shore.

Water samples and temperatures are required at various depths from the surface to the bottom and suitable equipment for this purpose is essential. Samples from the surface can be obtained in a wooden bucket, but at all other depths (and, preferably, for surface samples also) use is made of metal bottles which are lowered in an " open " state to the required depth and can be closed before being hauled up by means of a " messenger " despatched down the supporting wire. Two types of water bottle in general use are described below.

Insulating Water Bottle.—This is used for measuring temperatures and collecting samples of water at moderate depths, *i.e.*, from the surface down to a maximum of about 400 metres (220 fathoms, approx). The general appearance of the insulating bottle is shown in Plate D*.

The body of the instrument, consisting of a series of concentric cylinders A, slides freely on the tubular guides BB to which it is linked by holes in the top and bottom flanges PP. The guides are rigidly connected at the bottom by the cover-plate C, which carries a draw-off valve V and a number of rubber washers D, and at the top by the plate E.

The upper cover-plate F, provided with rubber washers O, slides freely on the guides BB and carries a thermometer guard G which is slotted so that the graduations of the thermometer N can be seen. G is attached to F by a screw-collar M and its upper end is spherical and shaped to engage in a catch (not visible) inside the head K. Springs in the guides BB press downwards on the sleeves HH and, when the catch is released, these force the upper cover plate F against the body A and the latter against the lower cover-plate C, thereby closing the bottle. The springs may be put out of action by lifting the sleeves HH until they butt against E and then giving them a half-turn.

The thermometer, which is divided in tenths of a degree centigrade and can be read by estimation to about 0·02°, has a long narrow bulb and is sealed up in a strong glass sheath to protect it against pressure. When the instrument is in use, the thermometer is kept permanently fixed in F with its bulb projecting some distance below it. A thick rubber washer O, through which the thermometer is pushed, makes a water-tight joint in F and the guard G can be unshipped when it is required to insert or remove it.

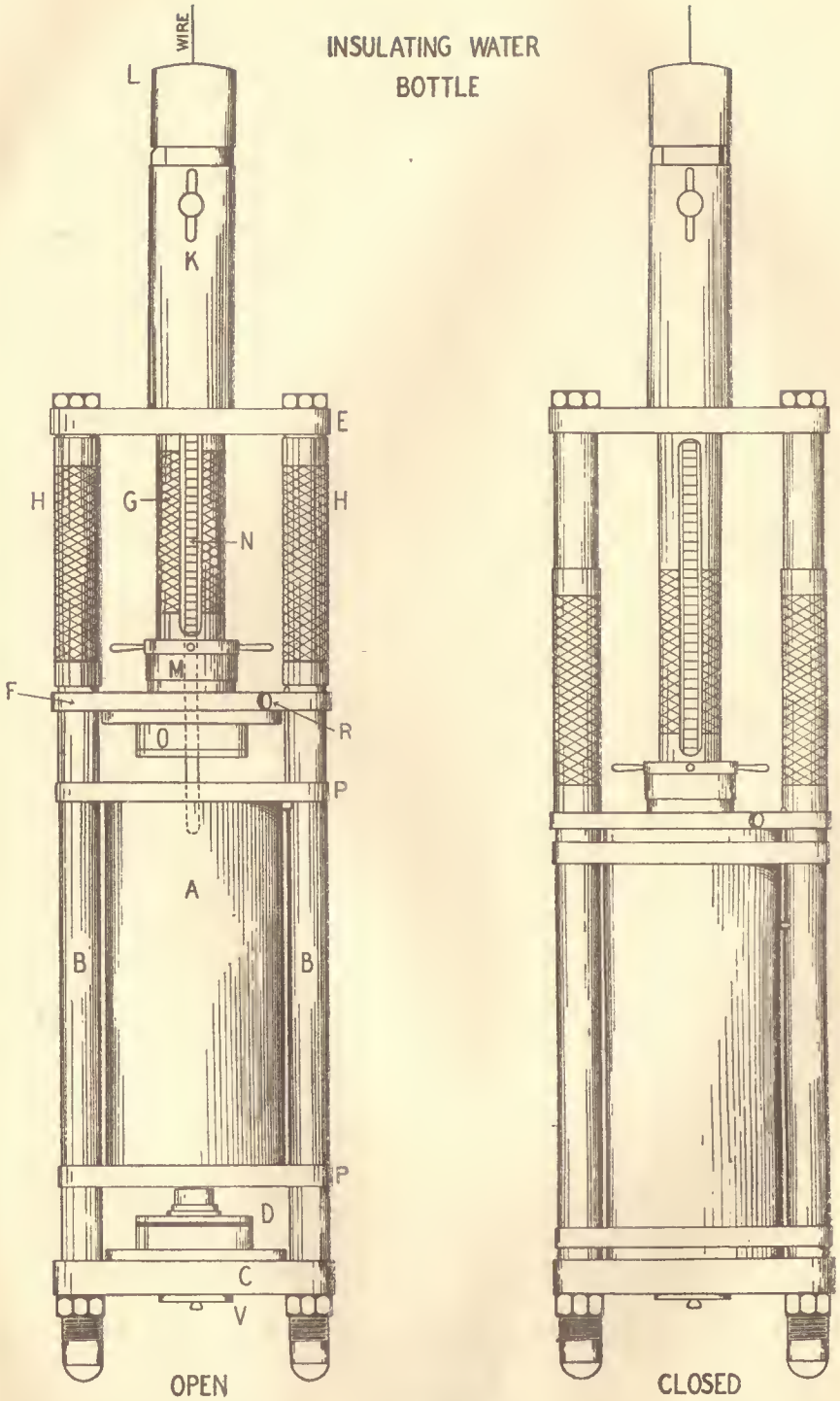

INSULATING WATER BOTTLE
WIRE
L
K
E
H
G
H
N
M
F
O
R
O
P
A
B
B
P
D
C
V
OPEN
CLOSED

PLATE D*

The bottle is conveniently lowered and hove in by the oceanographical winch (if fitted), using 6×7 strand, plough-steel wire cord, " True-lay " formation of 4 mm. diam.* ; but in view of the comparatively shallow depths in which this type of bottle is used, Kelvin sounding wire (rove on its own machine or on a hand-operated Lucas sounding machine) is equally suitable. The wire is led outboard over a measuring sheave (usually with a metre-reading dial) fitted at the head of a small davit or boom to keep it clear of the ship's side. An eye-splice round a small thimble must be worked in the outboard end of the wire to take over a screw pin in the head K ; care must be taken to see that the ends of the splice do not interfere with the free movement of the " messenger " which is used for dropping down the wire to close the bottle.

To prepare the bottle for lowering, the sleeves HH are raised against the springs and given a half-turn to lock them, and the guard G with the upper cover-plate F is lifted until the catch in the head of G is felt to engage. The sleeves are then turned back until they bear down again on F. The air valve R is now closed and the working of the draw-off valve V should be tested to see that it is not sticking. The bottle is then lowered to the surface and the measuring sheave dial set to zero.

When using the bottle, the ship must heave to or anchor ; if the former, every effort must be made to manoeuvre in such a way that the wire is kept vertical whilst the bottle is over the side. When the bottle has been lowered to the required depth, it is closed by means of a " messenger " which is hooked onto the wire and allowed to slide down. The messenger strikes the top of the instrument L, releases the catch and allows the springs to close the bottle. Before the bottle is closed a sufficient interval must be allowed to elapse to enable it to acquire the temperature of the surrounding water ; in depths of 100 metres or less, the messenger should be let go after an interval of a minute or two, but at greater depths the time taken for the messenger to fall will provide a sufficient interval and no waiting is necessary. The impact of the messenger on the bottle can be felt by holding a finger on the wire and, when this occurs, the instrument should be hove up as rapidly as possible and the temperature read as soon as the bottle can be brought inboard. The insulating system is sufficiently good to keep temperature changes under 0·01° Centigrade for a period of six minutes and this interval must not be exceeded. A small additive correction to the thermometer temperature is required to make allowance for the cooling of the water sample and bottle due to decompression. This correction varies with the depth and the temperature of the sample and will be found in Table XXII.

A water sample is drawn off by easing off the air valve R and pressing the sample bottle upwards with its head under the draw-off valve V. The sample bottle should be well rinsed with water from the insulating bottle before it is finally filled and corked.

The following points should be noted when using the insulating water bottle :

(1) Do *not* allow the bottle to close violently in air ; the shock may break the thermometer.

(2) Do *not* lower the bottle closed as it may collapse under pressure.

(3) When working from the ship at anchor in a strong current, the frame may be lengthened downwards by removing the lock-nuts at the bottom and fixing on metal extension pieces and a sinker which will help to keep the wire vertical.

(4) The working parts should be lightly oiled, but no oil or grease should be allowed to get onto the washers or inside the cylinders.

(5) When the bottle is not in use, the springs should be thrown out of action.

(6) The water-tightness of the bottle when closed should be tested periodically

* Weight, 3·7 lbs. per 100 ft.; breaking strain, 2,200 lbs.

by filling it and pressing up on the draw-off valve with the air valve shut ; no water should pass.

(7) Surface observations are better obtained with the bottle than by drawing up water in a bucket.

(8) The special thermometer should be used, not a " surface " thermometer which may collapse under pressure.

(9) The standard depths for observations with the insulating water bottle are surface, 5, 10, 20, 30, 50, 75, 100, 150, 200 and 300 metres.

Reversing Water Bottle.—In depths of more than 400 metres (about 220 fathoms) use is made of a reversing type of water bottle to which are attached thermometers of a special pattern. The appearance of a water bottle of this type is shown in Plate E*.

A is an open-ended metal cylinder pivoted at its centre in a rectangular frame B to which the lowering wire is secured by means of a screw clamp at the top and a guide near the bottom. The cylinder is fitted with two thermometer guards C and carries guides DDDD through which slide rods EE attached to the cover-plates FF. The latter are connected together by a strong spring which passes through the centre of the cylinder and are fitted with rubber washers so that, when closed, they make the cylinder watertight. The inner ends of the rods EE bear on cams GG which are secured to the frame and the bottle is " cocked," *i.e.*, put in the open position ready for lowering, by turning it in the frame until the drain tap (H) end is uppermost and the lug J engages in a toe fitted inside the top of the frame. In the act of cocking the bottle, the cams press outwards on the rods EE and push the cover-plates open.

The bottle is lowered to the required depth in this position, *i.e.* with the drain-tap end uppermost. It is closed by allowing a " messenger " K to slide down the wire and strike the lever L, thereby releasing J, and the spring, acting through EE on the cams, causes the bottle to turn upside down. The cover-plates then close and are held on by the spring and J engages in a second toe near the lower end of the frame to hold the bottle in the reversed position. When the bottle is cocked, an extension piece on one of the rods EE engages in the slot M and enables another messenger to be held by a small becket ; this will be automatically released when the bottle is closed, allowing the messenger to slide down the wire to operate a bottle secured at a greater depth. An air valve (not visible in the plate) is fitted at the opposite end to the drain tap so that a water sample can be drawn off into a sample bottle.

This type of bottle should always be operated from the oceanographical winch (or from a power-worked drum) rove with 4 mm. diam. plough-steel wire (see page 442). It is convenient to lead the wire outboard in the fore part of the ship over a measuring sheave secured at the head of a davit or boom ; and some arrangement should be fitted to take in the slack of the wire automatically when the ship is rolling or pitching. A lead of about 28 lb. on a few fathoms of hemp stray line should be bent on to the outboard end of the wire where the lowest bottle will be secured. As many as six bottle have been used simultaneously, but this involves a heavy strain on the wire, and a maximum of four is advisable. In great depths, *e.g.*, over 3,000 metres (1,640 fathoms), where the weight of the outboard wire itself is considerable, not more than two or three bottles should be used at a time.

Before first using the gear, it is advisable to test the action of the bottles by lowering each in turn to a depth of a few feet and noting if they close properly when the messenger is dropped. Cases have occurred where it has been found necessary to add sheet lead to the drain-tap end of the bottle to make it reverse completely.

The reversing water bottle is normally used in depths which do not permit anchoring and the ship should therefore heave to with the wind slightly on the bow on which the gear is rigged. Whilst the operation is in progress the ship

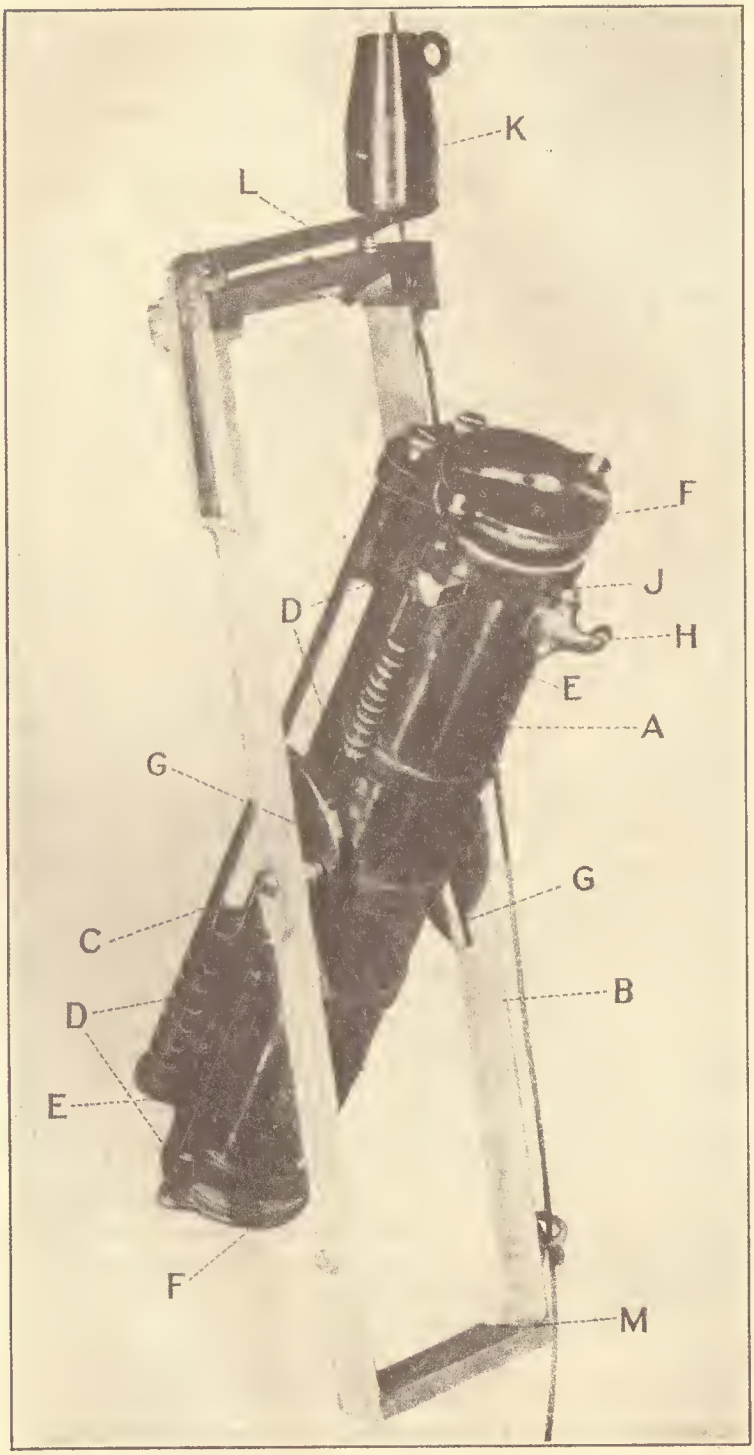


PLATE E*

must manoeuvre to keep the wire as nearly vertical as possible. Suppose that water samples and temperatures are required at depths of 3,000, 2,000 and 1,500 metres. Having lowered the sinker and stray line into the water, the lowest bottle is attached to the wire and the measuring sheave dial is set to zero when it is awash. The drum can now be unclutched and the wire allowed to run out until the dial shows 1,000 metres when the brake is applied and the second bottle, with a messenger tailed on, is attached. Similarly, the third bottle is attached when the dial reads 1,500 metres, after which the wire is allowed to run out until the dial reads 3,000. A messenger for closing the upper bottle may be let go after an interval which will ensure that the upper bottle has been at the required depth for not less than seven minutes before it is closed. Except at moderate depths, say, 1,000 metres or less, it is rarely possible to feel any indication on the wire when a bottle is closed, and the only method of procedure is to allow ample time for all the messengers to fall. The rate of fall depends on the shape of the messenger and will generally be about 300 to 400 metres a minute, so that in this example it will be advisable to allow an interval of about 15 minutes, to be on the safe side, before heaving in is commenced. A test run with a bottle lowered to, say, 500 metres (where it can be felt to close) will provide valuable data for estimating the necessary time intervals at greater depths. In heaving in, the motor must be stopped as each bottle reaches the upper deck level ; the bottle can then be detached and the thermometers should be read at once. A water sample may then be drawn off at leisure whilst heaving in is continued.

Measuring sheaves.—The accuracy of these should be tested by comparing soundings taken with them in fairly deep water with those taken by Echo sounding gear or a Lucas machine.

Reversing Thermometers.—These are used with the reversing water bottle for recording temperature at the required depth and they function by separating the mercury which stands above a certain point on the stem at that depth so that it can be read on a special scale after hauling up. The bulb is long and narrow ; above it there is an irregularity in the bore of the capillary, which may be either a simple contraction or " knife-edge," or more commonly a short branch ending blindly, the " appendix." Above this again there is an enlargement bent into an S-curve, or U-curve, or into a complete circle. A small thermometer of the common pattern is sealed up together with the reversing thermometer in a strong glass sheath, and the space between the main bulb and the sheath contains mercury to hasten the interchange of temperature.

With the bottle in the cocked position ready for lowering, two thermometers are placed in the guards with their main bulbs downward. When the bottle is closed, the thermometer is reversed with it and the mercury breaks at the appendix or knife-edge and runs to the other end of the capillary, where the temperature can be read off the scale which is graduated for the reversed position. As a rule the thread broken off will have expanded after reversal and the auxiliary thermometer must be read at the same time as the main thermometer in order that a correction for the expansion may be calculated.

CHAPTER XXII

CHART PRODUCTION FROM SURVEYS

Cartography.

The object of all hydrographic surveying is to assist the practice and increase the safety of navigation and, for that end, the surveyors' work must be translated into a navigational chart. A chart differs from a land map since it deals largely with things invisible to the eye, deeps and shoals that are concealed under the surface of the sea, whereas all the conditions and objects represented on a map may be verified by sight as they come into view. Hence the prime essential of a chart is accuracy of relative positions, lacking which it is worse than useless. A chart therefore is analogous to a scientific instrument that has been correctly designed and calibrated and is not merely a pictorial expression of natural objects. Whereas it is the business of the surveyor to record on his fair chart every detail that the limitations of drawing will allow, on the navigational chart a rigorous selection must ensure that while no necessary sounding or feature is omitted, there shall be no crowding of matter to obscure the legibility. Objects on sea and land must be represented with distinction and arranged with clarity, so that the primary essentials of navigation will be obvious but secondary needs will not be hidden. Also, the framework of the survey must be fixed into a suitable graduation with positions precisely related to points from other work. These are the business and science of cartography.

Planning of new charts.

In its inception, the chart precedes the survey, for it is designed solely with reference to the needs of navigation in a certain area ; if suitable materials are available the chart may be produced from them forthwith, but if they are either lacking or unreliable in whole or in part then the chart must wait until the surveyor can supply them. Two main factors control the scheming of a new chart ; the natural scale, and the size of the paper upon which the chart is to be printed. Experience has shown that it is advantageous to have as little variety as possible among the natural scales upon which charts are constructed and, whenever circumstances permit, charts are designed on certain standard natural scales ; as a matter of convenience, these are laid down as simple multiples or subdivisions of the proportion 1 : 100,000. The double-elephant sheet, giving a chart approximately 38 ins. by 25 ins., is usually regarded as the most suitable size for navigational charts. The detailed scheme of a new chart, once the area to be covered and the natural scale have been decided, must include suitable features near its borders for use as departure points and it must link up, by convenient overlaps, with adjacent charts (these conditions, however, do not apply to harbour plans). For these reasons it is customary to scheme charts in a complete and interlocking sequence for areas of seas or coasts that have well-defined physical or national limits ; so that, although immediate production may be contemplated for only one or two in such a series, it is assured that any charts adjacent to these, which may later become necessary, shall connect satisfactorily with those already published. In unknown waters it is not possible, of course, to scheme charts in any detail until the surveys are made ; whilst in more familiar regions, various factors may necessitate that the limits within which a survey is ordered do not correspond with the limits of the proposed navigational chart.

Data for constructing navigational charts

The materials from which the cartographer has to produce a chart consist of one or more fair charts drawn by the surveyor, with the relevant triangulation data and observations for positions, the original sailing directions, views and tidal data; frequently, earlier data of a similar though perhaps less precise nature must be consulted.

Graduation.

When the scheme is decided, the graduation into which the chart is to be placed can be set up. The projections normally used in hydrographic surveys have been discussed in Chapter VIII. Admiralty practice is to use the Mercator projection for navigational charts on scales of 1/50,000 and smaller, except in the case of those embracing the polar regions or those designed for plotting great circle tracks and wireless bearings, when the gnomonic projection is employed. Charts on scales larger than (and sometimes including) 1/50,000 are drawn on either Cassini's, Transverse Mercator or the gnomonic projections. Before any point can be plotted, it is necessary to compare the surveyor's observed position with positions that have been established previously in the vicinity and which, unlike modern atrolabe determinations, may depend on some local secondary meridian. Should the new chart be designed to take a place among existing charts of a particular area it may be necessary to shift the new survey bodily to the older positions and to record the later observations by means of a note on the chart which also defines the origin of the charted longitudes (latitudes are seldom in question).

Combining new and old surveys.

Into this graduation a framework of firmly connected points is plotted upon which the survey is to be laid down; the points of this framework are obtained from geographical positions given by the surveyor and co-ordinated points calculated through the triangulation. Difficulties, of a variety too great to be enumerated here, arise when several surveys have to be combined, especially when, as often happens, they are of different periods. Much of the work involved in solving these difficulties is unavoidable, but a full appreciation of the problems can lead, in many cases, to action being taken by the surveyor in the field which, even if not essential to the strict requirements of the survey, can materially assist the cartographer. Nowadays, whenever possible, the surveyor is supplied with details of the triangulation and other data of the surveys of areas adjacent to his work to facilitate proper connection, but as such data is not always available and since previously occupied stations are not always recoverable, the necessary adjustments may involve extensive investigations by the cartographer before the points can be plotted. On the drawings of old surveys the positions of the stations used are often discernible, and even an approximate connection of such positions to the stations selected for the new survey, if made on the spot, can give valuable aid. When an area is re-surveyed, serious confusion can arise if new stations are selected for triangulation without reference to those on which the older work was based; and although the modern practice of marking as permanently as possible and describing in detail the principal stations does much to remove these troubles, older stations must still be reckoned with if the final charting is to be satisfactory. These troubles of connection tend to become acute when a survey ends at a territorial limit opposite, but not connected with, the end of a survey by some other authority; or when objects in an area are fixed by two surveys with different results. When discrepancies have to be reconciled, care and forethought in providing connection between adjacent surveys will facilitate the work of the cartographer.

In an extensive triangulation it is often necessary to determine the astronomical positions of two or more stations and, as explained on pages 78 and 79, these seldom agree with the geodetic values calculated through the triangulation. The differences may be due to many causes, including errors of the plumb line (pages 325, 326), and it is only by elaborate checking of all relevant factors that a solution can be reached and the work laid down correctly on the chart.

Good compilation from old and modern surveys, maps, plans, and rough material, or whatever else is available, is the outcome of long experience only ; and the selection of data depends on a critical appraisement of all relevant material guided by a wide knowledge of survey work of all periods (as such knowledge grows, it brings a profound respect for the accuracy and extent of the work of the surveyors of a century or more ago). This search and trial for the task of fitting the old, the sketchy, or the doubtful, into a modern survey may be reversed when a new survey is used to supplement work on an existing chart ; and the preservation of scales and bearings or orientation provides the cartographer with some of his most interesting problems.

Drawing the chart.

The graduation of a new chart is usually drawn, for the sake of permanence, upon a sheet of heavy water-colour paper with a " hot-pressed " surface (a heavy-weight Whatman paper is satisfactory, having sufficiently stable properties). When all the materials for the chart have been selected and the framework of points plotted on the paper, the compilation stage of the chart is completed and the drawing begins. The drawing is usually made on a piece of tracing paper fitted over the graduation ; this not only has advantages in the later stages of reproduction, but its transparency and smooth surface are a convenience to the draughtsman. If the fair chart of the survey is on the same scale and projection as that of the chart to be produced, the tracing, onto which the plotted points have been transferred, may be laid directly over it and the work traced by fitting on the points, aided if necessary by a few bold rectangles of graduation. This, however, seldom happens and some measure of reduction is usually required. When the projection of fair sheet and chart are the same and the reduction does not affect legibility, a photographic reduction is made, in which the work is printed direct from the glass negative on to a lithographic zinc plate, from which paper proofs are pulled ; there is no damping of the paper at any stage of this process so there is no distortion or change of scale and work can be traced direct from the photo-reduction in the same way as from a fair sheet of the right scale. The more general method of reduction, by which any degree of difference in scale is made possible, is the use of similar rectangles. Fine pencil lines at right angles to one another are ruled on the original document, either through the graduation or else by scale, as circumstances dictate, and the tracing paper upon which the work is to be reduced is squared with exactly similar rectangles on the smaller scale (see pages 256, 257). In practice the sides of the smaller of such rectangles never exceed one-tenth of an inch.

Selection of soundings.

The selection of the soundings is one of the most important responsibilities in the composition of a chart. The sounding datum used by the surveyor must be compared with that used in earlier surveys and with that of adjacent charts, in case adjustments for uniformity should be necessary. From the fair sheet, on which soundings, especially in shoal areas, are often very close, those depths that are essential to the navigator must be selected and arranged with suitable regularity of spacing. It is usual to show soundings throughout the water area of a chart, and although there is no rigid rule for their spacing, tradition and experience have combined to standardise the practice ; in deep water they

are widely spaced but are closer on the continental shelf and in shoal water are spaced quite closely. In complex navigational passages, however, it is inadvisable to crowd the soundings for fear of confusion, and in channels and estuaries particularly, it is necessary to show the shoals without obscuring the passages through them. Since the size of an engraved figure is, more or less, constant there is need for increased discrimination in the selection and arrangement of the soundings as the scales of the charts decrease. The shoalest depths must always be shown, but they must not be allowed to give a false impression of the trend of the bottom; and for many conditions of navigation, practicability of anchoring, etc., it is necessary that maximum as well as minimum depths shall be given. The nature of the bottom must also be freely shown. In deep waters, a marked local decrease in depth is surrounded by more than the normal number of soundings, not only to attract attention to it, but to indicate that examination of a possible danger has been made.

The records of deep-sea soundings (which have not been shown on any fair chart) must be searched for material suitable for inclusion on any chart which embraces deep parts of an ocean.

A small scale chart is always drawn or corrected from the larger scale preceding it, and the same selection of soundings from the original survey is used throughout; so that it is frequently possible to follow a single sounding from an ocean chart, through all the coastal and approach sheets, to a large scale plan.

The practice now adopted of showing depths in fathoms and feet combined is not only more precise than the old method of showing fathoms and fractions, but has removed the need of the dual system whereby soundings appeared in feet on large scale charts and in fathoms on smaller scales. Fathom lines are taken direct from the fair sheet and are not sketched in among the selected soundings. The one, three, six and ten fathom lines are always shown, and in special cases, the two and five fathom contours may also be shown; the twenty and the hundred fathom lines are the contours shown in deeper waters, though others are used when occasion demands. For clarity in drawing, engraving and revising, fathom lines are usually drawn in distinctive coloured lines which are translated by the engraver into the conventional symbols.

Navigational aids and topographical detail.

Navigational aids, buoys, beacons, lights, etc., must also be shown and these are drawn boldly in order that they may be recognised at once. Topographical details, however, are not given great prominence unless directly useful to the navigator. The treatment of hills varies but the object, always, is that the navigator may be given a relief impression of what he will see on making the land; if accurate contours are available they are shown on the chart in combination with a form of shading designed to give this impression. However, in most parts of the world accurate contours are not available and a system of form lines, shown by horizontal hachures, has been generally adopted, since by this means the engraver or lithographic line-draughtsman can most readily imitate the effect of relief given by shading in wash or chalk. Any resemblance between these horizontal hachures and contour lines is purely accidental and, to prevent confusion, continuity in the hachure lines is always avoided.

Hachuring.—Hachuring is constructed of strips or bands of lines, each element of which is a short flexible line (following approximately the shape of the adjacent contour) drawn at right angles to an axis, which is normal to the contour curve at the middle point of the hachure (Plate Z, Fig. I). The successive lines, though not of precisely uniform length, imitate each other roughly in shape and decrease in thickness as they descend from the top to the bottom of a hill (Plate Z, Fig. II). In theory the lines of each strip are all the same distance apart and differ only in thickness, but in practice it is not always possible to adhere to this rule and some opening of the spaces may be found necessary at the base of a slope or along extended spurs (Plate Z, Fig. III).

Adjoining bands of hachures are drawn in a similar fashion but are spaced differently so that adjacent elements do not join up to form continuous lines. These joins, though slightly irregular, are not disguised since occasional dark or light joins suggest the characteristic " water course " appearance of mountain sides (Plate Z, Fig. IV). When the contours are convex the bands of hachures diverge, additional bands being inserted to fill the opening spaces ; when they are concave the bands converge and are tapered off and finished as necessary (Plate Z, Fig. V). (Fig. VI shows how these principles are applied to the sides of long spurs.) On a long slope, a closing of contours is indicated by a darkening, and an opening of contours by a lightening, of the hachures (Plate Z, Fig. VII). The sunlit side of a hill is treated like the shadow side except that the thickness of the hachures is lighter throughout, being relatively about 75 per cent of that on the sunlit side (Plate Z, Fig. VIII). Except in a small measure, relative heights are not indicated by the relative darkness of the shadow side. Obviously it is not possible to make a 10,000-foot mountain ten times as dark as one only 1,000 feet high, and in general, the greater size of the mountain mass is the only indication of its greater height. The intensity of the shadow indicates, almost entirely, the pitch of the slope at any point.

Some examples of the treatment of a local shelf and gully, a saddle between two peaks, a deep combe, etc., are shown in Plate Z, Fig. IX.

Lettering.

Every opportunity is taken to clarify the chart by utilising regular varieties of letter-forms in the names and other writing. By the use of upright and sloping, sanserif and serif, letters and figures, distinction is made between land and water features, between dangers that are covered at some state of the tide and those that are always exposed, between floating and fixed navigational aids, etc. Names, when not horizontal, must approach or leave the symbol representing a feature on a definite system of curvature (see Plate X, page 417) ; to avoid ambiguity they begin or end at the symbol and are not placed across it. Descriptions against objects in the water area are arranged to cause the least interference with the use of the chart ; channels, for instance, are left as clear of writing as possible. Although precise definitions of letter-styles and sizes are necessary to achieve uniformity yet, for the successful design of a chart, appropriate selection must be forthcoming in the balance of lettering, choice of curvature, and placing of principal names, title and notes. A chart may conform to all the set rules and yet, because of bad design, be ugly to look at and awkward to use.

Previously reported dangers.

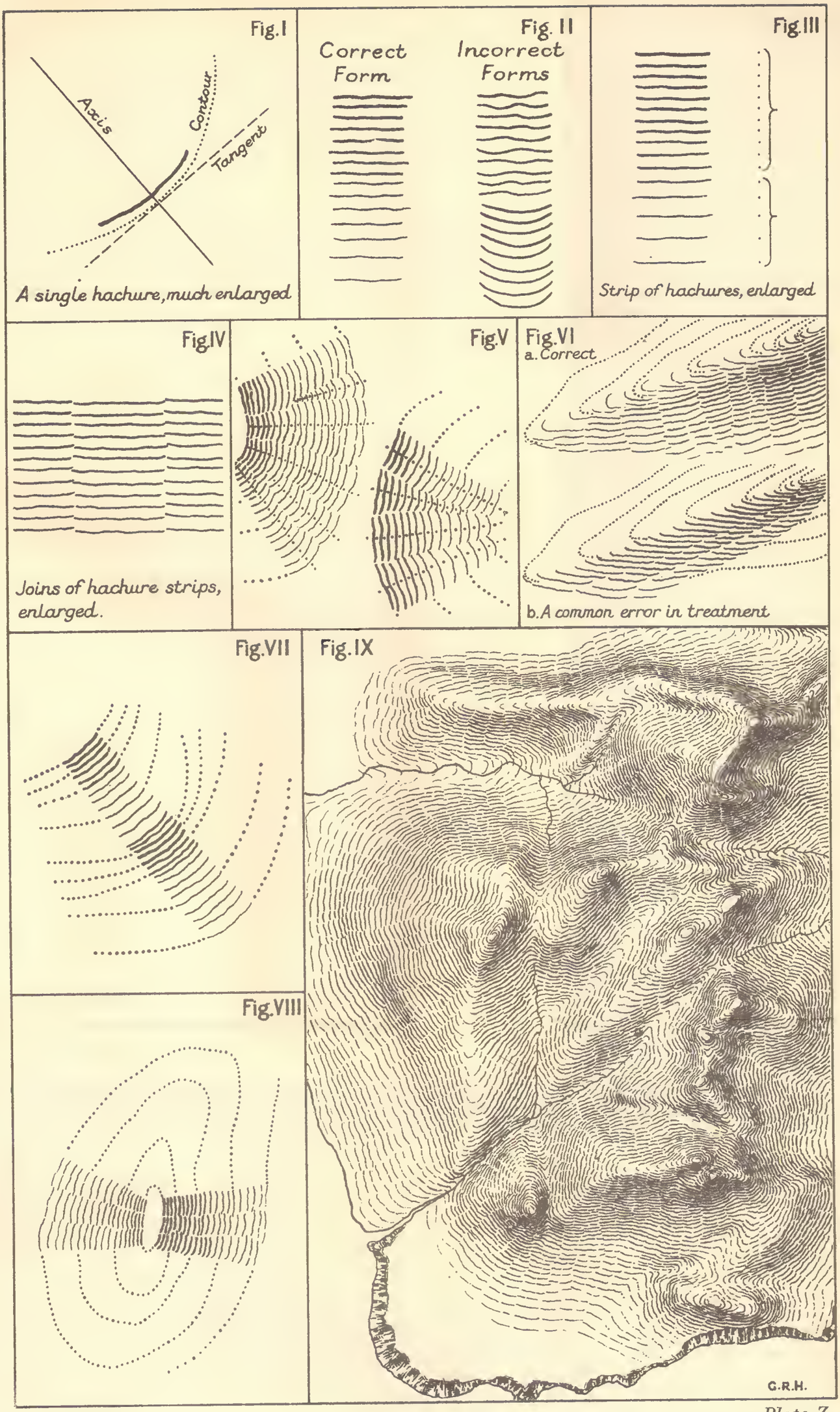
Existing charts are examined and, in particular, all dangers shown on them are compared with the corresponding work on the new survey so that no danger shall be omitted from the navigational chart which has not been entirely disproved. The absence from a fair chart of some important feature, which had appeared on charts previously, is not considered sufficfent authority for its omission unless it is confirmed by the surveyor's sailing directions or otherwise clearly shown to be non-existent.

Necessity of up-to-date correction before printing

Details of navigational aids, lights, buoys, beacons, fog signals, leading marks and the like given on the fair sheet may be out of date before the navgational chart is drawn, and reference to the appropriate authorities is necessary to ensure that the latest information is shown in every case.

Data from other authorities.

It is the duty of the cartographer to check in detail the work done by the surveyor in the field and it is only thus that the final accuracy of the chart can be guaranteed, since all the material required for such a purpose is not available



C.B.H.3386

Plate Z
(To face page 450)

to the surveyor. For instance, the orthography of the chart is dependent upon certain rules and authorities, which may call for amendments to the names given by the surveyor. Less tidal information is now given on charts than formerly and the details that are still shown are not taken directly from the fair sheet. Compass-roses (which are so placed that they escape the folds of the chart and can be used from any part of it), radio stations, cables and areas of magnetic disturbance are other details shown on a chart which are taken principally from sources outside the work of the surveyor.

Chart Histories

As soon as a chart is published, its history starts and a complete record of every correction, no matter how small, is meticulously preserved. From the history sheets it is possible to refer to the authority for any amendment made to the copper plate and thus all material wants in the life of a chart are available to the cartographer.

CHAPTER XXIII

REPRODUCTION AND PRINTING OF CHARTS

The reproduction and printing of Admiralty charts is undertaken in the Production Division of the Hydrographic Supplies Establishment from the manuscript drawings which are schemed and compiled in the Chart Branch of the Hydrographic Department (as described in the previous chapter). From these manuscript drawings the ordinary navigational charts used to be engraved on copperplates, which constituted the permanent originals and which were, and still are, kept corrected for the latest information. The basis of nearly all navigational charts is still the copperplate engraving, but, nowadays, not all new charts are reproduced as engravings.

Copperplate Engraving.

The manuscript drawings of a navigational chart are usually prepared in two (see Chapter XXII) ; the graduation on Whatman paper (which is practically immune from the dimensional changes caused by atmospheric variation), and the surveyed detail on tracing paper.

The graduation is first engraved on the copperplate. A copy of the surveyed detail is traced, with a fine steel point, onto a thin sheet of celluloid laid over the drawing ; the celluloid is dusted with a blue powder and wiped, so that the powder remains only in the scratches. The copperplate (bearing the graduation) is coated with wax and the prepared sheet of celluloid is placed, face down, in exact position on it. The back of the celluloid is gently burnished all over with a steel tool, which makes the blue powder adhere to the wax and thus reproduces the detail drawing, in reverse, on the copperplate.

After any adjustments, necessitated by distortion of the tracing paper (if this has occurred), have been made, the work is cut into the copper by hand, using steel tools called gravers. Features such as hills, low-water rocks, mangrove edging and views, however, necessitate more delicate treatment than gravers can produce, and these are etched on the plate, at a later stage, by the engravers. Compass roses, conversion tables, etc., which are common to different charts, are also etched directly onto the copperplate.

Etching.—Etching is both cheaper and quicker than hand engraving. For this, the plate is coated with a photographic emulsion, after which the appropriate detail, compass roses, etc., in the form of film positives, are positioned on it, and the whole is exposed in a vacuum frame to arc lamp illumination. In all places where the light penetrates through the transparent parts of the film, the emulsion is made insoluble both to water and to the etching solution, used later. The parts of the emulsion remaining soluble are washed away with water, leaving the copper exposed in a facsimile of the design on the film. In the last stage of the process the plate is treated with the etching solution, perchloride of iron, which attacks the exposed parts of the copper, and so etches on the copperplate all the detail which was shown on the film positive.

Steel coating.—When the engraving is completed in all respects, the copperplate is given a very thin coating of steel by electrical deposit, in order to protect the soft copper from wear.

Proofs.—Proofs of various sorts, for a variety of purposes, can be made from the copperplates. The simplest to make is the " wet " proof on ordinary chart paper. The plate is " rolled up " with the appropriate ink, which fills the engraving and, after surplus ink has been cleaned from its surface and edges, the plate is run through a press in contact with a sheet of thoroughly damped

paper. The paper receives the ink from the engraving and, as the work on the engraving was in reverse, the copy on the paper reads forward. These proofs are as solid and sharp as the state of the plate allows but, as the paper distorts substantially because of the thorough damping it has been given, such proofs are used only for purposes of revision and for checking the engraver's work.

In the past, all charts were printed by this laborious process ; about eight minutes being required to " pull " each copy. Nowadays, however, all Admiralty charts for ordinary use are printed by the lithographic process from zinc plates ; but, as explained later, the preliminary stages in making these plates vary considerably.

Lithography.

The basis of lithographic reproduction is that grease and water are antipathetic. This antipathy enables greasy marks and patches, which have been imposed on the surface of a suitably prepared lithographic material, to be recharged without losing or extending the detail of the work ; provided the correct amount of grease for recharging and the correct amount of water are used. The original lithographic material was Bavarian limestone, but zinc and aluminium are now used, and of these zinc is the better for chart and map reproduction.

In the lithographic process, a copy of the copperplate engraving has to be imposed on a zinc plate. This copy is transferred by means of a special greasy stick ink and paper coated with a mixture of plaster of paris and flour. The stick ink is rubbed over the copperplate, which is heated sufficiently to melt the ink and allow it to run into the engraving. After the surplus ink has been cleaned off its surface, but while it is still warm, the plate is run through the press, under heavy pressure, in contact with the previously damped transfer paper. The finest details of the engraving (and also any faults, scratches, etc., on the copperplate) are thus reproduced, in the greasy ink, on the transfer paper. Faults are erased with a knife and the transfer is placed between damp sheets of blotting paper for a short while.

In the meantime the surface of a sheet of zinc is matted with an abrasive which produces a grained surface that will hold greasy ink, and which has minute indentations that will retain water. The grained zinc is sensitised (*i.e.*, made chemically clean) by washing in a weak solution of nitric acid and alum.

When the zinc sheet is dry, the damp transfer is placed upon it and the two are run through a litho transfer press several times, the back of the transfer being damped between the runs till the water has penetrated the paper sufficiently to allow it to be lifted away, thus leaving the composition, with the greasy ink underneath, firmly fixed to the zinc plate. The plate is then washed clean of the composition, and is dried and, while the zinc is still sensitive to grease, any faults in the work are carefully made good.

The plate is de-sensitised by being sponged over with pure gum arabic, and dried. (The gum is repelled by the greasy design but adheres to the rest of the zinc and, provided the plate is suitably damped before ink is applied, it is protected from further grease deposits during subsequent operations.) The work on the plate can now be reinforced and charged, and a proof pulled, by running it in contact with a sheet of paper through a litho transfer press. If the proof is satisfactory the zinc plate can be sent to the printer.

Lithographic Printing Machines.—Two patterns are used, flatbed-direct machines and rotary-offset machines.

In a *Flatbed-direct* litho printing machine, the zinc plate is clamped on a flat iron bed, which moves backwards and forwards under a cylinder covered with japanned cloth. The sheets of paper to be printed are fed, by hand, above the cylinder, which revolves and causes the paper to come in contact, under

pressure, with the plate at each forward movement of the bed. Rollers automatically recharge the plate with ink and water (the former adhering only to the design) before each print is made. According to the size of the machine, from 250 to 400 copies an hour can be printed, in one colour only, by this method. Although about fifty times quicker than copperplate printing, this is too slow to cope with really long runs of charts.

Rotary-offset machines are used chiefly for the longer runs of charts. In these machines the zinc plate is secured round a cylinder (instead of to a flat bed) and does not come into direct contact with the paper. The design from the plate is offset to a rubber blanket, fixed round a second cylinder, and offset again, from the blanket to the paper, which is fed on to a third cylinder. In an offset machine, the design on the plate is offset twice (from the plate to the rubber blanket and from the blanket to the paper) and so must read forward, like the printed copy. Such a plate is made in a " reversing press " by offsetting the design from a copperplate (which, being made for direct printing, reads in reverse). Rotary offset machines are capable of 2,000 to 5,000 prints an hour.

Direct Reproduction.

Enamel Zincs.—New charts are not always engraved on copperplates. Fine finished drawings, made on enamel-coated zinc plates for subsequent photography, can be used instead. The enamel coating is applied to the zinc plate by spraying, and it is slightly roughened to provide a good drawing surface.

Similar to those made for copperplate engraving, the manuscript drawing for reproduction is usually on tracing paper, and a photographic copy of it is made, in a weak blue, on an enamel zinc. The copy is only a guide for the draughtsman and is not required when the drawing is finished, blue is used for it because this colour does not reproduce when photographed. The blue is drawn over in a high quality black ink, but title, notes and sometimes the place names, are " set up " in type, *i.e.*, proofs from the type are cut to shape and stuck on the enamel zinc. Copies of the compass roses, also, are made from a copperplate original, onto paper and are patched on the enamel in their correct positions. After revision of the completed drawing, the enamel zinc is passed to the photo section for a printing plate to be prepared from it (since it is not possible to print direct from an enamel zinc).

The enamel zinc drawing is photographed to the correct size (either true-to-scale or a reduction) onto a film or glass negative. It must be decided beforehand whether the chart is to be printed on a direct or offset machine, since different types of negative are required, reading forward for direct, and backwards for offset printing. For the former, a 45° prism is used in the camera to reverse the image, the fore-and-aft line of the camera being oriented parallel to the plate being photographed. Faults on the negative are covered over (" spotted out ") by applying an opaque, and lines, etc., which are not clear enough, are cut in.

The design on the negative has to be imposed on a zinc printing plate ; and to prepare for this, the plate is covered with a light-sensitive solution while it is clamped in a moving, whirling machine ; the solution being poured on the centre of the plate. The whirling ensures an even coating and throws off surplus solution ; electric elements fitted above it, dry the plate.

The coated zinc plate, with the negative face down on it, is placed in a vacuum frame, and exposed to arc lamp illumination for ten to twenty minutes. The emulsion which is immediately under the letters, figures and other clear parts of the negative is made insoluble in water by the light, but the rest of the emulsion on the plate is not affected.

The plate is then covered all over with thin black ink. It is fan-dried and put under running water, which washes away the soluble part of the (ink covered) emulsion but leaves the insoluble part, which represents the chart, ink covered,

on the plate. The plate is dried, examined for faults, which if found are made good, and is then de-sensitised with gum arabic. It is rolled up with ink and a proof is pulled in the transfer press ; if the proof is satisfactory the plate is ready for printing, as previously described.

Other methods of Direct Reproduction.—If speed of reproduction is very important, the chart may be drawn, as it will finally appear, though preferably on a larger scale, on any suitable medium such as tracing paper, baryta paper (unmounted or mounted on zinc), Whatman paper, chart paper, or kodatrace (and, of course, enamel zinc). The drawing is photographed, and a printing plate prepared from the negative as described above (under " *Enamel Zincs* "). Minor blemishes on the drawing become unnoticeable when it is reduced, at the photographic stage of the process, to the correct scale. This method produces a printed chart in a very short time, but it has a great disadvantage in the difficulty of keeping the original (in this case, the drawing from which the printing plate was made) corrected for up-to-date information (unless it is an enamel zinc, see later).

The normal way of transferring the design from a copperplate to a lithographic plate has already been described. Another method is by the direct reproduction, as described above, of a proof taken from the copperplate engraving. To achieve the best results, the proof is pulled on " baryta " paper, a specially coated paper on which very sharp prints are obtainable.

Alterations on Chart Plates.

Copperplate engravings.—Alterations are made by scraping or grinding away the work to be amended from the face of the copperplate, tapping up the hollowed area from the back as necessary, to level the working face, replanishing (*i.e.*, flattening) the surface, and then engraving the new work.

Another method of levelling off the surface of the copperplate, after the old work has been removed, is the *Galvino-Plastic*. In this, copper is deposited electrolytically in the hollow, the deposited copper being then ground down to the level of the original plate, ready for the new work to be engraved. This is preferable to, and is gradually superseding, the old method, since it causes less damage or distortion to the plate.

Zinc printing plates.—The work to be amended is removed by applying a saturated solution of caustic potash ; after which the patch is sensitised (chemically cleaned) with a weak solution of nitric acid and alum, and the new work is inserted by the lithographic draughtsman (using a brush for the finest work). Lastly, the patch is sponged over with gum arabic to de-sensitise the zinc background.

Enamel zincs.—Erasures are made easily by a damp rubber, a knife, or a suitable ink solvent ; and the new work is drawn with ink (like the original).

Glass negatives.—Erasures are made by knife, and the patch is coated with a mixture of lamp-black, gum and benzole. When this has dried, the new work is cut through the coating to the glass, with a needle.

Colour Printing.

The only colour shown on the ordinary navigational charts is the yellow used for the light flares, but in future, a land tint is to be used as well. Extensive use of colour has been made during the last few years, however, on Air charts, M charts and a variety of other special charts.

A separate lithographic plate is required for each colour. To ensure that the various colours fit together exactly on the finished chart it is necessary that one of the plates shall be the " master " and the others made as replicas of it ; normally the black plate is the master. An impression in black ink of the master is made on paper, and is dusted over completely with a non-greasy violet powder, which adheres to the ink work but can be blown off the rest of

the sheet. A clean zinc plate is damped and, with the paper sheet face down on it, is run through the press. The powder adheres to the zinc in the form of a violet facsimile of the black (master) plate, and enables the lithographic draughtsman, working with greasy ink, to insert in their correct positions all the details required on that particular colour plate, light flares or land tint, etc. Amongst the detail that must be inked in on each plate, are the reproductions of the small corner marks (registration marks) carried on the " master."

When the drawing is finished the plate is de-sensitised, as previously described ; is rolled up with the appropriate coloured ink, and is then ready for the printer.

With most printing machines, the sheet of paper must pass through the machine once for every colour (including the black) that has to be printed. However, on some rotary-offset machines, two colours can be impressed each time the paper passes through them.

The black plate is printed first, and the colour plates are then adjusted on the machine so that their registration marks are superimposed exactly over the impression of the black registration marks, thus ensuring that the colours are printed in their correct relative positions.

TABLE I

Elements of the Earth

Figure.	a Equatorial Radius	b Polar Radius	f Compression
Clarke (1858)	20,926,348 feet	20,855,233 feet	$\frac{1}{294{\cdot}3}$
,, (1880)	20,926,202 feet	20,854,895 feet	$\frac{1}{293{\cdot}5}$
Airy	20,923,713 feet	20,853,810 feet	$\frac{1}{299{\cdot}3}$
Everest (1st)	20,922,932 feet	20,853,375 feet	$\frac{1}{300{\cdot}8}$
Bessel	6,377,397 metres	6,356,079 metres	$\frac{1}{299{\cdot}2}$
Hayford (1910)	6,378,388 metres	6,356,912 metres	$\frac{1}{297{\cdot}0}$

(*Note.*—Clarke's 1866 figure does not differ much from his other figures.)

f = Compression = $\left(\frac{a-b}{a}\right)$

e = Eccentricity = $\left(\frac{a^2-b^2}{a^2}\right)^{\frac{1}{2}}$

*ν = Normal to meridian at lat. ϕ terminated by minor axis (or radius of curvature at right angles to meridian) = $\frac{a}{(1-e^2\sin^2\phi)^{\frac{1}{2}}}$

*ρ = Radius of curvature to meridian at lat. ϕ = $\frac{a(1-e^2)}{(1-e^2\sin^2\phi)^{\frac{3}{2}}}$

* See Tables III, IV, V and VI.

TABLE II

Spherical Excess

Area of Triangle in Square Nautical Miles.	Spherical Excess.	Area of Triangle in Square Nautical Miles.	Spherical Excess.
	″		″
5	0·1	70	1·2
10	0·2	80	1·4
15	0·3	90	1·6
20	0·3	100	1·7
30	0·5	200	3·5
40	0·7	300	5·2
50	0·9	400	6·9
60	1·0	500	8·7

This table is computed from the formula, spherical excess = $\frac{A}{r^2\sin 1''}$ where A = area of triangle in sq. feet and r = mean radius of curvature in feet.

(It is computed for latitude 30°, but may be used for all other latitudes without appreciable error.)

TABLE III

Linear value, in feet and its logarithm, of ρ sin 1″ and ν sin 1″

(Clarke 1858: compression $\frac{1}{294{\cdot}3}$)

Latitude		ρ sin 1″	Diff.	Log ρ sin 1″	Diff.	ν sin 1″	Diff.	Log ν sin 1″	Diff.
°	′								
0	0	100·7654	0	2·0033115	1	101·4538	0	2·0062683	0
	10	7654	1	3116	1	4538	0	2683	1
	20	7655	0	3117	2	4538	0	2684	0
	30	7655	1	3119	2	4538	0	2684	1
	40	7656	0	3121	4	4538	1	2685	1
	50	7656	1	3125	4	4539	0	2686	2
1	0	100·7657	1	2·0033129	4	101·4539	0	2·0062688	1
	10	7658	2	3133	6	4539	1	2689	2
	20	7660	1	3139	6	4540	0	2691	2
	30	7661	2	3145	8	4540	1	2693	3
	40	7663	2	3153	7	4541	0	2696	2
	50	7665	2	3160	9	4541	1	2698	3
2	0	100·7667	2	2·0033169	9	101·4542	1	2·0062701	3
	10	7669	2	3178	10	4543	1	2704	4
	20	7671	3	3188	11	4544	0	2708	3
	30	7674	2	3199	12	4544	1	2711	4
	40	7676	3	3211	12	4545	1	2715	4
	50	7679	3	3223	13	4546	1	2719	4
3	0	100·7682	3	2·0033236	14	101·4547	1	2·0062723	5
	10	7685	4	3250	15	4548	2	2728	5
	20	7689	3	3265	15	4550	1	2733	5
	30	7692	4	3280	16	4551	1	2738	5
	40	7696	4	3296	17	4552	1	2743	6
	50	7700	4	3313	17	4553	2	2749	6
4	0	100·7704	4	2·0033330	19	101·4555	1	2·0062755	6
	10	7708	5	3349	19	4556	2	2761	6
	20	7713	4	3368	19	4558	1	2767	7
	30	7717	5	3387	21	4559	2	2774	7
	40	7722	5	3408	21	4561	1	2781	7
	50	7727	5	3429	22	4562	2	2788	7
5	0	100·7732	5	2·0033451	23	101·4564	2	2·0062795	8
	10	7737	6	3474	23	4566	2	2803	7
	20	7743	5	3497	24	4568	2	2810	8
	30	7748	6	3521	25	4570	1	2818	9
	40	7754	6	3546	26	4571	2	2827	8
	50	7760	6	3572	26	4573	3	2835	9
6	0	100·7766		2·0033598		101·4576		2·0062844	

TABLE III

Linear value, in feet and its logarithm, of ρ sin 1″ and ν sin 1″

(Clarke 1858)

Latitude		ρ sin 1″	Diff.	Log ρ sin 1″	Diff.	ν sin 1″	Diff.	Log ν sin 1″	Diff.
°	′								
6	0	100·7766	7	2·0033598	27	101·4576	2	2·0062844	9
	10	7773	6	3625	28	4578	2	2853	9
	20	7779	7	3653	29	4580	2	2862	10
	30	7786	6	3682	29	4582	2	2872	10
	40	7792	7	3711	30	4584	3	2882	10
	50	7799	8	3741	31	4587	2	2892	10
7	0	100·7807	7	2·0033772	31	101·4589	3	2·0062902	10
	10	7814	7	3803	32	4592	2	2912	11
	20	7821	8	3835	33	4594	3	2923	11
	30	7829	8	3868	34	4597	2	2934	11
	40	7837	8	3902	34	4599	3	2945	12
	50	7845	8	3936	35	4602	3	2957	12
8	0	100·7853	8	2·0033971	36	101·4605	2	2·0062969	11
	10	7861	9	4007	37	4607	3	2980	13
	20	7870	8	4044	37	4610	3	2993	12
	30	7878	9	4081	38	4613	3	3005	13
	40	7887	9	4119	39	4616	3	3018	13
	50	7896	9	4158	39	4619	3	3031	13
9	0	100·7905	10	2·0034197	40	101·4622	3	2·0063044	13
	10	7915	9	4237	41	4625	3	3057	14
	20	7924	10	4278	41	4628	4	3071	13
	30	7934	9	4319	43	4632	3	3084	15
	40	7943	10	4362	42	4635	3	3099	14
	50	7953	11	4404	44	4638	4	3113	14
10	0	100·7964	10	2·0034448	44	101·4642	3	2·0063127	15
	10	7974	10	4492	46	4645	4	3142	15
	20	7984	11	4538	45	4649	3	3157	15
	30	7995	11	4583	47	4652	4	3172	16
	40	8006	11	4630	47	4656	4	3188	16
	50	8017	11	4677	48	4660	3	3204	16
11	0	100·8028	11	2·0034725	48	101·4663	4	2·0063220	16
	10	8039	11	4773	49	4667	4	3236	16
	20	8050	12	4822	50	4671	4	3252	17
	30	8062	12	4872	51	4675	4	3269	17
	40	8074	12	4923	51	4679	4	3286	17
	50	8086	12	4974	52	4683	4	3303	17
12	0	100·8098		2·0035026		101·4687		2·0063320	

TABLE III

Linear value, in feet and its logarithm, of ρ sin 1″ and ν sin 1″
(Clarke 1858)

Latitude		ρ sin 1″	Diff.	Log ρ sin 1″	Diff.	ν sin 1″	Diff.	Log ν sin 1″	Diff.
°	′								
12	0	100·8098	12	2·0035026	53	101·4687	4	2·0063320	18
	10	8110	12	5079	55	4691	4	3338	17
	20	8122	13	5132	54	4695	4	3355	18
	30	8135	13	5186	55	4699	4	3373	19
	40	8148	12	5241	55	4703	5	3392	18
	50	8160	13	5296	56	4708	4	3410	19
13	0	100·8173	14	2·0035352	57	101·4712	5	2·0063429	19
	10	8187	13	5409	57	4717	4	3448	19
	20	8200	13	5466	58	4721	5	3467	19
	30	8213	14	5524	59	4726	4	3486	20
	40	8227	14	5583	60	4730	5	3506	20
	50	8241	14	5643	60	4735	4	3526	20
14	0	100·8255	14	2·0035703	60	101·4739	5	2·0063546	20
	10	8269	14	5763	62	4744	5	3566	20
	20	8283	14	5825	62	4749	5	3586	21
	30	8297	15	5887	62	4754	5	3607	21
	40	8312	15	5949	64	4759	5	3628	21
	50	8327	15	6013	64	4764	5	3649	21
15	0	100·8342	15	2·0036077	64	101·4769	5	2·0063670	22
	10	8357	15	6141	66	4774	5	3692	22
	20	8372	15	6207	66	4779	5	3714	22
	30	8387	16	6273	66	4784	5	3736	22
	40	8403	15	6339	67	4789	5	3758	22
	50	8418	16	6406	68	4794	6	3780	23
16	0	100·8434	16	2·0036474	69	101·4800	5	2·0063803	23
	10	8450	16	6543	69	4805	5	3826	23
	20	8466	16	6612	70	4810	6	3849	23
	30	8482	16	6682	70	4816	5	3872	23
	40	8498	17	6752	71	4821	6	3895	24
	50	8515	16	6823	72	4827	5	3919	24
17	0	100·8531	17	2·0036895	72	101·4832	6	2·0063943	24
	10	8548	17	6967	73	4838	6	3967	24
	20	8565	17	7040	73	4844	5	3991	25
	30	8582	17	7113	74	4849	6	4016	24
	40	8599	18	7187	75	4855	6	4040	25
	50	8617	17	7262	75	4861	6	4065	26
18	0	100·8634		2·0037337		101·4867		2·0064091	

TABLE III

Linear value, in feet and its logarithm, of ρ sin 1″ and ν sin 1″
(Clarke 1858)

Latitude		ρ sin 1″	Diff.	Log ρ sin 1″	Diff.	ν sin 1″	Diff.	Log ν sin 1″	Diff.
°	′								
18	0	100·8634	18	2·0037337	76	101·4867	6	2·0064091	25
	10	8652	18	7413	77	4873	6	4116	25
	20	8670	18	7490	77	4879	6	4141	26
	30	8688	18	7567	78	4885	6	4167	26
	40	8706	18	7645	78	4891	6	4193	26
	50	8724	18	7723	79	4897	6	4219	26
19	0	100·8742	19	2·0037802	79	101·4903	6	2·0064245	27
	10	8761	18	7881	81	4909	6	4272	27
	20	8779	19	7962	80	4915	7	4299	26
	30	8798	19	8042	82	4922	6	4325	28
	40	8817	19	8124	81	4928	6	4353	27
	50	8836	19	8205	83	4934	7	4380	27
20	0	100·8855	19	2·0038288	83	101·4941	6	2·0064407	28
	10	8874	20	8371	83	4947	7	4435	28
	20	8894	19	8454	84	4954	6	4463	28
	30	8913	20	8538	85	4960	7	4491	28
	40	8933	20	8623	85	4967	7	4519	29
	50	8953	20	8708	86	4974	6	4548	28
21	0	100·8973	20	2·0038794	87	101·4980	7	2·0064576	29
	10	8993	20	8881	87	4987	7	4605	29
	20	9013	20	8968	87	4994	7	4634	29
	30	9033	21	9055	88	5001	6	4663	29
	40	9054	20	9143	89	5007	7	4692	30
	50	9074	21	9232	89	5014	7	4722	30
22	0	100·9095	21	2·0039321	90	101·5021	7	2·0064752	30
	10	9116	21	9411	90	5028	7	4782	30
	20	9137	21	9501	91	5035	7	4812	30
	30	9158	21	9592	91	5042	7	4842	30
	40	9179	22	9683	92	5049	8	4872	31
	50	9201	21	9775	92	5057	7	4903	31
23	0	100·9222	22	2·0039867	93	101·5064	7	2·0064934	31
	10	9244	21	2·0039960	93	5071	7	4965	31
	20	9265	22	2·0040053	94	5078	8	4996	31
	30	9287	22	0147	94	5086	7	5027	32
	40	9309	22	0241	95	5093	7	5059	31
	50	9331	22	0336	96	5100	8	5090	32
24	0	100·9353		2·0040432		101·5108		2·0065122	

TABLE III

Linear value, in feet and its logarithm, of ρ sin 1″ and ν sin 1″
(Clarke 1858)

Latitude		ρ sin 1″	Diff.	Log ρ sin 1″	Diff.	ν sin 1″	Diff.	Log ν sin 1″	Diff.
°	′								
24	0	100·9353	23	2·0040432	96	101·5108	7	2·0065122	32
	10	9376	22	0528	96	5115	8	5154	32
	20	9398	22	0624	97	5123	7	5186	32
	30	9420	23	0721	97	5130	8	5218	33
	40	9443	23	0818	98	5138	8	5251	32
	50	9466	23	0916	99	5146	7	5283	33
25	0	100·9489	23	2·0041015	98	101·5153	8	2·0065316	33
	10	9512	23	1113	100	5161	8	5349	33
	20	9535	23	1213	100	5169	7	5382	34
	30	9558	23	1313	100	5176	8	5416	33
	40	9581	24	1413	101	5184	8	5449	34
	50	9605	23	1514	101	5192	8	5483	33
26	0	100·9628	24	2·0041615	102	101·5200	8	2·0065516	34
	10	9652	24	1717	102	5208	8	5550	34
	20	9676	23	1819	102	5216	8	5584	34
	30	9699	24	1921	103	5224	8	5618	35
	40	9723	24	2024	104	5232	8	5653	34
	50	9747	25	2128	104	5240	8	5687	35
27	0	100·9772	24	2·0042232	104	101·5248	8	2·0065722	35
	10	9796	24	2336	105	5256	8	5757	35
	20	9820	25	2441	105	5264	9	5792	35
	30	9845	24	2546	106	5273	8	5827	35
	40	9869	25	2652	106	5281	8	5862	35
	50	9894	25	2758	107	5289	8	5897	36
28	0	100·9919	25	2·0042865	107	101·5297	9	2·0065933	36
	10	9944	25	2972	107	5306	8	5969	35
	20	9969	25	3079	108	5314	9	6004	36
	30	100·9994	25	3187	108	5323	8	6040	36
	40	101·0019	25	3295	109	5331	8	6076	37
	50	0044	25	3404	109	5339	9	6113	36
29	0	101·0069	26	2·0043513	109	101·5348	8	2·0066149	36
	10	0095	25	3622	110	5356	9	6185	37
	20	0120	26	3732	110	5365	9	6222	37
	30	0146	26	3842	111	5374	8	6259	37
	40	0172	26	3953	111	5382	9	6296	37
	50	0198	26	4064	111	5391	9	6333	37
30	0	101·0224		2·0044175		101·5400		2·0066370	

TABLE III

Linear value, in feet and its logarithm, of ρ sin 1″ and ν sin 1″
(Clarke 1858)

Latitude		ρ sin 1″	Diff.	Log ρ sin 1″	Diff.	ν sin 1″	Diff.	Log ν sin 1″	Diff.
°	′								
30	0	101·0224	26	2·0044175	112	101·5400	8	2·0066370	37
	10	0250	26	4287	112	5408	9	6407	37
	20	0276	26	4399	112	5417	9	6444	38
	30	0302	26	4511	113	5426	9	6482	37
	40	0328	26	4624	113	5435	8	6519	38
	50	0354	27	4737	114	5443	9	6557	38
31	0	101·0381	26	2·0044851	114	101·5452	9	2·0066595	38
	10	0407	27	4965	114	5461	9	6633	38
	20	0434	27	5079	115	5470	9	6671	38
	30	0461	26	5194	114	5479	9	6709	39
	40	0487	27	5308	116	5488	9	6748	38
	50	0514	27	5424	115	5497	9	6786	39
32	0	101·0541	27	2·0045539	116	101·5506	9	2·0066825	38
	10	0568	27	5655	117	5515	9	6863	39
	20	0595	27	5772	116	5524	9	6902	39
	30	0622	27	5888	117	5533	9	6941	39
	40	0649	28	6005	117	5542	9	6980	39
	50	0677	27	6122	118	5551	9	7019	39
33	0	101·0704	27	2·0046240	118	101·5560	10	2·0067058	39
	10	0731	28	6358	118	5570	9	7097	40
	20	0759	27	6476	118	5579	9	7137	39
	30	0786	27	6594	119	5588	9	7176	40
	40	0814	28	6713	119	5597	10	7216	39
	50	0842	28	6832	119	5607	9	7255	40
34	0	101·0870	27	2·0046951	120	101·5616	9	2·0067295	40
	10	0897	28	7071	120	5625	10	7335	40
	20	0925	28	7191	120	5635	9	7375	40
	30	0953	28	7311	121	5644	9	7415	40
	40	0981	28	7432	120	5653	10	7455	40
	50	1009	29	7552	121	5663	9	7495	41
35	0	101·1038	28	2·0047673	121	101·5672	10	2·0067536	40
	10	1066	28	7794	122	5682	9	7576	41
	20	1094	28	7916	122	5691	10	7617	40
	30	1122	29	8038	122	5701	9	7657	41
	40	1151	28	8160	122	5710	10	7698	41
	50	1179	29	8282	122	5720	9	7739	40
36	0	101·1208		2·0048404		101·5729		2·0067779	

TABLE III

Linear value, in feet and its logarithm, of ρ sin 1″ and ν sin 1″
(Clarke 1858)

Latitude		ρ sin 1″	Diff.	Log ρ sin 1″	Diff.	ν sin 1″	Diff.	Log ν sin 1″	Diff.
°	′								
36	0	101·1208	28	2·0048404	123	101·5729	10	2·0067779	41
	10	1236	29	8527	123	5739	9	7820	41
	20	1265	29	8650	123	5748	10	7861	41
	30	1294	28	8773	123	5758	10	7902	42
	40	1322	29	8896	124	5768	9	7944	41
	50	1351	29	9020	124	5777	10	7985	41
37	0	101·1380	29	2·0049144	124	101·5787	10	2·0068026	41
	10	1409	29	9268	124	5797	9	8067	42
	20	1438	29	9392	124	5806	10	8109	41
	30	1467	29	9516	125	5816	10	8150	42
	40	1496	29	9641	125	5826	9	8192	41
	50	1525	29	9766	125	5835	10	8233	42
38	0	101·1554	29	2·0049891	125	101·5845	10	2·0068275	42
	10	1583	29	2·0050016	125	5855	10	8317	41
	20	1612	30	0141	126	5865	9	8358	42
	30	1642	29	0267	125	5874	10	8400	42
	40	1671	29	0392	126	5884	10	8442	42
	50	1700	30	0518	126	5894	10	8484	42
39	0	101·1730	29	2·0050644	127	101·5904	10	2·0068526	42
	10	1759	29	0771	126	5914	10	8568	42
	20	1788	30	0897	126	5924	9	8610	43
	30	1818	29	1023	127	5933	10	8653	42
	40	1847	30	1150	127	5943	10	8695	42
	50	1877	29	1277	127	5953	10	8737	42
40	0	101·1906	30	2·0051404	127	101·5963	10	2·0068779	43
	10	1936	30	1531	127	5973	10	8822	42
	20	1966	29	1658	127	5983	10	8864	42
	30	1995	30	1785	128	5993	10	8906	43
	40	2025	30	1913	127	6003	10	8949	42
	50	2055	30	2040	128	6013	10	8991	43
41	0	101·2085	29	2·0052168	128	101·6023	10	2·0069034	43
	10	2114	30	2296	127	6033	10	9077	42
	20	2144	30	2423	128	6043	10	9119	43
	30	2174	30	2551	128	6053	10	9162	43
	40	2204	30	2679	129	6063	10	9205	42
	50	2234	30	2808	128	6073	10	9247	43
42	0	101·2264		2·0052936		101·6083		2·0069290	

TABLE III

Linear value, in feet and its logarithm, of ρ sin 1″ and ν sin 1″
(Clarke 1858)

Latitude		ρ sin 1″	Diff.	Log ρ sin 1″	Diff.	ν sin 1″	Diff.	Log ν sin 1″	Diff.
°	′								
42	0	101·2264	29	2·0052936	128	101·6083	10	2·0069290	43
	10	2293	30	3064	128	6093	10	9333	43
	20	2323	30	3192	129	6103	10	9376	42
	30	2353	30	3321	128	6113	10	9418	43
	40	2383	30	3449	129	6123	10	9461	43
	50	2413	30	3578	129	6133	10	9504	43
43	0	101·2443	30	2·0053707	128	101·6143	10	2·0069547	43
	10	2473	30	3835	129	6153	10	9590	43
	20	2503	30	3964	129	6163	10	9633	43
	30	2533	30	4093	129	6173	10	9676	43
	40	2563	30	4222	129	6183	10	9719	43
	50	2593	30	4351	128	6193	10	9762	43
44	0	101·2623	30	2·0054479	129	101·6203	10	2·0069805	43
	10	2653	31	4608	129	6213	10	9848	43
	20	2684	30	4737	129	6223	10	9891	43
	30	2714	30	4866	129	6233	10	9934	43
	40	2744	30	4995	129	6243	10	2·0069977	43
	50	2774	30	5124	129	6253	10	2·0070020	43
45	0	101·2804	30	2·0055253	129	101·6263	10	2·0070063	43
	10	2834	30	5382	129	6273	10	0106	43
	20	2864	30	5511	129	6283	11	0149	43
	30	2894	30	5640	129	6294	10	0192	43
	40	2924	30	5769	129	6304	10	0235	43
	50	2954	30	5898	129	6314	10	0278	43
46	0	101·2984	30	2·0056027	129	101·6324	10	2·0070321	42
	10	3014	30	6156	129	6334	10	0363	43
	20	3044	31	6285	129	6344	10	0406	43
	30	3075	30	6414	129	6354	10	0449	43
	40	3105	30	6543	129	6364	10	0492	43
	50	3135	30	6672	128	6374	10	0535	43
47	0	101·3165	30	2·0056800	129	101·6384	10	2·0070578	43
	10	3195	30	6929	129	6394	10	0621	43
	20	3225	30	7058	128	6404	10	0664	43
	30	3255	30	7186	129	6414	10	0707	43
	40	3285	30	7315	128	6424	10	0750	43
	50	3315	30	7443	129	6434	10	0793	42
48	0	101·3345		2·0057572		101·6444		2·0070835	

TABLE III

Linear value, in feet and its logarithm, of ρ sin 1″ and ν sin 1″
(Clarke 1858)

Latitude		ρ sin 1″	Diff.	Log ρ sin 1″	Diff.	ν sin 1″	Diff.	Log ν sin 1″	Diff.
°	′								
48	0	101·3345	30	2·0057572	128	101·6444	10	2·0070835	43
	10	3375	30	7700	128	6454	10	0878	43
	20	3405	29	7828	128	6464	10	0921	43
	30	3434	30	7956	129	6474	10	0964	42
	40	3464	30	8085	128	6484	10	1006	43
	50	3494	30	8213	127	6494	10	1049	43
49	0	101·3524	30	2·0058340	128	101·6504	10	2·0071092	42
	10	3554	30	8468	128	6514	10	1134	43
	20	3584	29	8596	127	6524	10	1177	42
	30	3613	30	8723	128	6534	10	1219	43
	40	3643	30	8851	127	6544	10	1262	42
	50	3673	30	8978	127	6554	10	1304	42
50	0	101·3703	29	2·0059105	127	101·6564	10	2·0071346	43
	10	3732	30	9232	127	6574	10	1389	42
	20	3762	29	9359	127	6584	10	1431	42
	30	3791	30	9486	127	6594	9	1473	43
	40	3821	30	9613	126	6603	10	1516	42
	50	3851	29	9739	127	6613	10	1558	42
51	0	101·3880	29	2·0059866	126	101·6623	10	2·0071600	42
	10	3909	30	2·0059992	126	6633	10	1642	42
	20	3939	29	2·0060118	126	6643	10	1684	42
	30	3968	30	0244	125	6653	9	1726	42
	40	3998	29	0369	126	6662	10	1768	42
	50	4027	29	0495	125	6672	10	1810	42
52	0	101·4056	29	2·0060620	126	101·6682	10	2·0071852	41
	10	4085	30	0746	125	6692	10	1893	42
	20	4115	29	0871	124	6702	9	1935	42
	30	4144	29	0995	125	6711	10	1977	41
	40	4173	29	1120	125	6721	10	2018	42
	50	4202	29	1245	124	6731	9	2060	41
53	0	101·4231	29	2·0061369	124	101·6740	10	2·0072101	41
	10	4260	29	1493	124	6750	10	2142	42
	20	4289	29	1617	123	6760	9	2184	41
	30	4318	29	1740	124	6769	10	2225	41
	40	4347	28	1864	123	6779	10	2266	41
	50	4375	29	1987	123	6789	9	2307	41
54	0	101·4404		2·0062110		101·6798		2·0072348	

TABLE III

Linear value, in feet and its logarithm, of ρ sin 1″ and ν sin 1″
(Clarke 1858)

Latitude		ρ sin 1″	Diff.	Log ρ sin 1″	Diff.	ν sin 1″	Diff.	Log ν sin 1″	Diff.
°	′								
54	0	101·4404	29	2·0062110	122	101·6798	10	2·0072348	41
	10	4433	28	2232	123	6808	9	2389	41
	20	4461	29	2355	122	6817	10	2430	40
	30	4490	28	2477	122	6827	9	2470	41
	40	4518	29	2599	122	6836	10	2511	41
	50	4547	28	2721	121	6846	9	2552	40
55	0	101·4575	29	2·0062842	122	101·6855	10	2·0072592	41
	10	4604	28	2964	121	6865	9	2633	40
	20	4632	28	3085	120	6874	10	2673	40
	30	4660	28	3205	121	6884	9	2713	40
	40	4688	28	3326	120	6893	10	2753	40
	50	4716	28	3446	120	6903	9	2793	40
56	0	101·4744	28	2·0063566	120	101·6912	9	2·0072833	40
	10	4772	28	3686	119	6921	10	2873	40
	20	4800	28	3805	119	6931	9	2913	40
	30	4828	28	3924	119	6940	9	2953	39
	40	4856	27	4043	118	6949	9	2992	40
	50	4883	28	4161	118	6958	10	3032	39
57	0	101·4911	27	2·0064279	118	101·6968	9	2·0073071	39
	10	4938	28	4397	118	6977	9	3110	40
	20	4966	27	4515	117	6986	9	3150	39
	30	4993	28	4632	117	6995	9	3189	39
	40	5021	27	4749	117	7004	9	3228	39
	50	5048	27	4866	116	7013	9	3267	38
58	0	101·5075	27	2·0064982	116	101·7022	10	2·0073305	39
	10	5102	27	5098	115	7032	9	3344	39
	20	5129	27	5213	116	7041	9	3383	38
	30	5156	27	5329	115	7050	9	3421	38
	40	5183	27	5444	114	7059	8	3459	39
	50	5210	27	5558	115	7067	9	3498	38
59	0	101·5237	26	2·0065673	114	101·7076	9	2·0073536	38
	10	5263	27	5787	113	7085	9	3574	37
	20	5290	26	5900	113	7094	9	3611	38
	30	5316	27	6013	113	7103	9	3649	38
	40	5343	26	6126	113	7112	9	3687	37
	50	5369	26	6239	112	7121	8	3724	38
60	0	101·5395		2·0066351		101·7129		2·0073762	

TABLE III

Linear value, in feet and its logarithm, of ρ sin 1″ and ν sin 1″
(Clarke 1858)

Latitude		ρ sin 1″	Diff.	Log ρ sin 1″	Diff.	ν sin 1″	Diff.	Log ν sin 1″	Diff.
°	′								
60	0	101·5395	26	2·0066351	111	101·7129	9	2·0073762	37
	10	5421	26	6462	112	7138	9	3799	37
	20	5447	26	6574	111	7147	8	3836	37
	30	5473	26	6685	110	7155	9	3873	37
	40	5499	26	6795	111	7164	9	3910	37
	50	5525	26	6906	109	7173	8	3947	36
61	0	101·5551	25	2·0067015	110	101·7181	9	2·0073983	37
	10	5576	26	7125	109	7190	8	4020	36
	20	5602	25	7234	108	7198	9	4056	36
	30	5627	25	7342	109	7207	8	4092	36
	40	5652	26	7451	107	7215	9	4128	36
	50	5678	25	7558	108	7224	8	4164	36
62	0	101·5703	25	2·0067666	107	101·7232	8	2·0074200	36
	10	5728	25	7773	106	7240	9	4236	35
	20	5753	24	7879	106	7249	8	4271	36
	30	5777	25	7985	106	7257	8	4307	35
	40	5802	25	8091	105	7265	8	4342	35
	50	5827	24	8196	105	7273	9	4377	35
63	0	101·5851	25	2·0068301	104	101·7282	8	2·0074412	35
	10	5876	24	8405	104	7290	8	4447	34
	20	5900	24	8509	104	7298	8	4481	35
	30	5924	24	8613	103	7306	8	4516	34
	40	5948	24	8716	102	7314	8	4550	34
	50	5972	24	8818	103	7322	8	4584	34
64	0	101·5996	24	2·0068921	101	101·7330	8	2·0074618	34
	10	6020	24	9022	101	7338	8	4652	34
	20	6044	23	9123	101	7346	8	4686	33
	30	6067	24	9224	100	7354	7	4719	34
	40	6091	23	9324	100	7361	8	4753	33
	50	6114	23	9424	99	7369	8	4786	33
65	0	101·6137	23	2·0069523	99	101·7377	8	2·0074819	33
	10	6160	23	9622	98	7385	7	4852	33
	20	6183	23	9720	98	7392	8	4885	32
	30	6206	23	9818	98	7400	8	4917	33
	40	6229	23	9916	96	7408	7	4950	32
	50	6252	22	2·0070012	97	7415	8	4982	32
66	0	101·6274		2·0070109		101·7423		2·0075014	

TABLE III

Linear value, in feet and its logarithm, of ρ sin 1″ and ν sin 1″
(Clarke 1858)

Latitude		ρ sin 1″	Diff.	Log ρ sin 1″	Diff.	ν sin 1″	Diff.	Log ν sin 1″	Diff.
°	′								
66	0	101·6274	23	2·0070109	96	101·7423	7	2·0075014	32
	10	6297	22	0205	95	7430	8	5046	32
	20	6319	22	0300	95	7438	7	5078	32
	30	6341	22	0395	94	7445	7	5110	31
	40	6363	22	0489	94	7452	8	5141	31
	50	6385	22	0583	93	7460	7	5172	31
67	0	101·6407	22	2·0070676	93	101·7467	7	2·0075203	31
	10	6429	21	0769	92	7474	8	5234	31
	20	6450	22	0861	92	7482	7	5265	31
	30	6472	21	0953	91	7489	7	5296	30
	40	6493	21	1044	91	7496	7	5326	30
	50	6514	21	1135	90	7503	7	5356	30
68	0	101·6535	21	2·0071225	89	101·7510	7	2·0075386	30
	10	6556	21	1314	90	7517	7	5416	30
	20	6577	21	1404	88	7524	7	5446	29
	30	6598	20	1492	88	7531	7	5475	30
	40	6618	21	1580	87	7538	6	5505	29
	50	6639	20	1667	87	7544	7	5534	29
69	0	101·6659	20	2·0071754	86	101·7551	7	2·0075563	29
	10	6679	21	1840	86	7558	7	5592	28
	20	6700	19	1926	85	7565	6	5620	28
	30	6719	20	2011	85	7571	7	5648	29
	40	6739	20	2096	84	7578	7	5677	28
	50	6759	19	2180	83	7585	6	5705	27
70	0	101·6778	20	2·0072263	83	101·7591	6	2·0075732	28
	10	6798	19	2346	82	7597	7	5760	28
	20	6817	19	2428	82	7604	6	5788	27
	30	6836	19	2510	81	7610	7	5815	27
	40	6855	19	2591	81	7617	6	5842	27
	50	6874	19	2672	80	7623	6	5869	26
71	0	101·6893	18	2·0072752	79	101·7629	6	2·0075895	27
	10	6911	19	2831	79	7635	7	5922	26
	20	6930	18	2910	78	7642	6	5948	26
	30	6948	18	2988	77	7648	6	5974	26
	40	6966	18	3065	77	7654	6	6000	26
	50	6984	18	3142	77	7660	6	6026	25
72	0	101·7002		2·0073219		101·7666		2·0076051	

TABLE IIIA

Linear value, in feet and its logarithm, of ρ sin 1″ and ν sin 1″

(Everest's 1st: compression $\frac{1}{300{\cdot}8}$)

Latitude		ρ sin 1″	Diff.	Log ρ sin 1″	Diff.	ν sin 1″	Diff.	Log ν sin 1″	Diff.
°	′								
0	0	100·7639	0	2·0033050	1	101·4372	0	2·0061974	0
	10	7639	0	3051	1	4372	0	1974	1
	20	7639	1	3052	1	4372	1	1975	0
	30	7640	0	3053	3	4373	0	1975	1
	40	7640	1	3056	3	4373	0	1976	1
	50	7641	1	3059	4	4373	0	1977	1
1	0	100·7642	1	2·0033063	5	101·4373	1	2·0061978	2
	10	7643	2	3068	6	4374	0	1980	2
	20	7645	1	3074	6	4374	1	1982	2
	30	7646	2	3080	7	4375	0	1984	2
	40	7648	1	3087	7	4375	1	1986	3
	50	7649	2	3094	9	4376	0	1989	3
2	0	100·7651	2	2·0033103	9	101·4376	1	2·0061992	3
	10	7653	3	3112	10	4377	1	1995	3
	20	7656	2	3122	10	4378	1	1998	3
	30	7658	3	3132	12	4379	1	2001	4
	40	7661	3	3144	12	4380	1	2005	4
	50	7664	3	3156	13	4381	1	2009	5
3	0	100·7667	3	2·0033169	13	101·4382	1	2·0062014	4
	10	7670	3	3182	14	4383	1	2018	5
	20	7673	4	3196	15	4384	1	2023	5
	30	7677	3	3211	16	4385	1	2028	5
	40	7680	4	3227	16	4386	1	2033	5
	50	7684	4	3243	18	4387	2	2038	6
4	0	100·7688	4	2·0033261	17	101·4389	1	2·0062044	6
	10	7692	4	3278	19	4390	2	2050	6
	20	7696	5	3297	19	4392	1	2056	7
	30	7701	5	3316	20	4393	2	2063	6
	40	7706	4	3336	21	4395	1	2069	7
	50	7710	5	3357	22	4396	2	2076	8
5	0	100·7715	5	2·0033379	22	101·4398	2	2·0062084	7
	10	7720	6	3401	23	4400	1	2091	8
	20	7726	5	3424	23	4401	2	2099	7
	30	7731	6	3447	25	4403	2	2106	9
	40	7737	6	3472	25	4405	2	2115	8
	50	7743	6	3497	26	4407	2	2123	9
6	0	100·7749		2·0033523		101·4409		2·0062132	

TABLE IIIA

Linear value, in feet and its logarithm, of ρ sin 1″ and ν sin 1″
(Everest's 1st)

Latitude		ρ sin 1″	Diff.	Log ρ sin 1″	Diff.	ν sin 1″	Diff.	Log ν sin 1″	Diff.
°	′								
6	0	100·7749	6	2·0033523	26	101·4409	2	2·0062132	8
	10	7755	6	3549	27	4411	2	2140	9
	20	7761	7	3576	28	4413	2	2149	10
	30	7768	6	3604	29	4415	3	2159	9
	40	7774	7	3633	29	4418	2	2168	10
	50	7781	7	3662	30	4420	2	2178	10
7	0	100·7788	7	2·0033692	31	101·4422	3	2·0062188	10
	10	7795	8	3723	32	4425	2	2198	11
	20	7803	7	3755	32	4427	3	2209	11
	30	7810	8	3787	33	4430	2	2220	11
	40	7818	8	3820	33	4432	3	2231	11
	50	7826	7	3853	35	4435	3	2242	11
8	0	100·7833	9	2·0033888	35	101·4438	2	2·0062253	12
	10	7842	8	3923	36	4440	3	2265	12
	20	7850	8	3959	36	4443	3	2277	12
	30	7858	9	3995	37	4446	3	2289	12
	40	7867	9	4032	38	4449	3	2301	13
	50	7876	9	4070	38	4452	3	2314	13
9	0	100·7885	9	2·0034108	40	101·4455	3	2·0062327	13
	10	7894	9	4148	40	4458	3	2340	13
	20	7903	9	4188	40	4461	3	2353	14
	30	7912	10	4228	41	4464	3	2367	14
	40	7922	10	4269	42	4467	4	2381	14
	50	7932	10	4311	43	4471	3	2395	14
10	0	100·7942	10	2·0034354	44	101·4474	3	2·0062409	14
	10	7952	10	4398	44	4477	4	2423	15
	20	7962	10	4442	44	4481	3	2438	15
	30	7972	11	4486	46	4484	4	2453	15
	40	7983	11	4532	46	4488	3	2468	15
	50	7994	11	4578	47	4491	4	2483	16
11	0	100·8005	11	2·0034625	47	101·4495	4	2·0062499	16
	10	8016	11	4672	48	4499	3	2515	16
	20	8027	11	4720	49	4502	4	2531	16
	30	8038	12	4769	50	4506	4	2547	17
	40	8050	11	4819	50	4510	4	2564	16
	50	8061	12	4869	51	4514	4	2580	17
12	0	100·8073		2·0034920		101·4518		2·0062597	

TABLE IIIA

Linear value, in feet and its logarithm, of ρ sin 1″ and ν sin 1″
(Everest's 1st)

Latitude		ρ sin 1″	Diff.	Log ρ sin 1″	Diff.	ν sin 1″	Diff.	Log ν sin 1″	Diff
°	′								
12	0	100·8073	12	2·0034920	51	101·4518	4	2·0062597	17
	10	8085	12	4971	52	4522	4	2614	18
	20	8097	12	5023	53	4526	4	2632	17
	30	8109	13	5076	54	4530	4	2649	18
	40	8122	12	5130	54	4534	4	2667	18
	50	8134	13	5184	55	4538	5	2685	19
13	0	100·8147	13	2·0035239	55	101·4543	4	2·0062704	18
	10	8160	13	5294	56	4547	4	2722	19
	20	8173	13	5350	57	4551	5	2741	19
	30	8186	13	5407	58	4556	4	2760	19
	40	8199	14	5465	58	4560	5	2779	19
	50	8213	14	5523	58	4565	4	2798	20
14	0	100·8227	13	2·0035581	60	101·4569	5	2·0062818	20
	10	8240	14	5641	60	4574	5	2838	20
	20	8254	14	5701	61	4579	4	2858	20
	30	8268	15	5762	61	4583	5	2878	20
	40	8283	14	5823	62	4588	5	2898	21
	50	8297	15	5885	62	4593	5	2919	21
15	0	100·8312	14	2·0035947	64	101·4598	5	2·0062940	21
	10	8326	15	6011	64	4603	5	2961	21
	20	8341	15	6075	64	4608	5	2982	22
	30	8356	15	6139	65	4613	5	3004	21
	40	8371	15	6204	66	4618	5	3025	22
	50	8386	16	6270	66	4623	5	3047	22
16	0	100·8402	15	2·0036336	67	101·4628	5	2·0063069	23
	10	8417	16	6403	68	4633	6	3092	22
	20	8433	16	6471	68	4639	5	3114	23
	30	8449	16	6539	69	4644	5	3137	23
	40	8465	16	6608	69	4649	6	3160	23
	50	8481	16	6677	71	4655	5	3183	24
17	0	100·8497	17	2·0036748	70	101·4660	6	2·0063207	23
	10	8514	16	6818	72	4666	5	3230	24
	20	8530	17	6890	71	4671	6	3254	24
	30	8547	17	6961	73	4677	6	3278	24
	40	8564	17	7034	73	4683	5	3302	24
	50	8581	17	7107	74	4688	6	3326	25
18	0	100·8598		2·0037181		101·4694		2·0063351	

TABLE IIIA

Linear value, in feet and its logarithm, of ρ sin 1″ and ν sin 1″
(Everest's 1st)

Latitude		ρ sin 1″	Diff.	Log ρ sin 1″	Diff.	ν sin 1″	Diff.	Log ν sin 1″	Diff.
°	′								
18	0	100·8598	17	2·0037181	74	101·4694	6	2·0063351	25
	10	8615	18	7255	75	4700	6	3376	25
	20	8633	17	7330	75	4706	5	3401	25
	30	8650	18	7405	76	4711	6	3426	25
	40	8668	18	7481	77	4717	6	3451	26
	50	8686	17	7558	77	4723	6	3477	25
19	0	100·8703	19	2·0037635	78	101·4729	6	2·0063502	26
	10	8722	18	7713	78	4735	7	3528	26
	20	8740	18	7791	79	4742	6	3554	27
	30	8758	19	7870	80	4748	6	3581	26
	40	8777	18	7950	80	4754	6	3607	27
	50	8795	19	8030	80	4760	6	3634	27
20	0	100·8814	19	2·0038110	82	101·4766	7	2·0063661	27
	10	8833	19	8192	81	4773	6	3688	27
	20	8852	19	8273	83	4779	7	3715	28
	30	8871	19	8356	83	4786	6	3743	27
	40	8890	19	8439	83	4792	6	3770	28
	50	8909	20	8522	84	4798	7	3798	28
21	0	100·8929	20	2·0038606	84	101·4805	7	2·0063826	28
	10	8949	19	8690	85	4812	6	3854	29
	20	8968	20	8775	86	4818	7	3883	28
	30	8988	20	8861	86	4825	7	3911	29
	40	9008	20	8947	87	4832	6	3940	29
	50	9028	21	9034	87	4838	7	3969	29
22	0	100·9049	20	2·0039121	88	101·4845	7	2·0063998	29
	10	9069	21	9209	88	4852	7	4027	29
	20	9090	20	9297	89	4859	7	4056	30
	30	9110	21	9386	89	4866	7	4086	30
	40	9131	21	9475	90	4873	7	4116	30
	50	9152	21	9565	90	4880	7	4146	30
23	0	100·9173	21	2·0039655	91	101·4887	7	2·0064176	30
	10	9194	21	9746	91	4894	7	4206	30
	20	9215	21	9837	92	4901	7	4236	31
	30	9236	22	2·0039929	93	4908	7	4267	31
	40	9258	21	2·0040022	92	4915	7	4298	31
	50	9279	22	0114	94	4922	8	4329	31
24	0	100·9301		2·0040208		101·4930		2·0064360	

TABLE IIIA

Linear value, in feet and its logarithm, of ρ sin 1″ and ν sin 1″
(Everest's 1st)

Latitude		ρ sin 1″	Diff.	Log ρ sin 1″	Diff.	ν sin 1″	Diff.	Log ν sin 1″	Diff.
°	′								
24	0	100·9301	22	2·0040208	94	101·4930	7	2·0064360	31
	10	9323	22	0302	94	4937	7	4391	32
	20	9345	22	0396	95	4944	8	4423	31
	30	9367	22	0491	95	4952	7	4454	32
	40	9389	22	0586	96	4959	8	4486	32
	50	9411	23	0682	96	4967	7	4518	32
25	0	100·9434	22	2·0040778	97	101·4974	8	2·0064550	32
	10	9456	23	0875	97	4982	7	4582	33
	20	9479	22	0972	97	4989	8	4615	32
	30	9501	23	1069	99	4997	8	4647	33
	40	9524	23	1168	98	5005	7	4680	33
	50	9547	23	1266	99	5012	8	4713	33
26	0	100·9570	23	2·0041365	100	101·5020	8	2·0064746	33
	10	9593	24	1465	100	5028	7	4779	33
	20	9617	23	1565	100	5035	8	4812	34
	30	9640	23	1665	101	5043	8	4846	33
	40	9663	24	1766	101	5051	8	4879	34
	50	9687	24	1867	102	5059	8	4913	34
27	0	100·9711	23	2·0041969	102	101·5067	8	2·0064947	34
	10	9734	24	2071	102	5075	8	4981	34
	20	9758	24	2173	103	5083	8	5015	34
	30	9782	24	2276	104	5091	8	5049	35
	40	9806	24	2380	104	5099	8	5084	35
	50	9830	24	2484	104	5107	8	5119	34
28	0	100·9854	25	2·0042588	104	101·5115	8	2·0065153	35
	10	9879	24	2692	105	5123	9	5188	35
	20	9903	25	2797	106	5132	8	5223	35
	30	9928	24	2903	106	5140	8	5258	36
	40	9952	25	3009	106	5148	8	5294	35
	50	100·9977	25	3115	107	5156	9	5329	36
29	0	101·0002	25	2·0043222	107	101·5165	8	2·0065365	35
	10	0027	25	3329	107	5173	8	5400	36
	20	0052	25	3436	108	5181	9	5436	36
	30	0077	25	3544	108	5190	8	5472	36
	40	0102	25	3652	109	5198	9	5508	36
	50	0127	26	3761	109	5207	8	5544	37
30	0	101·0153		2·0043870		101·5215		2·0065581	

TABLE IIIB

Linear value, in feet and its logarithm, of ρ sin 1″ and ν sin 1″

(Airy: compression $\frac{1}{299{\cdot}3}$)

Latitude		ρ sin 1″	Diff.	Log ρ sin 1″	Diff.	ν sin 1″	Diff.	Log ν sin 1″	Diff.
°	′								
49	0	101·3414	29	2·0057868	125	101·6343	10	2·0070402	42
	10	3443	29	7993	126	6353	9	0444	42
	20	3472	30	8119	125	6362	10	0486	42
	30	3502	29	8244	125	6372	10	0528	42
	40	3531	29	8369	126	6382	10	0570	41
	50	3560	29	8495	125	6392	10	0611	42
50	0	101·3589	29	2·0058620	125	101·6402	9	2·0070653	42
	10	3618	29	8745	124	6411	10	0695	41
	20	3647	30	8869	125	6421	10	0736	42
	30	3677	29	8994	125	6431	9	0778	41
	40	3706	29	9119	124	6440	10	0819	42
	50	3735	29	9243	124	6450	10	0861	41
51	0	101·3764	29	2·0059367	124	101·6460	9	2·0070902	41
	10	3793	29	9491	124	6469	10	0943	42
	20	3822	28	9615	124	6479	10	0985	41
	30	3850	29	9739	123	6489	9	1026	41
	40	3879	29	9862	124	6498	10	1067	41
	50	3908	29	2·0059986	123	6508	10	1108	41
52	0	101·3937	29	2·0060109	123	101·6518	9	2·0071149	41
	10	3966	28	0232	123	6527	10	1190	41
	20	3994	29	0355	123	6537	9	1231	41
	30	4023	29	0478	122	6546	10	1272	41
	40	4052	28	0600	123	6556	10	1313	41
	50	4080	29	0723	122	6566	9	1354	41
53	0	101·4109	28	2·0060845	122	101·6575	10	2·0071395	40
	10	4137	29	0967	121	6585	9	1435	41
	20	4166	28	1088	122	6594	10	1476	40
	30	4194	28	1210	121	6604	9	1516	41
	40	4222	28	1331	121	6613	10	1557	40
	50	4250	29	1452	121	6623	9	1597	40
54	0	101·4279	28	2·0061573	121	101·6632	9	2·0071637	41
	10	4307	28	1694	120	6641	10	1678	40
	20	4335	28	1814	120	6651	9	1718	40
	30	4363	28	1934	120	6660	9	1758	40
	40	4391	28	2054	120	6669	10	1798	40
	50	4419	28	2174	119	6679	9	1838	40
55	0	101·4447		2·0062293		101·6688		2·0071878	

TABLE IIIB

Linear value, in feet and its logarithm, of ρ sin 1″ and ν sin 1″
(Airy)

Latitude		ρ sin 1″	Diff.	Log ρ sin 1″	Diff.	ν sin 1″	Diff.	Log ν sin 1″	Diff.
°	′								
55	0	101·4447	28	2·0062293	120	101·6688	9	2·0071878	39
	10	4475	28	2413	118	6697	10	1917	40
	20	4503	27	2531	119	6707	9	1957	39
	30	4530	28	2650	119	6716	9	1996	40
	40	4558	28	2769	118	6725	9	2036	39
	50	4586	27	2887	118	6734	10	2075	40
56	0	101·4613	28	2·0063005	117	101·6744	9	2·0072115	39
	10	4641	27	3122	118	6753	9	2154	39
	20	4668	27	3240	117	6762	9	2193	39
	30	4695	28	3357	116	6771	9	2232	39
	40	4723	27	3473	117	6780	9	2271	39
	50	4750	27	3590	116	6789	9	2310	38
57	0	101·4777	27	2·0063706	116	101·6798	9	2·0072348	39
	10	4804	27	3822	115	6807	9	2387	39
	20	4831	27	3937	116	6816	9	2426	38
	30	4858	27	4053	115	6825	9	2464	38
	40	4885	27	4168	114	6834	9	2502	38
	50	4912	26	4282	115	6843	9	2540	39
58	0	101·4938	27	2·0064397	114	101·6852	9	2·0072579	38
	10	4965	27	4511	113	6861	9	2617	37
	20	4992	26	4624	114	6870	9	2654	38
	30	5018	26	4738	113	6879	9	2692	38
	40	5044	27	4851	112	6888	9	2730	37
	50	5071	26	4963	113	6897	8	2767	38
59	0	101·5097	26	2·0065076	112	101·6905	9	2·0072805	37
	10	5123	26	5188	111	6914	9	2842	37
	20	5149	26	5299	111	6923	8	2879	38
	30	5175	26	5410	111	6931	9	2917	37
	40	5201	26	5521	111	6940	9	2954	36
	50	5227	26	5632	110	6949	8	2990	37
60	0	101·5253	26	2·0065742	110	101·6957	9	2·0073027	37
	10	5279	25	5852	109	6966	8	3064	36
	20	5304	26	5961	110	6974	9	3100	37
	30	5330	25	6071	108	6983	8	3137	36
	40	5355	25	6179	109	6991	9	3173	36
	50	5380	26	6288	108	7000	8	3209	36
61	0	101·5406		2·0066396		101·7008		2·0073245	

TABLE IV

Values of Log $\frac{1}{6\nu^2}$ and Log $\frac{1}{24\nu^4}$. (Clarke 1858)

(These values may be used with other spheroids without appreciable error.)

Latitude	Log $\frac{1}{6\nu^2}$	Diff.	Log $\frac{1}{24\nu^4}$	Diff.
°				
1	$\overline{16}$·58046	1	$\overline{31}$·33701	1
3	045	1	700	3
5	044	2	697	4
7	042	3	693	6
9	039	4	687	7
11	035	4	680	8
13	031	5	672	10
15	026	5	662	11
17	021	6	651	12
19	015	7	639	13
21	008	7	626	15
23	001	8	611	15
25	·57993	8	596	16
27	985	8	580	17
29	977	9	563	18
31	968	9	545	18
33	959	10	527	19
35	949	10	508	20
37	939	10	488	20
39	929	10	468	21
41	919	10	447	20
43	909	10	427	21
45	899	11	406	20
47	888	10	386	21
49	878	10	365	20
51	868	10	345	20
53	858	10	325	20
55	848	10	305	19
57	838	9	286	18
59	829	9	268	18
61	820	8	250	17
63	812	8	233	17
65	804	8	216	15
67	796	7	201	15
69	789		186	

The formulæ for conversion of Cassini Eastings to Transverse Mercator Eastings and *vice versa* are :—

$$M = C + \frac{C^3}{6\nu^2} + \frac{C^5}{24\nu^4} + \cdots\cdot$$

$$C = M - \frac{M^3}{6\nu^2} + \frac{M^5}{24\nu^4} - \cdots\cdot$$

where ν is the normal at the Latitude of the point under consideration—

C is the True Cassini Easting.

M is the True Transverse Mercator Easting.

TABLE V

Logarithms for facilitating Computation of Latitudes, Longitudes, Reverse Azimuths and Transformation of Co-ordinates.

(Based on Clarke's 1858 figure ; compression $\frac{1}{294{\cdot}3}$)

Latitude		P=1/ρ sin 1″ log P	Diff.	Q=1/ν sin 1″ log Q	Diff.	Log $\frac{\tan\phi}{2\rho\nu \sin 1''}$	Diff.
°	′						
0	0	$\bar{3}{\cdot}9966885$	1	$\bar{3}{\cdot}9937317$	0	$\bar{13}{\cdot}83869$	30103
	10	6884	1	7317	1	$\bar{12}{\cdot}13972$	17610
	20	6883	2	7316	0	31582	12495
	30	6881	2	7316	1	44077	9692
	40	6879	4	7315	1	53769	7919
	50	6875	4	7314	2		
1	0	$\bar{3}{\cdot}9966871$	5	$\bar{3}{\cdot}9937312$	1	$\bar{12}{\cdot}61688$	6697
	10	6866	5	7311	2	68385	5801
	20	6861	6	7309	2	74186	5117
	30	6855	8	7307	3	79303	4578
	40	6847	7	7304	2	83881	4142
	50	6840	9	7302	3	88023	3781
2	0	$\bar{3}{\cdot}9966831$	9	$\bar{3}{\cdot}9937299$	3	$\bar{12}{\cdot}91804$	3480
	10	6822	10	7296	3	95284	3221
	20	6812	11	7293	4	$\bar{12}{\cdot}98505$	3000
	30	6801	12	7289	4	$\bar{11}{\cdot}01505$	2806
	40	6789	12	7285	4	04311	2637
	50	6777	13	7281	4	06948	2486
3	0	$\bar{3}{\cdot}9966764$	14	$\bar{3}{\cdot}9937277$	5	$\bar{11}{\cdot}09434$	2353
	10	6750	15	7272	5	11787	2232
	20	6735	15	7267	5	14019	2124
	30	6720	16	7262	5	16143	2026
	40	6704	17	7257	6	18169	1935
	50	6687	17	7251	6	20104	1854
4	0	$\bar{3}{\cdot}9966670$	19	$\bar{3}{\cdot}9937245$	6	$\bar{11}{\cdot}21958$	1779
	10	6651	19	7239	6	23737	1709
	20	6632	19	7233	7	25446	1646
	30	6613	21	7226	7	27092	1585
	40	6592	21	7219	7	28677	1531
	50	6571	22	7212	7	30208	1479
5	0	$\bar{3}{\cdot}9966549$	23	$\bar{3}{\cdot}9937205$	8	$\bar{11}{\cdot}31687$	1432
	10	6526	23	7197	7	33119	1386
	20	6503	24	7190	8	34505	1344
	30	6479	25	7182	9	35849	1304
	40	6454	26	7173	8	37153	1267
	50	6428	26	7165	9	38420	1233
6	0	$\bar{3}{\cdot}9966402$		$\bar{3}{\cdot}9937156$		$\bar{11}{\cdot}39653$	

TABLE V

Logarithms for facilitating Computation of Latitudes, Longitudes, Reverse Azimuths and Transformation of Co-ordinates.

(Clarke's 1858 figure.)

Latitude		P=1/ρ sin 1″ log P	Diff.	O=1/ν sin 1″ log O	Diff.	Log $\frac{\tan\phi}{2\rho\nu \sin 1''}$	Diff.
°	′						
6	0	$\bar{3}$·9966402	27	$\bar{3}$·9937156	9	$\bar{11}$·39653	1197
	10	6375	28	7147	9	40850	1168
	20	6347	29	7138	10	42018	1137
	30	6318	29	7128	10	43155	1108
	40	6289	30	7118	10	44263	1083
	50	6259	31	7108	10	45346	1056
7	0	$\bar{3}$·9966228	31	$\bar{3}$·9937098	10	$\bar{11}$·46402	1032
	10	6197	32	7088	11	47434	1009
	20	6165	33	7077	11	48443	987
	30	6132	34	7066	11	49430	964
	40	6098	34	7055	12	50394	946
	50	6064	35	7043	11	51340	926
8	0	$\bar{3}$·9966029	36	$\bar{3}$·9937032	12	$\bar{11}$·52266	906
	10	5993	37	7020	13	53172	889
	20	5956	37	7007	12	54061	873
	30	5919	38	6995	13	54934	855
	40	5881	39	6982	13	55789	840
	50	5842	39	6969	13	56629	825
9	0	$\bar{3}$·9965803	40	$\bar{3}$·9936956	13	$\bar{11}$·57454	809
	10	5763	41	6943	14	58263	796
	20	5722	41	6929	14	59059	782
	30	5681	43	6915	14	59841	769
	40	5638	43	6901	14	60610	756
	50	5595	43	6887	14	61366	745
10	0	$\bar{3}$·9965552	45	$\bar{3}$·9936873	15	$\bar{11}$·62111	732
	10	5507	45	6858	15	62843	721
	20	5462	45	6843	15	63564	710
	30	5417	47	6828	16	64274	699
	40	5370	47	6812	16	64973	689
	50	5323	48	6796	16	65662	678
11	0	$\bar{3}$·9965275	48	$\bar{3}$·9936780	16	$\bar{11}$·66340	670
	10	5227	49	6764	16	67010	659
	20	5178	50	6748	17	67669	651
	30	5128	51	6731	17	68320	641
	40	5077	51	6714	17	68961	633
	50	5026	52	6697	17	69594	624
12	0	$\bar{3}$·9964974		$\bar{3}$·9936680		$\bar{11}$·70218	

TABLE V

Logarithms for facilitating Computation of Latitudes, Longitudes, Reverse Azimuths and Transformation of Co-ordinates.

(Clarke's 1858 figure.)

Latitude		P=1/ρ sin 1″ log P	Diff.	O= 1/ν sin 1″ log O	Diff.	Log $\frac{\tan \phi}{2\rho\nu \sin 1''}$	Diff.
°	′						
12	0	$\bar{3}$·9964974	53	$\bar{3}$·9936680	18	$\bar{11}$·70218	617
	10	4921	53	6662	17	70835	608
	20	4868	54	6645	18	71443	602
	30	4814	55	6627	19	72045	593
	40	4759	55	6608	18	72638	586
	50	4704	56	6590	19	73224	579
13	0	$\bar{3}$·9964648	57	$\bar{3}$·9936571	19	$\bar{11}$·73803	572
	10	4591	57	6552	19	74375	566
	20	4534	58	6533	19	74941	559
	30	4476	59	6514	20	75500	553
	40	4417	60	6494	20	76053	546
	50	4357	60	6474	20	76599	540
14	0	$\bar{3}$·9964297	60	$\bar{3}$·9936454	20	$\bar{11}$·77139	535
	10	4237	62	6434	20	77674	528
	20	4175	62	6414	21	78202	524
	30	4113	62	6393	21	78726	517
	40	4051	64	6372	21	79243	512
	50	3987	64	6351	21	79755	508
15	0	$\bar{3}$·9963923	64	$\bar{3}$·9936330	22	$\bar{11}$·80263	501
	10	3859	66	6308	22	80764	497
	20	3793	66	6286	22	81261	492
	30	3727	66	6264	22	81753	487
	40	3661	67	6242	22	82240	483
	50	3594	68	6220	23	82723	479
16	0	$\bar{3}$·9963526	69	$\bar{3}$·9936197	23	$\bar{11}$·83202	473
	10	3457	69	6174	23	83675	469
	20	3388	70	6151	23	84144	466
	30	3318	70	6128	23	84610	460
	40	3248	71	6105	24	85070	457
	50	3177	72	6081	24	85527	453
17	0	$\bar{3}$·9963105	72	$\bar{3}$·9936057	24	$\bar{11}$·85980	449
	10	3033	73	6033	24	86429	445
	20	2960	73	6009	25	86874	441
	30	2887	74	5984	24	87315	439
	40	2813	75	5960	25	87754	433
	50	2738	75	5935	25	88187	431
18	0	$\bar{3}$·9962663		$\bar{3}$·9935910		$\bar{11}$·88618	

TABLE V

Logarithms for facilitating Computation of Latitudes, Longitudes, Reverse Azimuths and Transformation of Co-ordinates.

(Clarke's 1858 figure.)

Latitude		P= 1/ρ sin 1″ log P	Diff.	Q=1/ν sin 1″ log Q	Diff.	Log $\frac{\tan\phi}{2\rho\nu \sin 1''}$	Diff.
°	′						
18	0	$\bar{3}$·9962663	76	$\bar{3}$·9935910	26	$\bar{11}$·88618	427
	10	2587	77	5884	26	89045	424
	20	2510	77	5858	25	89469	420
	30	2433	78	5833	26	89889	417
	40	2355	78	5807	26	90306	414
	50	2277	79	5781	26	90720	411
19	0	$\bar{3}$·9962198	79	$\bar{3}$·9935755	27	$\bar{11}$·91131	408
	10	2119	81	5728	27	91539	405
	20	2038	80	5701	27	91944	402
	30	1958	82	5674	27	92346	399
	40	1876	81	5647	27	92745	396
	50	1795	83	5620	27	93141	393
20	0	$\bar{3}$·9961712	83	$\bar{3}$·9935593	28	$\bar{11}$·93534	391
	10	1629	83	5565	28	93925	388
	20	1546	85	5537	28	94313	385
	30	1461	84	5509	28	94698	382
	40	1377	86	5481	28	95080	380
	50	1291	85	5453	29	95460	379
21	0	$\bar{3}$·9961206	87	$\bar{3}$·9935424	29	$\bar{11}$·95839	375
	10	1119	87	5395	29	96214	373
	20	1032	87	5366	29	96587	370
	30	0945	88	5337	29	96957	368
	40	0857	89	5308	30	97325	366
	50	0768	89	5278	30	97691	363
22	0	$\bar{3}$·9960679	90	$\bar{3}$·9935248	30	$\bar{11}$·98054	362
	10	0589	90	5218	30	98416	360
	20	0499	91	5188	30	98776	357
	30	0408	91	5158	30	99133	355
	40	0317	92	5128	31	99488	353
	50	0225	92	5097	31	$\bar{11}$·99841	351
23	0	$\bar{3}$·9960133	93	$\bar{3}$·9935066	31	$\bar{10}$·00192	349
	10	$\bar{3}$·9960040	93	5035	31	00541	347
	20	$\bar{3}$·9959947	94	5004	31	00888	345
	30	9853	94	4973	31	01233	343
	40	9759	95	4942	32	01576	342
	50	9664	96	4910	32	01918	339
24	0	$\bar{3}$·9959568		$\bar{3}$·9934878		$\bar{10}$·02257	

TABLE V

Logarithms for facilitating Computation of Latitudes, Longitudes, Reverse Azimuths and Transformation of Co-ordinates.

(Clarke's 1858 figure.)

Latitude		P=1/ρ sin 1″ log P	Diff.	O=1/ν sin 1″ log O	Diff.	Log $\frac{\tan\phi}{2\rho\nu \sin 1''}$	Diff.
°	′						
24	0	$\bar{3}$·9959568	96	$\bar{3}$·9934878	32	$\bar{1}$0·02257	338
	10	9472	96	4846	32	02595	336
	20	9376	97	4814	32	02931	334
	30	9279	97	4782	33	03265	333
	40	9182	98	4749	32	03598	331
	50	9084	99	4717	33	03929	329
25	0	$\bar{3}$·9958985	98	$\bar{3}$·9934684	33	$\bar{1}$0·04258	328
	10	8887	100	4651	33	04586	326
	20	8787	100	4618	34	04912	324
	30	8687	100	4584	33	05236	323
	40	8587	101	4551	34	05559	322
	50	8486	101	4517	33	05881	320
26	0	$\bar{3}$·9958385	102	$\bar{3}$·9934484	34	$\bar{1}$0·06201	319
	10	8283	102	4450	34	06520	317
	20	8181	102	4416	34	06837	315
	30	8079	103	4382	35	07152	315
	40	7976	104	4347	34	07467	313
	50	7872	104	4313	35	07780	311
27	0	$\bar{3}$·9957768	104	$\bar{3}$·9934278	35	$\bar{1}$0·08091	311
	10	7664	105	4243	35	08402	309
	20	7559	105	4208	35	08711	307
	30	7454	106	4173	35	09018	307
	40	7348	106	4138	35	09325	305
	50	7242	107	4103	36	09630	304
28	0	$\bar{3}$·9957135	107	$\bar{3}$·9934067	36	$\bar{1}$0·09934	303
	10	7028	107	4031	35	10237	302
	20	6921	108	3996	36	10539	300
	30	6813	108	3960	36	10839	299
	40	6705	109	3924	37	11138	298
	50	6596	109	3887	36	11436	297
29	0	$\bar{3}$·9956487	109	$\bar{3}$·9933851	36	$\bar{1}$0·11733	296
	10	6378	110	3815	37	12029	295
	20	6268	110	3778	37	12324	294
	30	6158	111	3741	37	12618	293
	40	6047	111	3704	37	12911	291
	50	5936	111	3667	37	13202	291
30	0	$\bar{3}$·9955825		$\bar{3}$·9933630		$\bar{1}$0·13493	

TABLE V

Logarithms for facilitating Computation of Latitudes, Longitudes, Reverse Azimuths and Transformation of Co-ordinates.

(Clarke's 1858 figure.)

Latitude		P=1/ρ sin 1″ log P	Diff.	Q=1/ν sin 1″ log Q	Diff.	Log $\frac{\tan \phi}{2\rho\nu \sin 1''}$	Diff.
°	′						
30	0	$\bar{3}$·9955825	112	$\bar{3}$·9933630	37	$\bar{1}$0·13493	290
	10	5713	112	3593	37	13783	289
	20	5601	112	3556	38	14072	287
	30	5489	113	3518	37	14359	287
	40	5376	113	3481	38	14646	286
	50	5263	114	3443	38	14932	285
31	0	$\bar{3}$·9955149	114	$\bar{3}$·9933405	38	$\bar{1}$0·15217	285
	10	5035	114	3367	38	15502	283
	20	4921	115	3329	38	15785	283
	30	4806	115	3291	39	16068	281
	40	4691	115	3252	38	16349	281
	50	4576	115	3214	38	16630	280
32	0	$\bar{3}$·9954461	116	$\bar{3}$·9933176	39	$\bar{1}$0·16910	279
	10	4345	117	3137	39	17189	279
	20	4228	116	3098	39	17468	277
	30	4112	117	3059	39	17745	276
	40	3995	117	3020	39	18021	277
	50	3878	118	2981	39	18298	275
33	0	$\bar{3}$·9953760	118	$\bar{3}$·9932942	39	$\bar{1}$0·18573	275
	10	3642	118	2903	40	18848	274
	20	3524	118	2863	39	19122	273
	30	3406	119	2824	40	19395	272
	40	3287	119	2784	39	19667	272
	50	3168	119	2745	40	19939	272
34	0	$\bar{3}$·9953049	120	$\bar{3}$·9932705	40	$\bar{1}$0·20211	271
	10	2929	120	2665	40	20482	270
	20	2809	120	2625	40	20752	269
	30	2689	121	2585	40	21021	268
	40	2568	120	2545	40	21289	269
	50	2448	121	2505	41	21558	267
35	0	$\bar{3}$·9952327	121	$\bar{3}$·9932464	40	$\bar{1}$0·21825	267
	10	2206	122	2424	41	22092	266
	20	2084	122	2383	40	22358	266
	30	1962	122	2343	41	22624	266
	40	1840	122	2302	41	22890	265
	50	1718	122	2261	40	23155	263
36	0	$\bar{3}$·9951596		$\bar{3}$·9932221		$\bar{1}$0·23418	

TABLE V

Logarithms for facilitating Computation of Latitudes, Longitudes, Reverse Azimuths and Transformation of Co-ordinates.

(Clarke's 1858 figure.)

Latitude		P=1/ρ sin 1″ log P	Diff.	O= 1/ν sin 1″ log O	Diff.	Log $\frac{\tan\phi}{2\rho\nu \sin 1''}$	Diff.
°	′						
36	0	$\bar{3}$·9951596	123	$\bar{3}$·9932221	41	$\bar{10}$·23418	264
	10	1473	123	2180	41	23682	264
	20	1350	123	2139	41	23946	263
	30	1227	123	2098	41	24209	262
	40	1104	124	2057	42	24471	262
	50	0980	124	2015	41	24733	261
37	0	$\bar{3}$·9950856	124	$\bar{3}$·9931974	41	$\bar{10}$·24994	261
	10	0732	124	1933	42	25255	261
	20	0608	124	1891	41	25516	260
	30	0484	125	1850	42	25776	260
	40	0359	125	1808	41	26036	259
	50	0234	125	1767	42	26295	259
38	0	$\bar{3}$·9950109	125	$\bar{3}$·9931725	42	$\bar{10}$·26554	258
	10	$\bar{3}$·9949984	125	1683	41	26812	258
	20	9859	126	1642	42	27070	259
	30	9733	126	1600	42	27329	257
	40	9607	125	1558	42	27586	257
	50	9482	126	1516	42	27843	257
39	0	$\bar{3}$·9949356	127	$\bar{3}$·9931474	42	$\bar{10}$·28100	256
	10	9229	126	1432	42	28356	257
	20	9103	126	1390	42	28613	255
	30	8977	127	1348	43	28868	256
	40	8850	127	1305	42	29124	255
	50	8723	127	1263	42	29379	255
40	0	$\bar{3}$·9948596	127	$\bar{3}$·9931221	43	$\bar{10}$·29634	255
	10	8469	127	1178	42	29889	254
	20	8342	127	1136	42	30143	255
	30	8215	128	1094	43	30398	253
	40	8087	127	1051	42	30651	254
	50	7960	128	1009	43	30905	254
41	0	$\bar{3}$·9947832	128	$\bar{3}$·9930966	43	$\bar{10}$·31159	254
	10	7704	127	0923	42	31413	252
	20	7577	128	0881	43	31665	253
	30	7449	128	0838	42	31918	253
	40	7321	129	0796	43	32171	252
	50	7192	128	0753	43	32423	253
42	0	$\bar{3}$·9947064		$\bar{3}$·9930710		$\bar{10}$·32676	

TABLE V

Logarithms for facilitating Computation of Latitudes, Longitudes, Reverse Azimuths and Transformation of Co-ordinates.

(Clarke's 1858 figure.)

Latitude		$P=1/\rho \sin 1''$ log P	Diff.	$O=1/\nu \sin 1''$ log O	Diff.	Log $\frac{\tan \phi}{2\rho\nu \sin 1''}$	Diff.
°	′						
42	0	$\bar{3}$·9947064	128	$\bar{3}$·9930710	43	$\bar{1}$0·32676	252
	10	6936	129	0667	43	32928	251
	20	6807	128	0624	42	33179	253
	30	6679	128	0582	43	33432	252
	40	6551	129	0539	43	33684	252
	50	6422	129	0496	43	33936	251
43	0	$\bar{3}$·9946293	128	$\bar{3}$·9930453	43	$\bar{1}$0·34187	252
	10	6165	129	0410	43	34439	252
	20	6036	129	0367	43	34691	251
	30	5907	129	0324	43	34942	251
	40	5778	129	0281	43	35193	251
	50	5649	129	0238	43	35444	252
44	0	$\bar{3}$·9945520	128	$\bar{3}$·9930195	42	$\bar{1}$0·35696	250
	10	5392	129	0153	43	35946	251
	20	5263	129	0110	43	36197	252
	30	5134	129	0067	43	36449	251
	40	5005	129	0024	43	36700	250
	50	4876	129	$\bar{3}$·9929981	43	36950	251
45	0	$\bar{3}$·9944747	129	$\bar{3}$·9929938	43	$\bar{1}$0·37201	252
	10	4618	129	9895	43	37453	250
	20	4489	129	9852	43	37703	251
	30	4360	129	9809	43	37954	251
	40	4231	129	9766	43	38205	251
	50	4102	129	9723	43	38456	251
46	0	$\bar{3}$·9943973	129	$\bar{3}$·9929680	43	$\bar{1}$0·38707	252
	10	3844	129	9637	43	38959	251
	20	3715	129	9594	43	39210	251
	30	3586	129	9551	43	39461	251
	40	3457	129	9508	43	39712	252
	50	3328	129	9465	43	39964	251
47	0	$\bar{3}$·9943199	128	$\bar{3}$·9929422	43	$\bar{1}$0·40215	252
	10	3071	129	9379	43	40467	252
	20	2942	128	9336	43	40719	252
	30	2814	129	9293	43	40971	251
	40	2685	128	9250	43	41222	252
	50	2557	129	9207	42	41474	253
48	0	$\bar{3}$·9942428		$\bar{3}$·9929165		$\bar{1}$0·41727	

TABLE V

Logarithms for facilitating Computation of Latitudes, Longitudes, Reverse Azimuths and Transformation of Co-ordinates.

(Clarke's 1858 figure.)

Latitude		P=1/ρ sin 1″ log P	Diff.	Q=1/ν sin 1″ log Q	Diff.	Log $\frac{\tan\phi}{2\rho\nu \sin 1''}$	Diff.
°	′						
48	0	$\bar{3}\cdot9942428$	128	$\bar{3}\cdot9929165$	43	$\bar{10}\cdot41727$	252
	10	2300	128	9122	43	41979	252
	20	2172	129	9079	43	42231	254
	30	2043	128	9036	42	42485	253
	40	1915	128	8994	43	42738	252
	50	1787	127	8951	42	42990	254
49	0	$\bar{3}\cdot9941660$	128	$\bar{3}\cdot9928909$	43	$\bar{10}\cdot43244$	254
	10	1532	128	8866	43	43498	254
	20	1404	127	8823	42	43752	253
	30	1277	128	8781	43	44005	254
	40	1149	127	8738	42	44259	255
	50	1022	127	8696	42	44514	254
50	0	$\bar{3}\cdot9940895$	127	$\bar{3}\cdot9928654$	43	$\bar{10}\cdot44768$	255
	10	0768	127	8611	42	45023	256
	20	0641	127	8569	42	45279	256
	30	0514	127	8527	43	45535	255
	40	0387	126	8484	42	45790	256
	50	0261	127	8442	42	46046	257
51	0	$\bar{3}\cdot9940134$	126	$\bar{3}\cdot9928400$	42	$\bar{10}\cdot46303$	257
	10	$\bar{3}\cdot9940008$	126	8358	42	46560	257
	20	$\bar{3}\cdot9939882$	126	8316	42	46817	257
	30	9756	126	8274	42	47074	258
	40	9630	125	8232	42	47332	258
	50	9505	125	8190	42	47590	259
52	0	$\bar{3}\cdot9939380$	126	$\bar{3}\cdot9928148$	41	$\bar{10}\cdot47849$	259
	10	9254	125	8107	42	48108	259
	20	9129	124	8065	41	48367	260
	30	9005	125	8024	42	48627	260
	40	8880	125	7982	42	48887	260
	50	8755	124	7940	41	49147	261
53	0	$\bar{3}\cdot9938631$	124	$\bar{3}\cdot9927899$	41	$\bar{10}\cdot49408$	262
	10	8507	124	7858	42	49670	261
	20	8383	123	7816	41	49931	263
	30	8260	124	7775	41	50194	263
	40	8136	123	7734	41	50457	263
	50	8013	123	7693	41	50720	263
54	0	$\bar{3}\cdot9937890$		$\bar{3}\cdot9927652$		$\bar{10}\cdot50983$	

TABLE V

Logarithms for facilitating Computation of Latitudes, Longitudes, Reverse Azimuths and Transformation of Co-ordinates.

(Clarke's 1858 figure.)

Latitude		P= 1/ρ sin 1″ log P	Diff.	O=1/ν sin 1″ log O	Diff.	Log $\frac{\tan \phi}{2\rho\nu \sin 1''}$	Diff.
°	′						
54	0	$\bar{3}$·9937890	122	$\bar{3}$·9927652	41	$\bar{1}$0·50983	265
	10	7768	123	7611	41	51248	265
	20	7645	122	7570	40	51513	266
	30	7523	122	7530	41	51779	266
	40	7401	122	7489	41	52045	265
	50	7279	121	7448	40	52310	267
55	0	$\bar{3}$·9937158	122	$\bar{3}$·9927408	41	$\bar{1}$0·52577	268
	10	7036	121	7367	40	52845	268
	20	6915	120	7327	40	53113	268
	30	6795	121	7287	40	53381	270
	40	6674	120	7247	40	53651	270
	50	6554	120	7207	40	53921	271
56	0	$\bar{3}$·9936434	120	$\bar{3}$·9927167	40	$\bar{1}$0·54192	271
	10	6314	119	7127	40	54463	271
	20	6195	119	7087	40	54734	273
	30	6076	119	7047	39	55007	274
	40	5957	118	7008	40	55281	274
	50	5839	118	6968	39	55555	275
57	0	$\bar{3}$·9935721	118	$\bar{3}$·9926929	40	$\bar{1}$0·55830	275
	10	5603	118	6889	39	56105	275
	20	5485	117	6850	39	56380	277
	30	5368	117	6811	39	56657	278
	40	5251	117	6772	39	56935	279
	50	5134	116	6733	38	57214	279
58	0	$\bar{3}$·9935018	116	$\bar{3}$·9926695	39	$\bar{1}$0·57493	280
	10	4902	116	6656	39	57773	280
	20	4786	115	6617	38	58053	282
	30	4671	115	6579	38	58335	282
	40	4556	114	6541	39	58617	284
	50	4442	115	6502	38	58901	284
59	0	$\bar{3}$·9934327	114	$\bar{3}$·9926464	38	$\bar{1}$0·59185	285
	10	4213	113	6426	37	59470	286
	20	4100	113	6389	38	59756	287
	30	3987	113	6351	38	60043	288
	40	3874	113	6313	37	60331	289
	50	3761	112	6276	38	60620	289
60	0	$\bar{3}$·9933649		$\bar{3}$·9926238		$\bar{1}$0·60909	

TABLE V

Logarithms for facilitating Computation of Latitudes, Longitudes, Reverse Azimuths and Transformation of Co-ordinates.

(Clarke's 1858 figure.)

Latitude		P= 1/ρ sin 1″ log P	Diff.	O=1/ν sin 1″ log O	Diff.	Log $\frac{\tan \phi}{2\rho\,\nu \sin 1''}$	Diff.
°	′						
60	0	$\bar{3}$·9933649	111	$\bar{3}$·9926238	37	$\bar{1}$0·60909	291
	10	3538	112	6201	37	61200	292
	20	3426	111	6164	37	61492	293
	30	3315	110	6127	37	61785	294
	40	3205	111	6090	37	62079	294
	50	3094	109	6053	36	62373	296
61	0	$\bar{3}$·9932985	110	$\bar{3}$·9926017	37	$\bar{1}$0·62669	297
	10	2875	109	5980	36	62966	298
	20	2766	108	5944	36	63264	300
	30	2658	109	5908	36	63564	300
	40	2549	107	5872	36	63864	302
	50	2442	108	5836	36	64166	302
62	0	$\bar{3}$·9932334	107	$\bar{3}$·9925800	36	$\bar{1}$0·64468	304
	10	2227	106	5764	35	64772	305
	20	2121	106	5729	36	65077	307
	30	2015	106	5693	35	65384	308
	40	1909	105	5658	35	65692	309
	50	1804	105	5623	35	66001	310
63	0	$\bar{3}$·9931699	104	$\bar{3}$·9925588	35	$\bar{1}$0·66311	311
	10	1595	104	5553	34	66622	313
	20	1491	104	5519	35	66935	315
	30	1387	103	5484	34	67250	315
	40	1284	102	5450	34	67565	317
	50	1182	103	5416	34	67882	319
64	0	$\bar{3}$·9931079	101	$\bar{3}$·9925382	34	$\bar{1}$0·68201	320
	10	0978	101	5348	34	68521	321
	20	0877	101	5314	33	68842	323
	30	0776	100	5281	34	69165	325
	40	0676	100	5247	33	69490	326
	50	0576	99	5214	33	69816	328
65	0	$\bar{3}$·9930477		$\bar{3}$·9925181		$\bar{1}$0·70144	

TABLE Va

Logarithms for facilitating Computation of Latitudes, Longitudes, Reverse Azimuths and Transformation of Co-ordinates.

(Based on Everest's first figure : compression $\frac{1}{300{\cdot}8}$)

Latitude		P=1/ρ sin 1″ log P	Diff.	Q=1/ν sin 1″ log Q	Diff.	Log $\frac{\tan\phi}{2\rho\nu \sin 1''}$	Diff.
°	′						
0	0	$\bar{3}{\cdot}9966950$	1	$\bar{3}{\cdot}9938026$	0		
	10	6949	1	8026	1	$\bar{13}{\cdot}83877$	30103
	20	6948	1	8025	0	$\bar{12}{\cdot}13980$	17610
	30	6947	3	8025	1	31590	12495
	40	6944	3	8024	1	44085	9692
	50	6941	4	8023	1	53777	7919
1	0	$\bar{3}{\cdot}9966937$	5	$\bar{3}{\cdot}9938022$	2	$\bar{12}{\cdot}61696$	6696
	10	6932	6	8020	2	68392	5801
	20	6926	6	8018	2	74193	5118
	30	6920	7	8016	2	79311	4578
	40	6913	7	8014	3	83889	4141
	50	6906	9	8011	3	88030	3782
2	0	$\bar{3}{\cdot}9966897$	9	$\bar{3}{\cdot}9938008$	3	$\bar{12}{\cdot}91812$	3479
	10	6888	10	8005	3	95291	3222
	20	6878	10	8002	3	$\bar{12}{\cdot}98513$	2999
	30	6868	12	7999	4	$\bar{11}{\cdot}01512$	2807
	40	6856	12	7995	4	04319	2637
	50	6844	13	7991	5	06956	2486
3	0	$\bar{3}{\cdot}9966831$	13	$\bar{3}{\cdot}9937986$	4	$\bar{11}{\cdot}09442$	2353
	10	6818	14	7982	5	11795	2232
	20	6804	15	7977	5	14027	2124
	30	6789	16	7972	5	16151	2025
	40	6773	16	7967	5	18176	1936
	50	6757	18	7962	6	20112	1854
4	0	$\bar{3}{\cdot}9966739$	17	$\bar{3}{\cdot}9937956$	6	$\bar{11}{\cdot}21966$	1778
	10	6722	19	7950	6	23744	1710
	20	6703	19	7944	7	25454	1645
	30	6684	20	7937	6	27099	1586
	40	6664	21	7931	7	28685	1531
	50	6643	22	7924	8	30216	1479
5	0	$\bar{3}{\cdot}9966621$	22	$\bar{3}{\cdot}9937916$	7	$\bar{11}{\cdot}31695$	1431
	10	6599	23	7909	8	33126	1387
	20	6576	23	7901	7	34513	1344
	30	6553	25	7894	9	35857	1304
	40	6528	25	7885	8	37161	1267
	50	6503	26	7877	9	38428	1232
6	0	$\bar{3}{\cdot}9966477$		$\bar{3}{\cdot}9937868$		$\bar{11}{\cdot}39660$	

TABLE Va

Logarithms for facilitating Computation of Latitudes, Longitudes, Reverse Azimuths and Transformation of Co-ordinates.

(Everest's 1st figure.)

Latitude		P=1/ρ sin 1″ log P	Diff.	O=1/ν sin 1″ log O	Diff.	Log $\frac{\tan\phi}{2\rho\nu \sin 1''}$	Diff.
°	′						
6	0	$\bar{3}$·9966477	26	$\bar{3}$·9937868	8	$\bar{11}$·39660	1199
	10	6451	27	7860	9	40859	1167
	20	6424	28	7851	10	42026	1137
	30	6396	29	7841	9	43163	1109
	40	6367	29	7832	10	44272	1082
	50	6338	30	7822	10	45354	1056
7	0	$\bar{3}$·9966308	31	$\bar{3}$·9937812	10	$\bar{11}$·46410	1032
	10	6277	32	7802	11	47442	1009
	20	6245	32	7791	11	48451	986
	30	6213	33	7780	11	49437	966
	40	6180	33	7769	11	50403	945
	50	6147	35	7758	11	51348	925
8	0	$\bar{3}$·9966112	35	$\bar{3}$·9937747	12	$\bar{11}$·52273	907
	10	6077	36	7735	12	53180	889
	20	6041	36	7723	12	54069	873
	30	6005	37	7711	12	54942	855
	40	5968	38	7699	13	55797	840
	50	5930	38	7686	13	56637	824
9	0	$\bar{3}$·9965892	40	$\bar{3}$·9937673	13	$\bar{11}$·57461	810
	10	5852	40	7660	13	58271	796
	20	5812	40	7647	14	59067	782
	30	5772	41	7633	14	59849	769
	40	5731	42	7619	14	60618	757
	50	5689	43	7605	14	61375	744
10	0	$\bar{3}$·9965646	44	$\bar{3}$·9937591	14	$\bar{11}$·62119	732
	10	5602	44	7577	15	62851	721
	20	5558	44	7562	15	63572	710
	30	5514	46	7547	15	64282	699
	40	5468	46	7532	15	64981	689
	50	5422	47	7517	16	65670	678
11	0	$\bar{3}$·9965375	47	$\bar{3}$·9937501	16	$\bar{11}$·66348	670
	10	5328	48	7485	16	67018	659
	20	5280	49	7469	16	67677	651
	30	5231	50	7453	17	68328	641
	40	5181	50	7436	16	68969	633
	50	5131	51	7420	17	69602	625
12	0	$\bar{3}$·9965080		$\bar{3}$·9937403		$\bar{11}$·70227	

TABLE Va

Logarithms for facilitating Computation of Latitudes, Longitudes, Reverse Azimuths and Transformation of Co-ordinates.

(Everest's 1st figure.)

Latitude		P=1/ρ sin 1″ log P	Diff.	Q= 1/ν sin 1″ log Q	Diff.	Log $\frac{\tan\phi}{2\rho\nu \sin 1''}$	Diff.
°	′						
12	0	$\bar{3}$·9965080	51	$\bar{3}$·9937403	17	$\bar{11}$·70227	616
	10	5029	52	7386	18	70843	609
	20	4977	53	7368	17	71452	601
	30	4924	54	7351	18	72053	593
	40	4870	54	7333	18	72646	586
	50	4816	55	7315	19	73232	579
13	0	$\bar{3}$·9964761	55	$\bar{3}$·9937296	18	$\bar{11}$·73811	572
	10	4706	56	7278	19	74383	566
	20	4650	57	7259	19	74949	559
	30	4593	58	7240	19	75508	553
	40	4535	58	7221	19	76061	547
	50	4477	58	7202	20	76608	540
14	0	$\bar{3}$·9964419	60	$\bar{3}$·9937182	20	$\bar{11}$·77148	534
	10	4359	60	7162	20	77682	529
	20	4299	61	7142	20	78211	523
	30	4238	61	7122	20	78734	517
	40	4177	62	7102	21	79251	512
	50	4115	62	7081	21	79763	508
15	0	$\bar{3}$·9964053	64	$\bar{3}$·9937060	21	$\bar{11}$·80271	502
	10	3989	64	7039	21	80773	497
	20	3925	64	7018	22	81270	492
	30	3861	65	6996	21	81762	487
	40	3796	66	6975	22	82249	483
	50	3730	66	6953	22	82732	478
16	0	$\bar{3}$·9963664	67	$\bar{3}$·9936931	23	$\bar{11}$·83210	474
	10	3597	68	6908	22	83684	469
	20	3529	68	6886	23	84153	465
	30	3461	69	6863	23	84618	461
	40	3392	69	6840	23	85079	457
	50	3323	71	6817	24	85536	453
17	0	$\bar{3}$·9963252	70	$\bar{3}$·9936793	23	$\bar{11}$·85989	449
	10	3182	72	6770	24	86438	445
	20	3110	71	6746	24	86883	441
	30	3039	73	6722	24	87324	437
	40	2966	73	6698	24	87761	434
	50	2893	74	6674	25	88195	432
18	0	$\bar{3}$·9962819		$\bar{3}$·9936649		$\bar{11}$·88627	

TABLE Va

Logarithms for facilitating Computation of Latitudes, Longitudes, Reverse Azimuths and Transformation of Co-ordinates.

(Everest's 1st figure.)

Latitude		P=1/ρ sin 1″ log P	Diff.	O=1/ν sin 1″ log O	Diff.	Log $\frac{\tan\phi}{2\rho\nu \sin 1''}$	Diff.
°	′						
18	0	$\bar{3}$·9962819	74	$\bar{3}$·9936649	25	$\bar{11}$·88627	428
	10	2745	75	6624	25	89055	424
	20	2670	75	6599	25	89479	419
	30	2595	76	6574	25	89898	416
	40	2519	77	6549	26	90314	414
	50	2442	77	6523	25	90728	412
19	0	$\bar{3}$·9962365	78	$\bar{3}$·9936498	26	$\bar{11}$·91140	409
	10	2287	78	6472	26	91549	405
	20	2209	79	6446	27	91954	401
	30	2130	80	6419	26	92355	398
	40	2050	80	6393	27	92753	396
	50	1970	80	6366	27	93149	394
20	0	$\bar{3}$·9961890	82	$\bar{3}$·9936339	27	$\bar{11}$·93543	391
	10	1808	81	6312	27	93934	388
	20	1727	83	6285	28	94322	385
	30	1644	83	6257	27	94707	383
	40	1561	83	6230	28	95090	380
	50	1478	84	6202	28	95470	378
21	0	$\bar{3}$·9961394	84	$\bar{3}$·9936174	28	$\bar{11}$·95848	375
	10	1310	85	6146	29	96223	373
	20	1225	86	6117	28	96596	370
	30	1139	86	6089	29	96966	368
	40	1053	87	6060	29	97334	366
	50	0966	87	6031	29	97700	364
22	0	$\bar{3}$·9960879	88	$\bar{3}$·9936002	29	$\bar{11}$·98064	362
	10	0791	88	5973	29	98426	359
	20	0703	89	5944	30	98785	357
	30	0614	89	5914	30	99142	355
	40	0525	90	5884	30	99497	353
	50	0435	90	5854	30	$\bar{11}$·99850	351
23	0	$\bar{3}$·9960345	91	$\bar{3}$·9935824	30	$\bar{10}$·00201	349
	10	0254	91	5794	30	00550	347
	20	0163	92	5764	31	00897	346
	30	$\bar{3}$·9960071	93	5733	31	01243	344
	40	3·9959978	92	5702	31	01587	341
	50	9886	94	5671	31	01928	339
24	0	$\bar{3}$·9959792		$\bar{3}$·9935640		$\bar{10}$·02267	

TABLE Va

Logarithms for facilitating Computation of Latitudes, Longitudes, Reverse Azimuths and Transformation of Co-ordinates.

(Everest's 1st figure.)

Latitude		P=1/ρ sin 1″. log P	Diff.	O=1/ν sin 1″ log O	Diff.	Log $\frac{\tan \phi}{2\rho\nu \sin 1''}$	Diff.
°	′						
24	0	$\bar{3}$·9959792	94	$\bar{3}$·9935640	31	$\bar{1}$0·02267	338
	10	9698	94	5609	32	02605	336
	20	9604	95	5577	31	02941	334
	30	9509	95	5546	32	03275	333
	40	9414	96	5514	32	03608	331
	50	9318	96	5482	32	03939	329
25	0	$\bar{3}$·9959222		$\bar{3}$·9935450		$\bar{1}$0·04268	

TABLE Vb

Logarithms for facilitating Computation of Latitudes, Longitudes, Reverse Azimuths and Transformation of Co-ordinates.

(Based on Airy's figure ; compression $\frac{1}{299{\cdot}3}$)

Latitude		P=1/ρ sin 1″ log P	Diff.	O= 1/ν sin 1″ log O	Diff.	Log $\frac{\tan\phi}{2\rho\nu \sin 1''}$	Diff.
°	′						
49	0	$\bar{3}{\cdot}9942132$	125	$\bar{3}{\cdot}9929598$	42	$\bar{10}{\cdot}43255$	254
	10	2007	126	9556	42	43509	254
	20	1881	125	9514	42	43763	254
	30	1756	125	9472	42	44017	254
	40	1631	126	9430	41	44271	255
	50	1505	125	9389	42	44526	254
50	0	$\bar{3}{\cdot}9941380$	125	$\bar{3}{\cdot}9929347$	42	$\bar{10}{\cdot}44780$	255
	10	1255	124	9305	41	45035	256
	20	1131	125	9264	42	45291	255
	30	1006	125	9222	41	45546	256
	40	0881	124	9181	42	45802	256
	50	0757	124	9139	41	46058	257
51	0	$\bar{3}{\cdot}9940633$	124	$\bar{3}{\cdot}9929098$	41	$\bar{10}{\cdot}46315$	257
	10	0509	124	9057	42	46572	257
	20	0385	124	9015	41	46829	257
	30	0261	123	8974	41	47086	258
	40	0138	124	8933	41	47344	258
	50	$\bar{3}{\cdot}9940014$	123	8892	41	47602	259
52	0	$\bar{3}{\cdot}9939891$	123	$\bar{3}{\cdot}9928851$	41	$\bar{10}{\cdot}47861$	259
	10	9768	123	8810	41	48120	259
	20	9645	123	8769	41	48379	260
	30	9522	122	8728	41	48639	260
	40	9400	123	8687	41	48899	261
	50	9277	122	8646	41	49160	261
53	0	$\bar{3}{\cdot}9939155$	122	$\bar{3}{\cdot}9928605$	40	$\bar{10}{\cdot}49421$	261
	10	9033	121	8565	41	49682	262
	20	8912	122	8524	40	49944	262
	30	8790	121	8484	41	50206	263
	40	8669	121	8443	40	50469	263
	50	8548	121	8403	40	50732	264
54	0	$\bar{3}{\cdot}9938427$	121	$\bar{3}{\cdot}9928363$	41	$\bar{10}{\cdot}50996$	265
	10	8306	120	8322	40	51261	264
	20	8186	120	8282	40	51525	266
	30	8066	120	8242	40	51791	266
	40	7946	120	8202	40	52057	266
	50	7826	119	8162	40	52323	267
55	0	$\bar{3}{\cdot}9937707$		$\bar{3}{\cdot}9928122$		$\bar{10}{\cdot}52590$	

TABLE Vb

Logarithms for facilitating Computation of Latitudes, Longitudes, Reverse Azimuths and Transformation of Co-ordinates.

(Airy's figure.)

Latitude		P=1/ρ sin 1″ log P	Diff.	Q=1/ν sin 1″ log Q	Diff.	Log $\frac{\tan \phi}{2\rho\nu \sin 1''}$	Diff.
°	′						
55	0	$\bar{3}$·9937707	120	$\bar{3}$·9928122	39	$\bar{1}$0·52590	268
	10	7587	118	8083	40	52858	268
	20	7469	119	8043	39	53126	269
	30	7350	119	8004	40	53395	269
	40	7231	118	7964	39	53664	270
	50	7113	118	7925	40	53934	271
56	0	$\bar{3}$·9936995	117	$\bar{3}$·9927885	39	$\bar{1}$0·54205	271
	10	6878	118	7846	39	54476	272
	20	6760	117	7807	39	54748	272
	30	6643	116	7768	39	55020	274
	40	6527	117	7729	39	55294	274
	50	6410	116	7690	38	55568	274
57	0	$\bar{3}$·9936294	116	$\bar{3}$·9927652	39	$\bar{1}$0·55842	276
	10	6178	115	7613	39	56118	276
	20	6063	116	7574	38	56394	277
	30	5947	115	7536	38	56671	277
	40	5832	114	7498	38	56948	279
	50	5718	115	7460	39	57227	279
58	0	$\bar{3}$·9935603	114	$\bar{3}$·9927421	38	$\bar{1}$0·57506	280
	10	5489	113	7383	37	57786	281
	20	5376	114	7346	38	58067	281
	30	5262	113	7308	38	58348	283
	40	5149	112	7270	37	58631	283
	50	5037	113	7233	38	58914	284
59	0	$\bar{3}$·9934924	112	$\bar{3}$·9927195	37	$\bar{1}$0·59198	285
	10	4812	111	7158	37	59483	286
	20	4701	111	7121	38	59769	287
	30	4590	111	7083	37	60056	288
	40	4479	111	7046	36	60344	289
	50	4368	110	7010	37	60633	290
60	0	$\bar{3}$·9934258	110	$\bar{3}$·9926973	37	$\bar{1}$0·60923	291
	10	4148	109	6936	36	61214	291
	20	4039	110	6900	37	61505	293
	30	3929	108	6863	36	61798	294
	40	3821	109	6827	36	62092	295
	50	3712	108	6791	36	62387	296
61	0	$\bar{3}$·9933604		$\bar{3}$·9926755		$\bar{1}$0·62683	

TABLE VI

Table showing the value of p, q, r for the computation of rectangular spheroidal co-ordinates.

Argument 10000	Arguments			Argument 10000	Arguments.		
	$x' + x$	$x' - x$	$y' - y$		$x' + x$	$x' - x$	$y' - y$
	p	q	r		p	q	r
5	4	1	3	30	111	37	74
6	5	2	3	31	119	40	79
7	6	2	4	32	127	42	85
8	8	3	5	33	135	45	90
9	10	4	6	34	143	48	95
10	12	4	8	35	152	51	101
11	15	5	10	36	160	53	107
12	18	6	12	37	169	56	113
13	21	7	14	38	179	60	119
14	24	8	16	39	188	63	126
15	28	9	19	40	198	66	132
16	32	11	21	41	208	69	139
17	36	12	24	42	218	73	146
18	40	13	27	43	229	76	153
19	45	15	30	44	240	80	160
20	50	17	33	45	251	84	167
21	55	18	36	46	262	87	175
22	60	20	40	47	273	91	182
23	65	22	44	48	285	95	190
24	71	24	48	49	297	99	198
25	77	26	52	50	309	103	206
26	84	28	56	51	322	107	215
27	90	30	60	52	335	112	223
28	97	32	65	53	348	116	232
29	104	35	69	54	361	120	241
30	111	37	74	55	374	125	250

This table is made out for Latitude 52° 10′ but may be used for topographical work in other latitudes without any errors of consequence.

The formulæ for calculating p, q and r are :—

$$p = \text{log. sec.} \left[\frac{\frac{1}{2}(x'+x)}{\sqrt{\rho\,\nu}\,\sin 1''}\right]''$$

$$q = \text{log.} \left[\frac{\frac{1}{2}(x'-x)}{\sqrt{\rho\,\nu}}\right] - \text{log. sin.} \left[\frac{\frac{1}{2}(x'-x)}{\sqrt{\rho\,\nu}\,\sin 1''}\right]''$$

$$r = \text{log. tan} \left[\frac{\frac{1}{2}(y'-y)}{\sqrt{\rho\,\nu}\,\sin 1''}\right]'' - \text{log.} \left[\frac{\frac{1}{2}(y'-y)}{\sqrt{\rho\,\nu}}\right]$$

TABLE VII

Computation of Trigonometrical Heights

Angle K, for Use in Computing Heights when only One Angle has been Observed.

Distance between Stations in feet	K	
	′	″
1,000	0	4
2,000	0	9
3,000	0	13
4,000	0	17
5,000	0	21
6,000	0	26
7,000	0	30
8,000	0	34
9,000	0	38
10,000	0	43
20,000	1	25
30,000	2	8
40,000	2	50
50,000	3	33
60,000	4	15
70,000	4	58
80,000	5	40
90,000	6	23
100,000	7	5

The value of the coefficient of refraction has been found on the average to be about 0·07. It is with this value that the above table has been computed.

TABLE VIII

Dip of the Sea Horizon

Height of Eye	Dip		Height of Eye	Dip	
ft.	′	″	ft.	′	″
1	0	59	26	5	01
2	1	23	27	5	07
3	1	42	28	5	12
4	1	58	29	5	18
5	2	12	30	5	23
6	2	25	31	5	29
7	2	36	32	5	34
8	2	47	33	5	39
9	2	57	34	5	44
10	3	07	35	5	49
11	3	16	36	5	54
12	3	25	37	5	59
13	3	33	38	6	04
14	3	41	39	6	08
15	3	49	40	6	13
16	3	56	41	6	18
17	4	03	42	6	23
18	4	10	43	6	27
19	4	17	44	6	32
20	4	24	45	6	36
21	4	31	46	6	40
22	4	37	47	6	45
23	4	43	48	6	49
24	4	49	49	6	53
25	4	55	50	6	58

Calculated with a coefficient of refraction of 0·077.

Abnormal Refraction may alter these values considerably.

TABLE IX
Dip of a Shore Horizon

Distance of Land in Nautical Miles	Height of Eye in feet							
	5	10	15	20	25	30	35	40
	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
0·5	5 50	11 30	17 10	22 50	28 30	34 05	39 45	45 25
0·6	5 00	9 40	14 25	19 05	23 50	28 30	33 15	37 55
0·8	3 50	7 25	10 55	14 30	18 00	21 30	25 05	28 35
1·0	3 15	6 05	8 55	11 45	14 30	17 20	20 10	23 00
1·2	2 50	5 15	7 35	9 55	12 15	14 40	17 00	19 20
1·4	2 35	4 40	6 40	8 40	10 40	12 45	14 45	16 45
1·6	2 30	4 15	6 00	7 45	9 30	11 20	13 05	14 50
1·8	2 20	3 55	5 30	7 00	8 35	10 10	11 45	13 20
2·0	2 15	3 40	5 05	6 30	7 55	9 20	10 45	12 10
2·2	2 15	3 30	4 45	6 05	7 20	8 40	9 55	11 10
2·4	2 10	3 20	4 35	5 45	6 55	8 05	9 15	10 25
2·6	—	3 15	4 20	5 25	6 30	7 35	8 40	9 45
2·8	—	3 10	4 10	5 15	6 15	7 15	8 15	9 15
3·0	—	3 10	4 05	5 00	6 00	6 55	7 50	8 50
3·5	—	3 05	3 55	4 40	5 30	6 20	7 10	7 55
4·0	—	—	3 50	4 30	5 15	5 55	6 40	7 20
4·5	—	—	—	4 25	5 00	5 40	6 20	6 55
5·0	—	—	—	4 25	4 55	5 30	6 05	6 40
5·5	—	—	—	—	4 55	5 25	5 55	6 25
6·0	—	—	—	—	—	5 20	5 50	6 20
6·5	—	—	—	—	—	—	5 45	6 15
7·0	—	—	—	—	—	—	—	6 10

(Calculated to nearest 5″, using coefficient of refraction of 0·077)
Abnormal Refractions may alter these values considerably

TABLE X

Corrections for slope to be subtracted from distances measured along sloping ground in Theodolite Traverses, etc., to obtain corresponding horizontal distances.

(Measured distance = 1.)

Angle of slope.	0′	10′	20′	30′	40′	50′
°						
0					0·00007	0·00011
1	0·00015	0·00021	0·00027	0·00034	0·00042	0·00051
2	0·00061	0·00071	0·00083	0·00095	0·00108	0·00122
3	0·00137	0·00153	0·00169	0·00187	0·00205	0·00224
4	0·00244	0·00264	0·00286	0·00308	0·00332	0·00356
5	0·00381	0·00406	0·00433	0·00460	0·00489	0·00518
6	0·00548	0·00579	0·00610	0·00643	0·00676	0·00710
7	0·00745	0·00781	0·00818	0·00856	0·00894	0·00933
8	0·00973	0·01014	0·01056	0·01098	0·01142	0·01186
9	0·01231	0·01277	0·01324	0·01371	0·01420	0·01469
10	0·01519	0·01570	0·01622	0·01675	0·01728	0·01782
11	0·01837	0·01893	0·01950	0·02008	0·02066	0·02125
12	0·02185	0·02246	0·02308	0·02370	0·02434	0·02498
13	0·02563	0·02629	0·02696	0·02763	0·02831	0·02900
14	0·02970	0·03041	0·03113	0·03185	0·03258	0·03333
15	0·03407	0·03483	0·03560	0·03637	0·03715	0·03794
16	0·03874	0·03954	0·04036	0·04118	0·04201	0·04285
17	0·04370	0·04455	0·04541	0·04628	0·04716	0·04805
18	0·04894	0·04985	0·05076	0·05168	0·05260	0·05354
19	0·05448	0·05543	0·05639	0·05736	0·05833	0·05932
20	0·06031	0·06131	0·06231	0·06333	0·06435	0·06538
21	0·06642	0·06747	0·06852	0·06958	0·07065	0·07173
22	0·07282	0·07391	0·07501	0·07612	0·07724	0·07836
23	0·07950	0·08064	0·08178	0·08294	0·08410	0·08528
24	0·08645	0·08764	0·08884	0·09004	0·09125	0·09247
25	0·09369	0·09493	0·09617	0·09741	0·09867	0·09993
26	0·10121	0·10248	0·10377	0·10507	0·10637	0·10768
27	0·10899	0·11032	0·11165	0·11299	0·11434	0·11569
28	0·11705	0·11842	0·11980	0·12118	0·12257	0·12397
29	0·12538	0·12679	0·12822	0·12964	0·13108	0·13252
30	0·13397	0·13543	0·13690	0·13837	0·13985	0·14134

Example:—Measured distance = 749 ft.; angle of slope 7° 2′. The correction for slope = (749 × 0·00752) ft., *i.e.* 5·63 ft., and the horizontal distance, therefore, = (749 − 5·6) ft., *i.e.* 743·4 ft.

TABLE Xa.

Corrections to be subtracted from distances obtained by tachymeter readings of a staff held vertically.
(Measured distance = 1)

Angle of Slope	0′	10′	20′	30′	40′	50′
Deg.						
0		0·000 01	0·000 03	0·000 08	0·000 14	0·000 21
1	0·000 30	000 41	000 54	000 69	000 85	001 02
2	001 22	001 43	001 66	001 90	002 16	002 44
3	002 74	003 05	003 38	003 73	004 09	004 47
4	004 87	005 28	005 71	006 16	006 62	007 10
5	007 60	008 11	008 64	009 19	009 75	010 33
6	010 93	011 54	012 17	012 82	013 48	014 16
7	014 85	015 56	016 29	017 04	017 80	018 58
8	019 37	020 18	021 01	021 85	022 71	023 58
9	024 47	025 38	026 30	027 24	028 20	029 17
10	030 15	031 16	032 18	033 21	034 26	035 33
11	036 41	037 51	038 62	039 75	040 89	042 05
12	043 23	044 42	045 62	046 85	048 08	049 34
13	050 60	051 89	053 18	054 50	055 82	057 17
14	058 53	059 90	061 29	062 69	064 11	065 54
15	066 99	068 45	069 93	071 42	072 92	074 44
16	075 98	077 52	079 09	080 66	082 26	083 86
17	085 48	087 11	088 76	090 42	092 10	093 79
18	095 49	097 21	098 94	100 68	102 44	104 21
19	105 99	107 79	109 60	111 43	113 26	115 11
20	116 98	118 85	120 74	122 65	124 56	126 49
21	128 43	130 38	132 35	134 32	136 31	138 32
22	140 33	142 36	144 40	146 45	148 51	150 58
23	152 67	154 77	156 88	159 00	161 13	163 28
24	165 43	167 60	169 78	171 97	174 17	176 38
25	178 61	180 84	183 08	185 34	187 61	189 88
26	192 17	194 47	196 77	199 09	201 42	203 76
27	206 11	208 47	210 83	213 21	215 60	218 00
28	220 40	222 82	225 25	227 68	230 12	232 58
29	235 04	237 51	239 99	242 48	244 98	247 49
30	250 00	252 52	255 06	257 60	260 14	262 70

TABLE XI

Table of Natural Scales Expressed in Inches to the Mile of Mid-Latitude at 45°

Natural Scale.	Inches.	Natural Scale.	Inches.
1 : 1,000	= 72·92	1 : 45,000	= 1·62
1 : 2,000	= 36·46	1 : 50,000	= 1·46
1 : 3,000	= 24·31	1 : 55,000	= 1·33
1 : 4,000	= 18·23	1 : 60,000	= 1·22
1 : 5,000	= 14·58	1 : 65,000	= 1·12
1 : 6,000	= 12·15	1 : 70,000	= 1·04
1 : 7,000	= 10·42	1 : 75,000	= 0·97
1 : 8,000	= 9·12	1 : 80,000	= 0·91
1 : 9,000	= 8·10	1 : 85,000	= 0·86
1 : 10,000	= 7·29	1 : 90,000	= 0·81
1 : 11,000	= 6·63	1 : 95,000	= 0·77
1 : 12,000	= 6·08	1 : 100,000	= 0·73
1 : 13,000	= 5·61	1 : 150,000	= 0·49
1 : 14,000	= 5·21	1 : 200,000	= 0·36
1 : 15,000	= 4·86	1 : 250,000	= 0·29
1 : 16,000	= 4·56	1 : 300,000	= 0·24
1 : 17,000	= 4·29	1 : 350,000	= 0·21
1 : 18,000	= 4·05	1 : 400,000	= 0·18
1 : 19,000	= 3·84	1 : 500,000	= 0·15
1 : 20,000	= 3·65	1 : 1,000,000	= 0·073
1 : 25,000	= 2·92	1 : 2,000,000	= 0·037
1 : 30,000	= 2·43	1 : 3,000,000	= 0·024
1 : 35,000	= 2·08	1 : 4,000,000	= 0·018
1 : 40,000	= 1·82	1 : 5,000,000	= 0·015

TABLE XII—Astronomical Refraction

The Mean Refraction is for Barometer 30 in. and Temperature 50° Fahr.

Apparent Altitude.		Mean Refraction.		Diff. for 1′ Alt.	Diff. for + 1 in. Baro.	Diff. for − 1° Temp.	Apparent Altitude.		Mean Refraction.		Diff. for 1′ Alt.	Diff. for + 1 in. Baro.	Diff. for − 1° Temp.
°	′	′	″	″	″	″	°	′	′	″	″	″	″
0	0	33	51	11·7	74	8·1	**4**	0	11	52	2·2	24·1	1·70
	5	32	53	11·3	71	7·6		10	11	30	2·1	23·4	1·64
	10	31	58	10·9	69	7·3		20	11	10	2·0	22·7	1·58
	15	31	5	10·5	67	7·0		30	10	50	1·9	22·0	1·53
	20	30	13	10·1	65	6·7		40	10	32	1·8	21·3	1·48
	25	29	24	9·7	63	6·4		50	10	15	1·7	20·7	1·43
	30	28	37	9·4	61	6·1	**5**	0	9	58	1·6	20·1	1·38
	35	27	51	9·0	59	5·9		10	9	42	1·5	19·6	1·34
	40	27	6	8·7	58	5·6		20	9	27	1·5	19·1	1·30
	45	26	24	8·4	56	5·4		30	9	12	1·4	18·6	1·26
	50	25	43	8·0	55	5·1		40	8	58	1·3	18·1	1·22
	55	25	3	7·7	53	4·9		50	8	45	1·3	17·6	1·19
1	0	24	25	7·4	52	4·7	**6**	0	8	32	1·2	17·2	1·15
	5	23	48	7·1	50	4·6		10	8	20	1·2	16·8	1·12
	10	23	13	6·9	49	4·5		20	8	9	1·1	16·4	1·09
	15	22	40	6·6	48	4·4		30	7	58	1·1	16·0	1·06
	20	22	8	6·3	46	4·2		40	7	47	1·0	15·7	1·03
	25	21	37	6·1	45	4·0		50	7	37	1·0	15·3	1·00
	30	21	7	5·9	44	3·9	**7**	0	7	27	1·0	15·0	0·98
	35	20	38	5·7	43	3·8		10	7	17	0·9	14·6	0·95
	40	20	10	5·5	42	3·6		20	7	8	0·9	14·3	0·93
	45	19	43	5·3	40	3·5		30	6	59	0·8	14·1	0·91
	50	19	17	5·1	39	3·4		40	6	51	0·8	13·8	0·89
	55	18	52	4·9	39	3·3		50	6	43	0·8	13·5	0·87
2	0	18	28	4·8	38	3·2	**8**	0	6	35	0·7	13·3	0·85
	5	18	5	4·6	37	3·1		10	6	28	0·7	13·1	0·83
	10	17	43	4·4	36	3·0		20	6	21	0·7	12·8	0·82
	15	17	21	4·3	36	2·9		30	6	14	0·7	12·6	0·80
	20	17	0	4·1	35	2·8		40	6	7	0·7	12·3	0·79
	25	16	40	4·0	34	2·8		50	6	0	0·6	12·1	0·77
	30	16	21	3·9	33	2·7	**9**	0	5	54	0·6	11·9	0·76
	35	16	2	3·7	33	2·7		10	5	47	0·6	11·7	0·74
	40	15	43	3·6	32	2·6		20	5	41	0·6	11·5	0·73
	45	15	25	3·5	32	2·5		30	5	36	0·6	11·3	0·71
	50	15	8	3·4	31	2·4		40	5	30	0·5	11·1	0·71
	55	14	51	3·3	30	2·3		50	5	25	0·5	11·0	0·70
3	0	14	35	3·2	30	2·3	**10**	0	5	20	0·5	10·8	0·69
	5	14	19	3·1	29	2·2		10	5	15	0·5	10·6	0·67
	10	14	4	3·0	29	2·2		20	5	10	0·5	10·4	0·65
	15	13	50	2·9	28	2·1		30	5	5	0·5	10·2	0·64
	20	13	35	2·8	28	2·1		40	5	0	0·5	10·1	0·63
	25	13	21	2·7	27	2·0		50	4	56	0·4	9·9	0·62
	30	13	7	2·7	27	2·0	**11**	0	4	51	0·4	9·8	0·60
	35	12	54	2·6	26	2·0		10	4	47	0·4	9·6	0·59
	40	12	41	2·5	26	1·9		20	4	43	0·4	9·5	0·58
	45	12	28	2·4	25	1·9		30	4	39	0·4	9·4	0·57
	50	12	16	2·4	25	1·9		40	4	35	0·4	9·2	0·56
	55	12	3	2·3	25	1·8		50	4	31	0·4	9·1	0·55

NOTE.—If the Barometer is over 30 in. the Barometer correction is *additive* to the Mean Refraction; if the Thermometer is less than 50° the Thermometer correction is *additive* to the Mean Refraction; and *vice versa*.

TABLE XII—Astronomical Refraction

The Mean Refraction is for Barometer 30 in. and Temperature 50° Fahr.

Apparent Altitude.		Mean Refraction.	Diff. for 1′ Alt.	Diff. for + 1 in. Baro	Diff. for − 1° Temp.	Apparent Altitude.		Mean Refraction.	Diff. for 1′ Alt.	Diff. for + 1 in. Baro.	Diff. for − 1° Temp.
°	′	′ ″	″	″	″	°	′	′ ″	″	″	″
12	0	4 28·1	0·38	9·00	0·556	42	0	1 4·6	0·038	2·16	0·130
	10	4 24·4	0·37	8·86	0·548	43	0	1 2·4	0·036	2·09	0·125
	20	4 20·8	0·36	8·74	0·541	44	0	1 0·3	0·034	2·02	0·120
	30	4 17·3	0·35	8·63	0·533	45	0	0 58·2	0·034	1·94	0·117
	40	4 13·9	0·33	8·51	0·525	46	0	0 56·1	0·033	1·88	0·112
	50	4 10·7	0·32	8·41	0·517	47	0	0 54·2	0·032	1·81	0·108
13	0	4 7·5	0·31	8·30	0·509	48	0	0 52·3	0·031	1·75	0·104
	10	4 4·4	0·31	8·20	0·503	49	0	0 50·5	0·030	1·69	0·101
	20	4 1·4	0·30	8·10	0·496	50	0	0 48·8	0·029	1·63	0·097
	30	3 58·4	0·30	8·00	0·489	51	0	0 47·1	0·028	1·58	0·094
	40	3 55·5	0·29	7·89	0·482	52	0	0 45·4	0·027	1·52	0·090
	50	3 52·6	0·29	7·79	0·476	53	0	0 43·8	0·026	1·47	0·088
14	0	3 49·9	0·28	7·70	0·469	54	0	0 42·3	0·026	1·41	0·085
	10	3 47·1	0·28	7·61	0·464	55	0	0 40·8	0·025	1·36	0·082
	20	3 44·4	0·27	7·52	0·458	56	0	0 39·3	0·025	1·31	0·079
	30	3 41·8	0·26	7·43	0·453	57	0	0 37·8	0·025	1·26	0·076
	40	3 39·2	0·26	7·34	0·448	58	0	0 36·4	0·024	1·22	0·073
	50	3 36·7	0·25	7·26	0·444	59	0	0 35·0	0·024	1·17	0·070
15	0	3 34·3	0·24	7·18	0·439	60	0	0 33·6	0·023	1·12	0·067
	30	3 27·3	0·22	6·95	0·424	61	0	0 32·3	0·022	1·08	0·065
16	0	3 20·6	0·21	6·73	0·411	62	0	0 31·0	0·022	1·04	0·062
	30	3 14·4	0·20	6·51	0·399	63	0	0 29·7	0·021	0·99	0·060
17	0	3 8·5	0·19	6·31	0·386	64	0	0 28·4	0·021	0·95	0·057
	30	3 2·9	0·18	6·12	0·374	65	0	0 27·2	0·020	0·91	0·055
18	0	2 57·6	0·17	5·95	0·362	66	0	0 25·9	0·020	0·87	0·052
19	0	2 47·7	0·16	5·61	0·341	67	0	0 24·7	0·020	0·83	0·050
20	0	2 38·7	0·15	5·31	0·322	68	0	0 23·5	0·020	0·79	0·047
21	0	2 30·5	0·13	5·04	0·305	69	0	0 22·4	0·020	0·75	0·045
22	0	2 23·2	0·12	4·79	0·290	70	0	0 21·2	0·020	0·71	0·043
23	0	2 16·5	0·11	4·57	0·276	71	0	0 19·9	0·020	0·67	0·040
24	0	2 10·1	0·10	4·35	0·264	72	0	0 18·8	0·019	0·63	0·038
25	0	2 4·2	0·09	4·16	0·252	73	0	0 17·7	0·018	0·59	0·036
26	0	1 58·8	0·09	3·98	0·241	74	0	0 16·6	0·018	0·56	0·033
27	0	1 53·8	0·08	3·81	0·230	75	0	0 15·5	0·018	0·52	0·031
28	0	1 49·1	0·08	3·65	0·219	76	0	0 14·4	0·018	0·48	0·029
29	0	1 44·7	0·07	3·50	0·209	77	0	0 13·4	0·017	0·45	0·027
30	0	1 40·5	0·07	3·36	0·201	78	0	0 12·3	0·017	0·41	0·025
31	0	1 36·6	0·06	3·23	0·193	79	0	0 11·2	0·017	0·38	0·023
32	0	1 33·0	0·06	3·11	0·186	80	0	0 10·2	0·017	0·34	0·021
33	0	1 29·5	0·06	2·99	0·179	81	0	0 9·2	0·017	0·31	0·018
34	0	1 26·1	0·05	2·88	0·173	82	0	0 8·2	0·017	0·27	0·016
35	0	1 23·0	0·05	2·78	0·167	83	0	0 7·1	0·017	0·24	0·014
36	0	1 20·0	0·05	2·68	0·161	84	0	0 6·1	0·017	0·20	0·012
37	0	1 17·1	0·05	2·58	0·155	85	0	0 5·1	0·017	0·17	0·010
38	0	1 14·4	0·05	2·49	0·149	86	0	0 4·1	0·017	0·14	0·008
39	0	1 11·8	0·04	2·40	0·144	87	0	0 3·1	0·017	0·10	0·006
40	0	1 9·3	0·04	2·32	0·139	88	0	0 2·0	0·017	0·07	0·004
41	0	1 6·9	0·04	2·24	0·134	89	0	0 1·0	0·017	0·03	0·002

NOTE.—If the Barometer is over 30 in. the Barometer correction is *additive* to the Mean Refraction; if the Thermometer is less than 50° the Thermometer correction is *additive* to the Mean Refraction; and *vice versa*.

TABLE XIII

Ex-Meridian Table

For computing the reduction to the meridian in seconds, or $\frac{\text{Vers h}}{\text{Sin 1}''}$.

o HOURS

S.	0m	1m	2m	3m	4m	5m	6m	7m	8m	9m	10m	11m	12m	13m	14m	15m	16m	17m	18m	19m	20m	21m	22m	23m	24m
	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
0	0·0	2·0	7·8	17·7	31·4	49·1	70·7	96·2	125·7	159·0	196·3	237·5	282·7	331·7	384·7	441·6	502·5	567·2	635·9	708·4	784·9	865·3	949·6	1037·8	1129·9
1	0·0	2·0	8·0	17·9	31·7	49·4	71·1	96·7	126·2	159·6	197·0	238·3	283·5	332·6	385·6	442·6	503·5	568·3	637·0	709·7	786·2	866·6	951·0	1039·3	1131·4
2	0·0	2·1	8·1	18·1	31·9	49·7	71·5	97·1	126·7	160·2	197·6	239·0	284·3	333·4	386·6	443·6	504·6	569·4	638·2	710·9	787·5	868·0	952·4	1040·8	1133·0
3	0·0	2·2	8·2	18·3	32·2	50·1	71·9	97·6	127·2	160·8	198·3	239·7	285·0	334·3	387·5	444·6	505·6	570·5	639·4	712·1	788·8	869·4	953·8	1042·3	1134·6
4	0·0	2·2	8·4	18·5	32·5	50·4	72·3	98·0	127·8	161·4	198·9	240·4	285·8	335·2	388·4	445·6	506·7	571·6	640·6	713·4	790·1	870·8	955·3	1043·8	1136·2
5	0·0	2·3	8·5	18·7	32·7	50·7	72·7	98·5	128·3	162·0	199·6	241·2	286·6	336·0	389·3	446·5	507·7	572·8	641·7	714·6	791·4	872·1	956·7	1045·3	1137·8
6	0·0	2·4	8·7	18·9	33·0	51·1	73·1	99·0	128·8	162·6	200·3	241·9	287·4	336·9	390·2	447·5	508·8	573·9	642·9	715·9	792·7	873·5	958·2	1046·8	1139·3
7	0·0	2·4	8·8	19·1	33·3	51·4	73·5	99·4	129·3	163·2	200·9	242·6	288·2	337·7	391·2	448·5	509·8	575·0	644·1	717·1	794·0	874·9	959·6	1048·3	1140·9
8	0·0	2·5	8·9	19·3	33·5	51·7	73·9	99·9	129·9	163·8	201·6	243·3	289·0	338·6	392·1	449·5	510·9	576·1	645·3	718·4	795·4	876·3	961·1	1049·8	1142·5
9	0·0	2·6	9·1	19·5	33·8	52·1	74·3	100·4	130·4	164·4	202·2	244·1	289·8	339·4	393·0	450·5	511·9	577·2	646·5	719·6	796·7	877·6	962·5	1051·3	1144·0
10	0·0	2·7	9·2	19·7	34·1	52·4	74·7	100·8	130·9	165·0	202·9	244·8	290·6	340·3	393·9	451·5	513·0	578·4	647·7	720·9	798·0	879·0	963·9	1052·8	1145·6
11	0·1	2·7	9·4	19·9	34·4	52·7	75·1	101·3	131·5	165·6	203·6	245·5	291·4	341·2	394·9	452·5	514·0	579·5	648·9	722·1	799·3	880·4	965·4	1054·3	1147·2
12	0·1	2·8	9·5	20·1	34·6	53·1	75·5	101·8	132·0	166·2	204·2	246·2	292·2	342·0	395·8	453·5	515·1	580·6	650·0	723·4	800·7	881·8	966·9	1055·9	1148·8
13	0·1	2·9	9·6	20·3	34·9	53·4	75·9	102·3	132·6	166·8	204·9	247·0	293·0	342·9	396·7	454·5	516·1	581·7	651·2	724·6	802·0	883·2	968·3	1057·4	1150·4
14	0·1	3·0	9·8	20·5	35·2	53·8	76·3	102·7	133·1	167·4	205·6	247·7	293·8	343·7	397·6	455·5	517·2	582·9	652·4	725·9	803·3	884·6	969·8	1058·9	1152·0
15	0·1	3·1	9·9	20·7	35·5	54·1	76·7	103·2	133·6	168·0	206·3	248·5	294·6	344·6	398·6	456·5	518·3	584·0	653·6	727·2	804·6	886·0	971·2	1060·4	1153·6
16	0·1	3·1	10·1	20·9	35·7	54·5	77·1	103·7	134·2	168·6	206·9	249·2	295·4	345·5	399·5	457·5	519·3	585·1	654·8	728·4	806·0	887·4	972·7	1062·0	1155·2
17	0·2	3·2	10·2	21·2	36·0	54·8	77·5	104·2	134·7	169·2	207·6	249·9	296·2	346·4	400·5	458·5	520·4	586·2	656·0	729·7	807·3	888·8	974·1	1063·5	1156·8
18	0·2	3·3	10·4	21·4	36·3	55·1	77·9	104·6	135·3	169·8	208·3	250·7	297·0	347·2	401·4	459·5	521·5	587·4	657·2	730·9	808·6	890·2	975·5	1065·0	1158·3
19	0·2	3·4	10·5	21·6	36·6	55·5	78·3	105·1	135·8	170·4	208·9	251·4	297·8	348·1	402·3	460·5	522·5	588·5	658·4	732·2	809·9	891·6	977·0	1066·5	1159·9
20	0·2	3·5	10·7	21·8	36·9	55·8	78·8	105·6	136·3	171·0	209·6	252·2	298·6	349·0	403·3	461·5	523·6	589·6	659·6	733·5	811·3	893·0	978·5	1068·1	1161·5
21	0·2	3·6	10·8	22·0	37·2	56·2	79·2	106·1	136·9	171·6	210·3	252·9	299·4	349·8	404·2	462·5	524·7	590·8	660·8	734·7	812·6	894·4	979·9	1069·6	1163·1
22	0·3	3·7	11·0	22·3	37·4	56·5	79·6	106·6	137·4	172·2	211·0	253·6	300·2	350·7	405·1	463·5	525·7	591·9	662·0	736·0	813·9	895·8	981·4	1071·1	1164·7
23	0·3	3·8	11·1	22·5	37·7	56·9	80·0	107·0	138·0	172·9	211·7	254·4	301·0	351·6	406·1	464·5	526·8	593·0	663·2	737·3	815·2	897·2	982·9	1072·6	1166·3
24	0·3	3·8	11·3	22·7	38·0	57·3	80·4	107·5	138·5	173·5	212·3	255·1	301·8	352·5	407·0	465·5	527·9	594·2	664·4	738·5	816·6	898·6	984·4	1074·2	1167·9
25	0·3	3·9	11·5	22·9	38·3	57·6	80·8	108·0	139·1	174·1	213·0	255·9	302·6	353·3	408·0	466·5	529·0	595·3	665·6	739·8	817·9	900·0	985·8	1075·7	1169·5
26	0·4	4·0	11·6	23·1	38·6	58·0	81·3	108·5	139·6	174·7	213·7	256·6	303·5	354·2	408·9	467·5	530·0	596·5	666·8	741·1	819·2	901·4	987·3	1077·2	1171·1
27	0·4	4·1	11·8	23·4	38·9	58·3	81·7	109·0	140·2	175·3	214·4	257·4	304·3	355·1	409·8	468·5	531·1	597·6	668·0	742·3	820·5	902·8	988·8	1078·7	1172·7
28	0·4	4·2	11·9	23·6	39·2	58·7	82·1	109·5	140·7	175·9	215·1	258·1	305·1	356·0	410·8	469·5	532·2	598·7	669·2	743·6	821·9	904·2	990·3	1080·3	1174·3
29	0·5	4·3	12·1	23·8	39·5	59·0	82·5	110·0	141·3	176·6	215·8	258·9	305·9	356·9	411·7	470·5	533·3	599·9	670·4	744·9	823·2	905·6	991·8	1081·8	1175·9

0 Hours

S.	0m	1m	2m	3m	4m	5m	6m	7m	8m	9m	10m	11m	12m	13m	14m	15m	16m	17m	18m	19m	20m	21m	22m	23m	24m
	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
30	0·5	4·4	12·3	24·0	39·8	59·4	83·0	110·4	141·8	177·2	216·4	259·6	306·7	357·7	412·7	471·5	534·3	601·0	671·6	746·2	824·6	907·0	993·2	1083·3	1177·5
31	0·5	4·5	12·4	24·3	40·1	59·8	83·4	110·9	142·4	177·8	217·1	260·4	307·5	358·6	413·6	472·6	535·4	602·2	672·8	747·4	825·9	908·4	994·7	1084·8	1179·1
32	0·6	4·6	12·6	24·5	40·3	60·1	83·8	111·4	143·0	178·4	217·8	261·1	308·4	359·5	414·6	473·6	536·5	603·3	674·1	748·7	827·3	909·8	996·2	1086·4	1180·7
33	0·6	4·7	12·8	24·7	40·6	60·5	84·2	111·9	143·5	179·0	218·5	261·9	309·2	360·4	415·5	474·6	537·6	604·5	675·3	750·0	828·6	911·2	997·6	1087·9	1182·3
34	0·6	4·8	12·9	25·0	40·9	60·8	84·7	112·4	144·1	179·7	219·2	262·6	310·0	361·3	416·5	475·6	538·7	605·6	676·5	751·3	829·9	912·6	999·1	1089·5	1183·9
35	0·7	4·9	13·1	25·2	41·2	61·2	85·1	112·9	144·6	180·3	219·9	263·4	310·8	362·2	417·4	476·6	539·7	606·8	677·7	752·6	831·2	914·0	1000·6	1091·0	1185·5
36	0·7	5·0	13·3	25·4	41·5	61·6	85·5	113·4	145·2	180·9	220·6	264·1	311·6	363·1	418·4	477·6	540·8	607·9	678·9	753·8	832·6	915·5	1002·1	1092·6	1187·1
37	0·7	5·1	13·4	25·7	41·8	61·9	86·0	113·9	145·8	181·6	221·3	264·9	312·5	364·0	419·4	478·7	541·9	609·1	680·1	755·1	833·9	916·9	1003·5	1094·1	1188·7
38	0·8	5·2	13·6	25·9	42·1	62·3	86·4	114·4	146·3	182·2	222·0	265·7	313·3	364·8	420·3	479·7	543·0	610·2	681·3	756·4	835·3	918·3	1005·0	1095·7	1190·3
39	0·8	5·3	13·8	26·2	42·5	62·7	86·8	114·9	146·9	182·8	222·7	266·4	314·1	365·7	421·3	480·7	544·1	611·4	682·6	757·7	836·6	919·7	1006·5	1097·2	1191·9
40	0·9	5·4	14·0	26·4	42·8	63·0	87·3	115·4	147·5	183·5	223·4	267·2	315·0	366·6	422·2	481·7	545·2	612·5	683·8	759·0	838·0	921·1	1008·0	1098·8	1193·5
41	0·9	5·6	14·1	26·6	43·1	63·4	87·7	115·9	148·0	184·1	224·1	268·0	315·8	367·5	423·2	482·8	546·3	613·7	685·0	760·2	839·3	922·5	1009·4	1100·3	1195·1
42	1·0	5·7	14·3	26·9	43·4	63·8	88·1	116·4	148·6	184·7	224·8	268·7	316·6	368·4	424·2	483·8	547·4	614·8	686·2	761·5	840·7	923·9	1010·9	1101·9	1196·7
43	1·0	5·8	14·5	27·1	43·7	64·2	88·6	116·9	149·2	185·4	225·5	269·5	317·4	369·3	425·1	484·8	548·4	616·0	687·4	762·8	842·0	925·3	1012·4	1103·4	1198·3
44	1·1	5·9	14·7	27·4	44·0	64·5	89·0	117·4	149·7	186·0	226·2	270·3	318·3	370·2	426·1	485·8	549·5	617·2	688·7	764·1	843·4	926·8	1013·9	1105·0	1199·9
45	1·1	6·0	14·8	27·6	44·3	64·9	89·5	117·9	150·3	186·6	226·9	271·0	319·1	371·1	427·0	486·9	550·6	618·3	689·9	765·4	844·7	928·2	1015·4	1106·5	1201·5
46	1·2	6·1	15·0	27·9	44·6	65·3	89·9	118·4	150·9	187·3	227·6	271·8	319·9	372·0	428·0	487·9	551·7	619·5	691·1	766·7	846·1	929·6	1016·9	1108·1	1203·1
47	1·2	6·2	15·2	28·1	44·9	65·7	90·3	118·9	151·5	187·9	228·3	272·6	320·8	372·9	429·0	488·9	552·8	620·6	692·4	768·0	847·5	931·0	1018·4	1109·6	1204·7
48	1·3	6·4	15·4	28·3	45·2	66·0	90·8	119·5	152·0	188·5	229·0	273·3	321·6	373·8	430·0	490·0	553·9	621·8	693·6	769·3	848·9	932·4	1019·9	1111·2	1206·4
49	1·3	6·5	15·6	28·6	45·5	66·4	91·2	120·0	152·6	189·2	229·7	274·1	322·4	374·7	430·9	491·0	555·0	623·0	694·8	770·6	850·2	933·8	1021·4	1112·7	1208·0
50	1·4	6·6	15·8	28·8	45·9	66·8	91·7	120·5	153·2	189·8	230·4	274·9	323·3	375·6	431·9	492·0	556·1	624·1	696·0	771·9	851·6	935·2	1022·8	1114·3	1209·6
51	1·4	6·7	15·9	29·1	46·2	67·2	92·1	121·0	153·8	190·5	231·1	275·6	324·1	376·5	432·8	493·1	557·2	625·3	697·3	773·1	852·9	936·6	1024·3	1115·8	1211·2
52	1·5	6·8	16·1	29·4	46·5	67·6	92·6	121·5	154·4	191·1	231·8	276·4	325·0	377·4	433·8	494·1	558·3	626·5	698·5	774·5	854·3	938·1	1025·8	1117·4	1212·9
53	1·5	7·0	16·3	29·6	46·8	68·0	93·0	122·0	154·9	191·8	232·5	277·2	325·8	378·3	434·8	495·2	559·4	627·6	699·7	775·7	855·7	939·5	1027·3	1118·9	1214·5
54	1·6	7·1	16·5	29·9	47·1	68·3	93·5	122·5	155·5	192·4	233·2	278·0	326·7	379·3	435·8	496·2	560·5	628·8	701·0	777·1	857·1	940·9	1028·8	1120·5	1216·1
55	1·6	7·2	16·7	30·1	47·5	68·7	93·9	123·1	156·1	193·1	234·0	278·8	327·5	380·2	436·7	497·2	561·6	630·0	702·2	778·4	858·4	942·3	1030·3	1122·0	1217·7
56	1·7	7·3	16·9	30·4	47·8	69·1	94·4	123·6	156·7	193·7	234·7	279·5	328·4	381·1	437·7	498·3	562·8	631·2	703·5	779·7	859·8	943·8	1031·8	1123·6	1219·4
57	1·8	7·5	17·1	30·6	48·1	69·5	94·8	124·1	157·3	194·4	235·4	280·3	329·2	382·0	438·7	499·3	563·9	632·3	704·7	781·0	861·1	945·2	1033·3	1125·1	1221·0
58	1·8	7·6	17·3	30·9	48·4	69·9	95·3	124·6	157·8	195·0	236·1	281·1	330·0	382·9	439·7	500·4	565·0	633·5	705·9	782·3	862·5	946·6	1034·8	1126·7	1222·6
59	1·9	7·7	17·5	31·2	48·8	70·3	95·7	125·1	158·4	195·7	236·8	281·9	330·9	383·8	440·6	501·4	566·1	634·7	707·1	783·6	863·9	948·1	1036·3	1128·3	1224·2
60	2·0	7·8	17·7	31·4	49·1	70·7	96·2	125·7	159·0	196·3	237·5	282·7	331·7	384·7	441·6	502·5	567·2	635·9	708·4	784·9	865·3	949·6	1037·8	1129·9	1225·9

***TABLE XIV—Table for the Graduation**

Latitude.		Abscissæ and Ordinates for					
		5′ Longitude.		10′ Longitude.		15′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
0	0	5·034	0·000	10·068	0·000	15·102	0·000
	10	5·034	0·000	10·068	0·000	15·102	0·000
	20	5·034	0·000	10·068	0·000	15·101	0·000
	30	5·034	0·000	10·068	0·000	15·101	0·000
	40	5·034	0·000	10·068	0·000	15·101	0·001
	50	5·033	0·000	10·067	0·000	15·100	0·001
1	0	5·033	0·000	10·067	0·000	15·100	0·001
	10	5·033	0·000	10·066	0·000	15·099	0·001
	20	5·033	0·000	10·065	0·000	15·098	0·001
	30	5·032	0·000	10·064	0·000	15·097	0·001
	40	5·032	0·000	10·064	0·001	15·096	0·001
	50	5·031	0·000	10·063	0·001	15·095	0·001
2	0	5·031	0·000	10·062	0·001	15·093	0·001
	10	5·030	0·000	10·061	0·001	15·091	0·001
	20	5·030	0·000	10·059	0·001	15·089	0·001
	30	5·029	0·000	10·058	0·001	15·087	0·001
	40	5·028	0·000	10·057	0·001	15·085	0·001
	50	5·028	0·000	10·056	0·001	15·084	0·002
3	0	5·027	0·000	10·054	0·001	15·082	0·002
	10	5·026	0·000	10·053	0·001	15·079	0·002
	20	5·025	0·000	10·051	0·001	15·076	0·002
	30	5·024	0·000	10·049	0·001	15·073	0·002
	40	5·023	0·000	10·047	0·001	15·070	0·002
	50	5·022	0·000	10·045	0·001	15·067	0·003
4	0	5·022	0·000	10·043	0·001	15·064	0·003
	10	5·020	0·000	10·040	0·001	15·061	0·003
	20	5·019	0·000	10·038	0·001	15·058	0·003
	30	5·018	0·000	10·036	0·001	15·055	0·003
	40	5·017	0·000	10·034	0·001	15·051	0·003
	50	5·016	0·000	10·032	0·001	15·048	0·003
5	0	5·015	0·000	10·029	0·001	15·044	0·003

* Notes.

(1) These abscissæ and ordinates are given in inches and decimals of an inch for a scale of 1 inch to a nautical mile (approx. 1/73,000). For any other scale the figures must be multiplied by that scale as given in inches to a mile.

Latitude.	Abscissæ and Ordinates for					
	20′ Longitude.		25′ Longitude.		30′ Longitude.	
	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
° ′						
0 0	20·136	0·000	25·170	0·000	30·204	0·000
10	20·136	0·000	25·170	0·000	30·204	0·000
20	20·135	0·000	25·169	0·001	30·203	0·001
30	20·135	0·001	25·169	0·001	30·202	0·001
40	20·135	0·001	25·168	0·001	30·202	0·002
50	20·134	0·001	25·167	0·002	30·201	0·002
1 0	20·133	0·001	25·167	0·002	30·200	0·003
10	20·132	0·001	25·166	0·002	30·198	0·003
20	20·131	0·001	25·164	0·002	30·196	0·003
30	20·130	0·001	25·162	0·002	30·193	0·003
40	20·128	0·001	25·160	0·003	30·191	0·004
50	20·126	0·002	25·158	0·003	30·189	0·004
2 0	20·124	0·002	25·155	0·003	30·186	0·004
10	20·121	0·002	25·152	0·003	30·182	0·004
20	20·118	0·002	25·148	0·003	30·178	0·005
30	20·116	0·002	25·145	0·003	30·174	0·005
40	20·114	0·002	25·142	0·004	30·170	0·006
50	20·111	0·003	25·139	0·004	30·167	0·007
3 0	20·108	0·003	25·136	0·004	30·163	0·007
10	20·105	0·003	25·132	0·004	30·158	0·007
20	20·102	0·003	25·127	0·004	30·152	0·008
30	20·098	0·003	25·122	0·005	30·146	0·008
40	20·094	0·004	25·117	0·005	30·140	0·009
50	20·090	0·004	25·112	0·006	30·135	0·009
4 0	20·086	0·004	25·107	0·006	30·129	0·010
10	20·081	0·004	25·102	0·006	30·123	0·010
20	20·076	0·004	25·096	0·006	30·116	0·010
30	20·072	0·004	25·090	0·007	30·109	0·010
40	20·068	0·005	25·085	0·007	30·103	0·011
50	20·064	0·005	25·080	0·008	30·096	0·011
5 0	20·059	0·005	25·074	0·008	30·088	0·011

(2) The table is calculated for the gnomonic projection but may be used without appreciable error for graduating charts on Cassini's projection on scales of 1/100,000 and larger.

TABLE XIV—Table for the Graduation

Latitude.		Abscissæ and Ordinates for					
		5′ Longitude.		10′ Longitude.		15′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
5	0	5·015	0·000	10·029	0·001	15·044	0·003
	10	5·013	0·000	10·027	0·001	15·040	0·003
	20	5·012	0·000	10·024	0·001	15·035	0·003
	30	5·010	0·000	10·021	0·001	15·031	0·003
	40	5·009	0·000	10·018	0·002	15·027	0·003
	50	5·007	0·000	10·015	0·002	15·022	0·003
6	0	5·006	0·000	10·012	0·002	15·018	0·004
	10	5·004	0·000	10·009	0·002	15·013	0·004
	20	5·003	0·000	10·006	0·002	15·008	0·004
	30	5·001	0·000	10·002	0·002	15·003	0·004
	40	4·999	0·000	9·999	0·002	14·998	0·004
	50	4·998	0·000	9·996	0·002	14·993	0·004
7	0	4·996	0·000	9·992	0·002	14·988	0·004
	10	4·994	0·000	9·988	0·002	14·982	0·004
	20	4·992	0·000	9·984	0·002	14·976	0·004
	30	4·990	0·000	9·981	0·002	14·971	0·004
	40	4·988	0·001	9·977	0·002	14·965	0·004
	50	4·986	0·001	9·973	0·002	14·959	0·004
8	0	4·984	0·001	9·969	0·002	14·953	0·004
	10	4·982	0·001	9·965	0·002	14·947	0·004
	20	4·980	0·001	9·961	0·002	14·941	0·004
	30	4·978	0·001	9·956	0·002	14·934	0·005
	40	4·976	0·001	9·952	0·002	14·928	0·005
	50	4·974	0·001	9·948	0·003	14·921	0·005
9	0	4·971	0·001	9·943	0·003	14·914	0·005
	10	4·969	0·001	9·938	0·003	14·907	0·005
	20	4·967	0·001	9·933	0·003	14·900	0·005
	30	4·964	0·001	9·928	0·003	14·892	0·005
	40	4·962	0·001	9·923	0·003	14·885	0·005
	50	4·960	0·001	9·918	0·003	14·878	0·005
10	0	4·957	0·001	9·913	0·003	14·870	0·005
	10	4·955	0·001	9·908	0·003	14·862	0·005
	20	4·952	0·001	9·903	0·003	14·854	0·005
	30	4·949	0·001	9·897	0·003	14·846	0·005
	40	4·946	0·001	9·892	0·003	14·838	0·006
	50	4·943	0·001	9·887	0·003	14·830	0·006
11	0	4·941	0·001	9·881	0·003	14·822	0·006

Latitude.	Abscissæ and Ordinates for					
	20′ Longitude.		25′ Longitude.		30′ Longitude.	
	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
° ′						
5 0	20·059	0·005	25·074	0·008	30·088	0·011
10	20·054	0·005	25·067	0·008	30·080	0·012
20	20·048	0·005	25·059	0·008	30·071	0·012
30	20·042	0·005	25·052	0·009	30·063	0·013
40	20·036	0·006	25·045	0·009	30·054	0·013
50	20·030	0·006	25·037	0·010	30·045	0·014
6 0	20·024	0·006	25·030	0·010	30·036	0·014
10	20·018	0·006	25·022	0·010	30·027	0·014
20	20·010	0·006	25·014	0·010	30·017	0·015
30	20·004	0·007	25·005	0·011	30·007	0·015
40	19·998	0·007	24·997	0·011	29·997	0·015
50	19·991	0·007	24·989	0·011	29·987	0·016
7 0	19·984	0·007	24·980	0·011	29·976	0·016
10	19·977	0·007	24·970	0·011	29·965	0·016
20	19·969	0·007	24·961	0·011	29·954	0·017
30	19·962	0·007	24·952	0·012	29·942	0·017
40	19·954	0·007	24·942	0·012	29·931	0·018
50	19·946	0·008	24·932	0·012	29·919	0·018
8 0	19·938	0·008	24·922	0·012	29·907	0·018
10	19·930	0·008	24·912	0·012	29·894	0·018
20	19·921	0·008	24·901	0·013	29·881	0·019
30	19·912	0·008	24·890	0·013	29·868	0·019
40	19·904	0·008	24·879	0·014	29·855	0·020
50	19·895	0·009	24·868	0·014	29·842	0·020
9 0	19·886	0·009	24·857	0·014	29·828	0·020
10	19·876	0·009	24·845	0·014	29·813	0·020
20	19·866	0·009	24·833	0·015	29·799	0·021
30	19·856	0·009	24·820	0·015	29·784	0·021
40	19·846	0·009	24·808	0·015	29·770	0·022
50	19·836	0·009	24·796	0·016	29·755	0·022
10 0	19·826	0·010	24·783	0·016	29·740	0·023
10	19·816	0·010	24·770	0·016	29·724	0·023
20	19·806	0·010	24·757	0·017	29·708	0·024
30	19·795	0·011	24·744	0·017	29·691	0·024
40	19·785	0·011	24·731	0·017	29·675	0·024
50	19·774	0·011	24·718	0·017	29·659	0·024
11 0	19·763	0·011	24·703	0·017	29·643	0·024

TABLE XIV—Table for the Graduation

Latitude.		Abscissæ and Ordinates for					
		5′ Longitude.		10′ Longitude.		15′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
11	0	4·941	0·001	9·881	0·003	14·822	0·006
	10	4·938	0·001	9·875	0·003	14·813	0·006
	20	4·935	0·001	9·869	0·003	14·804	0·006
	30	4·932	0·001	9·863	0·003	14·795	0·006
	40	4·929	0·001	9·857	0·003	14·786	0·007
	50	4·926	0·001	9·851	0·003	14·777	0·007
12	0	4·922	0·001	9·845	0·003	14·767	0·007
	10	4·919	0·001	9·838	0·003	14·758	0·007
	20	4·916	0·001	9·832	0·003	14·749	0·007
	30	4·913	0·001	9·826	0·003	14·739	0·007
	40	4·910	0·001	9·820	0·003	14·730	0·007
	50	4·907	0·001	9·813	0·003	14·720	0·007
13	0	4·903	0·001	9·807	0·004	14·710	0·007
	10	4·900	0·001	9·800	0·004	14·700	0·007
	20	4·897	0·001	9·793	0·004	14·690	0·007
	30	4·893	0·001	9·786	0·004	14·679	0·007
	40	4·890	0·001	9·779	0·004	14·669	0·008
	50	4·887	0·001	9·772	0·004	14·659	0·008
14	0	4·883	0·001	9·765	0·004	14·648	0·008
	10	4·879	0·001	9·758	0·004	14·637	0·008
	20	4·875	0·001	9·750	0·004	14·626	0·008
	30	4·871	0·001	9·743	0·004	14·614	0·008
	40	4·868	0·001	9·736	0·004	14·603	0·008
	50	4·864	0·001	9·729	0·004	14·592	0·008
15	0	4·860	0·001	9·721	0·004	14·581	0·008
	10	4·856	0·001	9·713	0·004	14·569	0·008
	20	4·852	0·001	9·705	0·004	14·557	0·008
	30	4·848	0·001	9·697	0·004	14·545	0·008
	40	4·844	0·001	9·689	0·004	14·533	0·009
	50	4·840	0·001	9·681	0·004	14·522	0·009
16	0	4·837	0·001	9·673	0·004	14·510	0·009
	10	4·833	0·001	9·665	0·004	14·497	0·009
	20	4·829	0·001	9·657	0·004	14·484	0·009
	30	4·824	0·001	9·648	0·004	14·472	0·009
	40	4·820	0·001	9·640	0·004	14·459	0·009
	50	4·816	0·001	9·632	0·004	14·447	0·009
17	0	4·811	0·001	9·623	0·004	14·434	0·010

Latitude.		Abscissæ and Ordinates for					
		20′ Longitude.		25′ Longitude.		30′ Longitude	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
11	0	19·763	0·011	24·703	0·017	29·643	0·024
	10	19·751	0·011	24·689	0·017	29·626	0·025
	20	19·739	0·011	24·674	0·017	29·608	0·025
	30	19·727	0·012	24·659	0·018	29·590	0·026
	40	19·715	0·012	24·643	0·018	29·572	0·026
	50	19·703	0·012	24·628	0·018	29·553	0·027
12	0	19·690	0·012	24·613	0·018	29·534	0·027
	10	19·678	0·012	24·597	0·018	29·515	0·027
	20	19·665	0·012	24·582	0·019	29·497	0·028
	30	19·652	0·013	24·565	0·019	29·478	0·028
	40	19·639	0·013	24·549	0·020	29·459	0·029
	50	19·626	0·013	24·533	0·020	29·440	0·029
13	0	19·613	0·013	24·517	0·020	29·421	0·029
	10	19·600	0·013	24·499	0·020	29·400	0·029
	20	19·586	0·013	24·482	0·021	29·379	0·029
	30	19·572	0·013	24·465	0·021	29·358	0·030
	40	19·558	0·014	24·448	0·021	29·337	0·030
	50	19·544	0·014	24·431	0·022	29·316	0·030
14	0	19·530	0·014	24·413	0·022	29·295	0·031
	10	19·515	0·014	24·395	0·022	29·273	0·031
	20	19·501	0·014	24·376	0·022	29·251	0·032
	30	19·486	0·014	24·357	0·022	29·229	0·032
	40	19·471	0·015	24·339	0·022	29·207	0·033
	50	19·457	0·015	24·321	0·023	29·185	0·033
15	0	19·442	0·015	24·302	0·023	29·162	0·033
	10	19·426	0·015	24·282	0·023	29·138	0·033
	20	19·410	0·015	24·262	0·024	29·114	0·034
	30	19·394	0·015	24·242	0·024	29·091	0·034
	40	19·378	0·016	24·222	0·024	29·067	0·035
	50	19·362	0·016	24·202	0·024	29·044	0·035
16	0	19·346	0·016	24·182	0·024	29·020	0·035
	10	19·330	0·016	24·162	0·024	28·995	0·035
	20	19·313	0·016	24·141	0·024	28·969	0·035
	30	19·296	0·016	24·120	0·024	28·944	0·036
	40	19·280	0·016	24·099	0·024	28·919	0·036
	50	19·263	0·016	24·078	0·024	28·894	0·036
17	0	19·246	0·017	24·057	0·025	28·868	0·036

TABLE XIV—Table for the Graduation

Latitude.		Abscissæ and Ordinates for					
		5′ Longitude.		10′ Longitude.		15′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
17	0	4·811	0·001	9·623	0·004	14·434	0·010
	10	4·807	0·001	9·614	0·004	14·421	0·010
	20	4·802	0·001	9·605	0·004	14·407	0·010
	30	4·798	0·001	9·596	0·004	14·394	0·010
	40	4·793	0·001	9·587	0·004	14·380	0·010
	50	4·789	0·001	9·578	0·004	14·367	0·010
18	0	4·785	0·001	9·569	0·004	14·354	0·010
	10	4·780	0·001	9·560	0·004	14·340	0·010
	20	4·775	0·001	9·551	0·004	14·316	0·010
	30	4·770	0·001	9·541	0·004	14·311	0·010
	40	4·765	0·001	9·531	·0004	14·297	0·010
	50	4·760	0·001	9·522	0·004	14·283	0·010
19	0	4·756	0·001	9·513	0·004	14·269	0·010
	10	4·751	0·001	9·503	0·004	14·255	0·010
	20	4·746	0·001	9·493	0·004	14·240	0·010
	30	4·742	0·001	9·483	0·004	14·225	0·010
	40	4·737	0·001	9·473	0·004	14·210	0·010
	50	4·732	0·001	9·463	0·004	14·195	0·010
20	0	4·727	0·001	9·453	0·004	14·180	0·010
	10	4·721	0·001	9·442	0·004	14·165	0·010
	20	4·716	0·001	9·432	0·004	14·149	0·010
	30	4·711	0·001	9·422	0·004	14·133	0·011
	40	4·706	0·001	9·412	0·005	14·118	0·011
	50	4·701	0·001	9·402	0·005	14·103	0·011
21	0	4·696	0·001	9·391	0·005	14·087	0·011
	10	4·690	0·001	9·381	0·005	14·071	0·011
	20	4·685	0·001	9·370	0·005	14·055	0·011
	30	4·679	0·001	9·359	0·005	14·038	0·011
	40	4·674	0·001	9·348	0·005	14·021	0·011
	50	4·668	0·001	9·337	0·005	14·005	0·011
22	0	4·663	0·001	9·326	0·005	13·989	0·011
	10	4·657	0·001	9·315	0·005	13·973	0·011
	20	4·651	0·001	9·304	0·005	13·956	0·012
	30	4·646	0·001	9·292	0·005	13·939	0·012
	40	4·640	0·001	9·281	0·005	13·922	0·012
	50	4·634	0·001	9·270	0·005	13·905	0·012
23	0	4·629	0·001	9·258	0·005	13·887	0·012

Latitude.		Abscissæ and Ordinates for					
		20′ Longitude.		25′ Longitude.		30′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
17	0	19·246	0·017	24·057	0·025	28·868	0·036
	10	19·228	0·017	24·035	0·025	28·842	0·036
	20	19·210	0·017	24·012	0·026	28·815	0·037
	30	19·192	0·017	23·990	0·026	28·788	0·037
	40	19·174	0·017	23·967	0·027	28·761	0·037
	50	19·156	0·017	23·945	0·027	28·735	0·038
18	0	19·138	0·017	23·923	0·027	28·708	0·038
	10	19·120	0·017	23·900	0·027	28·680	0·038
	20	19·101	0·017	23·876	0·027	28·651	0·039
	30	19·082	0·017	23·852	0·027	28·623	0·039
	40	19·063	0·017	23·829	0·027	28·595	0·040
	50	19·044	0·018	23·806	0·028	28·567	0·040
19	0	19·025	0·018	23·782	0·028	28·539	0·040
	10	19·006	0·018	23·757	0·028	28·509	0·040
	20	18·986	0·018	23·732	0·029	28·479	0·041
	30	18·966	0·019	23·707	0·029	28·450	0·041
	40	18·946	0·019	23·683	0·029	28·420	0·042
	50	18·926	0·019	23·658	0·030	28·390	0·042
20	0	18·906	0·019	23·633	0·030	28·360	0·043
	10	18·886	0·019	23·607	0·030	28·329	0·043
	20	18·865	0·019	23·581	0·030	28·298	0·043
	30	18·844	0·019	23·555	0·030	28·267	0·043
	40	18·824	0·019	23·529	0·030	28·236	0·044
	50	18·803	0·019	23·503	0·030	28·205	0·044
21	0	18·782	0·019	23·477	0·030	28·174	0·044
	10	18·761	0·019	23·451	0·030	28·142	0·044
	20	18·739	0·020	23·424	0·030	28·109	0·044
	30	18·717	0·020	23·397	0·031	28·076	0·045
	40	18·695	0·020	23·370	0·031	28·043	0·045
	50	18·674	0·020	23·343	0·031	28·011	0·045
22	0	18·653	0·020	23·315	0·031	27·978	0·045
	10	18·631	0·020	23·287	0·031	27·945	0·045
	20	18·608	0·020	23·259	0·032	27·911	0·046
	30	18·585	0·020	23·231	0·032	27·877	0·046
	40	18·562	0·021	23·203	0·032	27·843	0·046
	50	18·540	0·021	23·175	0·033	27·809	0·047
23	0	18·517	0·021	23·146	0·033	27·775	0·047

TABLE XIV—Table for the graduation

Latitude.		Abscissæ and Ordinates for					
		5′ Longitude.		10′ Longitude.		15′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
23	0	4·629	0·001	9·258	0·005	13·887	0·012
	10	4·623	0·001	9·247	0·005	13·870	0·012
	20	4·617	0·002	9·235	0·005	13·852	0·012
	30	4·611	0·002	9·223	0·005	13·834	0·012
	40	4·605	0·002	9·211	0·005	13·816	0·012
	50	4·599	0·002	9·199	0·005	13·799	0·012
24	0	4·594	0·002	9·187	0·005	13·781	0·012
	10	4·588	0·002	9·175	0·005	13·763	0·012
	20	4·582	0·002	9·163	0·005	13·745	0·012
	30	4·576	0·002	9·151	0·005	13·727	0·012
	40	4·570	0·002	9·139	0·005	13·709	0·012
	50	4·564	0·002	9·127	0·005	13·690	0·012
25	0	4·557	0·002	9·114	0·005	13·671	0·012
	10	4·551	0·002	9·102	0·005	13·652	0·012
	20	4·545	0·002	9·089	0·005	13·633	0·012
	30	4·538	0·002	9·076	0·006	13·613	0·013
	40	4·532	0·002	9·063	0·006	13·594	0·013
	50	4·526	0·002	9·050	0·006	13·575	0·013
26	0	4·519	0·002	9·037	0·006	13·556	0·013
	10	4·513	0·002	9·024	0·006	13·537	0·013
	20	4·506	0·002	9·011	0·006	13·517	0·013
	30	4·499	0·002	8·998	0·006	13·497	0·013
	40	4·493	0·002	8·985	0·006	13·477	0·013
	50	4·486	0·002	8·972	0·006	13·457	0·013
27	0	4·479	0·002	8·958	0·006	13·437	0·013
	10	4·473	0·002	8·945	0·006	13·417	0·013
	20	4·466	0·002	8·931	0·006	13·397	0·013
	30	4·459	0·002	8·917	0·006	13·376	0·013
	40	4·452	0·002	8·904	0·006	13·356	0·014
	50	4·445	0·002	8·890	0·006	13·336	0·014
28	0	4·438	0·002	8·876	0·006	13·315	0·014
	10	4·431	0·002	8·862	0·006	13·294	0·014
	20	4·424	0·002	8·848	0·006	13·273	0·014
	30	4·417	0·002	8·834	0·006	13·251	0·014
	40	4·410	0·002	8·820	0·006	13·230	0·014
	50	4·403	0·002	8·806	0·006	13·209	0·014
29	0	4·396	0·002	8·792	0·006	13·187	0·014

Latitude.		Abscissæ and Ordinates for					
		20′ Longitude.		25′ Longitude.		30′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
23	0	18·517	0·021	23·146	0·033	27·775	0·047
	10	18·494	0·021	23·117	0·033	27·740	0·047
	20	18·470	0·021	23·087	0·033	27·704	0·048
	30	18·446	0·022	23·057	0·034	27·669	0·048
	40	18·422	0·022	23·027	0·034	27·633	0·048
	50	18·398	0·022	22·998	0·034	27·598	0·049
24	0	18·375	0·022	22·968	0·034	27·562	0·049
	10	18·351	0·022	22·938	0·034	27·526	0·049
	20	18·327	0·022	22·908	0·034	27·490	0·049
	30	18·303	0·022	22·878	0·034	27·454	0·050
	40	18·278	0·023	22·847	0·035	27·417	0·050
	50	18·253	0·023	22·816	0·035	27·380	0·050
25	0	18·228	0·023	22·785	0·035	27·342	0·050
	10	18·203	0·023	22·753	0·035	27·304	0·050
	20	18·177	0·023	22·721	0·035	27·266	0·051
	30	18·151	0·023	22·689	0·036	27·227	0·051
	40	18·126	0·023	22·657	0·036	27·189	0·052
	50	18·101	0·023	22·625	0·036	27·151	0·052
26	0	18·075	0·023	22·593	0·036	27·112	0·052
	10	18·049	0·023	22·560	0·036	27·073	0·052
	20	18·023	0·023	22·527	0·036	27·033	0·052
	30	17·996	0·023	22·494	0·036	26·993	0·052
	40	17·970	0·023	22·461	0·036	26·954	0·053
	50	17·943	0·023	22·428	0·036	26·915	0·053
27	0	17·916	0·024	22·395	0·037	26·875	0·053
	10	17·889	0·024	22·361	0·037	26·834	0·053
	20	17·862	0·024	22·327	0·037	26·793	0·054
	30	17·835	0·024	22·293	0·037	26·752	0·054
	40	17·808	0·024	22·259	0·037	26·711	0·054
	50	17·781	0·024	22·225	0·037	26·670	0·055
28	0	17·753	0·024	22·191	0·037	26·629	0·055
	10	17·725	0·024	22·156	0·037	26·587	0·055
	20	17·697	0·024	22·121	0·037	26·545	0·055
	30	17·668	0·024	22·085	0·038	26·502	0·055
	40	17·640	0·024	22·050	0·038	26·460	0·055
	50	17·612	0·024	22·015	0·038	26·418	0·056
29	0	17·583	0·024	21·979	0·038	26·375	0·056

TABLE XIV—Table for the Graduation

Latitude.		Abscissæ and Ordinates for					
		5′ Longitude.		10′ Longitude.		15′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
29	0	4·396	0·002	8·792	0·006	13·187	0·014
	10	4·389	0·002	8·778	0·006	13·166	0·014
	20	4·382	0·002	8·763	0·006	13·144	0·014
	30	4·374	0·002	8·748	0·006	13·122	0·014
	40	4·367	0·002	8·734	0·006	13·101	0·014
	50	4·360	0·002	8·719	0·006	13·079	0·014
30	0	4·352	0·002	8·705	0·006	13·057	0·014
	10	4·345	0·002	8·690	0·006	13·035	0·014
	20	4·338	0·002	8·675	0·006	13·012	0·014
	30	4·330	0·002	8·659	0·006	12·989	0·014
	40	4·323	0·002	8·644	0·006	12·967	0·015
	50	4·315	0·002	8·629	0·006	12·944	0·015
31	0	4·307	0·002	8·614	0·006	12·921	0·015
	10	4·300	0·002	8·599	0·006	12·898	0·015
	20	4·292	0·002	8·584	0·006	12·875	0·015
	30	4·284	0·002	8·568	0·006	12·852	0·015
	40	4·277	0·002	8·553	0·006	12·829	0·015
	50	4·269	0·002	8·538	0·006	12·806	0·015
32	0	4·261	0·002	8·522	0·006	12·783	0·015
	10	4·253	0·002	8·506	0·006	12·759	0·015
	20	4·245	0·002	8·490	0·006	12·735	0·015
	30	4·237	0·002	8·474	0·007	12·711	0·015
	40	4·229	0·002	8·459	0·007	12·688	0·015
	50	4·221	0·002	8·443	0·007	12 664	0·015
33	0	4·213	0·002	8·427	0·007	12·640	0·015
	10	4·205	0·002	8·411	0·007	12·616	0·015
	20	4·197	0·002	8·395	0·007	12·592	0·015
	30	4·189	0·002	8·378	0·007	12·567	0·015
	40	4·181	0·002	8·362	0·007	12·543	0·015
	50	4·173	0·002	8·346	0·007	12·519	0·015
34	0	4·165	0·002	8·329	0·007	12·494	0·015
	10	4·157	0·002	8·313	0·007	12·469	0·015
	20	4·148	0·002	8·296	0·007	12·444	0·015
	30	4·139	0·002	8·279	0·007	12·418	0·015
	40	4·131	0·002	8·263	0·007	12·393	0·015
	50	4·123	0·002	8·246	0·007	12·368	0·016
35	0	4·114	0·002	8·229	0·007	12·343	0·016

Latitude.		Abscissæ and Ordinates for					
		20′ Longitude.		25′ Longitude.		30′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
29	0	17·583	0·024	21·979	0·038	26·375	0·056
	10	17·555	0·024	21·943	0·038	26·332	0·056
	20	17·526	0·024	21·907	0·038	26·288	0·056
	30	17·497	0·025	21·871	0·039	26·245	0·056
	40	17·468	0·025	21·835	0·039	26·202	0·056
	50	17·439	0·025	21·799	0·039	26·158	0·057
30	0	17·409	0·025	21·762	0·039	26·114	0·057
	10	17·379	0·025	21·724	0·039	26·069	0·057
	20	17·349	0·025	21·686	0·039	26·024	0·057
	30	17·319	0·026	21·648	0·039	25·978	0·058
	40	17·289	0·026	21·611	0·040	25·933	0·058
	50	17·259	0·026	21·573	0·040	25·888	0·058
31	0	17·228	0·026	21·535	0·040	25·843	0·058
	10	17·198	0·026	21·497	0·040	25·797	0·058
	20	17·167	0·026	21·459	0·040	25·751	0·058
	30	17·136	0·026	21·420	0·040	25·705	0·059
	40	17·106	0·026	21·382	0·041	25·659	0·059
	50	17·075	0·026	21·344	0·041	25·613	0·059
32	0	17·044	0·026	21·305	0·041	25·566	0·059
	10	17·013	0·026	21·266	0·041	25·519	0·059
	20	16·981	0·026	21·226	0·041	25·471	0·059
	30	16·949	0·027	21·186	0·041	25·423	0·060
	40	16·918	0·027	21·147	0·041	25·376	0·060
	50	16·886	0·027	21·107	0·042	25·329	0·060
33	0	16·854	0·027	21·067	0·042	25·281	0·060
	10	16·822	0·027	21·027	0·042	25·233	0·060
	20	16·789	0·027	20·986	0·042	25·184	0·060
	30	16·756	0·027	20·945	0·042	25·135	0·060
	40	16·724	0·027	20·905	0·042	25·086	0·061
	50	16·691	0·027	20·864	0·043	25·037	0·061
34	0	16·658	0·027	20·823	0·043	24·988	0·061
	10	16·625	0·027	20·781	0·043	24·938	0·061
	20	16·592	0·027	20·739	0·043	24·888	0·061
	30	16·558	0·027	20·697	0·043	24·837	0·061
	40	16·525	0·027	20·656	0·043	24·787	0·061
	50	16·492	0·027	20·614	0·043	24·737	0·062
35	0	16·458	0·027	20·572	0·043	24·687	0·062

TABLE XIV—Table for the Graduation

Latitude.		Abscissæ and Ordinates for					
		5′ Longitude.		10′ Longitude.		15′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
35	0	4·114	0·002	8·229	0·007	12·343	0·016
	10	4·106	0·002	8·212	0·007	12·318	0·016
	20	4·098	0·002	8·195	0·007	12·292	0·016
	30	4·089	0·002	8·178	0·007	12·266	0·016
	40	4·081	0·002	8·161	0·007	12·241	0·016
	50	4·072	0·002	8·144	0·007	12·215	0·016
36	0	4·063	0·002	8·126	0·007	12·189	0·016
	10	4·055	0·002	8·109	0·007	12·163	0·016
	20	4·046	0·002	8·092	0·007	12·137	0·016
	30	4·037	0·002	8·074	0·007	12·110	0·016
	40	4·028	0·002	8·057	0·007	12·084	0·016
	50	4·019	0·002	8·039	0·007	12·058	0·016
37	0	4·010	0·002	8·021	0·007	12·031	0·016
	10	4·001	0·002	8·003	0·007	12·004	0·016
	20	3·992	0·002	7·985	0·007	11·977	0·016
	30	3·983	0·002	7·967	0·007	11·950	0·016
	40	3·975	0·002	7·949	0·007	11·924	0·016
	50	3·966	0·002	7·931	0·007	11·897	0·016
38	0	3·957	0·002	7·913	0·007	11·870	0·016
	10	3·948	0·002	7·895	0·007	11·843	0·016
	20	3·939	0·002	7·877	0·007	11·815	0·016
	30	3·929	0·002	7·858	0·007	11·787	0·016
	40	3·920	0·002	7·840	0·007	11·760	0·016
	50	3·911	0·002	7·822	0·007	11·733	0·016
39	0	3·902	0·002	7·803	0·007	11·705	0·016
	10	3·893	0·002	7·785	0·007	11·677	0·016
	20	3·884	0·002	7·766	0·007	11·649	0·016
	30	3·874	0·002	7·747	0·007	11·621	0·016
	40	3·865	0·002	7·729	0·007	11·593	0·016
	50	3·855	0·002	7·710	0·007	11·565	0·016
40	0	3·845	0·002	7·691	0·007	11·536	0·017
	10	3·836	0·002	7·672	0·007	11·508	0·017
	20	3·827	0·002	7·653	0·007	11·479	0·017
	30	3·817	0·002	7·634	0·007	11·450	0·017
	40	3·808	0·002	7·615	0·007	11·422	0·017
	50	3·798	0·002	7·596	0·007	11·393	0·017
41	0	3·788	0·002	7·576	0·007	11·364	0·017

Latitude.		Abscissæ and Ordinates for					
		20′ Longitude.		25′ Longitude.		30′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
35	0	16·458	0·027	20·572	0·043	24·687	0·062
	10	16·424	0·027	20·530	0·043	24·636	0·062
	20	16·390	0·027	20·487	0·043	24·585	0·062
	30	16·355	0·027	20·444	0·043	24·533	0·062
	40	16·321	0·028	20·402	0·043	24·482	0·062
	50	16·287	0·028	20·359	0·043	24·431	0·063
36	0	16·253	0·028	20·316	0·044	24·379	0·063
	10	16·218	0·028	20·272	0·044	24·327	0·063
	20	16·183	0·028	20·228	0·044	24·274	0·063
	30	16·147	0·028	20·184	0·044	24·221	0·063
	40	16·112	0·028	20·140	0·044	24·169	0·063
	50	16·077	0·028	20·096	0·044	24·116	0·063
37	0	16·042	0·028	20·052	0·044	24·063	0·064
	10	16·006	0·028	20·007	0·044	24·009	0·064
	20	15·970	0·028	19·962	0·044	23·955	0·064
	30	15·934	0·028	19·917	0·044	23·901	0·064
	40	15·898	0·029	19·873	0·044	23·848	0·064
	50	15·862	0·029	19·828	0·044	23·794	0·064
38	0	15·826	0·029	19·783	0·044	23·740	0·064
	10	15·790	0·029	19·737	0·044	23·685	0·064
	20	15·753	0·029	19·691	0·044	23·630	0·064
	30	15·716	0·029	19·645	0·044	23·575	0·064
	40	15·680	0·029	19·600	0·044	23·520	0·064
	50	15·643	0·029	19·554	0·044	23·465	0·064
39	0	15·606	0·029	19·508	0·044	23·410	0·064
	10	15·569	0·029	19·462	0·044	23·354	0·064
	20	15·532	0·029	19·415	0·044	23·298	0·064
	30	15·494	0·029	19·368	0·045	23·242	0·064
	40	15·457	0·029	19·321	0·045	23·186	0·064
	50	15·420	0·029	19·274	0·045	23·130	0·064
40	0	15·382	0·029	19·227	0·045	23·073	0·064
	10	15·344	0·029	19·180	0·045	23·016	0·064
	20	15·306	0·029	19·132	0·045	22·959	0·064
	30	15·268	0·029	19·084	0·045	22·901	0·065
	40	15·230	0·029	19·037	0·045	22·844	0·065
	50	15·192	0·029	18·989	0·045	22·787	0·065
41	0	15·153	0·029	18·941	0·045	22·729	0·065

TABLE XIV—Table for the Graduation

Latitude.		Abscissæ and Ordinates for					
		5′ Longitude.		10′ Longitude.		15′ Longitude.	
		Abstissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
41	0	3·788	0·002	7·576	0·007	11·364	0·017
	10	3·779	0·002	7·557	0·007	11·335	0·017
	20	3·769	0·002	7·538	0·007	11·306	0·017
	30	3·759	0·002	7·518	0·007	11·277	0·017
	40	3·750	0·002	7·499	0·007	11·248	0·017
	50	3·740	0·002	7·479	0·007	11·219	0·017
42	0	3·730	0·002	7·459	0·007	11·189	0·017
	10	3·720	0·002	7·439	0·007	11·160	0·017
	20	3·710	0·002	7·420	0·007	11·130	0·017
	30	3·700	0·002	7·400	0·007	11·100	0·017
	40	3·690	0·002	7·380	0·007	11·070	0·017
	50	3·680	0·002	7·360	0·007	11·040	0·017
43	0	3·670	0·002	7·340	0·007	11·010	0·017
	10	3·660	0·002	7·320	0·007	10·980	0·017
	20	3·650	0·002	7·300	0·007	10·950	0·017
	30	3·640	0·002	7·279	0·007	10·919	0·017
	40	3·630	0·002	7·259	0·007	10·889	0·017
	50	3·620	0·002	7·239	0·007	10·859	0·017
44	0	3·609	0·002	7·219	0·007	10·828	0·017
	10	3·599	0·002	7·199	0·007	10·797	0·017
	20	3·589	0·002	7·178	0·007	10·766	0·017
	30	3·578	0·002	7·157	0·007	10·735	0·017
	40	3·568	0·002	7·137	0·007	10·704	0·017
	50	3·558	0·002	7·116	0·007	10·673	0·017
45	0	3·547	0·002	7·095	0·007	10·642	0·017
	10	3·537	0·002	7·074	0·007	10·611	0·017
	20	3·527	0·002	7·053	0·007	10·580	0·017
	30	3·516	0·002	7·032	0·007	10·548	0·017
	40	3·506	0·002	7·011	0·007	10·517	0·017
	50	3·496	0·002	6·990	0·007	10·486	0·017
46	0	3·485	0·002	6·969	0·007	10·454	0·017
	10	3·475	0·002	6·948	0·007	10·422	0·017
	20	3·464	0·002	6·927	0·007	10·390	0·017
	30	3·453	0·002	6·906	0·007	10·358	0·017
	40	3·443	0·002	6·884	0·007	10·326	0·017
	50	3·432	0·002	6·863	0·007	10·294	0·017
47	0	3·421	0·002	6·842	0·007	10·262	0·017

Latitude.		Abscissæ and Ordinates for					
		20′ Longitude.		25′ Longitude.		30′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
41	0	15·153	0·029	18·941	0·045	22·729	0·065
	10	15·114	0·029	18·893	0·045	22·671	0·065
	20	15·075	0·029	18·844	0·045	22·613	0·065
	30	15·036	0·029	18·795	0·045	22·554	0·065
	40	14·997	0·029	18·746	0·045	22·496	0·065
	50	14·958	0·029	18·697	0·045	22·437	0·065
42	0	14·918	0·029	18·648	0·045	22·378	0·065
	10	14·879	0·029	18·599	0·045	22·319	0·065
	20	14·839	0·029	18·549	0·045	22·259	0·065
	30	14·799	0·029	18·499	0·045	22·199	0·065
	40	14·760	0·029	18·450	0·045	22·140	0·065
	50	14·720	0·029	18·400	0·045	22·080	0·065
43	0	14·680	0·029	18·350	0·045	22·020	0·065
	10	14·640	0·029	18·300	0·045	21·960	0·065
	20	14·600	0·029	18·249	0·045	21·899	0·065
	30	14·559	0·029	18·198	0·045	21·838	0·065
	40	14·519	0·029	18·148	0·045	21·778	0·065
	50	14·479	0·030	18·098	0·045	21·717	0·065
44	0	14·438	0·030	18·047	0·045	21·656	0·065
	10	14·397	0·030	17·996	0·045	21·595	0·065
	20	14·356	0·030	17·944	0·045	21·533	0·065
	30	14·314	0·030	17·892	0·045	21·471	0·065
	40	14·273	0·030	17·841	0·046	21·409	0·066
	50	14·232	0·030	17·789	0·046	21·347	0·066
45	0	14·190	0·030	17·737	0·046	21·285	0·066
	10	14·148	0·030	17·685	0·046	21·223	0·066
	20	14·106	0·030	17·633	0·046	21·160	0·066
	30	14·064	0·030	17·580	0·046	21·097	0·066
	40	14·022	0·030	17·528	0·046	21·034	0·066
	50	13·980	0·030	17·476	0·046	20·971	0·066
46	0	13·938	0·030	17·423	0·046	20·908	0·066
	10	13·896	0·030	17·370	0·045	20·845	0·066
	20	13·854	0·030	17·317	0·045	20·781	0·065
	30	13·811	0·030	17·264	0·045	20·717	0·065
	40	13·769	0·030	17·211	0·045	20·653	0·065
	50	13·726	0·030	17·158	0·045	20·589	0·065
47	0	13·683	0·030	17·105	0·045	20·525	0·065

TABLE XIV—Table for the Graduation

Latitude.		Abscissæ and Ordinates for					
		5′ Longitude.		10′ Longitude.		15′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
47	0	3·421	0·002	6·842	0·007	10·262	0·017
	10	3·411	0·002	6·821	0·007	10·230	0·017
	20	3·400	0·002	6·799	0·007	10·198	0·017
	30	3·389	0·002	6·777	0·007	10·165	0·017
	40	3·378	0·002	6·755	0·007	10·133	0·017
	50	3·367	0·002	6·733	0·007	10·101	0·017
48	0	3·356	0·002	6·712	0·007	10·068	0·017
	10	3·345	0·002	6·690	0·007	10·035	0·017
	20	3·334	0·002	6·668	0·007	10·002	0·017
	30	3·323	0·002	6·646	0·007	9·969	0·017
	40	3·312	0·002	6·624	0·007	9·936	0·017
	50	3·301	0·002	6·602	0·007	9·903	0·017
49	0	3·290	0·002	6·580	0·007	9·869	0·017
	10	3·279	0·002	6·558	0·007	9·836	0·017
	20	3·268	0·002	6·536	0·007	9·803	0·017
	30	3·257	0·002	6·513	0·007	9·769	0·017
	40	3·246	0·002	6·491	0·007	9·736	0·017
	50	3·235	0·002	6·469	0·007	9·702	0·017
50	0	3·223	0·002	6·446	0·007	9·668	0·017
	10	3·212	0·002	6·424	0·007	9·635	0·017
	20	3·201	0·002	6·401	0·007	9·601	0·017
	30	3·189	0·002	6·378	0·007	9·567	0·017
	40	3·178	0·002	6·356	0·007	9·533	0·017
	50	3·167	0·002	6·333	0·007	9·499	0·017
51	0	3·155	0·002	6·310	0·007	9·465	0·017
	10	3·144	0·002	6·287	0·007	9·431	0·017
	20	3·133	0·002	6·264	0·007	9·397	0·017
	30	3·121	0·002	6·241	0·007	9·362	0·016
	40	3·110	0·002	6·218	0·007	9·328	0·016
	50	3·098	0·002	6·195	0·007	9·293	0·016
52	0	3·086	0·002	6·172	0·007	9·258	0·016
	10	3·075	0·002	6·149	0·007	9·224	0·016
	20	3·063	0·002	6·126	0·007	9·189	0·016
	30	3·051	0·002	6·103	0·007	9·154	0·016
	40	3·040	0·002	6·079	0·007	9·119	0·016
	50	3·028	0·002	6·056	0·007	9·084	0·016
53	0	3·016	0·002	6·033	0·007	9·049	0·016

Latitude.		Abscissæ and Ordinates for					
		20′ Longitude.		25′ Longitude.		30′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
47	0	13·683	0·030	17·105	0·045	20·525	0·065
	10	13·640	0·029	17·051	0·045	20·460	0·065
	20	13·596	0·029	16·997	0·045	20·395	0·065
	30	13·553	0·029	16·943	0·045	20·330	0·065
	40	13·510	0·029	16·889	0·045	20·265	0·065
	50	13·467	0·029	16·835	0·045	20·200	0·065
48	0	13·423	0·029	16·780	0·045	20·135	0·065
	10	13·380	0·029	16·725	0·045	20·069	0·065
	20	13·336	0·029	16·670	0·045	20·003	0·065
	30	13·292	0·029	16·615	0·045	19·937	0·065
	40	13·248	0·029	16·560	0·045	19·871	0·065
	50	13·204	0·029	16·505	0·045	19·805	0·065
49	0	13·160	0·029	16·450	0·045	19·739	0·065
	10	13·116	0·029	16·394	0·045	19·672	0·065
	20	13·071	0·029	16·338	0·045	19·605	0·065
	30	13·026	0·029	16·282	0·045	19·538	0·065
	40	12·982	0·029	16·226	0·045	19·471	0·065
	50	12·937	0·029	16·170	0·045	19·404	0·065
50	0	12·892	0·029	16·114	0·045	19·337	0·065
	10	12·847	0·029	16·058	0·045	19·270	0·065
	20	12·802	0·029	16·002	0·044	19·202	0·065
	30	12·756	0·029	15·945	0·044	19·134	0·064
	40	12·711	0·029	15·889	0·044	19·066	0·064
	50	12·666	0·029	15·832	0·044	18·998	0·064
51	0	12·620	0·029	15·775	0·044	18·930	0·064
	10	12·575	0·029	15·718	0·044	18·862	0·064
	20	12·529	0·029	15·661	0·044	18·793	0·064
	30	12·483	0·029	15·603	0·044	18·724	0·063
	40	12·437	0·029	15·546	0·044	18·655	0·063
	50	12·391	0·029	15·489	0·044	18·586	0·063
52	0	12·345	0·029	15·431	0·044	18·517	0·063
	10	12·299	0·029	15·373	0·044	18·448	0·063
	20	12·253	0·028	15·315	0·044	18·378	0·063
	30	12·206	0·028	15·257	0·043	18·308	0·063
	40	12·159	0·028	15·198	0·043	18·238	0·063
	50	12·112	0·028	15·140	0·043	18·168	0·063
53	0	12·065	0·028	15·082	0·043	18·098	0·063

TABLE XIV—Table for the Graduation

Latitude.		Abscissæ and Ordinates for					
		5′ Longitude.		10′ Longitude.		15′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
53	0	3·016	0·002	6·033	0·007	9·049	0·016
	10	3·005	0·002	6·009	0·007	9·014	0·016
	20	2·993	0·002	5·985	0·007	8·978	0·016
	30	2·981	0·002	5·962	0·007	8·943	0·016
	40	2·969	0·002	5·938	0·007	8·907	0·016
	50	2·957	0·002	5·914	0·007	8·872	0·016
54	0	2·946	0·002	5·891	0·007	8·837	0·016
	10	2·934	0·002	5·867	0·007	8·801	0·016
	20	2·922	0·002	5·844	0·007	8·765	0·016
	30	2·910	0·002	5·820	0·007	8·730	0·016
	40	2·898	0·002	5·796	0·007	8·695	0·016
	50	2·886	0·002	5·773	0·007	8·659	0·016
55	0	2·874	0·002	5·749	0·007	8·623	0·016
	10	2·862	0·002	5·725	0·007	8·587	0·016
	20	2·850	0·002	5·701	0·007	8·551	0·016
	30	2·838	0·002	5·676	0·007	8·514	0·016
	40	2·826	0·002	5·652	0·007	8·478	0·016
	50	2·814	0·002	5·628	0·007	8·442	0·016
56	0	2·802	0·002	5·604	0·007	8·405	0·016
	10	2·790	0·002	5·580	0·007	8·369	0·015
	20	2·778	0·002	5·556	0·007	8·333	0·015
	30	2·765	0·002	5·531	0·007	8·296	0·015
	40	2·753	0·002	5·507	0·007	8·260	0·015
	50	2·741	0·002	5·482	0·007	8·223	0·015
57	0	2·729	0·002	5·457	0·007	8·186	0·015
	10	2·717	0·002	5·433	0·007	8·149	0·015
	20	2·705	0·002	5·408	0·007	8·112	0·015
	30	2·692	0·002	5·383	0·007	8·075	0·015
	40	2·680	0·002	5·359	0·007	8·038	0·015
	50	2·668	0·002	5·334	0·007	8·001	0·015
58	0	2·655	0·002	5·309	0·007	7·964	0·015
	10	2·643	0·002	5·284	0·007	7·927	0·015
	20	2·630	0·002	5·259	0·006	7·889	0·015
	30	2·617	0·002	5·234	0·006	7·851	0·015
	40	2·605	0·002	5·210	0·006	7·814	0·015
	50	2·593	0·002	5·185	0·006	7·777	0·015
59	0	2·580	0·002	5·160	0·006	7·739	0·015

Latitude.		Abscissæ and Ordinates for					
		20′ Longitude.		25′ Longitude.		30′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
53	0	12·065	0·028	15·082	0·043	18·098	0·063
	10	12·018	0·028	15·023	0·043	18·028	0·063
	20	11·971	0·028	14·964	0·043	17·958	0·063
	30	11·924	0·028	14·905	0·043	17·887	0·062
	40	11·877	0·028	14·846	0·043	17·816	0·062
	50	11·830	0·028	14·787	0·043	17·745	0·062
54	0	11·783	0·028	14·728	0·043	17·674	0·062
	10	11·735	0·028	14·668	0·043	17·602	0·062
	20	11·687	0·028	14·609	0·043	17·531	0·062
	30	11·640	0·028	14·550	0·042	17·460	0·062
	40	11·593	0·028	14·491	0·042	17·389	0·062
	50	11·545	0·028	14·432	0·042	17·318	0·062
55	0	11·497	0·028	14·372	0·042	17·246	0·062
	10	11·449	0·027	14·312	0·042	17·174	0·061
	20	11·401	0·027	14·251	0·042	17·101	0·061
	30	11·352	0·027	14·190	0·042	17·028	0·061
	40	11·304	0·027	14·130	0·042	16·955	0·061
	50	11·256	0·027	14·070	0·042	16·883	0·061
56	0	11·207	0·027	14·009	0·042	16·811	0·061
	10	11·159	0·027	13·949	0·042	16·738	0·060
	20	11·110	0·027	13·888	0·042	16·665	0·060
	30	11·061	0·027	13·827	0·042	16·592	0·060
	40	11·013	0·027	13·766	0·042	16·519	0·060
	50	10·964	0·027	13·705	0·042	16·446	0·060
57	0	10·915	0·027	13·644	0·042	16·372	0·060
	10	10·866	0·027	13·582	0·042	16·298	0·060
	20	10·817	0·026	13·520	0·041	16·224	0·060
	30	10·767	0·026	13·458	0·041	16·150	0·059
	40	10·718	0·026	13·397	0·041	16·076	0·059
	50	10·669	0·026	13·335	0·041	16·002	0·059
58	0	10·619	0·026	13·273	0·041	15·928	0·059
	10	10·569	0·026	13·211	0·040	15·853	0·059
	20	10·519	0·026	13·149	0·040	15·778	0·059
	30	10·469	0·026	13·086	0·040	15·703	0·058
	40	10·419	0·026	13·024	0·040	15·628	0·058
	50	10·369	0·026	12·962	0·040	15·553	0·058
59	0	10·319	0·026	12·899	0·040	15·479	0·058

TABLE XIV—Table for the Graduation

Latitude.		Abscissæ and Ordinates for					
		5′ Longitude.		10′ Longitude.		15′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
59	0	2·580	0·002	5·160	0·006	7·739	0·015
	10	2·568	0·002	5·135	0·006	7·702	0·015
	20	2·555	0·002	5·110	0·006	7·664	0·014
	30	2·542	0·002	5·084	0·006	7·626	0·014
	40	2·530	0·002	5·059	0·006	7·588	0·014
	50	2·517	0·002	5·034	0·006	7·550	0·014
60	0	2·504	0·002	5·008	0·006	7·512	0·014
	10	2·492	0·002	4·983	0·006	7·474	0·014
	20	2·479	0·002	4·958	0·006	7·436	0·014
	30	2·466	0·002	4·932	0·006	7·398	0·014
	40	2·454	0·002	4·907	0·006	7·360	0·014
	50	2·441	0·002	4·882	0·006	7·322	0·014
61	0	2·428	0·002	4·856	0·006	7·283	0·014
	10	2·415	0·002	4·831	0·006	7·245	0·014
	20	2·402	0·002	4·805	0·006	7·207	0·014
	30	2·389	0·002	4·779	0·006	7·168	0·014
	40	2·377	0·002	4·754	0·006	7·130	0·014
	50	2·364	0·002	4·728	0·006	7·091	0·014
62	0	2·351	0·002	4·702	0·006	7·052	0·014
	10	2·338	0·002	4·676	0·006	7·014	0·014
	20	2·325	0·002	4·650	0·006	6·975	0·014
	30	2·312	0·002	4·624	0·006	6·936	0·013
	40	2·299	0·002	4·598	0·006	6·897	0·013
	50	2·286	0·002	4·572	0·006	6·858	0·013
63	0	2·273	0·002	4·546	0·006	6·819	0·013
	10	2·260	0·002	4·520	0·006	6·780	0·013
	20	2·247	0·002	4·494	0·006	6·741	0·013
	30	2·234	0·002	4·468	0·006	6·701	0·013
	40	2·221	0·002	4·442	0·006	6·662	0·013
	50	2·208	0·002	4·416	0·006	6·623	0·013
64	0	2·195	0·002	4·389	0·006	6·584	0·013
	10	2·182	0·002	4·363	0·005	6·545	0·013
	20	2·169	0·002	4·337	0·005	6·505	0·013
	30	2·155	0·002	4·310	0·005	6·465	0·012
	40	2·142	0·002	4·284	0·005	6·426	0·012
	50	2·129	0·002	4·258	0·005	6·387	0·012
65	0	2·116	0·002	4·231	0·005	6·347	0·012

Latitude.		Abscissæ and Ordinates for					
		20′ Longitude.		25′ Longitude.		30′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°							
59	0	10·319	0·026	12·899	0·040	15·479	0·058
	10	10·269	0·026	12·836	0·040	15·404	0·058
	20	10·219	0·026	12·773	0·040	15·328	0·057
	30	10·168	0·025	12·710	0·039	15·252	0·057
	40	10·118	0·025	12·647	0·039	15·177	0·057
	50	10·067	0·025	12·584	0·039	15·101	0·057
60	0	10·016	0·025	12·520	0·039	15·025	0·056
	10	9·966	0·025	12·457	0·039	14·949	0·056
	20	9·915	0·025	12·394	0·039	14·873	0·055
	30	9·864	0·025	12·330	0·039	14·796	0·055
	40	9·813	0·025	12·267	0·039	14·720	0·055
	50	9·762	0·025	12·203	0·039	14·644	0·055
61	0	9·711	0·025	12·139	0·039	14·567	0·055
	10	9·660	0·025	12·075	0·039	14·490	0·055
	20	9·609	0·025	12·011	0·038	14·413	0·055
	30	9·557	0·024	11·947	0·038	14·336	0·055
	40	9·506	0·024	11·883	0·038	14·259	0·055
	50	9·455	0·024	11·819	0·038	14·182	0·055
62	0	9·403	0·024	11·754	0·038	14·105	0·055
	10	9·352	0·024	11·690	0·038	14·028	0·054
	20	9·300	0·024	11·625	0·037	13·950	0·054
	30	9·248	0·023	11·560	0·037	13·872	0·054
	40	9·196	0·023	11·495	0·037	13·795	0·054
	50	9·144	0·023	11·430	0·037	13·717	0·054
63	0	9·092	0·023	11·365	0·037	13·639	0·053
	10	9·040	0·023	11·300	0·036	13·561	0·053
	20	8·988	0·023	11·235	0·036	13·482	0·053
	30	8·935	0·023	11·169	0·036	13·403	0·053
	40	8·883	0·023	11·104	0·036	13·325	0·052
	50	8·831	0·023	11·039	0·036	13·247	0·052
64	0	8·779	0·022	10·973	0·035	13·168	0·052
	10	8·727	0·022	10·908	0·035	13·089	0·051
	20	8·674	0·022	10·842	0·035	13·010	0·051
	30	8·621	0·022	10·776	0·035	12·931	0·051
	40	8·569	0·022	10·710	0·035	12·852	0·050
	50	8·516	0·022	10·644	0·035	12·773	0·050
65	0	8·463	0·022	10·578	0·035	12·694	0·050

TABLE XIV—Table for the Graduation

Latitude.		Abscissæ and Ordinates for					
		5′ Longitude.		10′ Longitude.		15′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
65	0	2·116	0·002	4·231	0·005	6·347	0·012
	10	2·103	0·002	4·205	0·005	6·307	0·012
	20	2·090	0·002	4·179	0·005	6·267	0·012
	30	2·076	0·002	4·152	0·005	6·227	0·012
	40	2·063	0·002	4·126	0·005	6·187	0·012
	50	2·050	0·002	4·099	0·005	6·147	0·012
66	0	2·036	0·002	4·072	0·005	6·107	0·012
	10	2·023	0·001	4·046	0·005	6·067	0·012
	20	2·010	0·001	4·019	0·005	6·027	0·012
	30	1·996	0·001	3·992	0·005	5·987	0·012
	40	1·983	0·001	3·965	0·005	5·947	0·012
	50	1·970	0·001	3·938	0·005	5·907	0·012
67	0	1·956	0·001	3·911	0·005	5·867	0·012
	10	1·943	0·001	3·884	0·005	5·827	0·012
	20	1·929	0·001	3·857	0·005	5·786	0·012
	30	1·915	0·001	3·830	0·005	5·745	0·011
	40	1·902	0·001	3·803	0·005	5·705	0·011
	50	1·889	0·001	3·776	0·005	5·665	0·011
68	0	1·875	0·001	3·749	0·005	5·624	0·011
	10	1·862	0·001	3·723	0·005	5·584	0·011
	20	1·848	0·001	3·696	0·005	5·543	0·011
	30	1·834	0·001	3·668	0·005	5·502	0·011
	40	1·821	0·001	3·641	0·005	5·462	0·011
	50	1·807	0·001	3·614	0·005	5·421	0·011
69	0	1·793	0·001	3·587	0·005	5·380	0·011
	10	1·780	0·001	3·560	0·005	5·339	0·011
	20	1·766	0·001	3·533	0·005	5·298	0·011
	30	1·752	0·001	3·505	0·005	5·257	0·011
	40	1·739	0·001	3·478	0·005	5·216	0·010
	50	1·725	0·001	3·451	0·004	5·175	0·010
70	0	1·711	0·001	3·423	0·004	5·134	0·010
	10	1·698	0·001	3·396	0·004	5·093	0·010
	20	1·684	0·001	3·368	0·004	5·052	0·010
	30	1·670	0·001	3·340	0·004	5·010	0·010
	40	1·657	0·001	3·313	0·004	4·969	0·010
	50	1·643	0·001	3·286	0·004	4·928	0·010
71	0	1·629	0·001	3·258	0·004	4·886	0·010

Latitude.		Abscissæ and Ordinates for					
		20′ Longitude.		25′ Longitude.		30′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
65	0	8·463	0·022	10·578	0·035	12·694	0·050
	10	8·410	0·022	10·512	0·035	12·615	0·050
	20	8·357	0·022	10·446	0·035	12·535	0·049
	30	8·303	0·022	10·379	0·034	12·455	0·049
	40	8·250	0·022	10·313	0·034	12·375	0·049
	50	8·197	0·022	10·247	0·034	12·295	0·048
66	0	8·143	0·022	10·180	0·034	12·215	0·048
	10	8·090	0·021	10·113	0·034	12·135	0·048
	20	8·037	0·021	10·046	0·034	12·055	0·048
	30	7·983	0·021	9·979	0·033	11·975	0·048
	40	7·930	0·021	9·912	0·033	11·895	0·048
	50	7·877	0·021	9·845	0·033	11·815	0·047
67	0	7·823	0·021	9·778	0·033	11·734	0·047
	10	7·769	0·021	9·711	0·033	11·653	0·047
	20	7·715	0·021	9·644	0·032	11·572	0·047
	30	7·661	0·021	9·576	0·032	11·491	0·046
	40	7·607	0·021	9·509	0·032	11·410	0·046
	50	7·553	0·020	9·442	0·031	11·329	0·046
68	0	7·499	0·020	9·374	0·031	11·248	0·046
	10	7·445	0·020	9·306	0·031	11·167	0·046
	20	7·391	0·020	9·238	0·031	11·086	0·045
	30	7·336	0·020	9·170	0·031	11·004	0·045
	40	7·282	0·020	9·103	0·030	10·923	0·045
	50	7·228	0·019	9·035	0·030	10·842	0·044
69	0	7·173	0·019	8·967	0·030	10·760	0·044
	10	7·119	0·019	8·899	0·030	10·678	0·044
	20	7·064	0·019	8·831	0·030	10·596	0·043
	30	7·009	0·019	8·762	0·030	10·514	0·043
	40	6·955	0·019	8·694	0·029	10·433	0·043
	50	6·901	0·019	8·626	0·029	10·351	0·042
70	0	6·846	0·019	8·557	0·029	10·269	0·042
	10	6·791	0·019	8·489	0·029	10·187	0·042
	20	6·736	0·019	8·420	0·029	10·104	0·041
	30	6·681	0·019	8·351	0·029	10·021	0·041
	40	6·626	0·018	8·282	0·029	9·939	0·041
	50	6·571	0·018	8·213	0·028	9·856	0·041
71	0	6·515	0·018	8·144	0·028	9·773	0·041

TABLE XIV—Table for the Graduation

Latitude.		Abscissæ and Ordinates for					
		5′ Longitude.		10′ Longitude.		15′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
71	0	1·629	0·001	3·258	0·004	4·886	0·010
	10	1·615	0·001	3·231	0·004	4·845	0·010
	20	1·601	0·001	3·203	0·004	4·804	0·010
	30	1·587	0·001	3·175	0·004	4·762	0·010
	40	1·574	0·001	3·148	0·004	4·721	0·009
	50	1·560	0·001	3·120	0·004	4·680	0·009
72	0	1·546	0·001	3·092	0·004	4·638	0·009
	10	1·532	0·001	3·065	0·004	4·597	0·009
	20	1·518	0·001	3·037	0·004	4·555	0·009
	30	1·504	0·001	3·009	0·004	4·513	0·009
	40	1·491	0·001	2·981	0·004	4·472	0·009
	50	1·477	0·001	2·953	0·004	4·430	0·009
73	0	1·463	0·001	2·925	0·004	4·388	0·009
	10	1·449	0·001	2·897	0·004	4·346	0·009
	20	1·435	0·001	2·869	0·004	4·304	0·009
	30	1·421	0·001	2·841	0·004	4·262	0·009
	40	1·407	0·001	2·814	0·004	4·220	0·009
	50	1·393	0·001	2·786	0·003	4·178	0·009
74	0	1·379	0·001	2·758	0·003	4·136	0·009
	10	1·365	0·001	2·730	0·003	4·094	0·009
	20	1·351	0·001	2·702	0·003	4·052	0·009
	30	1·337	0·001	2·674	0·003	4·010	0·009
	40	1·323	0·001	2·646	0·003	3·968	0·008
	50	1·309	0·001	2·618	0·003	3·926	0·008
75	0	1·295	0·001	2·589	0·003	3·884	0·008
	10	1·281	0·001	2·561	0·003	3·842	0·008
	20	1·267	0·001	2·533	0·003	3·800	0·008
	30	1·252	0·001	2·505	0·003	3·757	0·008
	40	1·238	0·001	2·477	0·003	3·715	0·008
	50	1·224	0·001	2·449	0·003	3·673	0·008
76	0	1·210	0·001	2·420	0·003	3·630	0·008
	10	1·196	0·001	2·392	0·003	3·588	0·008
	20	1·182	0·001	2·364	0·003	3·545	0·008
	30	1·167	0·001	2·335	0·003	3·502	0·008
	40	1·153	0·001	2·307	0·003	3·460	0·007
	50	1·139	0·001	2·279	0·003	3·418	0·007
77	0	1·125	0·001	2·250	0·003	3·375	0·007

Latitude.		Abscissæ and Ordinates for					
		20′ Longitude.		25′ Longitude.		30′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
71	0	6·515	0·018	8·144	0·028	9·773	0·041
	10	6·460	0·018	8·075	0·028	9·691	0·040
	20	6·405	0·018	8·006	0·027	9·608	0·040
	30	6·350	0·018	7·937	0·027	9·525	0·039
	40	6·295	0·017	7·869	0·027	9·443	0·039
	50	6·240	0·017	7·800	0·027	9·360	0·038
72	0	6·185	0·017	7·731	0·027	9·277	0·038
	10	6·130	0·017	7·662	0·026	9·194	0·037
	20	6·074	0·017	7·592	0·026	9·110	0·037
	30	6·018	0·017	7·522	0·026	9·026	0·037
	40	5·963	0·017	7·453	0·026	8·943	0·036
	50	5·907	0·016	7·383	0·025	8·860	0·036
73	0	5·851	0·016	7·313	0·025	8·776	0·036
	10	5·795	0·016	7·243	0·025	8·692	0·036
	20	5·739	0·016	7·173	0·025	8·608	0·035
	30	5·683	0·016	7·103	0·025	8·524	0·035
	40	5·627	0·016	7·034	0·024	8·441	0·035
	50	5·571	0·015	6·964	0·024	8·357	0·035
74	0	5·515	0·015	6·894	0·024	8·273	0·035
	10	5·459	0·015	6·824	0·024	8·189	0·034
	20	5·403	0·015	6·754	0·023	8·105	0·034
	30	5·347	0·015	6·684	0·023	8·021	0·034
	40	5·291	0·015	6·614	0·023	7·937	0·034
	50	5·235	0·015	6·544	0·022	7·853	0·033
75	0	5·179	0·015	6·473	0·022	7·768	0·033
	10	5·123	0·015	6·403	0·022	7·684	0·032
	20	5·066	0·015	6·333	0·022	7·599	0·032
	30	5·009	0·015	6·262	0·022	7·514	0·032
	40	4·953	0·015	6·192	0·022	7·430	0·031
	50	4·897	0·014	6·121	0·022	7·345	0·031
76	0	4·840	0·014	6·050	0·022	7·260	0·031
	10	4·784	0·014	5·979	0·022	7·175	0·031
	20	4·727	0·014	5·908	0·021	7·090	0·030
	30	4·670	0·014	5·837	0·021	7·005	0·030
	40	4·614	0·014	5·767	0·021	6·920	0·030
	50	4·557	0·013	5·696	0·020	6·853	0·029
77	0	4·500	0·013	5·625	0·020	6·750	0·029

TABLE XIV—Table for the Graduation

Latitude.		Abscissæ and Ordinates for					
		5′ Longitude.		10′ Longitude.		15′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
77	0	1·125	0·001	2·250	0·003	3·375	0·007
	10	1·111	0·001	2·222	0·003	3·333	0·007
	20	1·097	0·001	2·194	0·003	3·290	0·007
	30	1·082	0·001	2·165	0·003	3·247	0·007
	40	1·068	0·001	2·137	0·003	3·205	0·007
	50	1·054	0·001	2·109	0·003	3·163	0·007
78	0	1·040	0·001	2·080	0·003	3·120	0·007
	10	1·026	0·001	2·052	0·002	3·077	0·007
	20	1·012	0·001	2·023	0·002	3·034	0·007
	30	0·997	0·001	1·994	0·002	2·991	0·006
	40	0·983	0·001	1·966	0·002	2·949	0·006
	50	0·969	0·001	1·938	0·002	2·906	0·006
79	0	0·954	0·001	1·909	0·002	2·863	0·006
	10	0·940	0·001	1·881	0·002	2·820	0·006
	20	0·926	0·001	1·852	0·002	2·777	0·006
	30	0·911	0·001	1·823	0·002	2·734	0·005
	40	0·897	0·001	1·795	0·002	2·691	0·005
	50	0·883	0·001	1·766	0·002	2·648	0·005
80	0	0·868	0·001	1·737	0·002	2·605	0·005

Latitude.		Abscissæ and Ordinates for					
		20′ Longitude.		25′ Longitude.		30′ Longitude.	
		Abscissæ.	Ordinates.	Abscissæ.	Ordinates.	Abscissæ.	Ordinates.
°	′						
77	0	4·500	0·013	5·625	0·020	6·750	0·029
	10	4·444	0·013	5·554	0·020	6·665	0·028
	20	4·387	0·013	5·483	0·019	6·580	0·028
	30	4·330	0·012	5·412	0·019	6·495	0·028
	40	4·273	0·012	5·341	0·019	6·410	0·027
	50	4·216	0·012	5·270	0·019	6·325	0·027
78	0	4·159	0·012	5·199	0·018	6·239	0·027
	10	4·102	0·012	5·128	0·018	6·154	0·026
	20	4·045	0·012	5·057	0·018	6·069	0·026
	30	3·988	0·011	4·986	0·017	5·983	0·025
	40	3·931	0·011	4·915	0·017	5·898	0·025
	50	3·874	0·011	4·844	0·017	5·812	0·025
79	0	3·817	0·011	4·772	0·017	5·726	0·024
	10	3·760	0·010	4·701	0·017	5·640	0·024
	20	3·703	0·010	4·629	0·016	5·554	0·023
	30	3·645	0·010	4·557	0·016	5·468	0·023
	40	3·588	0·010	4·486	0·016	5·383	0·023
	50	3·531	0·009	4·414	0·015	5·297	0·022
80	0	3·474	0·009	4·342	0·015	5·211	0·022

TABLE XV

TEN FOOT POLE TABLE

Sextant Angles subtended at different distances by a pole ten feet in length

Feet	Angle	Feet	Angle	Feet	Angle	Feet	Angle
	° ′		° ′		° ′		° ′
50	11 13·1	275	2 02·8	500	1 07·5	725	0 46·6
55	10 12·3	280	2 00·6	505	1 06·8	730	0 46·3
60	9 21·6	285	1 58·5	510	1 06·2	735	0 46·0
65	8 38·5	290	1 56·5	515	1 05·6	740	0 45·6
70	8 01·6	295	1 54·5	520	1 04·9	745	0 45·3
75	7 29·6	300	1 52·6	525	1 04·3	750	0 45·0
80	7 01·6	305	1 50·7	530	1 03·7	755	0 44·7
85	6 36·8	310	1 48·9	535	1 03·1	760	0 44·4
90	6 14·8	315	1 47·2	540	1 02·5	765	0 44·1
95	5 55·1	320	1 45·5	545	1 02·0	770	0 43·8
100	5 37·4	325	1 43·9	550	1 01·4	775	0 43·6
105	5 21·4	330	1 42·3	555	1 00·8	780	0 43·3
110	5 06·8	335	1 40·8	560	1 00·3	785	0 43·0
115	4 53·5	340	1 39·3	565	0 59·8	790	0 42·8
120	4 41·3	345	1 37·9	570	0 59·3	795	0 42·5
125	4 30·0	350	1 36·5	575	0 58·7	800	0 42·2
130	4 19·6	355	1 35·1	580	0 58·2	805	0 42·0
135	4 10·0	360	1 33·8	585	0 57·7	810	0 41·7
140	4 01·1	365	1 32·5	590	0 57·2	815	0 41·4
145	3 52·8	370	1 31·3	595	0 56·8	820	0 41·2
150	3 45·1	375	1 30·1	600	0 56·3	825	0 40·9
155	3 37·8	380	1 28·9	605	0 55·8	830	0 40·7
160	3 31·0	385	1 27·7	610	0 55·4	835	0 40·4
165	3 24·6	390	1 26·6	615	0 54·9	840	0 40·2
170	3 18·6	395	1 25·5	620	0 54·5	845	0 40·0
175	3 12·9	400	1 24·4	625	0 54·0	850	0 39·7
180	3 07·6	405	1 23·4	630	0 53·6	855	0 39·5
185	3 02·5	410	1 22·4	635	0 53·2	860	0 39·3
190	2 57·7	415	1 21·4	640	0 52·8	865	0 39·1
195	2 53·2	420	1 20·4	645	0 52·4	870	0 38·8
200	2 48·8	425	1 19·4	650	0 52·0	875	0 38·6
205	2 44·7	430	1 18·5	655	0 51·6	880	0 38·4
210	2 40·8	435	1 17·6	660	0 51·2	885	0 38·2
215	2 37·1	440	1 16·7	665	0 50·8	890	0 38·0
220	2 33·5	445	1 15·9	670	0 50·4	895	0 37·7
225	2 30·1	450	1 15·0	675	0 50·0	900	0 37·5
230	2 26·9	455	1 14·2	680	0 49·7	905	0 37·3
235	2 23·7	460	1 13·4	685	0 49·3	910	0 37·1
240	2 20·7	465	1 12·6	690	0 48·9	915	0 36·9
245	2 17·8	470	1 11·8	695	0 48·6	920	0 36·7
250	2 15·1	475	1 11·1	700	0 48·3	925	0 36·5
255	2 12·4	480	1 10·4	705	0 47·9	930	0 36·3
260	2 09·9	485	1 09·6	710	0 47·6	935	0 36·1
265	2 07·4	490	1 08·9	715	0 47·2	940	0 35·9
270	2 05·1	495	1 08·2	720	0 46·9	945	0 35·7

The above angles are corrected for the parallax due to a separation of 0·177 feet between the index mirror and the line of collimation

TABLE XVI

Table of Subtended Angles

Angles subtended by various lengths at different distances.

Feet	Distance in Nautical Miles.																	
	0·25	0·5	0·75	1	1·25	1·5	1·75	2	2·5	3	3·5	4	4·5	5	5·5	6	7	8
	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
1	2 16	1 08	0 45	0 34	0 27	0 23	0 19	0 17	0 14	0 11	0 10	0 09	0 08	0 07	0 06	0 06	0 05	0 04
2	4 31	2 16	1 30	1 08	0 54	0 45	0 39	0 34	0 27	0 23	0 19	0 17	0 15	0 14	0 12	0 11	0 10	0 09
3	6 47	3 24	2 16	1 42	1 21	1 08	0 58	0 51	0 41	0 34	0 29	0 25	0 23	0 20	0 19	0 17	0 15	0 12
4	9 03	4 31	3 01	2 16	1 49	1 30	1 18	1 08	0 54	0 45	0 39	0 34	0 30	0 27	0 25	0 23	0 19	0 17
5	11 19	5 40	3 46	2 50	2 16	1 53	1 37	1 25	1 08	0 57	0 48	0 42	0 38	0 34	0 31	0 28	0 24	0 21
6	13 34	6 47	4 31	3 24	2 43	2 16	1 57	1 42	1 21	1 08	0 58	0 51	0 45	0 41	0 37	0 34	0 29	0 25
7	15 50	7 55	5 17	3 58	3 10	2 38	2 16	1 59	1 35	1 19	1 08	0 59	0 53	0 48	0 43	0 40	0 34	0 30
8	18 06	9 03	6 02	4 31	3 37	3 01	2 35	2 16	1 49	1 30	1 18	1 08	1 00	0 54	0 49	0 45	0 39	0 34
9	20 21	10 11	6 47	5 05	4 04	3 24	2 57	2 33	2 02	1 42	1 27	1 16	1 08	1 01	0 55	0 51	0 44	0 38
10	22 37	11 19	7 32	5 39	4 32	3 46	3 14	2 50	2 16	1 53	1 37	1 25	1 15	1 08	1 02	0 57	0 48	0 42
11	24 53	12 26	8 18	6 13	4 59	4 09	3 33	3 07	2 29	2 04	1 47	1 33	1 25	1 15	1 08	1 02	0 53	0 47
12	27 08	13 34	9 03	6 47	5 26	4 31	3 53	3 24	2 43	2 16	1 56	1 42	1 30	1 21	1 14	1 08	0 58	0 51
13	29 24	14 42	9 48	7 21	5 53	4 54	4 12	3 40	2 56	2 27	2 06	1 50	1 38	1 28	1 20	1 14	1 03	0 55
14	31 40	15 50	10 33	7 55	6 20	5 17	4 31	3 58	3 10	2 38	2 16	1 59	1 46	1 35	1 26	1 19	1 08	0 59
15	33 56	16 58	11 19	8 29	6 43	5 39	4 51	4 14	3 24	2 50	2 25	2 07	1 53	1 42	1 33	1 25	1 13	1 04

TABLE XVII

(a) Conversion of Metres and Centimetres into Feet and Inches

Metres	Feet	Centimetres	Inches
1	3·2808 4275	1	0·3937 0113
2	6·5616 8550	2	0·7874 0226
3	9·8425 2825	3	1·1811 0339
4	13·1233 7100	4	1·5748 0452
5	16·4042 1375	5	1·9685 0565
6	19·6850 5650	6	2·3622 0678
7	22·9658 9925	7	2·7559 0791
8	26·2467 4200	8	3·1496 0904
9	29·5275 8475	9	3·5433 1017

1 millimetre = 0·0393 70113 inches (approx. $\frac{1}{25}$ inch)
1 metre = 39·3701 13 inches (Log. 1·595 1666)
= 3·2808 4275 feet (,, 0·515 9854)
= 1·0936 1425 yards (,, 0·038 8642)
1 kilometre = 1093·61425 yards (approx. $\frac{5}{8}$ of a land mile)

(b) Conversion of Feet and Inches into Metres and Centimetres

Feet	Metres	Inches	Centimetres
1	0·3047 9973	1	2·5399 9779
2	0·6095 9947	2	5·0799 956
3	0·9143 9920	3	7·6199 934
4	1·2191 9894	4	10·1599 912
5	1·5239 9867	5	12·6999 889
6	1·8287 9841	6	15·2399 867
7	2·1335 9814	7	17·7799 845
8	2·4383 9787	8	20·3199 823
9	2·7431 9761	9	22·8599 801
10	3·0479 9735	10	25·3999 779
11	3·3527 9708	11	27·9399 757
12	3·6575 9682	12	30·4799 735

1 inch = 2·5399 9779 centimetres
1 foot = 30·4799 735 centimetres (Log. 1·484 0146)
1 yard = 0·9143 99204 metres (,, $\bar{1}$·961 1358)
1 fathom = 1·8287 9841 metres (,, 0·262 1658)

Note.—The above values depend on the determination of Benoit and Chaney in 1896, but the latest determination made in 1927 by Sears, Johnson and Jolly gives: One International Prototype metre = 39·370 147 ins. = 3·280 8456 ft. (Log 0·515 9858) or, One foot = 0·3047 9947 metres (Log $\bar{1}$·484 0142).

TABLE XVIII—Chords of Arcs from 0° to 60°; radius=10

Min.	0°	Parts for "	1°	Parts for "	2°	Parts for "	3°	Parts for "	4°	Parts for "
0	0·00000	0	0·17453	0	0·34905	0	0·52354	0	0·69799	0
1	0·00291	5	0·17744	5	0·35196	5	0·52645	5	0·70090	5
2	0·00582	10	0·18035	10	0·35486	10	0·52935	10	0·70380	10
3	0·00873	15	0·18326	15	0·35777	15	0·53226	15	0·70671	15
4	0·01164	19	0·18617	19	0·36068	19	0·53517	19	0·70962	19
5	0·01454	24	0·18907	24	0·36359	24	0·53808	24	0·71252	24
6	0·01745	29	0·19198	29	0·36650	29	0·54099	29	0·71543	29
7	0·02036	34	0·19489	34	0·36941	34	0·54389	34	0·71834	34
8	0·02327	39	0·19780	39	0·37232	39	0·54680	39	0·72125	39
9	0·02618	44	0·20071	44	0·37522	44	0·54971	44	0·72415	44
10	0·02909	49	0·20362	49	0·37813	49	0·55262	49	0·72706	49
11	0·03200	53	0·20653	53	0·38104	53	0·55552	53	0·72997	53
12	0·03491	58	0·20944	58	0·38395	58	0·55843	58	0·73287	58
13	0·03781	63	0·21234	63	0·38686	63	0·56134	63	0·73578	63
14	0·04072	68	0·21525	68	0·38977	68	0·56425	68	0·73869	68
15	0·04363	73	0·21816	73	0·39267	73	0·56716	73	0·74159	73
16	0·04654	78	0·22107	78	0·39558	78	0·57006	78	0·74450	78
17	0·04945	83	0·22398	83	0·39849	83	0·57297	83	0·74741	83
18	0·05236	87	0·22689	87	0·40140	87	0·57588	87	0·75032	87
19	0·05527	92	0·22980	92	0·40431	92	0·57879	92	0·75322	92
20	0·05818	97	0·23271	97	0·40722	97	0·58169	97	0·75613	97
21	0·06109	102	0·23561	102	0·41012	102	0·58460	102	0·75904	102
22	0·06400	107	0·23852	107	0·41303	107	0·58751	107	0·76194	107
23	0·06690	112	0·24143	112	0·41594	112	0·59042	112	0·76485	112
24	0·06981	116	0·24434	116	0·41885	116	0·59332	116	0·76776	116
25	0·07272	121	0·24725	121	0·42176	121	0·59623	121	0·77066	121
26	0·07563	126	0·25016	126	0·42466	126	0·59914	126	0·77357	126
27	0·07854	131	0·25307	131	0·42757	131	0·60205	131	0·77648	131
28	0·08145	136	0·25597	136	0·43048	136	0·60496	136	0·77938	136
29	0·08436	141	0·25888	141	0·43339	141	0·60786	141	0·78229	141
30	0·08727	145	0·26179	145	0·43630	145	0·61077	145	0·78520	145
31	0·09017	150	0·26470	150	0·43921	150	0·61368	150	0·78810	150
32	0·09308	155	0·26761	155	0·44211	155	0·61659	155	0·79101	155
33	0·09599	160	0·27052	160	0·44502	160	0·61949	160	0·79392	160
34	0·09890	165	0·27343	165	0·44793	165	0·62240	165	0·79682	165
35	0·10181	170	0·27633	170	0·45084	170	0·62531	170	0·79973	170
36	0·10472	175	0·27924	175	0·45375	175	0·62822	175	0·80264	175
37	0·10763	179	0·28215	179	0·45665	179	0·63112	179	0·80554	179
38	0·11054	184	0·28506	184	0·45956	184	0·63403	184	0·80845	184
39	0·11344	189	0·28797	189	0·46247	189	0·63694	189	0·81135	189
40	0·11635	194	0·29088	194	0·46538	194	0·63984	194	0·81426	194
41	0·11926	199	0·29379	199	0·46829	199	0·64275	199	0·81717	199
42	0·12217	204	0·29670	204	0·47120	204	0·64566	204	0·82007	204
43	0·12508	208	0·29960	208	0·47410	208	0·64857	208	0·82298	208
44	0·12799	213	0·30251	213	0·47701	213	0·65147	213	0·82589	213
45	0·13090	218	0·30542	218	0·47992	218	0·65438	218	0·82879	218
46	0·13381	223	0·30833	223	0·48283	223	0·65729	223	0·83170	223
47	0·13672	228	0·31124	228	0·48574	228	0·66020	228	0·83461	228
48	0·13963	233	0·31415	233	0·48864	233	0·66310	233	0·83751	233
49	0·14253	238	0·31705	238	0·49155	238	0·66601	238	0·84042	238
50	0·14544	242	0·31996	242	0·49446	242	0·66892	242	0·84333	242
51	0·14835	247	0·32287	247	0·49737	247	0·67182	247	0·84623	247
52	0·15126	252	0·32578	252	0·50028	252	0·67473	252	0·84914	252
53	0·15417	257	0·32869	257	0·50318	257	0·67764	257	0·85204	257
54	0·15708	262	0·33160	262	0·50609	262	0·68055	262	0·85495	262
55	0·15999	267	0·33451	267	0·50900	267	0·68345	267	0·85786	267
56	0·16290	272	0·33741	272	0·51191	272	0·68636	272	0·86076	272
57	0·16580	276	0·34032	276	0·51482	276	0·68927	276	0·86367	276
58	0·16871	281	0·34323	281	0·51772	281	0·69218	281	0·86658	281
59	0·17162	286	0·34614	286	0·52063	286	0·69508	286	0·86948	286
60	0·17453	291	0·34905	291	0·52354	291	0·69799	291	0·87239	291

TABLE XVIII—Chords of Arcs from 0° to 60°; radius=10

Min.	5°	Parts for "	6°	Parts for "	7°	Parts for "	8°	Parts for "	9°	Parts for "
0	0·87239	0	1·04672	0	1·22097	0	1·39513	0	1·56918	0
1	0·87529	5	1·04962	5	1·22387	5	1·39803	5	1·57208	5
2	0·87820	10	1·05253	10	1·22678	10	1·40093	10	1·57498	10
3	0·88111	15	1·05543	15	1·22968	15	1·40383	15	1·57788	15
4	0·88401	19	1·05834	19	1·23258	19	1·40674	19	1·58078	19
5	0·88692	24	1·06124	24	1·23549	24	1·40964	24	1·58368	24
6	0·88982	29	1·06415	29	1·23839	29	1·41254	29	1·58658	29
7	0·89273	34	1·06705	34	1·24129	34	1·41544	34	1·58948	34
8	0·89564	39	1·06996	39	1·24420	39	1·41834	39	1·59238	39
9	0·89854	44	1·07286	44	1·24710	44	1·42124	44	1·59528	44
10	0·90145	49	1·07577	48	1·25000	48	1·42415	48	1·59818	48
11	0·90435	53	1·07867	53	1·25291	53	1·42705	53	1·60108	53
12	0·90726	58	1·08158	58	1·25581	58	1·42995	58	1·60398	58
13	0·91017	63	1·08448	63	1·25871	63	1·43285	63	1·60688	63
14	0·91307	68	1·08739	68	1·26162	68	1·43575	68	1·60978	68
15	0·91598	73	1·09029	73	1·26452	73	1·43865	73	1·61268	73
16	0·91888	78	1·09319	77	1·26742	77	1·44155	77	1·61558	77
17	0·92179	83	1·09610	82	1·27032	82	1·44446	82	1·61847	82
18	0·92469	87	1·09900	87	1·27323	87	1·44736	87	1·62137	87
19	0·92760	92	1·10191	92	1·27613	92	1·45026	92	1·62427	92
20	0·93051	97	1·10481	97	1·27903	97	1·45316	97	1·62717	97
21	0·93341	102	1·10772	102	1·28194	102	1·45606	102	1·63007	102
22	0·93632	107	1·11062	106	1·28484	106	1·45896	106	1·63297	106
23	0·93922	112	1·11353	111	1·28774	111	1·46186	111	1·63587	111
24	0·94213	116	1·11643	116	1·29065	116	1·46476	116	1·63877	116
25	0·94503	121	1·11933	121	1·29355	121	1·46767	121	1·64167	121
26	0·94794	126	1·12224	126	1·29645	126	1·47057	126	1·64457	126
27	0·95085	131	1·12514	131	1·29935	131	1·47347	131	1·64747	131
28	0·95375	136	1·12805	135	1·30226	135	1·47637	135	1·65037	135
29	0·95666	141	1·13095	140	1·30516	140	1·47927	140	1·65326	140
30	0·95956	145	1·13386	145	1·30806	145	1·48217	145	1·65616	145
31	0·96247	150	1·13676	150	1·31096	150	1·48507	150	1·65906	150
32	0·96537	155	1·13966	155	1·31387	155	1·48797	155	1·66196	155
33	0·96828	160	1·14257	159	1·31677	159	1·49087	159	1·66486	159
34	0·97118	165	1·14547	164	1·31967	164	1·49377	164	1·66776	164
35	0·97409	170	1·14838	169	1·32257	169	1·49667	169	1·67066	169
36	0·97700	175	1·15128	174	1·32548	174	1·49957	174	1·67356	174
37	0·97990	179	1·15418	179	1·32838	179	1·50248	179	1·67645	179
38	0·98281	184	1·15709	184	1·33128	184	1·50538	184	1·67935	184
39	0·98571	189	1·15999	189	1·33418	189	1·50828	189	1·68225	189
40	0·98862	194	1·16290	193	1·33709	193	1·51118	193	1·68515	193
41	0·99152	199	1·16580	198	1·33999	198	1·51408	198	1·68805	198
42	0·99443	204	1·16870	203	1·34289	203	1·51698	203	1·69095	203
43	0·99733	208	1·17161	208	1·34579	208	1·51988	208	1·69385	208
44	1·00024	213	1·17451	213	1·34870	213	1·52278	213	1·69675	213
45	1·00314	218	1·17742	218	1·35160	218	1·52568	218	1·69964	218
46	1·00605	223	1·18032	222	1·35450	222	1·52858	222	1·70254	222
47	1·00895	228	1·18322	227	1·35740	227	1·53148	227	1·70544	227
48	1·01186	233	1·18613	232	1·36031	232	1·53438	232	1·70834	232
49	1·01476	238	1·18903	237	1·36321	237	1·53728	237	1·71124	237
50	1·01767	242	1·19193	242	1·36611	242	1·54018	242	1·71413	242
51	1·02057	247	1·19484	246	1·36901	246	1·54308	246	1·71703	246
52	1·02348	252	1·19774	251	1·37191	251	1·54598	251	1·71993	251
53	1·02638	257	1·20065	256	1·37482	256	1·54888	256	1·72283	256
54	1·02929	262	1·20355	261	1·37772	261	1·55178	261	1·72573	261
55	1·03219	267	1·20645	266	1·38062	266	1·55468	266	1·72863	266
56	1·03610	272	1·20936	271	1·38352	271	1·55758	271	1·73152	271
57	1·03800	276	1·21226	275	1·38642	275	1·56048	275	1·73442	275
58	1·04091	281	1·21516	280	1·38933	280	1·56338	280	1·73732	280
59	1·04381	286	1·21807	285	1·39223	285	1·56628	285	1·74022	285
60	1·04672	291	1·22097	290	1·39513	290	1·56918	290	1·74311	290

TABLE XVIII—Chords of Arcs from 0° to 60° ; radius=10

Min.	10°	Parts for ″	11°	Parts for ″	12°	Parts for ″	13°	Parts for ″	14°	Parts for ″
0	1·74311	0	1·91692	0	2·09057	0	2·26406	0	2·43739	0
1	1·74601	5	1·91981	5	2·09346	5	2·26695	5	2·44027	5
2	1·74891	10	1·92271	10	2·09636	10	2·26984	10	2·44316	10
3	1·75181	15	1·92560	14	2·09925	14	2·27273	14	2·44605	14
4	1·75471	19	1·92850	19	2·10214	19	2·27562	19	2·44894	19
5	1·75760	24	1·93139	24	2·10503	24	2·27851	24	2·45182	24
6	1·76050	29	1·93429	29	2·10793	29	2·28140	29	2·45471	29
7	1·76340	34	1·93718	34	2·11082	34	2·28429	34	2·45760	34
8	1·76630	39	1·94008	39	2·11371	39	2·28718	39	2·46048	39
9	1·76919	44	1·94297	43	2·11660	43	2·29007	43	2·46337	43
10	1·77209	48	1·94587	48	2·11950	48	2·29296	48	2·46626	48
11	1·77499	53	1·94876	53	2·12239	53	2·29585	53	2·46914	53
12	1·77789	58	1·95166	58	2·12528	58	2·29874	58	2·47203	58
13	1·78078	63	1·95455	63	2·12817	63	2·30163	63	2·47492	63
14	1·78368	68	1·95745	67	2·13107	67	2·30452	67	2·47780	67
15	1·78658	73	1·96034	72	2·13396	72	2·30741	72	2·48069	72
16	1·78948	77	1·96324	77	2·13685	77	2·31030	77	2·48358	77
17	1·79237	82	1·96613	82	2·13974	82	2·31319	82	2·48646	82
18	1·79527	87	1·96903	87	2·14264	87	2·31608	87	2·48935	87
19	1·79817	92	1·97192	92	2·14553	92	2·31897	92	2·49223	92
20	1·80106	97	1·97482	96	2·14842	96	2·32186	96	2·49512	96
21	1·80396	102	1·97771	101	2·15131	101	2·32475	101	2·49801	101
22	1·80686	106	1·98061	106	2·15420	106	2·32764	106	2·50089	106
23	1·80975	111	1·98350	111	2·15710	111	2·33053	111	2·50378	111
24	1·81265	116	1·98639	116	2·15999	116	2·33341	116	2·50666	116
25	1·81555	121	1·98929	120	2·16288	120	2·33630	120	2·50955	120
26	1·81845	126	1·99218	125	2·16577	125	2·33919	125	2·51244	125
27	1·82134	131	1·99508	130	2·16866	130	2·34208	130	2·51532	130
28	1·82424	135	1·99797	135	2·17155	135	2·34497	135	2·51821	135
29	1·82713	140	2·00087	140	2·17445	140	2·34786	140	2·52109	140
30	1·83003	145	2·00376	144	2·17734	144	2·35075	144	2·52398	144
31	1·83293	150	2·00666	149	2·18023	149	2·35364	149	2·52687	149
32	1·83583	155	2·00955	154	2·18312	154	2·35653	154	2·52975	154
33	1·83872	159	2·01244	159	2·18601	159	2·35941	159	2·53264	159
34	1·84162	164	2·01534	164	2·18890	164	2·36230	164	2·53552	164
35	1·84451	169	2·01823	169	2·19179	169	2·36519	169	2·53841	169
36	1·84741	174	2·02113	173	2·19469	173	2·36808	173	2·54129	173
37	1·85031	179	2·02402	178	2·19758	178	2·37097	178	2·54418	178
38	1·85320	184	2·02691	183	2·20047	183	2·37386	183	2·54706	183
39	1·85610	189	2·02981	188	2·20336	188	2·37674	188	2·54995	188
40	1·85900	193	2·03270	193	2·20625	193	2·37963	193	2·55283	193
41	1·96189	198	2·03560	197	2·20914	197	2·38252	197	2·55572	197
42	1·86479	203	2·03849	202	2·21203	202	2·38541	202	2·55860	202
43	1·86769	208	2·04138	207	2·21492	207	2·38830	207	2·56149	207
44	1·87058	213	2·04428	212	2·21782	212	2·39119	212	2·56437	212
45	1·87348	218	2·04717	217	2·22071	217	2·39407	217	2·56726	217
46	1·87637	222	2·05006	222	2·22360	222	2·39696	222	2·57014	222
47	1·87927	227	2·05296	226	2·22649	226	2·39985	226	2·57303	226
48	1·88217	232	2·05585	231	2·22938	231	2·40274	231	2·57591	231
49	1·88506	237	2·05874	236	2·23227	236	2·40562	236	2·57880	236
50	1·88796	242	2·06164	241	2·23516	241	2·40851	241	2·58168	241
51	1·89085	246	2·06453	246	2·23805	246	2·41140	246	2·58457	246
52	1·89375	251	2·06742	250	2·24094	250	2·41429	250	2·58745	250
53	1·89665	256	2·07032	255	2·24383	255	2·41717	255	2·59033	255
54	1·89954	261	2·07321	260	2·24672	260	2·42006	260	2·59322	260
55	1·90244	266	2·07610	265	2·24961	265	2·42295	265	2·59610	265
56	1·90533	271	2·07900	270	2·25250	270	2·42584	270	2·59899	270
57	1·90823	275	2·08189	275	2·25539	275	2·42872	275	2·60187	275
58	1·91112	280	2·08478	279	2·25828	279	2·43161	279	2·60476	279
59	1·91402	285	2·08768	284	2·26117	284	2·43450	284	2·60764	284
60	1·91692	290	2·09057	289	2·26406	289	2·43739	289	2·61052	289

TABLE XVIII—Chords of Arcs from 0° to 60° ; radius=10

Min.	15°	Parts for "	16°	Parts for "	17°	Parts for "	18°	Parts for "	19°	Parts for "
0	2·61052	0	2·78346	0	2·95619	0	3·12869	0	3·30095	0
1	2·61341	5	2·78634	5	2·95906	5	3·13156	5	3·30382	5
2	2·61629	10	2·78922	10	2·96194	10	3·13444	10	3·30669	10
3	2·61918	14	2·79210	14	2·96482	14	3·13731	14	3·30956	14
4	2·62206	19	2·79498	19	2·96770	19	3·14018	19	3·31243	19
5	2·62494	24	2·79786	24	2·97057	24	3·14305	24	3·31530	24
6	2·62783	29	2·80074	29	2·97345	29	3·14593	29	3·31816	29
7	2·63071	34	2·80362	34	2·97632	34	3·14880	34	3·32103	34
8	2·63359	38	2·80650	38	2·97920	38	3·15167	38	3·32390	38
9	2·63648	43	2·80938	43	2·98208	43	3·15454	43	3·32677	43
10	2·63936	48	2·81226	48	2·98495	48	3·15742	48	3·32964	48
11	2·64224	53	2·81514	53	2·98783	53	3·16029	53	3·33251	53
12	2·64513	58	2·81802	58	2·99071	57	3·16316	57	3·33537	57
13	2·64801	62	2·82090	62	2·99358	62	3·16603	62	3·33824	62
14	2·65089	67	2·82378	67	2·99646	67	3·16891	67	3·34111	67
15	2·65378	72	2·82666	72	2·99934	72	3·17178	72	3·34398	72
16	2·65666	77	2·82954	77	3·00221	77	3·17465	77	3·34685	77
17	2·65954	82	2·83242	82	3·00509	81	3·17752	81	3·34971	81
18	2·66243	86	2·83530	86	3·00796	86	3·18039	86	3·35258	86
19	2·66531	91	2·83818	91	3·01084	91	3·18327	91	3·35545	91
20	2·66819	96	2·84106	96	3·01371	96	3·18614	96	3·35832	95
21	2·67107	101	2·84394	101	3·01659	101	3·18901	101	3·36118	100
22	2·67396	106	2·84682	106	3·01947	105	3·19188	105	3·36405	105
23	2·67684	110	2·84970	110	3·02234	110	3·19475	110	3·36692	110
24	2·67972	115	2·85258	115	3·02522	115	3·19762	115	3·36979	115
25	2·68261	120	2·85546	120	3·02809	120	3·20049	120	3·37265	120
26	2·68549	125	2·85834	125	3·03097	124	3·20337	124	3·37552	124
27	2·68837	130	2·86122	130	3·03384	129	3·20624	129	3·37839	129
28	2·69125	134	2·86409	134	3·03672	134	3·20911	134	3·38126	134
29	2·69414	139	2·86697	139	3·03959	139	3·21198	139	3·38412	139
30	2·69702	144	2·86985	144	3·04247	143	3·21485	143	3·38699	143
31	2·69990	149	2·87273	149	3·04534	148	3·21772	148	3·38986	148
32	2·70278	154	2·87561	154	3·04822	153	3·22059	153	3·39272	153
33	2·70567	158	2·87849	158	3·05109	158	3·22346	158	3·39559	158
34	2·70855	163	2·88137	163	3·05397	163	3·22633	163	3·39846	163
35	2·71143	168	2·88425	168	3·05684	167	3·22920	167	3·40132	167
36	2·71431	173	2·88712	173	3·05972	172	3·23208	172	3·40419	172
37	2·71719	178	2·89000	178	3·06259	177	3·23495	177	3·40706	177
38	2·72008	182	2·89288	182	3·06547	182	3·23782	182	3·40992	182
39	2·72296	187	2·89576	187	3·06834	186	3·24069	186	3·41279	186
40	2·72584	192	2·89864	192	3·07121	191	3·24356	191	3·41566	191
41	2·72872	197	2·90152	197	3·07409	196	3·24643	196	3·41852	196
42	2·73160	202	2·90439	202	3·07696	201	3·24930	201	3·42139	201
43	2·73448	206	2·90727	206	3·07984	206	3·25217	206	3·42425	206
44	2·73737	211	2·91015	211	3·08271	210	3·25504	210	3·42712	210
45	2·74025	216	2·91303	216	3·08559	215	3·25791	215	3·42998	215
46	2·74313	221	2·91591	221	3·08846	220	3·26078	220	3·43285	220
47	2·74601	226	2·91878	226	3·09133	225	3·26365	225	3·43572	225
48	2·74889	230	2·92166	230	3·09421	230	3·26652	230	3·43858	230
49	2·75177	235	2·92454	235	3·09708	234	3·26939	234	3·44145	234
50	2·75465	240	2·92742	240	3·09996	239	3·27226	239	3·44431	239
51	2·75753	245	2·93029	245	3·10283	244	3·27513	244	3·44718	243
52	2·76042	250	2·93317	250	3·10570	249	3·27800	249	3·45004	249
53	2·76330	254	2·93605	254	3·10858	253	3·28087	253	3·45291	253
54	2·76618	259	2·93893	259	3·11145	258	3·28374	258	3·45577	258
55	2·76906	264	2·94180	264	3·11432	263	3·28661	263	3·45864	263
56	2·77194	269	2·94468	269	3·11720	268	3·28948	268	3·46150	268
57	2·77482	274	2·94756	274	3·12007	273	3·29234	273	3·46437	273
58	2·77770	278	2·95043	278	3·12294	277	3·29521	277	3·46723	277
59	2·78058	283	2·95331	283	3·12582	282	3·29808	282	3·47010	282
60	2·78346	288	2·95619	288	3·12869	287	3·30095	287	3·47296	287

TABLE XVIII—Chords of Arcs from 0° to 60° ; radius=10

Min.	20°	Parts for ″	21°	Parts for ″	22°	Parts for ″	23°	Parts for ″	24°	Parts for ″
0	3·47296	0	3·64471	0	3·81618	0	3·98736	0	4·15823	0
1	3·47583	5	3·64757	5	3·81904	5	3·99021	5	4·16108	5
2	3·47869	10	3·65043	10	3·82189	9	3·99306	9	4·16392	9
3	3·48156	14	3·65329	14	3·82475	14	3·99591	14	4·16677	14
4	3·48442	19	3·65615	19	3·82760	19	3·99876	19	4·16961	19
5	3·48729	24	3·65901	24	3·83046	24	4·00161	24	4·17246	24
6	3·49015	29	3·66187	29	3·83331	28	4·00446	28	4·17530	28
7	3·49301	34	3·66473	34	3·83617	33	4·00731	33	4·17815	33
8	3·49588	38	3·66759	38	3·83902	38	4·01016	38	4·18099	38
9	3·49874	43	3·67045	43	3·84188	43	4·01301	43	4·18384	43
10	3·50161	48	3·67331	48	3·84473	48	4·01586	48	4·18668	47
11	3·50447	52	3·67617	52	3·84758	52	4·01871	52	4·18953	52
12	3·50733	57	3·67903	57	3·85044	57	4·02156	57	4·19237	57
13	3·51020	62	3·68189	62	3·85329	62	4·02441	62	4·19522	62
14	3·51306	67	3·68475	67	3·85615	66	4·02726	66	4·19806	66
15	3·51593	72	3·68761	72	3·85900	71	4·03011	71	4·20090	71
16	3·51879	76	3·69046	76	3·86186	76	4·03296	76	4·20375	76
17	3·52165	81	3·69332	81	3·86471	81	4·03580	81	4·20659	80
18	3·52452	86	3·69618	86	3·86756	85	4·03865	85	4·20944	85
19	3·52738	91	3·69904	91	3·87042	90	4·04150	90	4·21228	90
20	3·53024	95	3·70190	95	3·87327	95	4·04435	95	4·21512	95
21	3·53310	100	3·70476	100	3·87613	100	4·04720	100	4·21797	99
22	3·53597	105	3·70762	105	3·87898	104	4·05004	104	4·22081	104
23	3·53883	110	3·71047	110	3·88183	109	4·05290	109	4·22365	109
24	3·54169	114	3·71333	114	3·88469	114	4·05575	114	4·22650	114
25	3·54456	119	3·71619	119	3·88754	119	4·05859	119	4·22934	118
26	3·54742	124	3·71905	124	3·89039	123	4·06144	123	4·23218	123
27	3·55028	129	3·72191	129	3·89325	128	4·06429	128	4·23503	128
28	3·55315	134	3·72476	134	3·89610	133	4·06714	133	4·23787	133
29	3·55601	138	3·72762	138	3·89895	138	4·06999	138	4·24071	137
30	3·55887	143	3·73048	143	3·90181	143	4·07284	143	4·24355	142
31	3·56173	148	3·73334	148	3·90466	147	4·07568	147	4·24640	147
32	3·56460	152	3·73620	152	3·90751	152	4·07853	152	4·24924	151
33	3·56746	157	3·73905	157	3·91036	157	4·08138	157	4·25208	156
34	3·57032	162	3·74191	162	3·91322	162	4·08423	162	4·25492	161
35	3·57318	167	3·74477	167	3·91607	166	4·08707	166	4·25777	166
36	3·57604	172	3·74763	172	3·91892	171	4·08992	171	4·26061	170
37	3·57891	176	3·75048	176	3·92177	176	4·09277	176	4·26345	175
38	3·58177	181	3·75334	181	3·92463	181	4·09562	181	4·26620	180
39	3·58463	186	3·75620	186	3·92748	185	4·09846	185	4·26913	185
40	3·58749	191	3·75906	191	3·93033	190	4·10131	190	4·27198	189
41	3·59035	195	3·76191	195	3·93318	195	4·10416	195	4·27482	194
42	3·59321	200	3·76477	200	3·93604	199	4·10700	199	4·27766	199
43	3·59608	205	3·76763	205	3·93889	204	4·10985	204	4·28050	204
44	3·59894	210	3·77048	210	3·94174	209	4·11270	209	4·28334	208
45	3·60180	214	3·77334	214	3·94459	214	4·11554	214	4·28618	213
46	3·60466	219	3·77620	219	3·94744	219	4·11839	219	4·28902	218
47	3·60752	224	3·77905	224	3·95029	223	4·12124	223	4·29187	222
48	3·61038	229	3·78191	229	3·95315	228	4·12408	228	4·29471	227
49	3·61324	234	3·78476	234	3·95600	233	4·12693	233	4·29755	232
50	3·61610	238	3·78762	238	3·95885	238	4·12978	238	4·30039	237
51	3·61896	243	3·79048	243	3·96170	242	4·13262	242	4·30323	241
52	3·62183	248	3·79333	248	3·96455	247	4·13547	247	4·30607	246
53	3·62469	252	3·79619	252	3·96740	252	4·13831	252	4·30891	251
54	3·62755	257	3·79905	257	3·97025	257	4·14116	257	4·31175	256
55	3·63041	262	3·80190	262	3·97310	261	4·14401	261	4·31459	260
56	3·63327	267	3·80476	267	3·97596	266	4·14685	266	4·31743	265
57	3·63613	272	3·80761	272	3·97881	271	4·14970	271	4·32027	270
58	3·63899	276	3·81047	276	3·98166	276	4·15254	276	4·32311	275
59	3·64185	281	3·81332	281	3·98451	280	4·15539	280	4·32595	279
60	3·64471	286	3·81618	286	3·98736	285	4·15823	285	4·32879	284

TABLE XVIII—Chords of Arcs from 0° to 60° ; radius=10

Min.	25°	Parts for ″	26°	Parts for ″	27°	Parts for ″	28°	Parts for ″	29°	Parts for ″
0	4·32879	0	4·49902	0	4·66891	0	4·83844	0	5·00760	0
1	4·33163	5	4·50186	5	4·67174	5	4·84126	5	5·01042	5
2	4·33447	9	4·50469	9	4·67456	9	4·84408	9	5·01323	9
3	4·33731	14	4·50752	14	4·67739	14	4·84690	14	5·01605	14
4	4·34015	19	4·51036	19	4·68022	19	4·84973	19	5·01886	19
5	4·34299	24	4·51319	24	4·68305	24	4·85255	23	5·02168	23
6	4·34583	28	4·51603	28	4·68588	28	4·85537	28	5·02450	28
7	4·34867	33	4·51886	33	4·68870	33	4·85819	33	5·02731	33
8	4·35151	38	4·52169	38	4·69153	38	4·86101	38	5·03013	37
9	4·35435	43	4·52453	42	4·69436	42	4·86384	42	5·03294	42
10	4·35719	47	4·52736	47	4·69719	47	4·86666	47	5·03576	47
11	4·36002	52	4·53019	52	4·70001	52	4·86948	52	5·03857	52
12	4·36286	57	4·53303	57	4·70284	57	4·87230	56	5·04139	56
13	4·36570	62	4·53586	61	4·70567	61	4·87512	61	5·04420	61
14	4·36854	66	4·53869	66	4·70850	66	4·87794	66	5·04702	66
15	4·37138	71	4·54152	71	4·71132	71	4·88076	70	5·04983	70
16	4·37422	76	4·54436	76	4·71415	76	4·88358	75	5·05265	75
17	4·37706	80	4·54719	80	4·71698	80	4·88640	80	5·05546	80
18	4·37990	85	4·55002	85	4·71980	85	4·88923	85	5·05827	84
19	4·38273	90	4·55286	90	4·72263	90	4·89205	89	5·06109	89
20	4·38557	95	4·55569	94	4·72546	94	4·89487	94	5·06390	94
21	4·38841	99	4·55852	99	4·72828	99	4·89769	99	5·06672	98
22	4·39125	104	4·56135	104	4·73111	104	4·90051	103	5·06953	103
23	4·39409	109	4·56419	109	4·73394	109	4·90333	108	5·07235	108
24	4·39692	114	4·56702	113	4·73676	113	4·90615	113	5·07516	113
25	4·39976	118	4·56985	118	4·73959	118	4·90897	117	5·07797	117
26	4·40260	123	4·57268	123	4·74241	123	4·91179	122	5·08079	122
27	4·40544	128	4·57551	127	4·74524	127	4·91461	127	5·08360	126
28	4·40827	133	4·57834	132	4·74807	132	4·91743	132	5·08641	131
29	4·41111	137	4·58118	137	4·75089	137	4·92025	136	5·08923	136
30	4·41395	142	4·58401	141	4·75372	141	4·92307	141	5·09204	140
31	4·41678	147	4·58684	146	4·75654	146	4·92588	145	5·09485	145
32	4·41962	151	4·58967	151	4·75937	151	4·92870	150	5·09766	150
33	4·42246	156	4·59250	156	4·76219	156	4·93152	155	5·10048	155
34	4·42530	161	4·59533	160	4·76502	160	4·93434	160	5·10329	159
35	4·42813	166	4·59816	165	4·76784	165	4·93716	165	5·10610	164
36	4·43097	170	4·60099	170	4·77067	170	4·93998	169	5·10892	169
37	4·43381	175	4·60382	174	4·77349	174	4·94280	174	5·11173	173
38	4·43664	180	4·60666	179	4·77632	179	4·94562	179	5·11454	178
39	4·43948	185	4·60949	184	4·77914	184	4·94844	183	5·11735	183
40	4·44232	189	4·61232	189	4·78197	189	4·95125	188	5·12016	187
41	4·44515	194	4·61515	193	4·79479	193	4·95407	192	5·12298	192
42	4·44799	199	4·61798	198	4·78762	198	4·95689	197	5·12579	197
43	4·45082	204	4·62081	203	4·79044	203	4·95971	202	5·12860	201
44	4·45366	208	4·62364	207	4·79327	207	4·96253	207	5·13141	206
45	4·45650	213	4·62647	212	4·79609	212	4·96534	212	5·13422	211
46	4·45933	218	4·62930	217	4·79891	217	4·96816	216	5·13703	215
47	4·46217	222	4·63213	222	4·80174	222	4·97098	221	5·13984	220
48	4·46500	227	4·63496	227	4·80456	227	4·97380	226	5·14266	225
49	4·46784	232	4·63779	231	4·80738	231	4·97661	230	5·14547	229
50	4·47067	237	4·64062	236	4·81021	236	4·97943	235	5·14828	234
51	4·47351	241	4·64345	241	4·81303	241	4·98225	240	5·15109	239
52	4·47634	246	4·64628	245	4·81585	245	4·98507	244	5·15390	244
53	4·47918	251	4·64910	250	4·81868	250	4·98788	249	5·15671	248
54	4·48201	256	4·65193	255	4·82150	255	4·99070	254	5·15952	253
55	4·48485	260	4·65476	259	4·82432	259	4·99352	259	5·16233	258
56	4·48768	265	4·65759	264	4·82715	264	4·99633	263	5·16514	262
57	4·49052	270	4·66042	269	4·82997	269	4·99915	268	5·16795	267
58	4·49335	275	4·66325	274	4·83279	274	5·00197	273	5·17076	272
59	4·49619	279	4·66608	278	4·83561	278	5·00478	277	5·17357	276
60	4·49902	284	4·66891	283	4·83844	283	5·00760	282	5·17638	281

TABLE XVIII—Chords of Arcs from 0° to 60° ; radius=10

Min.	30°	Parts for ″	31°	Parts for ″	32°	Parts for ″	33°	Parts for ″	34°	Parts for ″
0	5·17638	0	5·34477	0	5·51275	0	5·68031	0	5·84743	0
1	5·17919	5	5·34757	5	5·51554	5	5·68309	5	5·85022	5
2	5·18200	9	5·35037	9	5·51834	9	5·68588	9	5·85300	9
3	5·18481	14	5·35318	14	5·52114	14	5·68867	14	5·85578	14
4	5·18762	19	5·35598	19	5·52393	19	5·69146	19	5·85856	19
5	5·19043	23	5·35878	23	5·52673	23	5·69425	23	5·86134	23
6	5·19324	28	5·36158	28	5·52952	28	5·69704	28	5·86412	28
7	5·19605	33	5·36439	33	5·53232	32	5·69983	32	5·86690	32
8	5·19886	37	5·36719	37	5·53512	37	5·70262	37	5·86968	37
9	5·20167	42	5·36999	42	5·53791	42	5·70540	42	5·87246	42
10	5·20447	47	5·37279	47	5·54070	46	5·70819	46	5·87525	46
11	5·20728	52	5·37559	51	5·54350	51	5·71098	51	5·87803	51
12	5·21009	56	5·37840	56	5·54629	56	5·71377	56	5·88081	56
13	5·21290	61	5·38120	61	5·54909	60	5·71655	60	5·88359	60
14	5·21571	66	5·38400	65	5·55188	65	5·71934	65	5·88637	65
15	5·21851	70	5·38680	70	5·55468	70	5·72213	70	5·88915	69
16	5·22132	75	5·38960	75	5·55747	74	5·72492	74	5·89193	74
17	5·22413	80	5·39240	79	5·56027	79	5·72770	79	5·89471	79
18	5·22694	84	5·39520	84	5·56306	84	5·73049	84	5·89749	83
19	5·22975	89	5·39800	89	5·56585	88	5·73328	88	5·90026	88
20	5·23255	94	5·40081	93	5·56865	93	5·73606	93	5·90304	93
21	5·23536	98	5·40361	98	5·57144	98	5·73885	98	5·90582	97
22	5·23817	103	5·40641	103	5·57424	102	5·74164	102	5·90860	102
23	5·24098	108	5·40921	107	5·57703	107	5·74442	107	5·91138	107
24	5·24378	113	5·41201	112	5·57982	112	5·74721	112	5·91416	111
25	5·24659	117	5·41481	117	5·58261	116	5·75000	116	5·91694	116
26	5·24940	122	5·41761	121	5·58541	121	5·75278	121	5·91972	120
27	5·25221	126	5·42041	126	5·58820	126	5·75557	126	5·92250	125
28	5·25501	131	5·42321	131	5·59099	130	5·75835	130	5·92528	130
29	5·25782	136	5·42601	135	5·59379	135	5·76114	135	5·92805	134
30	5·26062	140	5·42881	140	5·59658	139	5·76393	139	5·93083	139
31	5·26343	145	5·43161	145	5·59937	144	5·76671	144	5·93361	144
32	5·26624	150	5·43441	149	5·60217	149	5·76950	149	5·93639	148
33	5·26904	155	5·43721	154	5·60496	153	5·77228	153	5·93916	153
34	5·27185	159	5·44001	159	5·60775	158	5·77507	158	5·94194	158
35	5·27466	164	5·44280	163	5·61054	163	5·77785	163	5·94472	162
36	5·27746	169	5·44560	168	5·61333	167	5·78064	167	5·94750	167
37	5·28027	173	5·44840	173	5·61613	172	5·78342	172	5·95028	171
38	5·28307	178	5·45120	177	5·61892	177	5·78621	177	5·95305	176
39	5·28588	183	5·45400	182	5·62171	181	5·78899	181	5·95583	181
40	5·28868	187	5·45680	187	5·62450	186	5·79177	186	5·95861	185
41	5·29149	192	5·45960	191	5·62729	191	5·79456	191	5·96138	190
42	5·29429	197	5·46240	196	5·63008	195	5·79734	195	5·96416	194
43	5·29710	201	5·46520	201	5·63287	200	5·80013	200	5·96694	199
44	5·29990	206	5·46799	205	5·63567	205	5·80291	205	5·96971	204
45	5·30271	211	5·47079	210	5·63846	209	5·80569	209	5·97249	209
46	5·30551	215	5·47359	215	5·64125	214	5·80848	214	5·97526	213
47	5·30832	220	5·47639	219	5·64404	219	5·81126	219	5·97804	218
48	5·31112	225	5·47918	224	5·64683	223	5·81404	223	5·98082	222
49	5·31393	229	5·48198	229	5·64962	228	5·81683	228	5·98359	227
50	5·31673	234	5·48478	233	5·65241	233	5·81961	233	5·98637	232
51	5·31954	239	5·48758	238	5·65520	237	5·82239	237	5·98914	236
52	5·32234	244	5·49037	243	5·65799	242	5·82518	242	5·99192	241
53	5·32514	248	5·49317	247	5·66078	247	5·82796	247	5·99469	246
54	5·32795	253	5·49597	252	5·66357	251	5·83074	251	5·99747	250
55	5·33075	258	5·49876	257	5·66636	256	5·83352	256	6·00024	255
56	5·33355	262	5·50156	261	5·66915	260	5·83631	260	6·00302	259
57	5·33636	267	5·50436	266	5·67194	265	5·83909	265	6·00579	264
58	5·33916	272	5·50715	271	5·67473	270	5·84187	270	6·00857	269
59	5·34196	276	5·50995	275	5·67752	274	5·84465	274	6·01134	273
60	5·34477	281	5·51275	280	5·68031	279	5·84743	279	6·01412	278

TABLE XVIII—Chords of Arcs from 0° to 60° ; radius=10

Min.	35°	Parts for ″	36°	Parts for ″	37°	Parts for ″	38°	Parts for ″	39°	Parts for ″
0	6·01412	0	6·18034	0	6·34609	0	6·51136	0	6·67614	0
1	6·01689	5	6·18311	5	6·34885	5	6·51411	5	6·67888	5
2	6·01966	9	6·18587	9	6·35161	9	6·51686	9	6·68162	9
3	6·02244	14	6·18864	14	6·35437	14	6·51961	14	6·68436	14
4	6·02521	18	6·19140	18	6·35713	18	6·52236	18	6·68710	18
5	6·02799	23	6·19417	23	6·35988	23	6·52511	23	6·68984	23
6	6·03076	28	6·19694	28	6·36264	28	6·52786	28	6·69259	27
7	6·03353	32	6·19970	32	6·36540	32	6·53061	32	6·69533	32
8	6·03631	37	6·20247	37	6·36816	37	6·53336	37	6·69807	37
9	6·03908	42	6·20523	41	6·37092	41	6·53611	41	6·70081	41
10	6·04185	46	6·20800	46	6·37367	46	6·53886	46	6·70355	46
11	6·04463	51	6·21076	51	6·37643	51	6·54161	51	6·70729	50
12	6·04740	55	6·21353	55	6·37919	55	6·54436	55	6·70903	55
13	6·05017	60	6·21629	60	6·38194	60	6·54711	60	6·71177	59
14	6·05294	65	6·21906	64	6·38470	64	6·54986	64	6·71451	64
15	6·05571	69	6·22182	69	6·38746	69	6·55260	69	6·71725	68
16	6·05849	74	6·22459	74	6·39021	73	6·55535	73	6·71999	73
17	6·06126	78	6·22735	78	6·39297	78	6·55810	78	6·72273	78
18	6·06403	83	6·23012	83	6·39573	83	6·56085	83	6·72547	82
19	6·06680	88	6·23288	87	6·39848	87	6·56360	87	6·72821	87
20	6·06958	92	6·23564	92	6·40124	92	6·56634	92	6·73095	91
21	6·07235	97	6·23841	97	6·40399	96	6·56909	96	6·73369	96
22	6·07512	102	6·24117	101	6·40675	101	6·57184	101	6·73643	100
23	6·07789	106	6·24394	106	6·40950	105	6·57459	105	6·73917	105
24	6·08066	111	6·24670	110	6·41226	110	6·57733	110	6·74191	110
25	6·08343	116	6·24946	115	6·41502	115	6·58008	115	6·74464	114
26	6·08620	120	6·25222	120	6·41777	119	6·58283	119	6·74738	119
27	6·08897	125	6·25499	124	6·42053	124	6·58557	124	6·75012	123
28	6·09174	129	6·25775	129	6·42328	128	6·58832	128	6·75286	128
29	6·09451	134	6·26051	133	6·42604	133	6·59107	133	6·75560	132
30	6·09729	138	6·26328	138	6·42879	137	6·59381	137	6·75833	137
31	6·10006	143	6·26604	143	6·43154	142	6·59656	142	6·76107	142
32	6·10283	148	6·26880	147	6·43430	147	6·59931	147	6·76381	146
33	6·10560	152	6·27156	152	6·43706	151	6·60205	151	6·76655	151
34	6·10837	157	6·27433	156	6·43980	156	6·60480	156	6·76928	155
35	6·11114	161	6·27709	161	6·44256	160	6·60754	160	6·77202	160
36	6·11391	166	6·27985	166	6·44531	165	6·61029	165	6·77476	164
37	6·11668	171	6·28261	170	6·44807	170	6·61303	170	6·77750	169
38	6·11945	175	6·28537	175	6·45082	174	6·61578	174	6·78023	174
39	6·12221	180	6·28813	179	6·45357	179	6·61852	179	6·78297	178
40	6·12498	185	6·29090	184	6·45633	183	6·62127	183	6·78570	183
41	6·12775	189	6·29366	189	6·45908	188	6·62401	188	6·78844	187
42	6·13052	194	6·29642	193	6·46183	193	6·62676	193	6·79118	192
43	6·13329	199	6·29918	198	6·46459	197	6·62950	197	6·79391	196
44	6·13606	203	6·30194	202	6·46734	202	6·63225	202	6·79665	201
45	6·13883	208	6·30470	207	6·47009	206	6·63499	206	6·79939	206
46	6·14160	212	6·30746	212	6·47284	211	6·63773	211	6·80212	210
47	6·14436	217	6·31022	216	6·47560	215	6·64048	215	6·80486	215
48	6·14713	222	6·31298	221	6·47835	220	6·64322	220	6·80759	219
49	6·14990	226	6·31574	225	6·48110	224	6·64597	224	6·81033	224
50	6·15267	231	6·31850	230	6·48385	229	6·64871	229	6·81306	228
51	6·15544	235	6·32126	235	6·48660	234	6·65145	234	6·81580	233
52	6·15820	240	6·32402	239	6·48936	238	6·65420	238	6·81853	237
53	6·16097	245	6·32678	244	6·49211	243	6·65694	243	6·82127	242
54	6·16374	249	6·32954	248	6·49486	248	6·65968	248	6·82400	247
55	6·16651	254	6·33230	253	6·49761	252	6·66242	252	6·82673	251
56	6·16927	259	6·33506	258	6·50036	257	6·66517	257	6·82947	256
57	6·17204	263	6·33782	262	6·50311	261	6·66791	261	6·83220	260
58	6·17481	268	6·34058	267	6·50586	266	6·67065	266	6·83494	265
59	6·17757	272	6·34334	271	6·50861	270	6·67340	270	6·83767	269
60	6·18034	277	6·34609	276	6·51136	275	6·67614	275	6·84040	274

TABLE XVIII—Chords of Arcs from 0° to 60° ; radius=10

Min.	40°	Parts for ″	41°	Parts for ″	42°	Parts for ″	43°	Parts for ″	44°	Parts for ″
0	6·84040	0	7·00415	0	7·16736	0	7·33002	0	7·49213	0
1	6·84314	5	7·00687	5	7·17007	5	7·33273	4	7·49483	4
2	6·84587	9	7·00960	9	7·17279	9	7·33544	9	7·49753	9
3	6·84860	14	7·01232	14	7·17551	14	7·33814	13	7·50022	13
4	6·85134	18	7·01505	18	7·17822	18	7·34085	18	7·50292	18
5	6·85407	23	7·01777	23	7·18094	23	7·34356	22	7·50562	22
6	6·85680	27	7·02049	27	7·18365	27	7·34626	27	7·50831	27
7	6·85953	32	7·02322	32	7·18637	32	7·34897	31	7·51101	31
8	6·86227	36	7·02594	36	7·18908	36	7·35167	36	7·51370	36
9	6·86500	41	7·02866	41	7·19179	41	7·35438	41	7·51640	40
10	6·86773	45	7·03139	45	7·19451	45	7·35708	45	7·51909	45
11	6·87046	50	7·03411	50	7·19722	50	7·35979	49	7·52179	49
12	6·87319	55	7·03683	54	7·19994	54	7·36249	54	7·52449	54
13	6·87593	59	7·03956	59	7·20265	59	7·36520	58	7·52718	58
14	6·87866	64	7·04228	63	7·20536	63	7·36790	63	7·52988	63
15	6·88139	68	7·04500	68	7·20808	68	7·37060	67	7·53257	67
16	6·88412	73	7·04772	72	7·21079	72	7·37331	72	7·53526	72
17	6·88685	77	7·05045	77	7·21350	77	7·37601	76	7·53796	76
18	6·88958	82	7·05317	82	7·21622	81	7·37872	81	7·54065	81
19	6·89231	87	7·05589	86	7·21893	86	7·38142	85	7·54335	85
20	6·89504	91	7·05861	91	7·22164	90	7·38412	90	7·54604	90
21	6·89777	96	7·06133	95	7·22435	95	7·38683	95	7·54873	94
22	6·90050	100	7·06405	100	7·22707	99	7·38953	99	7·55143	99
23	6·90323	105	7·06678	104	7·22978	104	7·39223	103	7·55412	103
24	6·90596	109	7·06950	109	7·23249	108	7·39494	108	7·55682	108
25	6·90869	114	7·07222	113	7·23520	113	7·39764	112	7·55951	112
26	6·91142	118	7·07494	118	7·23792	117	7·40034	117	7·56220	117
27	6·91415	123	7·07766	122	7·24063	122	7·40304	122	7·56489	121
28	6·91688	127	7·08038	127	7·24334	126	7·40574	126	7·56759	126
29	6·91961	132	7·08310	131	7·24605	131	7·40845	130	7·57028	130
30	6·92234	136	7·08582	136	7·24876	135	7·41115	135	7·57297	134
31	6·92507	141	7·08854	141	7·25147	140	7·41385	140	7·57566	139
32	6·92780	146	7·09126	145	7·25418	145	7·41655	144	7·57836	143
33	6·93053	150	7·09398	150	7·25689	149	7·41925	148	7·58105	148
34	6·93326	155	7·09670	154	7·25960	154	7·42195	153	7·58374	152
35	6·93599	159	7·09942	159	7·26231	158	7·42466	158	7·58643	157
36	6·93871	164	7·10214	163	7·26502	163	7·42736	162	7·58912	161
37	6·94144	168	7·10486	168	7·26773	167	7·43006	167	7·59181	166
38	6·94417	173	7·10758	172	7·27044	172	7·43276	171	7·59451	170
39	6·94690	177	7·11030	177	7·27315	176	7·43546	175	7·59720	175
40	6·94962	182	7·11302	181	7·27586	181	7·43816	180	7·59989	179
41	6·95235	186	7·11573	186	7·27857	185	7·44086	185	7·60258	184
42	6·95508	191	7·11845	190	7·28128	190	7·44356	189	7·60527	188
43	6·95781	196	7·12117	195	7·28399	194	7·44626	194	7·60796	193
44	6·96053	200	7·12389	199	7·28670	199	7·44896	198	7·61065	197
45	6·96326	205	7·12661	204	7·28941	203	7·45166	203	7·61334	202
46	6·96599	209	7·12932	209	7·29212	208	7·45436	207	7·61603	206
47	6·96871	214	7·13204	213	7·29483	212	7·45706	212	7·61872	211
48	6·97144	218	7·13476	218	7·29754	217	7·45976	216	7·62141	215
49	6·97417	223	7·13748	222	7·30024	221	7·46246	221	7·62410	220
50	6·97689	228	7·14019	227	7·30295	226	7·46515	225	7·62679	224
51	6·97962	232	7·14291	231	7·30566	230	7·46785	229	7·62947	229
52	6·98235	237	7·14563	236	7·30837	235	7·47055	234	7·63216	233
53	6·98507	241	7·14834	240	7·31107	239	7·47325	239	7·63485	238
54	6·98780	246	7·15106	245	7·31378	244	7·47595	243	7·63754	242
55	6·99052	250	7·15378	249	7·31649	248	7·47864	248	7·64023	247
56	6·99325	255	7·15650	254	7·31920	253	7·48134	252	7·64292	251
57	6·99597	259	7·15921	258	7·32191	257	7·48404	257	7·64561	256
58	6·99870	264	7·16193	263	7·32461	262	7·48674	261	7·64829	260
59	7·00142	268	7·16464	267	7·32732	266	7·48944	266	7·65098	265
60	7·00415	273	7·16736	272	7·33002	271	7·49213	270	7·65367	269

TABLE XVIII—Chords of Arcs from 0° to 60° ; radius=10

Min.	45°	Parts for ″	46°	Parts for ″	47°	Parts for ″	48°	Parts for ″	49°	Parts for ″
0	7·65367	0	7·81462	0	7·97498	0	8·13473	0	8·29386	0
1	7·65636	4	7·81730	4	7·97765	4	8·13739	4	8·29651	4
2	7·65904	9	7·81998	9	7·98032	9	8·14005	9	8·29916	9
3	7·66173	13	7·82266	13	7·98298	13	8·14270	13	8·30181	13
4	7·66442	18	7·82533	18	7·98565	18	8·14536	18	8·30445	18
5	7·66710	22	7·82801	22	7·98832	22	8·14802	22	8·30710	22
6	7·66979	27	7·83069	27	7·99098	27	8·15067	26	8·30974	26
7	7·67248	31	7·83336	31	7·99365	31	8·15333	31	8·31239	31
8	7·67516	36	7·83604	36	7·99632	35	8·15599	35	8·31503	35
9	7·67785	40	7·83871	40	7·99898	40	8·15864	40	8·31768	40
10	7·68054	45	7·84139	45	8·00165	44	8·16130	44	8·32033	44
11	7·68322	49	7·84407	49	8·00432	49	8·16395	49	8·32297	48
12	7·68591	54	7·84674	53	8·00698	53	8·16661	53	8·32562	53
13	7·68859	58	7·84942	58	8·00965	58	8·16927	57	8·32826	57
14	7·69128	63	7·85209	62	8·01231	62	8·17192	62	8·33091	62
15	7·69396	67	7·85477	67	8·01498	66	8·17458	66	8·33355	66
16	7·69665	71	7·85744	71	8·01764	71	8·17723	71	8·33619	70
17	7·69933	76	7·86012	76	8·02031	75	8·17988	75	8·33884	75
18	7·70202	80	7·86279	80	8·02297	80	8·18254	79	8·34148	79
19	7·70470	85	7·86547	85	8·02564	84	8·18519	84	8·34413	84
20	7·70739	89	7·86814	89	8·02830	89	8·18785	88	8·34677	88
21	7·71007	94	7·87082	93	8·03096	93	8·19050	93	8·34941	92
22	7·71275	98	7·87349	98	8·03363	98	8·19315	97	8·35206	97
23	7·71544	103	7·87616	102	8·03629	102	8·19581	102	8·35470	101
24	7·71812	107	7·87884	107	8·03896	106	8·19846	106	8·35734	106
25	7·72080	112	7·88151	111	8·04162	111	8·20111	110	8·35998	110
26	7·72349	116	7·88419	116	8·04428	115	8·20377	115	8·36263	114
27	7·72617	121	7·88686	120	8·04694	120	8·20642	119	8·36527	119
28	7·72885	125	7·88953	125	8·04961	124	8·20907	124	8·36791	123
29	7·73154	130	7·89221	129	8·05227	129	8·21173	128	8·37055	128
30	7·73422	134	7·89488	133	8·05493	133	8·21438	132	8·37319	132
31	7·73690	138	7·89755	138	8·05760	137	8·21703	137	8·37584	136
32	7·73958	143	7·90022	142	8·06026	142	8·21968	141	8·37848	141
33	7·74227	147	7·90289	147	8·06292	146	8·22233	146	8·38112	145
34	7·74495	152	7·90557	151	8·06558	151	8·22498	150	8·38376	150
35	7·74763	156	7·90824	156	8·06824	155	8·22764	155	8·38640	154
36	7·75031	161	7·91091	160	8·07091	160	8·23029	159	8·38904	158
37	7·75299	165	7·91358	165	8·07357	164	8·23294	163	8·39168	163
38	7·75567	170	7·91625	169	8·07623	168	8·23559	168	8·39432	167
39	7·75836	174	7·91893	173	8·07889	173	8·23824	172	8·39696	172
40	7·76104	179	7·92160	178	8·08155	177	8·24089	177	8·39960	176
41	7·76372	183	7·92427	182	8·08421	182	8·24354	181	8·40224	180
42	7·76640	188	7·92694	187	8·08687	186	8·24619	186	8·40488	185
43	7·76908	192	7·92961	191	8·08953	191	8·24884	190	8·40752	189
44	7·77176	197	7·93228	196	8·09219	195	8·25149	194	8·41016	194
45	7·77444	201	7·93495	200	8·09485	200	8·25414	199	8·41280	198
46	7·77712	205	7·93762	205	8·09751	204	8·25679	203	8·41544	202
47	7·77980	210	7·94029	209	8·10017	208	8·25944	208	8·41808	207
48	7·78248	214	7·94296	214	8·10283	213	8·26209	212	8·42072	211
49	7·78516	219	7·94563	218	8·10549	217	8·26474	216	8·42335	216
50	7·78784	223	7·94830	222	8·10815	222	8·26739	221	8·42599	220
51	7·79052	228	7·95097	227	8·11081	226	8·27003	225	8·42863	224
52	7·79320	232	7·95364	231	8·11347	231	8·27268	230	8·43127	229
53	7·79588	237	7·95630	236	8·11613	235	8·27533	234	8·43391	233
54	7·79855	241	7·95897	240	8·11879	239	8·27798	239	8·43654	238
55	7·80123	246	7·96164	245	8·12144	244	8·28063	243	8·43918	242
56	7·80391	250	7·96431	249	8·12411	248	8·28328	247	8·44182	246
57	7·80659	255	7·96698	254	8·12676	253	8·28592	252	8·44446	251
58	7·80927	259	7·96965	258	8·12942	257	8·28857	256	8·44709	255
59	7·81194	264	7·97231	263	8·13208	262	8·29122	261	8·44973	260
60	7·81462	268	7·97498	267	8·13473	266	8·29386	265	8·45237	264

TABLE XVIII—Chords of Arcs from 0° to 60° ; radius=10

Min.	50°	Parts for "	51°	Parts for "	52°	Parts for "	53°	Parts for "	54°	Parts for "
0	8·45237	0	8·61022	0	8·76742	0	8·92396	0	9·07981	0
1	8·45500	4	8·61285	4	8·77004	4	8·92656	4	9·08240	4
2	8·45764	9	8·61547	9	8·77265	9	8·92916	9	9·08499	9
3	8·46027	13	8·61810	13	8·77527	13	8·93176	13	9·08759	13
4	8·46291	17	8·62072	17	8·77788	17	8·93437	17	9·09018	17
5	8·46555	22	8·62335	22	8·78049	22	8·93697	22	9·09277	22
6	8·46818	26	8·62597	26	8·78311	26	8·93957	26	9·09536	26
7	8·47082	31	8·62860	31	8·78572	30	8·94217	30	9·09795	30
8	8·47345	35	8·63122	35	8·78833	35	8·94478	35	9·10054	35
9	8·47609	39	8·63384	39	8·79095	39	8·94738	39	9·10313	39
10	8·47872	44	8·63647	44	8·79356	43	8·94998	43	9·10572	43
11	8·48135	48	8·63909	48	8·79617	48	8·95258	48	9·10831	47
12	8·48399	53	8·64171	52	8·79878	52	8·95518	52	9·11090	52
13	8·48662	57	8·64434	57	8·80140	57	8·95778	56	9·11349	56
14	8·48926	61	8·64696	61	8·80401	61	8·96038	61	9·11608	60
15	8·49189	66	8·64958	65	8·80662	65	8·96298	65	9·11867	65
16	8·49452	70	8·65221	70	8·80923	70	8·96558	69	9·12125	69
17	8·49716	75	8·65483	74	8·81184	74	8·96818	74	9·12384	73
18	8·49979	79	8·65745	79	8·81445	78	8·97078	78	9·12643	78
19	8·50242	83	8·66007	83	8·81707	83	8·97338	82	9·12902	82
20	8·50506	88	8·66270	87	8·81968	87	8·97598	87	9·13161	86
21	8·50769	92	8·66532	92	8·82229	91	8·97858	91	9·13420	91
22	8·51032	96	8·66794	96	8·82490	96	8·98118	95	9·13678	95
23	8·51295	101	8·67056	100	8·82751	100	8·98378	100	9·13937	99
24	8·51559	105	8·67318	105	8·83012	104	8·98638	104	9·14196	104
25	8·51822	110	8·67580	109	8·83273	109	8·98898	108	9·14455	108
26	8·52085	114	8·67842	114	8·83534	113	8·99158	113	9·14713	112
27	8·52348	118	8·68104	118	8·83795	117	8·99418	117	9·14972	117
28	8·52611	123	8·68366	122	8·84056	122	8·99677	121	9·15231	121
29	8·52874	127	8·68628	127	8·84317	126	8·99937	126	9·15489	125
30	8·53137	131	8·68891	131	8·84577	130	9·00197	130	9·15748	129
31	8·53401	136	8·69152	135	8·84838	135	9·00457	134	9·16006	134
32	8·53664	140	8·69414	140	8·85099	139	9·00716	139	9·16265	138
33	8·53927	145	8·69676	144	8·85360	144	9·00976	143	9·16524	142
34	8·54190	149	8·69938	148	8·85621	148	9·01236	147	9·16782	147
35	8·54453	153	8·70200	153	8·85882	152	9·01495	152	9·17041	151
36	8·54716	158	8·70462	157	8·86142	157	9·01755	156	9·17299	155
37	8·54979	162	8·70724	162	8·86403	161	9·02015	160	9·17558	160
38	8·55242	167	8·70986	166	8·86664	165	9·02274	165	9·17816	164
39	8·55505	171	8·71248	170	8·86925	170	9·02534	169	9·18075	168
40	8·55768	175	8·71510	175	8·87185	174	9·02793	173	9·18333	173
41	8·56031	180	8·71771	179	8·87446	178	9·03053	177	9·18592	177
42	8·56293	184	8·72033	183	8·87707	183	9·03313	182	9·18850	181
43	8·56556	188	8·72297	188	8·87967	187	9·03572	186	9·19108	186
44	8·56819	193	8·72556	192	8·88228	191	9·03832	191	9·19366	190
45	8·57082	197	8·72818	197	8·88489	196	9·04091	195	9·19625	194
46	8·57345	202	8·73080	201	8·88749	200	9·04351	199	9·19883	199
47	8·57607	206	8·73342	205	8·89010	204	9·04610	203	9·20141	203
48	8·57870	210	8·73604	210	8·89270	209	9·04869	208	9·20400	207
49	8·58133	215	8·73865	214	8·89531	213	9·05129	212	9·20658	212
50	8·58396	219	8·74127	218	8·89791	218	9·05388	217	9·20916	216
51	8·58659	224	8·74388	223	8·90052	222	9·05647	221	9·21174	220
52	8·58921	228	8·74650	227	8·90312	226	9·05907	225	9·21432	224
53	8·59184	232	8·74912	231	8·90573	231	9·06166	230	9·21691	229
54	8·59447	237	8·75173	236	8·90833	235	9·06426	234	9·21949	233
55	8·59709	241	8·75435	240	8·91094	239	9·06685	238	9·22207	237
56	8·59972	246	8·75696	245	8·91354	244	9·06944	243	9·22465	242
57	8·60234	250	8·75958	249	8·91615	248	9·07203	247	9·22723	246
58	8·60497	254	8·76219	253	8·91875	252	9·07463	251	9·22981	250
59	8·60760	259	8·76481	258	8·92135	257	9·07722	256	9·23239	255
60	8·61022	263	8·76742	262	8·92396	261	9·07981	260	9·23497	259

TABLE XVIII—Chords of Arcs from 0° to 60° ; radius=10

Min.	55°	Parts for "	56°	Parts for "	57°	Parts for "	58°	Parts for "	59°	Parts for "
0	9·23497	0	9·38943	0	9·54318	0	9·69619	0	9·84847	0
1	9·23755	4	9·39200	4	9·54573	4	9·69874	4	9·85100	4
2	9·24013	9	9·39457	9	9·54829	8	9·70128	8	9·85353	8
3	9·24271	13	9·39714	13	9·55084	13	9·70382	13	9·85607	13
4	9·24529	17	9·39970	17	9·55340	17	9·70637	17	9·85860	17
5	9·24787	21	9·40227	21	9·55596	21	9·70891	21	9·89113	21
6	9·25045	26	9·40484	26	9·55851	25	9·71145	25	9·86366	25
7	9·25303	30	9·40741	30	9·56107	30	9·71400	30	9·86619	30
8	9·25561	34	9·40997	34	9·56362	34	9·71654	34	9·86872	34
9	9·25819	38	9·41254	38	9·56617	38	9·71908	38	9·87125	38
10	9·26076	43	9·41511	43	9·56873	42	9·72162	42	9·87378	42
11	9·26334	47	9·41767	47	9·57128	47	9·72417	47	9·87631	46
12	9·26592	51	9·42024	51	9·57384	51	9·72671	51	9·87884	51
13	9·26850	56	9·42280	55	9·57639	55	9·72925	55	9·88137	55
14	9·27108	60	9·42537	60	9·57894	59	9·73179	59	9·88390	59
15	9·27365	64	9·42794	64	9·58150	64	9·73433	63	9·88642	63
16	9·27623	69	9·43050	68	9·58405	68	9·73687	68	9·88895	67
17	9·27881	73	9·43307	73	9·58661	72	9·73941	72	9·89148	72
18	9·28138	77	9·43563	77	9·58916	76	9·74195	76	9·89401	76
19	9·28396	81	9·43820	81	9·59171	81	9·74449	80	9·89654	80
20	9·28654	86	9·44076	85	9·59426	85	9·74703	85	9·89906	84
21	9·28911	90	9·44332	90	9·59681	89	9·74957	89	9·90159	89
22	9·29169	94	9·44589	94	9·59937	93	9·75211	93	9·90412	93
23	9·29427	99	9·44845	98	9·60192	98	9·75465	97	9·90665	97
24	9·29684	103	9·45102	102	9·60447	102	9·75719	102	9·90917	101
25	9·29942	107	9·45358	107	9·60702	106	9·75973	106	9·91170	106
26	9·30199	111	9·45614	111	9·60957	110	9·76227	110	9·91423	110
27	9·30457	116	9·45870	115	9·61212	115	9·76481	114	9·91675	114
28	9·30714	120	9·46127	119	9·61467	119	9·76735	119	9·91928	118
29	9·30972	124	9·46383	124	9·61723	123	9·76989	123	9·92180	122
30	9·31229	128	9·46639	128	9·61978	127	9·77242	127	9·92433	126
31	9·31486	133	9·46896	132	9·62233	132	9·77496	131	9·92686	131
32	9·31744	137	9·47152	137	9·62488	136	9·77750	135	9·92938	135
33	9·32001	141	9·47408	141	9·62743	140	9·78004	140	9·93191	139
34	9·32259	146	9·47664	145	9·62997	145	9·78258	144	9·93443	143
35	9·32516	150	9·47920	149	9·63252	149	9·78511	148	9·93696	147
36	9·32773	154	9·48176	154	9·63507	153	9·78765	152	9·93948	152
37	9·33031	158	9·48432	158	9·63762	157	9·79019	157	9·94200	156
38	9·33288	163	9·48689	162	9·64017	162	9·79272	161	9·94453	160
39	9·33545	167	9·48945	166	9·64272	166	9·79526	165	9·94705	164
40	9·33802	171	9·49201	171	9·64527	170	9·79779	169	9·94957	169
41	9·34060	176	9·49457	175	9·64782	174	9·80033	174	9·95210	173
42	9·34317	180	9·49713	179	9·65036	179	9·80287	178	9·95462	177
43	9·34574	184	9·49969	183	9·65291	183	9·80540	182	9·95714	181
44	9·34831	188	9·50225	188	9·65546	187	9·80794	186	9·95967	186
45	9·35088	193	9·50481	192	9·65801	191	9·81047	191	9·96219	190
46	9·35345	197	9·50737	196	9·66055	196	9·81301	195	9·96471	194
47	9·35603	201	9·50992	201	9·66310	200	9·81554	199	9·96723	198
48	9·35860	206	9·51248	205	9·66565	204	9·81808	203	9·96975	202
49	9·36117	210	9·51504	209	9·66819	208	9·82061	207	9·97228	207
50	9·36374	214	9·51760	213	9·67074	213	9·82314	212	9·97480	211
51	9·36631	218	9·52016	218	9·67329	217	9·82568	216	9·97732	215
52	9·36888	223	9·52272	222	9·67583	221	9·82821	220	9·97984	219
53	9·37145	227	9·52528	226	9·67838	225	9·83074	224	9·98236	223
54	9·37402	231	9·52783	230	9·68092	230	9·83328	229	9·98488	228
55	9·37659	236	9·53039	235	9·68347	234	9·83581	233	9·98740	232
56	9·37916	240	9·53295	239	9·68601	238	9·83834	237	9·98992	236
57	9·38173	244	9·53550	243	9·68856	242	9·84087	241	9·99244	240
58	9·38429	249	9·53806	247	9·69110	247	9·84341	246	9·99496	245
59	9·38686	253	9·54062	252	9·69365	251	9·84594	250	9·99748	249
60	9·38943	257	9·54318	256	9·69619	255	9·84848	254	10·00000	253

TABLE XIX

Corrections to be applied to observed angular distance to obtain the true horizontal angle between an elevated object and an object on the horizon.

Apparent altitude of elevated object	Observed Angular Distance														
	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°	130°	140°	150°
	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′		° ′	° ′	° ′	° ′	° ′	° ′
1°	− 03	− 02	− 01	− 01	− 01	− 00	− 00	− 00	0	+ 00	+ 00	+ 00	+0 01	+ 01	+ 01
2°	− 12	− 06	− 04	− 02	− 02	− 01	− 01	− 00	0	+ 00	+ 01	+ 01	+ 02	+ 02	+ 04
3°	− 28	− 13	− 08	− 06	− 04	− 03	− 02	− 01	0	+ 01	+ 02	+ 03	+ 04	+ 06	+ 08
4°	− 50	− 23	− 15	− 10	− 07	− 05	− 03	− 01	0	+ 01	+ 03	+ 05	+ 07	+ 10	+ 15
5°	−1 20	− 37	− 23	− 16	− 11	− 08	− 05	− 02	0	+ 02	+ 05	+ 08	+ 11	+ 16	+ 23
6°	−1 59	− 53	− 33	− 23	− 16	− 11	− 07	− 03	0	+ 03	+ 07	+ 11	+ 16	+ 23	+ 33
7°	−2 50	− 1 13	− 45	− 31	− 22	− 15	− 09	− 05	0	+ 05	+ 09	+ 15	+ 22	+ 31	+ 45
8°	−3 59	− 1 37	− 59	− 41	− 28	− 20	− 12	− 06	0	+ 06	+ 12	+ 20	+ 28	+ 41	+ 59
9°	−5 37	− 2 04	− 1 16	− 52	− 36	− 25	− 16	− 08	0	+ 08	+ 16	+ 25	+ 36	+ 52	+ 1 16
10°		− 2 36	− 1 34	− 1 04	− 45	− 31	− 19	− 09	0	+ 09	+ 19	+ 31	+ 45	+ 1 04	+ 1 34
12°		− 3 53	− 2 18	− 1 33	−1 05	− 45	− 28	− 14	0	+ 14	+ 28	+ 45	+1 05	+ 1 33	+ 2 18
14°		− 5 35	− 3 12	− 2 08	−1 29	−1 01	− 38	− 19	0	+ 19	+ 38	+1 01	+1 29	+ 2 08	+ 3 12
16°		− 7 51	− 4 17	− 2 50	−1 58	−1 20	− 51	− 24	0	+ 24	+ 51	+1 20	+1 58	+ 2 50	+ 4 17
18°		−11 08	− 5 35	− 3 39	−2 31	−1 43	−1 05	− 31	0	+ 31	+1 05	+1 43	+2 31	+ 3 39	+ 5 35
20°			− 7 10	− 4 36	−3 10	−2 09	−1 21	− 39	0	+ 39	+1 21	+2 09	+3 10	+ 4 36	+ 7 10
25°			−12 51	− 7 42	−5 10	−3 29	−2 10	−1 03	0	+1 03	+2 10	+3 29	+5 10	+ 7 42	+12 51
30°				−12 12	−7 55	−5 16	−3 16	−1 34	0	+1 34	+3 16	+5 16	+7 55	+12 12	

This table is computed from the formula, Cos. horizontal angle = Cos. angular distance × Sec. apparent altitude

TABLE XX

Rhythmic Time Signals

Corrections, in decimals of a second, to be added to the Chronometer time of the tick, whole or half second, immediately preceding the first dash of a rhythmic time signal (modified system) to give the exact *Chronometer time of the first dash*

Elapsed time	Corrn. (secs.)	Elapsed time	Corrn. (secs.)	Elapsed time	Corrn. (secs.)	Elapsed time	Corrn. (secs.)
1	0·016	16	0·262	30·5	0·008	45·5	0·254
2	0·033	17	0·279	31·5	0·025	46·5	0·270
3	0·049	18	0·295	32·5	0·041	47·5	0·287
4	0·066	19	0·311	33·5	0·057	48·5	0·303
5	0·082	20	0·328	34·5	0·074	49·5	0·320
6	0·098	21	0·344	35·5	0·090	50·5	0·336
7	0·115	22	0·361	36·5	0·107	51·5	0·352
8	0·131	23	0·377	37·5	0·123	52·5	0·369
9	0·148	24	0·393	38·5	0·139	53·5	0·385
10	0·164	25	0·410	39·5	0·156	54·5	0·402
11	0·180	26	0·426	40·5	0·172	55·5	0·418
12	0·197	27	0·443	41·5	0·188	56·5	0·434
13	0·213	28	0·459	42·5	0·205	57·5	0·451
14	0·230	29	0·475	43·5	0·221	58·5	0·467
15	0·246	30	0·492	44·5	0·238	59·5	0·484

NOTES

(1) Elapsed time=chron. time of coincidence—chron. time of tick preceding first dash of time signal.

(All times being given to the nearest half-second and the hours and minutes ignored.)

(2) When the start of the dash very nearly coincides with a tick of the chronometer, it may be difficult to decide the exact chronometer time of the preceding tick. All doubt may be dispelled by inspection; for, if the *first* coincidence after each dash is a whole second, then the tick preceding the dash must have been a whole second, and vice versa. In other words, the elapsed times are always whole seconds for the first coincidence and half-seconds for the other.

(3) Example on page 334.

TABLE XXI

Constant for Pierce's Criterion

For use with the "least square" method of finding the probable value of the mean of a series

CONSTANT FOR PIERCE'S CRITERION

n	m		
	1.	2.	3.
8	0·494	0·341	0·263
9	0·453	0·317	0·247
10	0·422	0·298	0·233
11	0·395	0·281	0·223
12	0·373	0·268	0·213
13	0·353	0·256	0·204
14	0·335	0·245	0·197
15	0·320	0·235	0·190
16	0·307	0·227	0·184
17	0·295	0·219	0·178
18	0·284	0·212	0·173
19	0·274	0·205	0·168
20	0·264	0·199	0·163
21	0·255	0·193	0·159
22	0·248	0·188	0·155
23	0·240	0·183	0·151
24	0·232	0·178	0·148

Note.—Pierce's Criterion is applied first with $m=1$, then, if any of the series have to be rejected, with $m=2$, and so on.

See explanation on page 349 *et seq.*

TABLE XXII

Ekman's Table of Corrections (in degrees Centigrade) to be *added* to temperatures measured in the Insulating Water Bottle

(See page 442)

Temperature of Water Sample (Centigrade)	Depth											
	100 *55*	200 *109*	300 *164*	400 *219*	500 *273*	600 *328*	700 *383*	800 *437*	900 *492*	1000 *547*	1100 *601*	1200 metres *656 fathoms*
°												
— 2	·00	·01	·01	·02	·02	·03	·03	·04	·05	·05	·06	·07
— 1	·01	·01	·02	·02	·03	·03	·04	·05	·05	·06	·07	·08
0	·01	·01	·02	·02	·03	·04	·05	·05	·06	·07	·08	·09
1	·01	·01	·02	·03	·04	·04	·05	·06	·07	·08	·09	·10
2	·01	·02	·02	·03	·04	·05	·06	·07	·08	·09	·10	·11
3	·01	·02	·03	·03	·04	·05	·06	·07	·09	·10	·11	·12
4	·01	·02	·03	·04	·05	·06	·07	·08	·09	·10	·11	·13
5	·01	·02	·03	·04	·05	·06	·07	·09	·10	·11	·12	·14
6	·01	·02	·03	·04	·06	·07	·08	·09	·11	·12	·13	·14
7	·01	·02	·04	·05	·06	·07	·09	·10	·11	·12	·14	·15
8	·01	·03	·04	·05	·06	·08	·09	·10	·12	·13	·15	·16
9	·01	·03	·04	·05	·07	·08	·10	·11	·12	·14	·15	·17
10	·01	·03	·04	·06	·07	·09	·10	·12	·13	·15	·16	·18
11	·01	·03	·04	·06	·07	·09	·10	·12	·14	·15	·17	·19
12	·02	·03	·05	·06	·08	·09	·11	·13	·14	·16	·18	·19
13	·02	·03	·05	·06	·08	·10	·11	·13	·15	·17	·18	·20
14	·02	·03	·05	·07	·08	·10	·12	·14	·15	·17	·19	·21
15	·02	·03	·05	·07	·09	·10	·12	·14	·16	·18	·20	·22
16	·02	·04	·05	·07	·09	·11	·13	·15	·17	·19	·20	·22
17	·02	·04	·06	·07	·09	·11	·13	·15	·17	·19	·21	·23
18	·02	·04	·06	·08	·10	·12	·14	·16	·18	·20	·22	·24
19	·02	·04	·06	·08	·10	·12	·14	·16	·18	·20	·22	·25
20	·02	·04	·06	·08	·10	·12	·14	·17	·19	·21	·23	·25

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Printed in England for Her Majesty's Stationery Office
by the Hydrographic Supplies Establishment.

C.B.H.3386 - Wt.30352-Dd.H.0241-600-10/63

18′ 33° 19′

21′

35° 20′

Increasing about 3′ annually 28 Var.ⁿ 1° 10′ E. (1936)

MAGNETIC

KYRENIA

Quarries

Ch.

Fort (conspic.)

English Church

Club

Cemetery

Government Offices

Hospital

White Ho.

Old Ch.

Quarries

Cemetery

Cemetery

Ch.

From Ayios Yeoryios

coloured with Scrub

Metres 500 0 1000 Metres

Feet 1000 500 0 1000 2000 3000 4000 5000 Feet

0 5 10 Cables or 1 Sea Mile (6,066·6 ft.)

NORTH COAST
KYRENIA

SURVEYED BY COMMANDER A. DAY, R.N.

Assisted by the officers of H.M. SURVEYING SHIP "ORMONDE" 1934.

Kyrenia Fort △ Lat. 35° 20′ 15″·5 N., Long. 33° 19′ 17″·9 E.

Natural Scale $\frac{1}{12,500}$

Conspicuous Object
Kyrenia Fort

Longitude 33° 19′ East from Greenwich

C.B.H. 3386

Hydro. Dept.

MEDITERRANEAN

CYPRUS - NORTH COAST

KYRENIA

SURVEYED BY COMMANDER A. DAY R.N.

Assisted by

Lieutenant - Commander K.F. Boxall, Lieutenants W. M. Passmore, H.J.C. Stokes, R.E. Terry, C.P.W. Marshall, Sub-Lieutenant H.S.M. Davenport R.N. and Surveying Recorders W. Lovejoy P.O. B.Ward P.O. & H. Jennings A.B.

H. M. SURVEYING SHIP ORMONDE

May and June, 1934

Kyrenia Fort Δ - Latitude 35° 20' 15"45 N., Longitude 33° 20' 17"91 E.

Figures in Red denote heights in feet above H.W.S.

SOUNDINGS IN FATHOMS

(under eleven, in fathoms and feet)

Soundings are reduced to 6·8 feet below a ⊼ cut near the top of the harbour wall close northward of the inshore end of the Custom House pier or 11·1 feet below a ⊼ cut on the south east corner of the Custom House or approximately L.W. Springs.

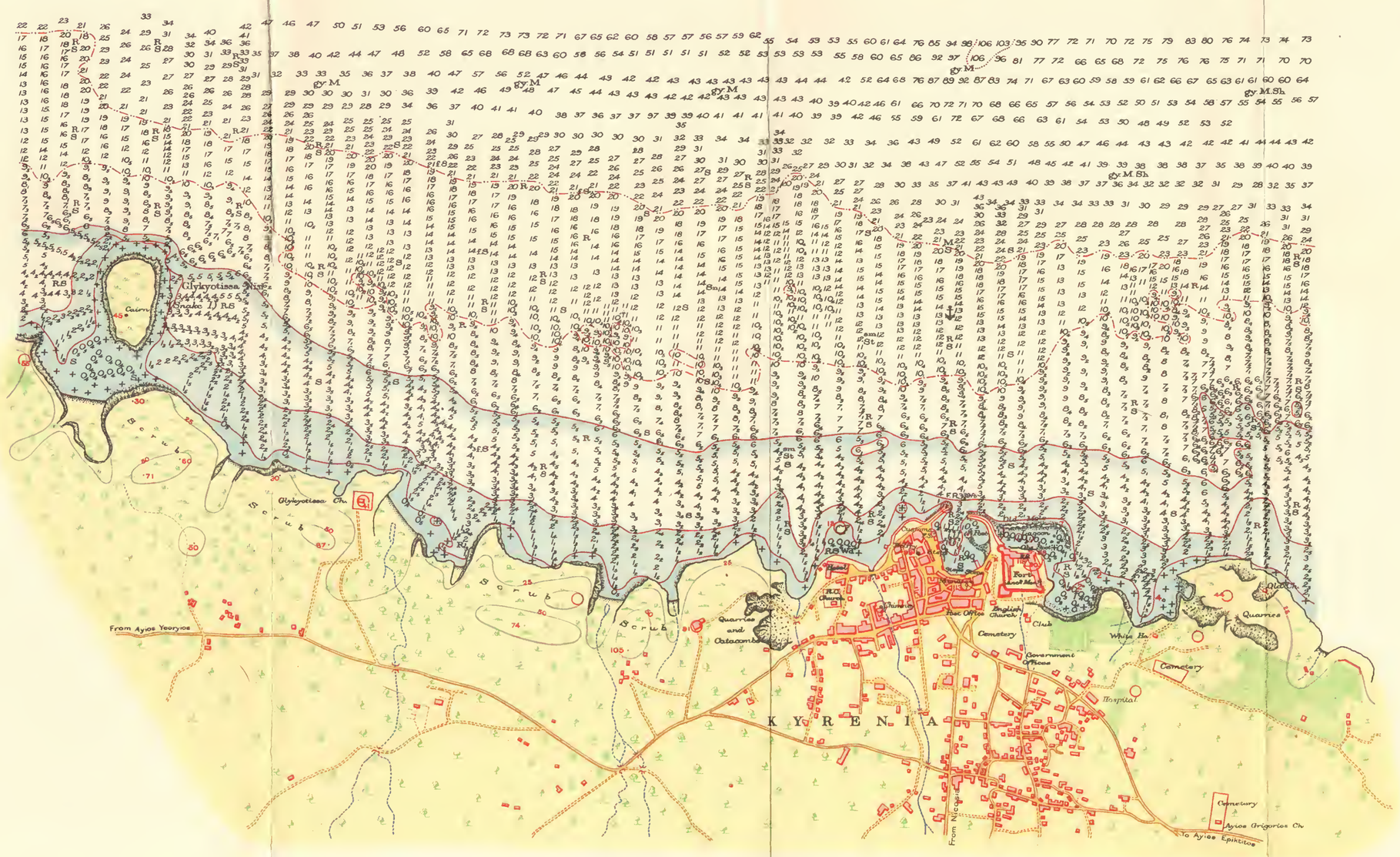
Natural Scale 1/2500

Object on this chart conspicuous to the navigator

Kyrenia Fort

MEMOIR

1. *This chart has been constructed from the Triangulation of the Survey of Cyprus 1913-14.*
2. *See Original Document F.43. "Cyprus. Latitudes, Longitudes and co-ordinates of Stations of the Major Triangulation."*
3. *Kyrenia Fort Δ Latitude 35° 20' 15"45 N, Longitude 33° 20' 17"91 E, depending on the Point of Origin of the Survey of Cyprus (q.v.) being in Lat. 35° 00' 00" N. Long. 33° 20' 00" E.*
4. *See also Returns of Geographical Position, Famagusta 1931, Nos 3, 4, 5 & 6 of 1932 and Nos 5 & 6 of 1934 and communication No 1/37/2 of 2nd Nov. 1934.*
5. *The chart and its graduation were plotted by rectangular co-ordinates in accordance with H.D. 284.*
6. *The detail of the topography is from Cyprus Survey 6" to a mile maps, the contouring having been sketched in on the ground by plane table.*
7. *No tidal observations were made other than 48 hours continuous tide pole readings to establish a local sounding datum.*
8. *No tidal stream observations were made.*
9. *No special current observations were made.*
10. *The nature of the bottom in anchorage waters of from 6 to 15 fathoms is mud with weed affording good holding ground.*



Scale of Metres

Metres 100 0 100 1000 Metres

Scale of Feet

Feet 1000 500 0 1000 2000 3000 4000 5000 Feet

Scale of Latitude and Distance

Cables 10 5 0 1 Sea Mile

(6067 ft.)

Drawn by Lieutenant W.M. Passmore R.N.

Longitude 33° 20' East from Greenwich

C.B.H. 3386

Hydro. Dept.